Spatial Models in R

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15 May 2015

Agenda

1. Spatial models
2. Geographically weighted regression (GWR)
3. Interpretation of parameters
4. GWR in R
5. References

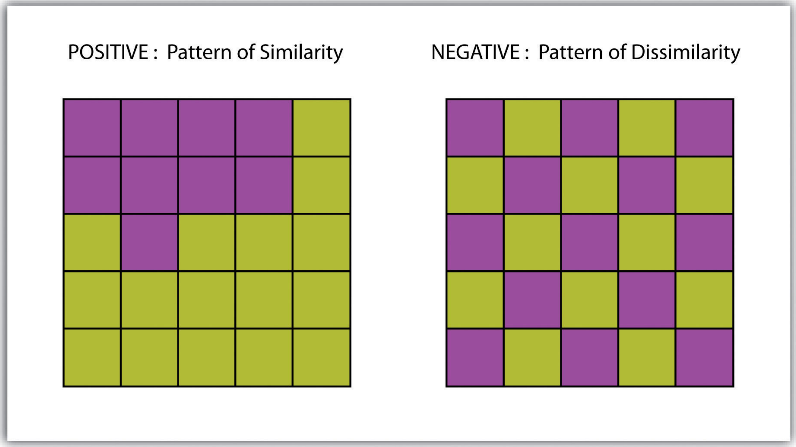
Spatial analysis

Waldo Tobler (1970)

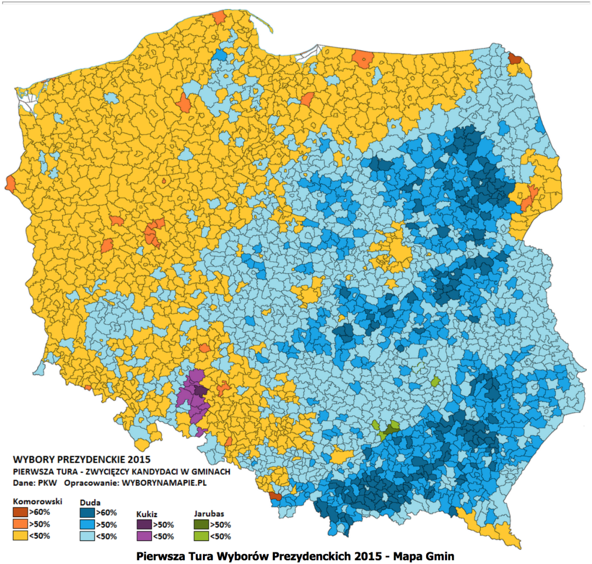
> *Everything is related to everything else, but near things are more related than distant things.*

1. Polygon/Area data analysis – requires information about membership in area (eg. voievodships, provinces)
2. Point data analysis – requires information about location of each point

Spatial Correlation

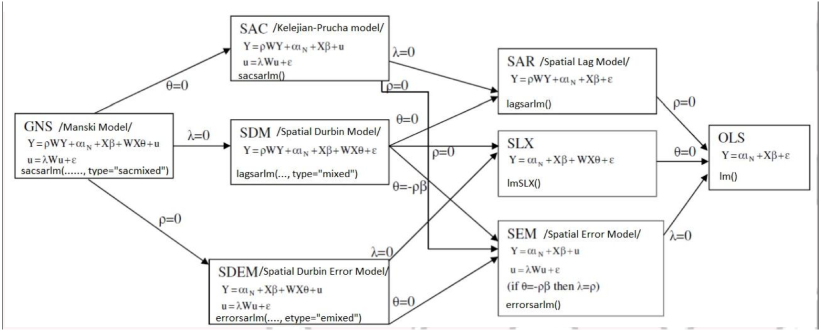


Spatial Correlation - elections in Poland



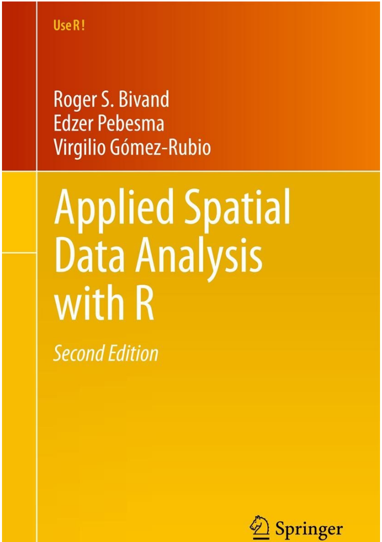
Basic spatial models

- Spatial Autocorrelation Model (SAR Lag) – assuming autocorrelation in space
- Spatial Error Model (SAR Error) – assuming autocorrelation of errors in space
- Spatial Autocorrelation and Error Model (SAR Lag and Error) – assuming both autocorrelations

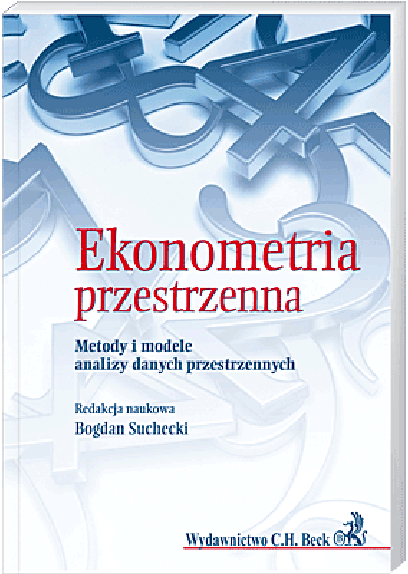


Basic spatial models - literature

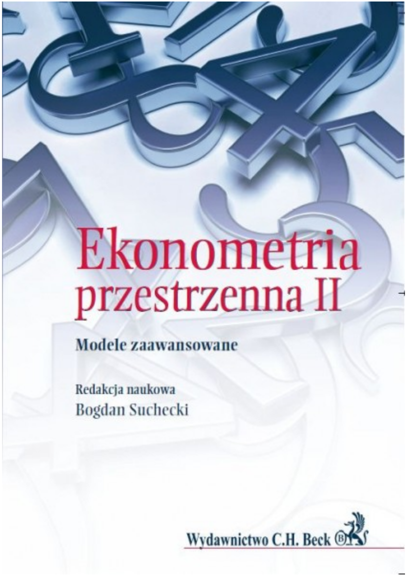
Please refer to book Bivand, R. S., Pebesma, E., & Gómez-Rubio, V. (2013). Applied spatial data analysis with R (Vol. 10). Springer Science & Business Media.



Basic spatial models - literature (in Polish)



Basic spatial models - literature (in Polish)



Geographically weighted regression

We can divide models into two groups:

1. Assuming spatial stationarity - model parameters are fixed in space
2. Assuming spatial non-stationarity - model parameters vary in space

- Geographically weighted regression (GWR) is an exploratory technique mainly intended to indicate where non-stationarity is taking place on the map, that is where locally weighted regression coefficients move away from their global values.
- Its basis is the concern that the fitted coefficient values of a global model, fitted to all the data, may not represent detailed local variations in the data adequately – in this it follows other local regression implementations.
- **We use it to explore spatial non-stationarity**

Geographically weighted regression - notation

Consider a global regression model written as:

$$y_i = \beta_0 + \sum_k \beta_k x_{ik} + \epsilon_i$$

We can consider the following cases:

1. parameters are constant for all observations (fixed)
2. parameters can be different for each observation (random)

Therefore, we can consider situatio when β_k depends on location (u_i, v_i) which can be written as $\beta_k(u_i, v_i)$.

Geographically weighted regression - notation

In general we can written the model by

$$\mathbf{Y} = \beta \mathbf{X} + \epsilon$$

and we can estimate β by standard OLS estimator given by

$$\hat{\beta} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{Y}.$$

Geographically weighted regression - notation

The GWR equivalent is

$$\mathbf{Y} = (\beta \otimes \mathbf{X})\mathbf{1} + \epsilon.$$

β and $\mathbf{X}$ will have dimensions $n \times (k+1)$ and $\mathbf{1}$ is a $(k+1) \times 1$ vector of 1. The matrix β now consists of n sets of local parameters and has the following structure

$$\beta = \begin{pmatrix} \beta_0(u_1, v_1) & \beta_1(u_1, v_1) & \cdots & \beta_k(u_1, v_1) \\ \beta_0(u_2, v_2) & \beta_1(u_2, v_2) & \cdots & \beta_k(u_2, v_2) \\ \vdots & \vdots & \ddots & \vdots \\ \beta_0(u_n, v_n) & \beta_1(u_n, v_n) & \cdots & \beta_k(u_n, v_n) \end{pmatrix}$$

Parameters for each row of the above matrix are estimated by

$$\hat{\beta}(i) = (\mathbf{X}^T\mathbf{W}(i)\mathbf{X})^{-1}\mathbf{X}^T\mathbf{W}(\mathbf{i})\mathbf{Y},$$

Where $\mathbf{W}(i) = diag(\mathbf{w})$ and $\mathbf{w} = (w_{i1}, w_{i2}, \ldots, w_{in})^T$. w_{in} is a

Geographically weighted regression - local standard errors

We can rewrite estimator for β of GWR model as:

$$\hat{\beta}(u_i, v_i) = \mathbf{C}\mathbf{y},$$

where $\mathbf{C} = (\mathbf{X}^T\mathbf{W}(i)\mathbf{X})^{-1}\mathbf{X}^T\mathbf{W}(i)$. The variance of the parameter estimates is given by

$$Var(\hat{\beta}(u_i, v_i)) = \mathbf{C}\mathbf{C}^T\sigma^2$$

where $\sigma^2 = \sum_i (y_i - \hat{y}_i)/(n - 2\nu_1 + \nu_2)$ where $\nu_1 = tr(\mathbf{S})$, $\nu_1 = tr(\mathbf{S}^T\mathbf{S})$ and $\mathbf{S}$ is a hat matrix which maps $\hat{\mathbf{y}}$ on $\mathbf{y}$ in the following manner:

$$\hat{\mathbf{y}} = \mathbf{S}\mathbf{y}$$

where each row of $\mathbf{S}$ is given by

$$\mathbf{r}_i = \mathbf{X}_i\mathbf{X}^T\mathbf{W}(i)\mathbf{X})^{-1}\mathbf{X}^T\mathbf{W}(\mathbf{i}).$$

Geographically weighted regression - weight matrix

We can consider the following settings for weights

- $w_{ij} = 1, \forall i, j$
- $w_{ij} = 1$ if $d_{ij} < d$, otherwise $w_{ij} = 0$
- $w_{ij} = exp(-0.5(d_{ij}/b)^2)$ - Gaussian weighting function
- $w_{ij} = [1 - (d_{ij}/b)^2]^2$ if $d_{ij} < d$, otherwise $w_{ij} = 0$ - bi-square function

Geographically weighted regression - *b* selection criterion

We can consider the following methods for model selection

- Cross-Validation Criterion (CV)

$$CV = \sum_{i=1}^{n} (y_i - \hat{y}_i(b))^2$$

- Generalized Cross-Validation Criterion (GCV)

$$GCV = n \sum_{i=1}^{n} (y_i - \hat{y}_i(b))^2 (n - \nu_1)^2$$

- the Akaike Information Criterion (AIC)

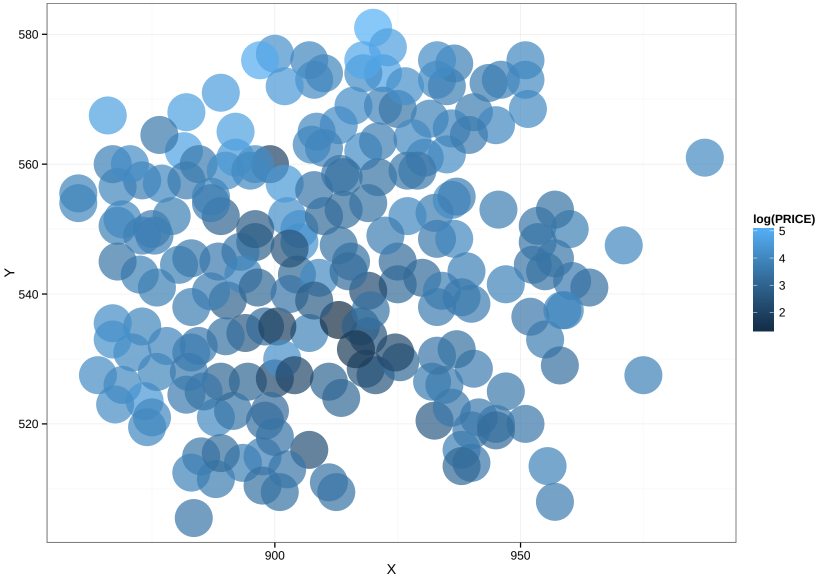
$$AIC_c = 2nlog(\hat{\sigma}) + nlog(2\pi) + n \times \frac{n + tr(\mathbf{S})}{n - 2 - str(\mathbf{S})}$$

- Bayesian Information Criterion (BIC)

Geographically weighted regression – highlights

- GWR allows to take into account spatial heterogenity
- GWR allows to take into account assumption that coefficient vary in space
- GWR will give information about possible

Visualisation of results



Information about dataset

- Topic: House sales price and characteristics for a spatial hedonic regression, Baltimore, MD 1978.
- Source : Original data made available by Robin Dubin, Weatherhead School of Management, Case Western Research University, Cleveland, OH, `Robin.Dubin@weatherhead.cwru.edu`.
- Reference: Dubin, Robin A. (1992). Spatial autocorrelation and neighborhood quality. Regional * Science and Urban Economics 22(3), 433-452.

Information about dataset

- STATION ID variable
- PRICE - sales price of house iin $1,000 (MLS)
- NROOM - number of rooms
- DWELL - 1 if detached unit, 0 otherwise
- NBATH - number of bathrooms
- PATIO - 1 if patio, 0 otherwise
- FIREPL - 1 if fireplace, 0 otherwise
- AC - 1 if air conditioning, 0 otherwise
- BMENT - 1 if basement, 0 otherwise
- NSTOR - number of stories

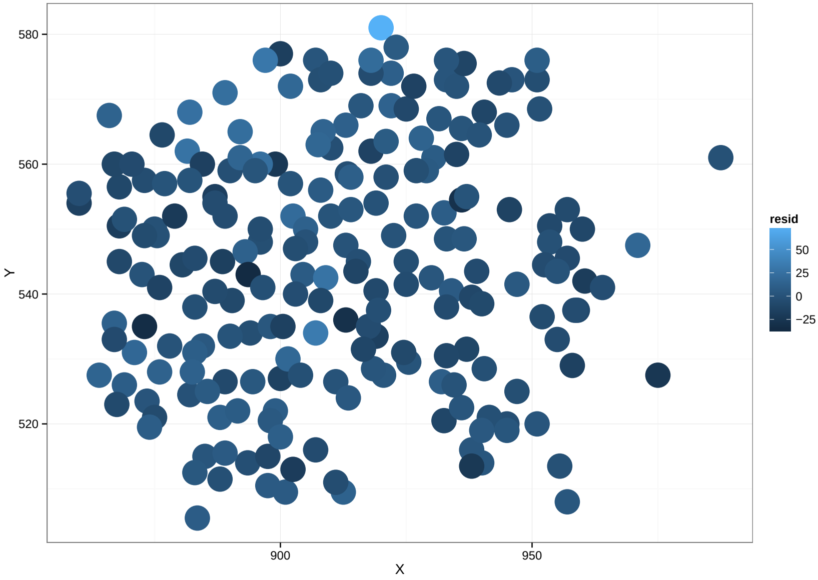
Information about dataset

- GAR - number of car spaces in garage ($0 =$ no garage)
- AGE - age of dwelling in years
- CITCOU - 1 if dwelling is in Baltimore County, 0 otherwise
- LOTSZ - lot size in hundreds of square feet
- SQFT - interior living space in hundreds of square feet
- X - x coordinate on the Maryland grid
- Y - y coordinate on the Maryland grid

Simple linear regression

```
##
## Call:
## lm(formula = PRICE ~ DWELL + NBATH + PATIO + FIREPL + AC
##     NSTOR + GAR + CITCOU + LOTSZ + SQFT, data = baltimor
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -35.351  -6.600  -0.896   6.124  74.261
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  8.00529    5.50218   1.455 0.147264
## DWELL        6.36037    2.55389   2.490 0.013576 *
## NBATH        6.18702    1.83627   3.369 0.000905 ***
## PATIO        9.43528    2.76449   3.413 0.000778 ***
## FIREPL      10.76620    2.44020   4.412 1.68e-05 ***
## AC           8.24456    2.30796   3.572 0.000444 ***
## BMENT        3.81140    1.00446   3.794 0.000196 ***
```

Visualisation of residuals



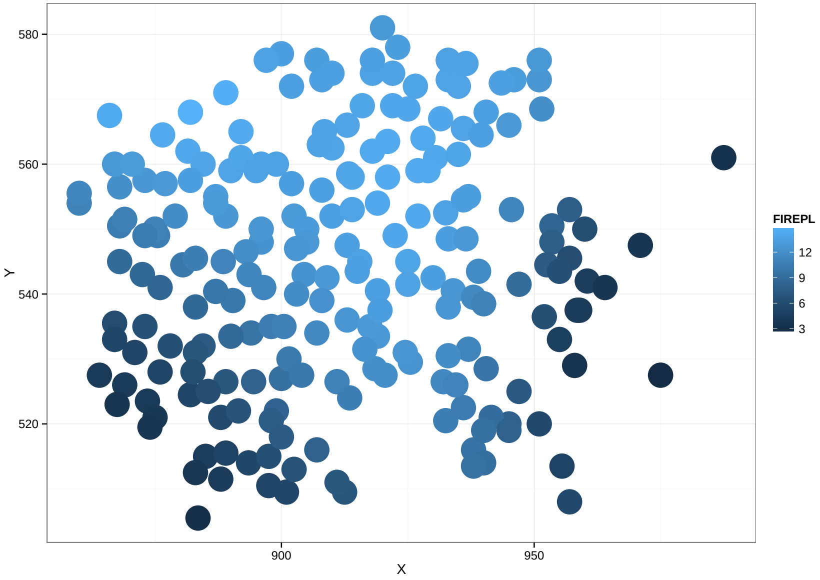
GWR in R

```
## Bandwidth: 56.69026 CV score: 36351.51
## Bandwidth: 91.63518 CV score: 37637.86
## Bandwidth: 35.0931 CV score: 34314.3
## Bandwidth: 21.74533 CV score: 32481.94
## Bandwidth: 13.49595 CV score: 32949
## Bandwidth: 20.77383 CV score: 32394.68
## Bandwidth: 19.02757 CV score: 32288.25
## Bandwidth: 16.91468 CV score: 32280.53
## Bandwidth: 17.84809 CV score: 32263.26
## Bandwidth: 17.87388 CV score: 32263.3
## Bandwidth: 17.82144 CV score: 32263.24
## Bandwidth: 17.8171 CV score: 32263.24
## Bandwidth: 17.8172 CV score: 32263.24
## Bandwidth: 17.81725 CV score: 32263.24
## Bandwidth: 17.81716 CV score: 32263.24
## Bandwidth: 17.8172 CV score: 32263.24
```

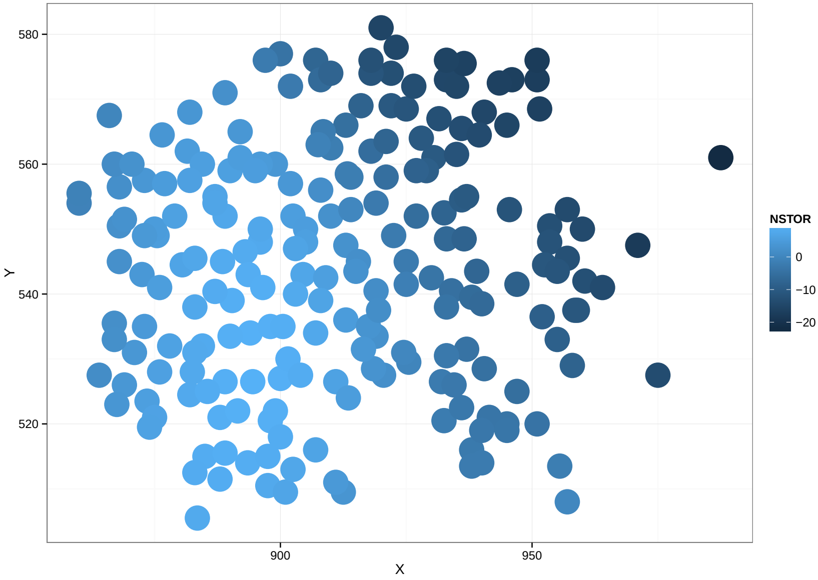
GWR in R

```
## Call:
## gwr(formula = PRICE ~ DWELL + NBATH + PATIO + FIREPL + A
##     NSTOR + GAR + CITCOU + LOTSZ + SQFT, data = baltimor
##     baltimore$Y), bandwidth = col.bw, hatmatrix = TRUE)
## Kernel function: gwr.Gauss
## Fixed bandwidth: 17.8172
## Summary of GWR coefficient estimates at data points:
##                    Min.    1st Qu.     Median     3rd Qu
## X.Intercept. -13.050000  -9.548000   0.274000  13.95000
## DWELL         -8.052000   4.674000   6.793000   8.95100
## NBATH          0.667400   5.116000   6.342000   6.94600
## PATIO         -5.532000   3.697000   6.452000   9.64800
## FIREPL         2.690000   7.705000  11.800000  13.41000
## AC             2.880000   5.978000   7.174000   8.94200
## BMENT         -0.098190   3.314000   4.060000   5.33200
## NSTOR        -22.410000  -7.504000   1.893000   6.14200
## GAR           -0.406400   1.710000   3.275000   5.64300
## CITCOU         5.435000  10.690000  13.850000  15.10000
```

GWR in R



GWR in R



References

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