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Problem 2

1.

The probability of jth observation being the first bootstrap observation is equal to $\frac{1}{n}$ because of the uniform distribution. The probability of a complementary event is then equal to $1 - \frac{1}{n}$

2.

The probability of jth observation being in the bootstrap sample k times can be obtained with binomial distribution:

$P = \binom{n}{k} p^k (1-p)^{n-k}$

In this case $k = 0$ and $p = \frac{1}{n}$ (as we use uniform distribution), so the probability of the given event can be expressed as:

$P = \frac{n!}{0! * n!} (\frac{1}{n})^0 (1 - \frac{1}{n})^n =$
$= (1 - \frac{1}{n})^n$

Which was to be demonstrated.

3.

As the value of n increases, the probability from 2.1 converges to 1: $\lim_{n \to \inf} 1 - \frac{1}{n} = 1$ Whereas the probability obtained in 2.2 converges to $e^{-1} \approx 0.368$ $\lim_{n \to \inf} (1 - \frac{1}{n})^n = e^{-1}$