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## Problem 3 - Generalized Additive Models

1. When $j = 1$:

$$\hat{\beta}_1 = argmin_\beta \sum_i (y_i - \alpha - \hat{f}_2(x_{2i}) - \beta x_{1i})^2 + \lambda\beta^2$$

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We find the derivative with respect to β and set it to 0:

$$-2x_{1i}\sum_i (y_i - \alpha - \hat{\beta}_2 x_{2i} - \beta x_{1i}) + 2\lambda\beta = 0$$

$$-x_{1i}\sum_i y_i + x_{1i}N\alpha + x_{1i}\hat{\beta}_2\sum_i x_{2i} + x_{1i}\beta\sum_i x_{1i} + \lambda\beta = 0$$

$$\hat{\beta}_1 = \beta = \frac{x_{1i}\sum_i y_i - x_{1i}N\alpha - x_{1i}\hat{\beta}_2\sum_i x_{2i}}{x_{1i}\sum_i x_{1i} + \lambda}$$

When $j = 2$:

$$\hat{\beta}_2 = argmin_\beta \sum_i (y_i - \alpha - \hat{f}_1(x_{1i}) - \beta x_{2i})^2 + \lambda\beta^2$$

$$\hat{\beta}_2 = argmin_\beta \sum_i (y_i - \alpha - \hat{\beta}_1 x_{1i} - \beta x_{2i})^2 + \lambda\beta^2$$

Similarly we find:

$$\hat{\beta}_2 = \frac{x_{2i}\sum_i y_i - x_{2i}N\alpha - x_{2i}\hat{\beta}_1\sum_i x_{1i}}{x_{2i}\sum_i x_{2i} + \lambda}$$

2. Yes, it is possible for it to produce the same result as cubic Smoothing Splines. The reason for that is the objective of cubic Smoothing Splines which is to find function g(x) that minimizes:

$$\sum_{i=1}^{n} (y_i - g(x_i))^2 + \lambda\int g''(t)^2 dt$$

Somehow we're trying to minimize a similar form, so after convergence we may end up with betas that more or less give the same result as Smoothing Splines.

3. Since $E(f_j(X_j)) = 0$, if we divide the formula for the optimal – say $\widehat{\beta}_1$ – in some iteration by N it becomes independent from $\widehat{\beta}_2$. Should this condition be altered then that term remains and thus we have update order dependency.
4. The smoothing operator based on cubic smoothing splines:

$$\widehat{g}_\lambda = S_\lambda y = argmin_\beta \sum_i (y_i - g(x_i))^2 + \lambda \int g''(t)^2 dt$$

$$S_\lambda = argmin_\lambda \frac{1}{y} \sum_i (y_i - g(x_i))^2 + \lambda \int g''(t)^2 dt$$