Rotations

Wojciech Pachowiak

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Coordinate systems

What orientations do the following mathematical rotation representations express?

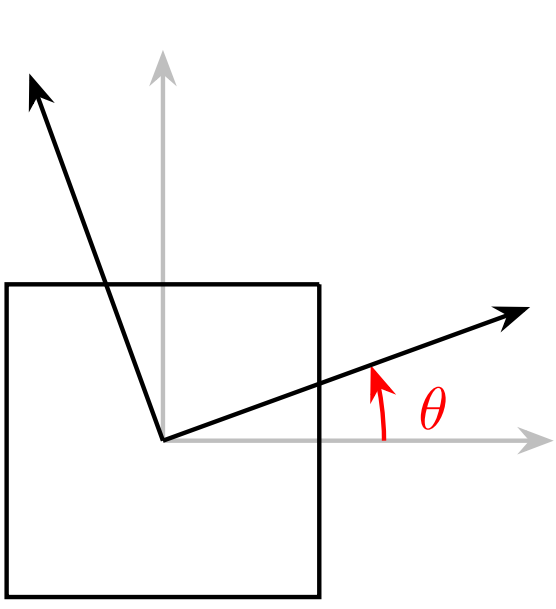
- Euler angles $(41°, -18°, -83°)$ in XYZ extrinsic order
- Quaternion $(0.73, (0.16, -0.34, -0.57))$
- Axis angle tuple $(86°, [0.24, -0.5, -0.84])$
- Axis angle vector $[0.36, -0.75, -1.26]$
- 3D rotation matrix

$$\begin{bmatrix} 0.116 & 0.725 & -0.68 \\ -0.944 & 0.3 & 0.15 \\ 0.31 & 0.624 & 0.718 \end{bmatrix}$$

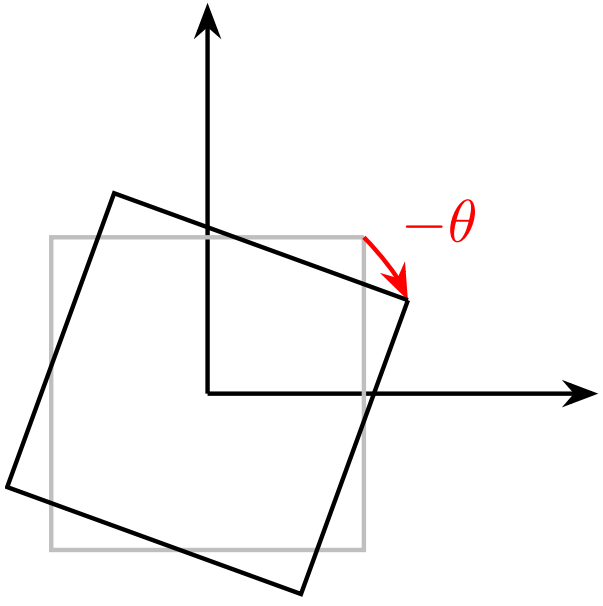
The same orientation but with respect to what coordinate system (frame of reference)?

Active and passive rotations

Which is moving? Objects within a coordinate system or the coordinate system itself?



(a) Passive rotation

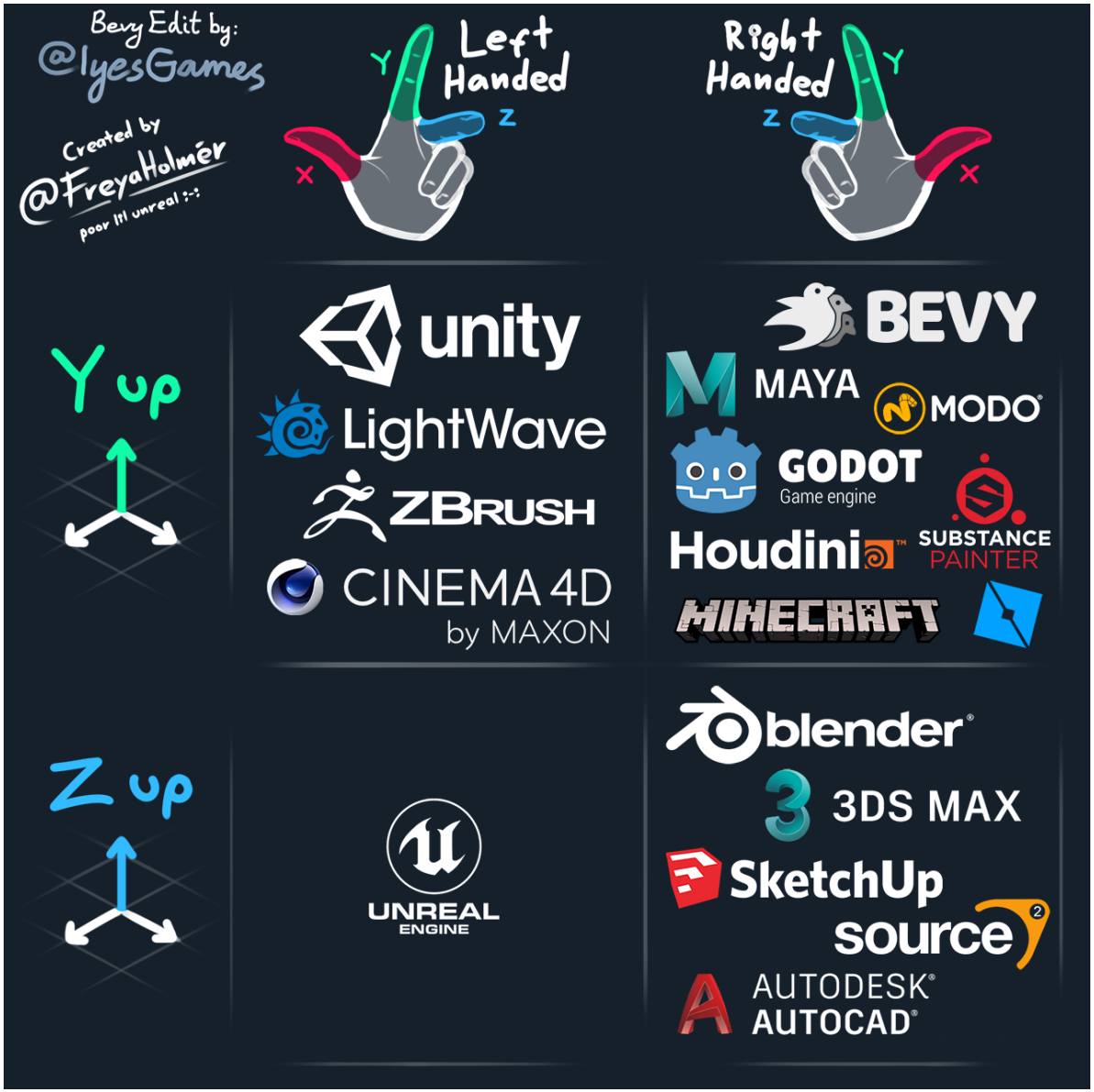


(b) Active rotation

To convert between these two perspectives, invert the rotation:

$$\mathbf{R}_{active} = \mathbf{R}^{-1}_{passive}$$

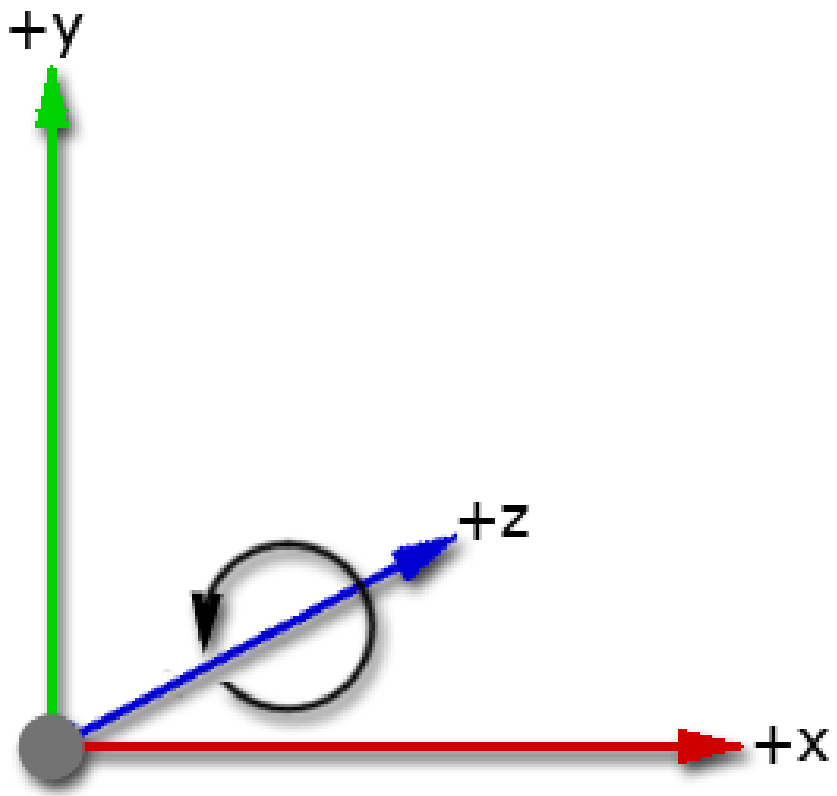
Handedness of coordinate system



https://bevy-cheatbook.github.io/fundamentals/coords.html

Positive angle

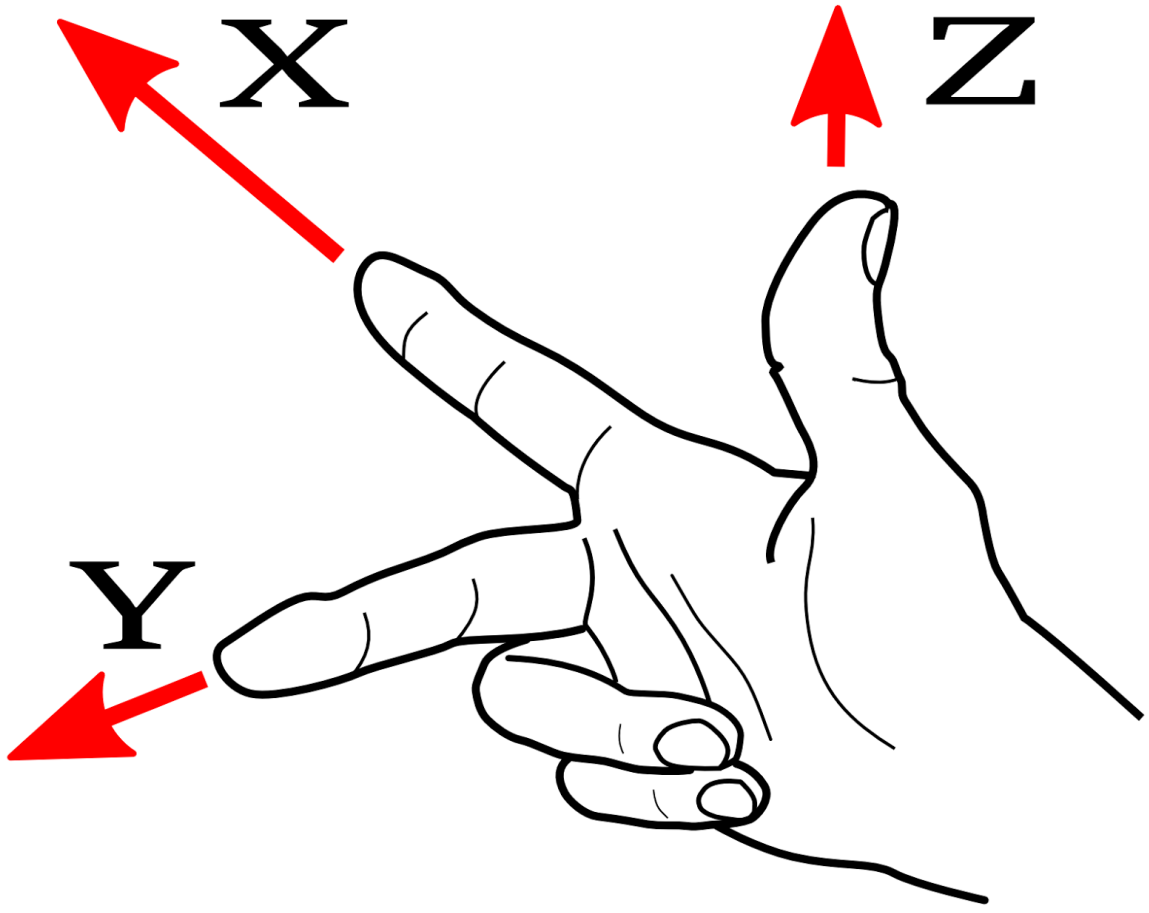
Is positive angle clockwise or counterclockwise when looking along some axis?



https://docs.unity3d.com/Manual/QuaternionAndEulerRotationsInUnity.html

Forward, up, right vectors

Which vectors are considered forward, upward, rightward, leftward, etc.?



https://stackoverflow.com/a/34068511

Intrinsic and extrinsic rotations

When it comes to Euler angles, are rotations performed around the global axes or some local axes?
TODO formula for conversion between them

See the pictures for a visual explanation:
`https://en.wikipedia.org/w/index.php?title=Davenport_chained_rotations&oldid=1222677779#Conversion_between_intrinsic_and_extrinsic_rotations`

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2D rotations

Euler angles

3D rotation matrices: Euler-angle-like

3D rotation matrices: axis-angle-like

Quaternions

Rodriguez formula

Axis-angle, exponential map

In SciPy

`https://docs.scipy.org/doc/scipy/reference/generated/scipy.spatial.transform.Rotation.as_quat.html`

Note the *seq* argument in *as_euler* method. Also note the *canonical* and *scalar_first* arguments in *as_quat*.

We didn't cover:

- Davenport angles — like Euler angles but generalized to nonperpendicular axes.
- Modified Rodriguez Parameters (MRP) — like axis-angle vectors but the angle of rotation is transformed.

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Gimbal lock

A-to-B rotation interpolation

A-to-C-through-B rotation interpolation