

# **Neural Networks**

## **Lecture 3 - Convolutional Neural Networks**

**Igor Janos**

# Agenda

## 1. Convolution Operation

- Examples of hand-crafted convolutional filters

## 2. Convolutional Neural Network

- Convolutional layer, assembling multiple layers together
- Convolutional architectures

## 3. Siamese Networks

- Image similarity, Face recognition

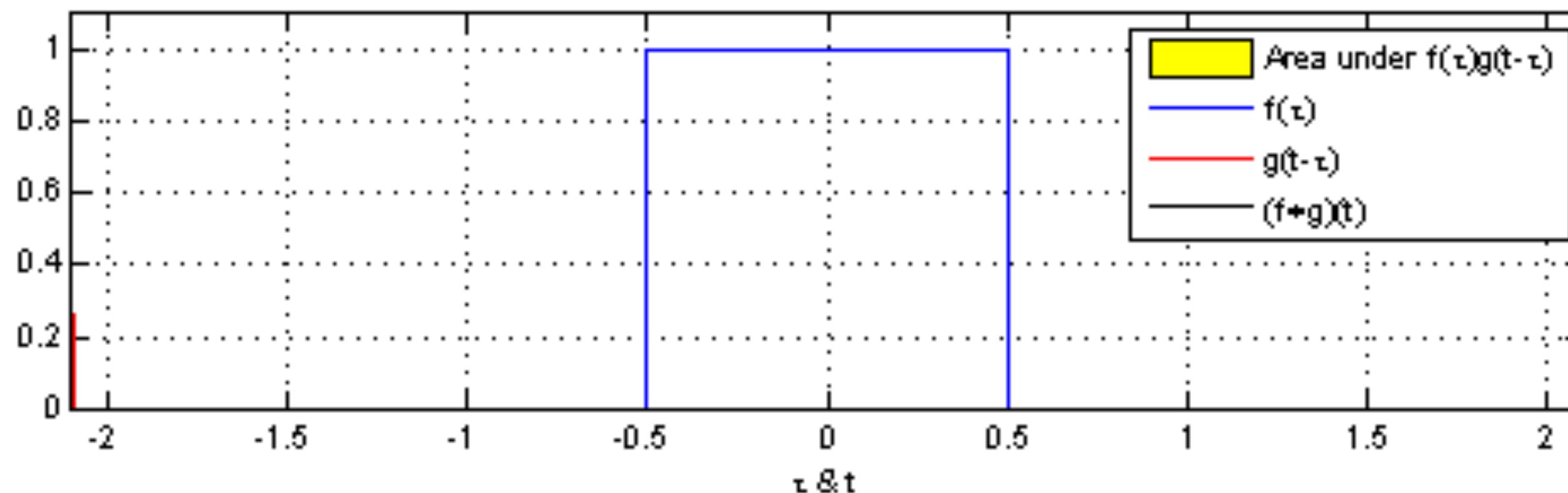
# 1 Convolution Operation

# Convolution

- Integral transform

$$(f * g)(t) := \int_{-\infty}^{\infty} f(\tau)g(t - \tau)d\tau$$

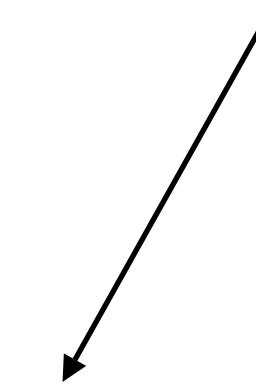
Reflected about  $y$ -axis and shifted



# Discrete Convolution

$$(f * g)[n] := \sum_{m=-\infty}^{\infty} f[m]g[n - m]$$

Reflected about  $y$ -axis and shifted



# Discrete Convolution

## Commutativity

$$(f * g) = (g * f)$$

$$(f * g)(t) := \int_{-\infty}^{\infty} f(\tau)g(t - \tau)d\tau$$

$$(f * g)[n] := \sum_{m=-\infty}^{\infty} f[m]g[n - m]$$

$$(f * g)(t) := \int_{-\infty}^{\infty} f(t - \tau)g(\tau)d\tau$$

$$(f * g)[n] := \sum_{m=-\infty}^{\infty} f[n - m]g[m]$$

# Discrete Convolution

- Let's define  $g$  only on a small interval  $m \in \{-1,0,1\}$

$$(f * g)[n] := \sum_{m=-1}^1 f[n - m]g[m]$$

 Kernel with size 3

# Discrete Convolution

## Image Processing

- Extend for 2-dimensional data

$$(f * g)[x, y] := \sum_{i=-a}^a \sum_{j=-b}^b f[x - i, y - j]g[i, j]$$



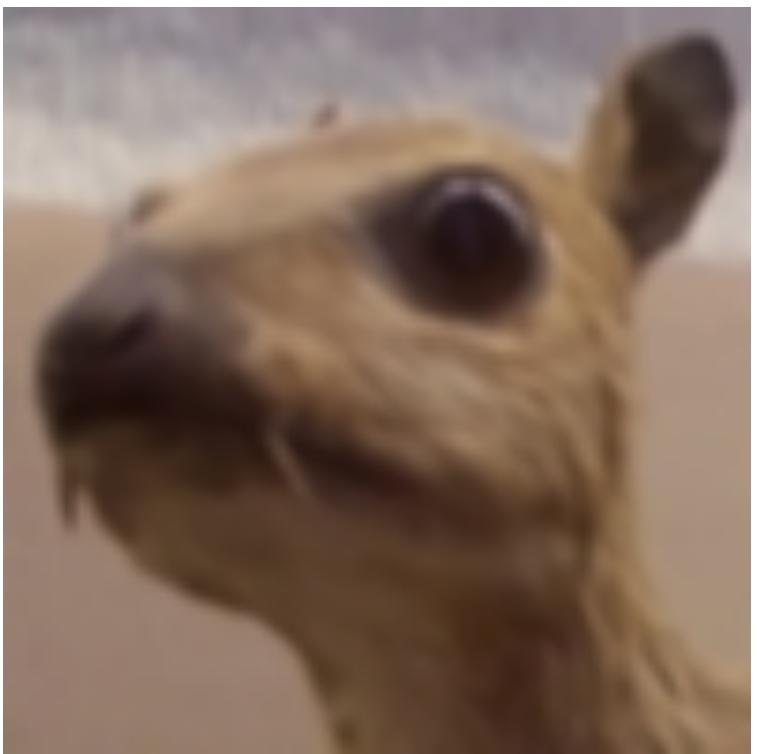
Kernel size =  $(2a + 1) \times (2b + 1)$

# Discrete Convolution

## Image Processing - Examples

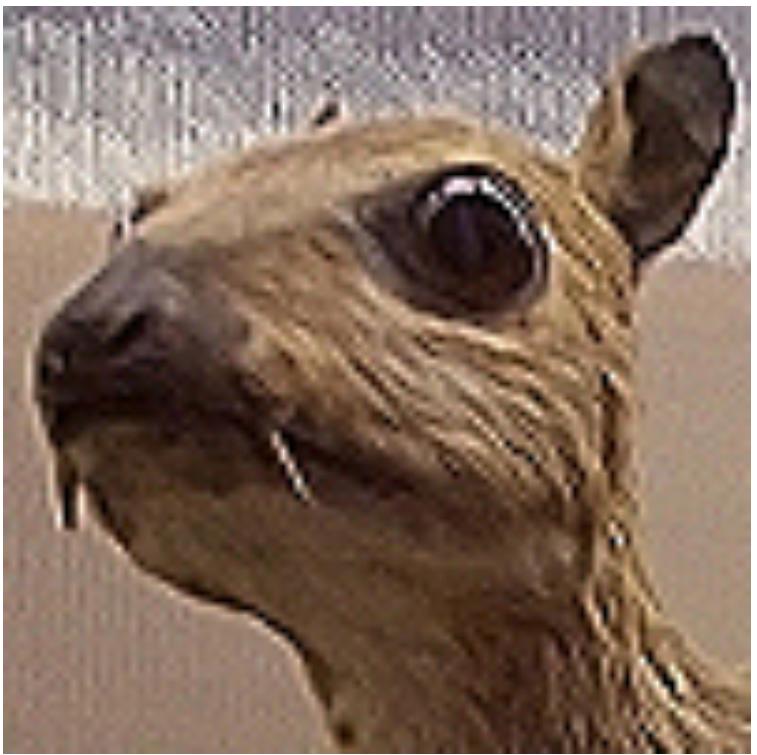
$$\frac{1}{16} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

Gaussian Blur



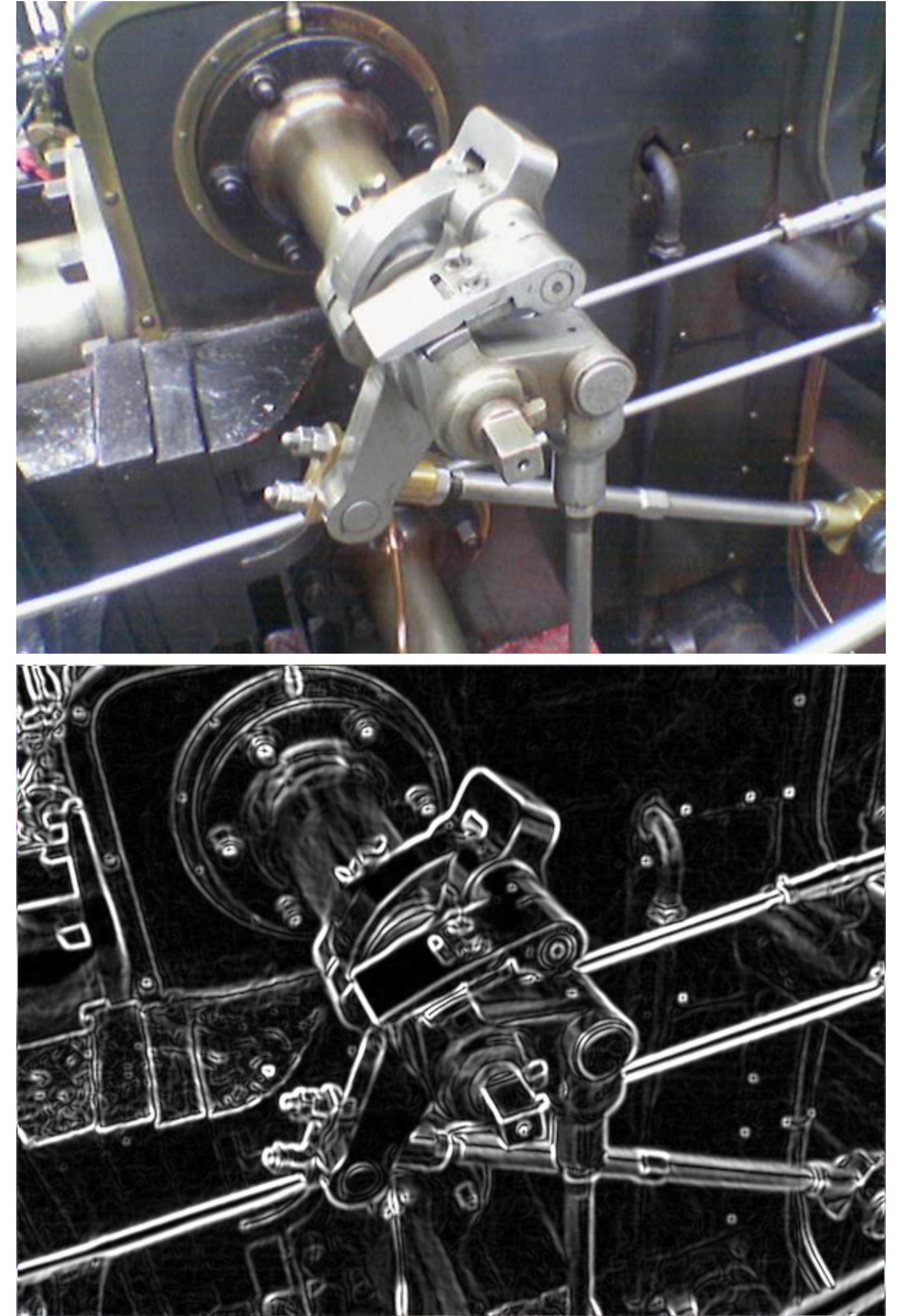
$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

Sharpen



$$\begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

Sobel Operator



# Discrete Convolution

## Image Processing - Examples

- Convolution gives us **representations**
- Representations are specified by the **kernel matrix**
- Hand-crafted or **learned**

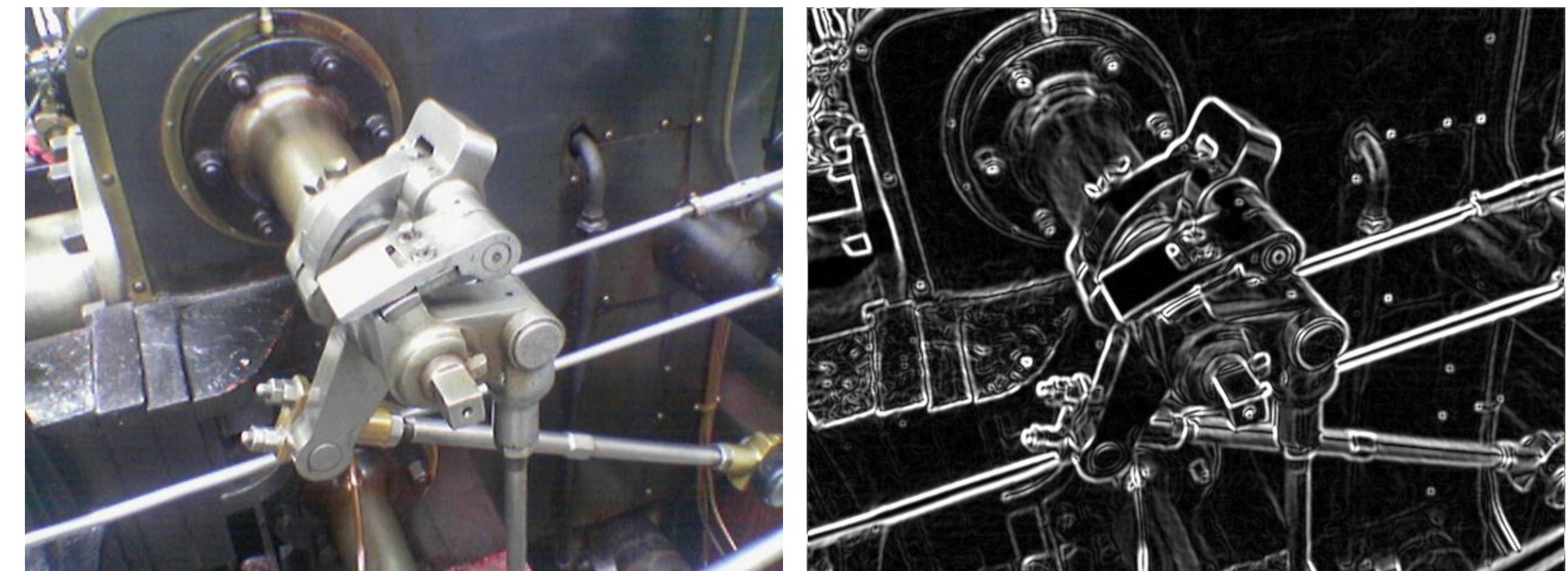
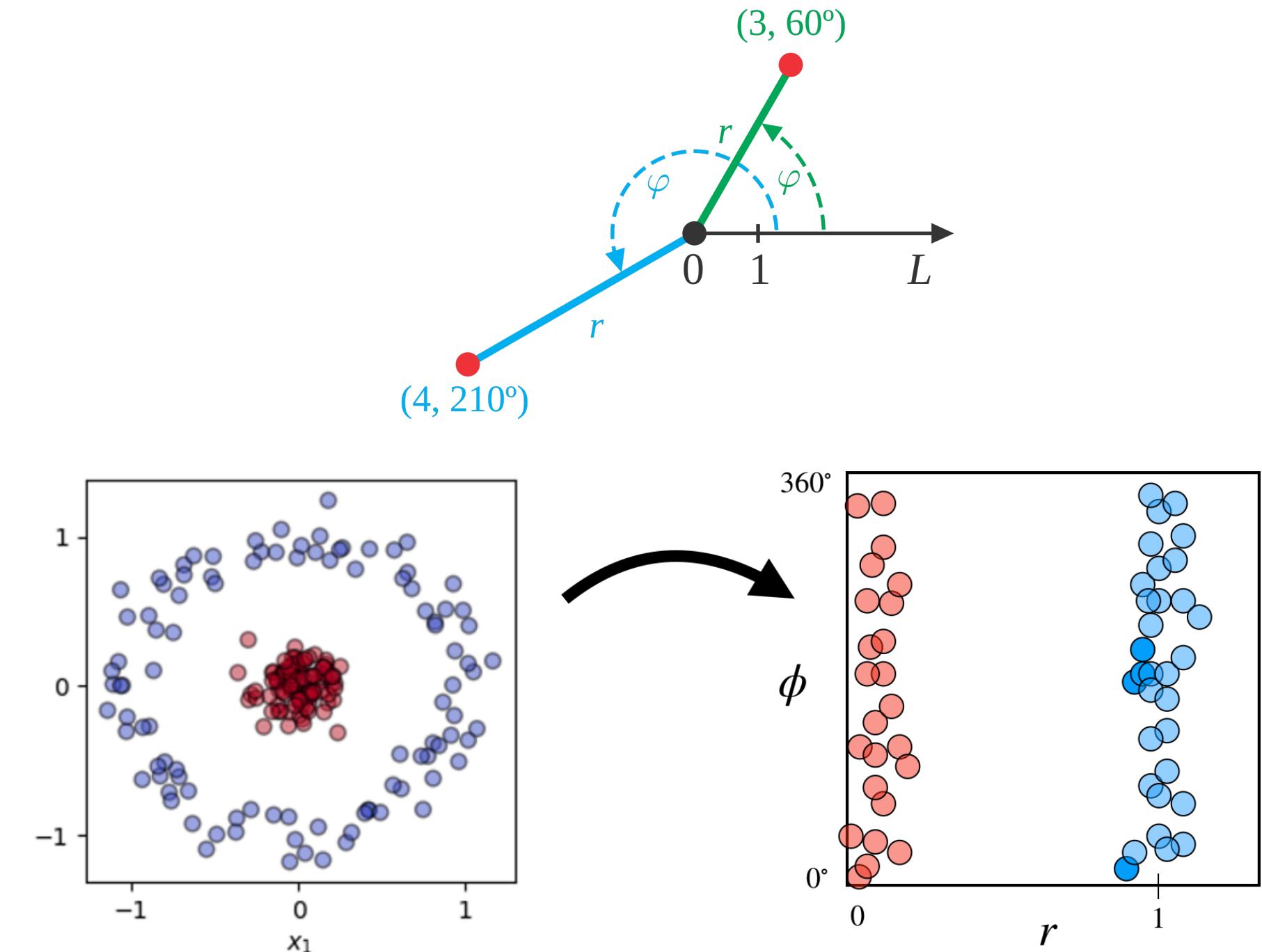


Image - [https://en.wikipedia.org/wiki/Sobel\\_operator](https://en.wikipedia.org/wiki/Sobel_operator)

# **2 Convolutional Neural Network**

# Convolutional Neural Network

- **Inspired by biology**
  - Visual cortex - topographical representation of the field of vision
  - Reception field of neighbouring cells corresponds to adjacent portions of the field of vision
  - Sequence of cells can perceive complex shapes
- **Uses convolution as building blocks**

# Convolutional Neural Network

## Convolution vs. Cross-Correlation

Convolution

$$(f * g)[n] := \sum_{m=-1}^1 f[n - m]g[m]$$

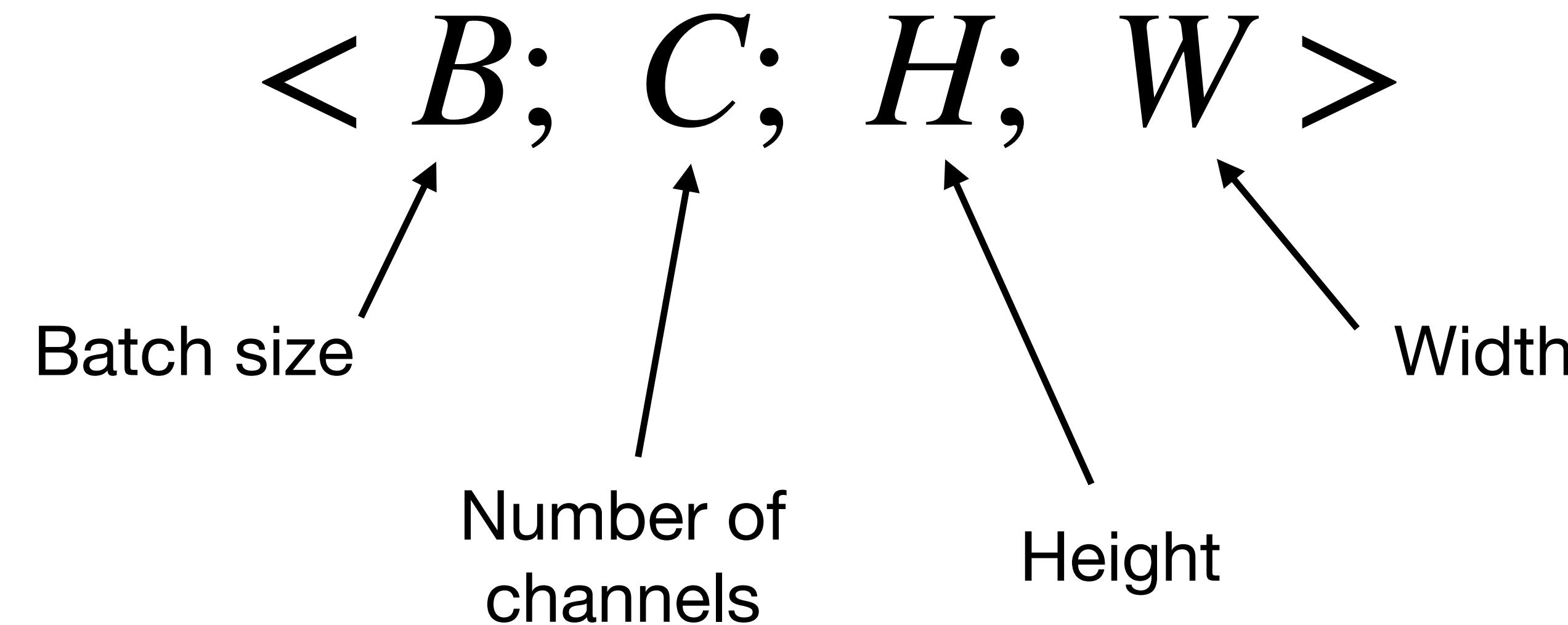
Cross-Correlation

$$(f \circ g)[n] := \sum_{m=-1}^1 f[n + m]g[m]$$

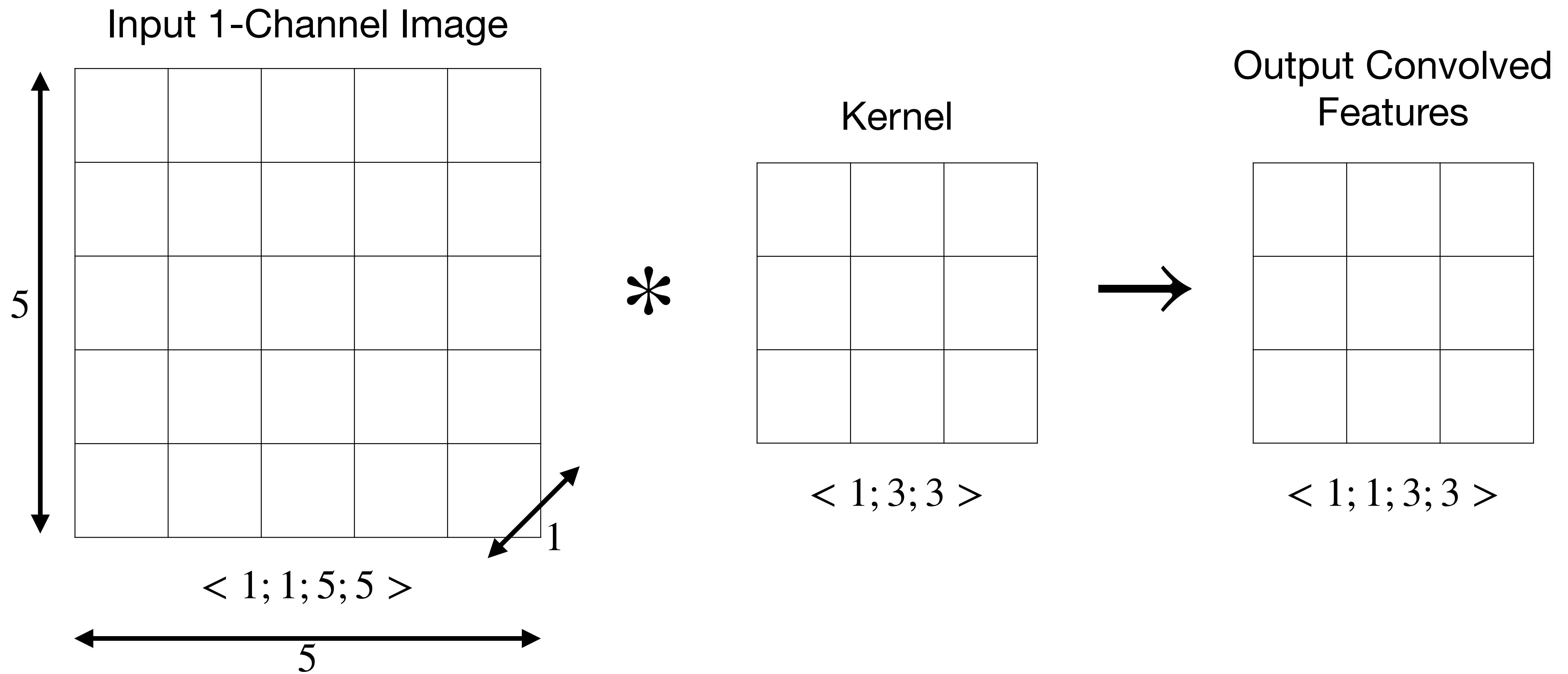
- The **kernel is learned** - we do not flip the kernel
- Cross-correlation is easier to implement
- We call it “**Convolution**”

# Convolutional Layer

- Assume we are working with 2D data (images)
- Our data (tensors) come in shape:



# Convolutional Layer Example

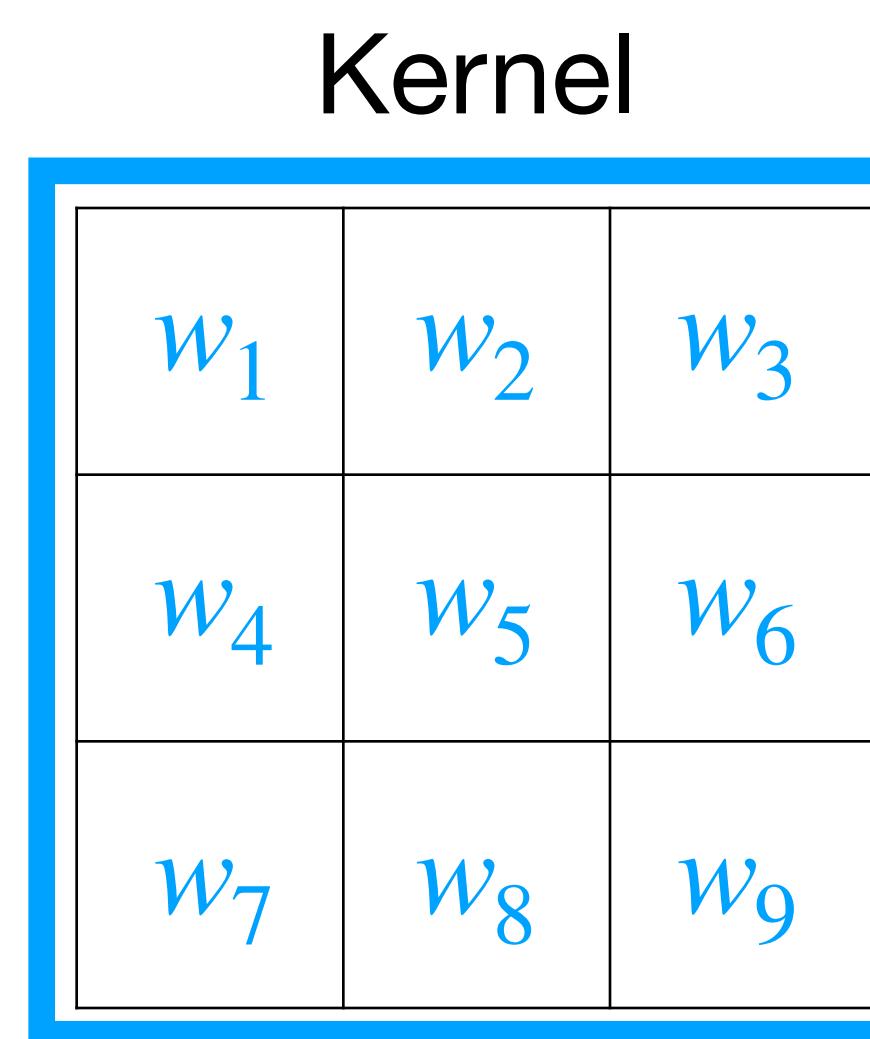


# Convolutional Layer Example

$$(f * g)[x, y] := \sum_{i=-1}^1 \sum_{j=-1}^1 f[x + i, y + j]g[i, j]$$

Input 1-Channel Image


\*



$< 1; 3; 3 >$

$< 1; 1; 5; 5 >$

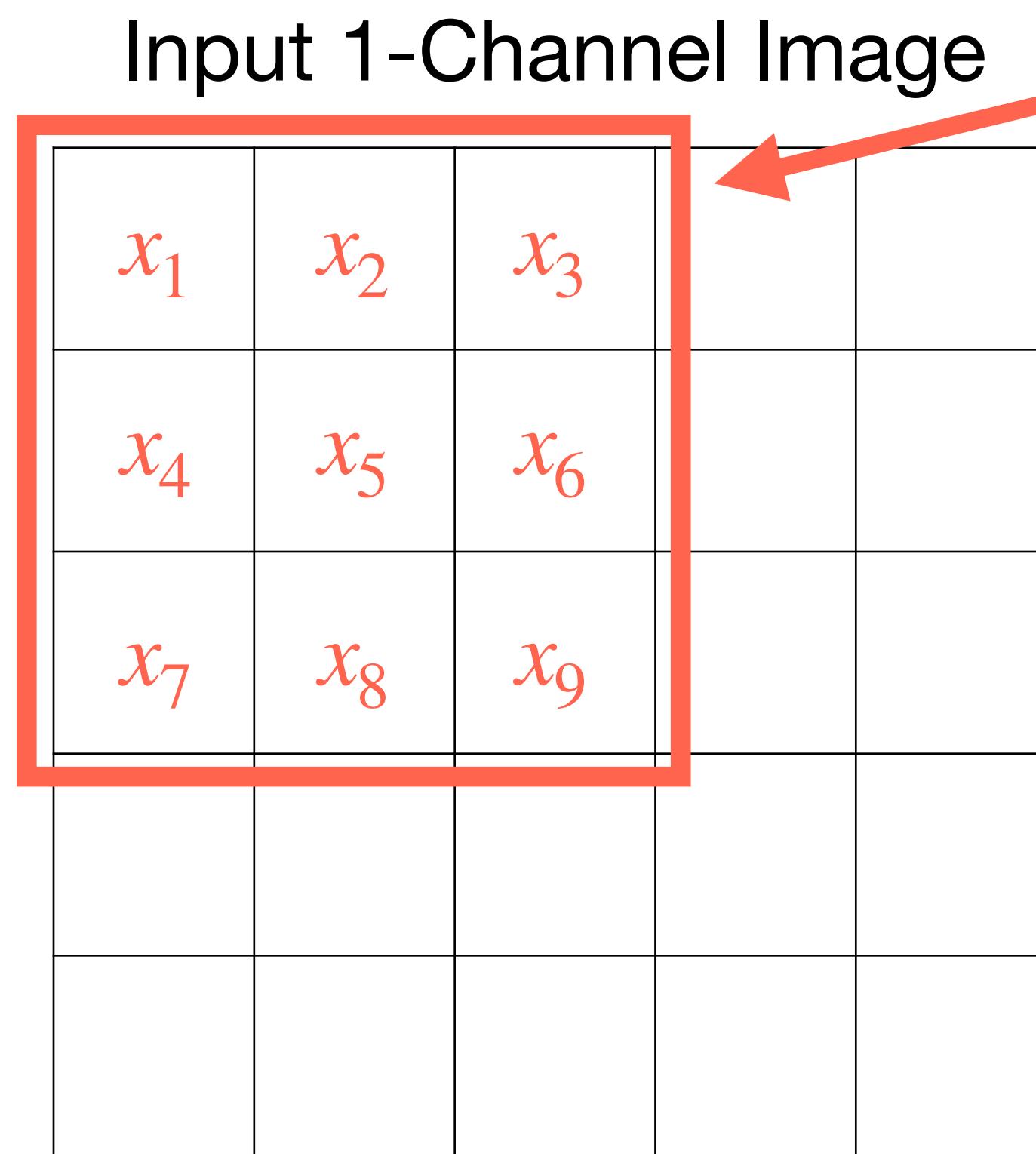
Output Convolved Features



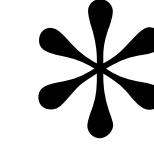

$< 1; 1; 3; 3 >$

# Convolutional Layer

## Example



Kernel



$w_1$	$w_2$	$w_3$
$w_4$	$w_5$	$w_6$
$w_7$	$w_8$	$w_9$

$< 1; 3; 3 >$

$< 1; 1; 5; 5 >$

$$(f * g)[x, y] := \sum_{i=-1}^1 \sum_{j=-1}^1 f[x + i, y + j]g[i, j]$$

Output Convolved  
Features




$< 1; 1; 3; 3 >$

# Convolutional Layer

## Example

Input 1-Channel Image

$x_1$	$x_2$	$x_3$		
$x_4$	$x_5$	$x_6$		
$x_7$	$x_8$	$x_9$		

\*

Kernel

$w_1$	$w_2$	$w_3$
$w_4$	$w_5$	$w_6$
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$< 1; 3; 3 >$

$$(f * g)[x, y] := \sum_{i=-1}^1 \sum_{j=-1}^1 f[x + i, y + j]g[i, j]$$

Output Convolved  
Features

$z_1$		

$< 1; 1; 3; 3 >$

# Convolutional Layer Example

$$(f * g)[x, y] := \sum_{i=-1}^1 \sum_{j=-1}^1 f[x + i, y + j]g[i, j]$$

Input 1-Channel Image

$x_1$	$x_2$	$x_3$		
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Kernel

$w_1$	$w_2$	$w_3$
$w_4$	$w_5$	$w_6$
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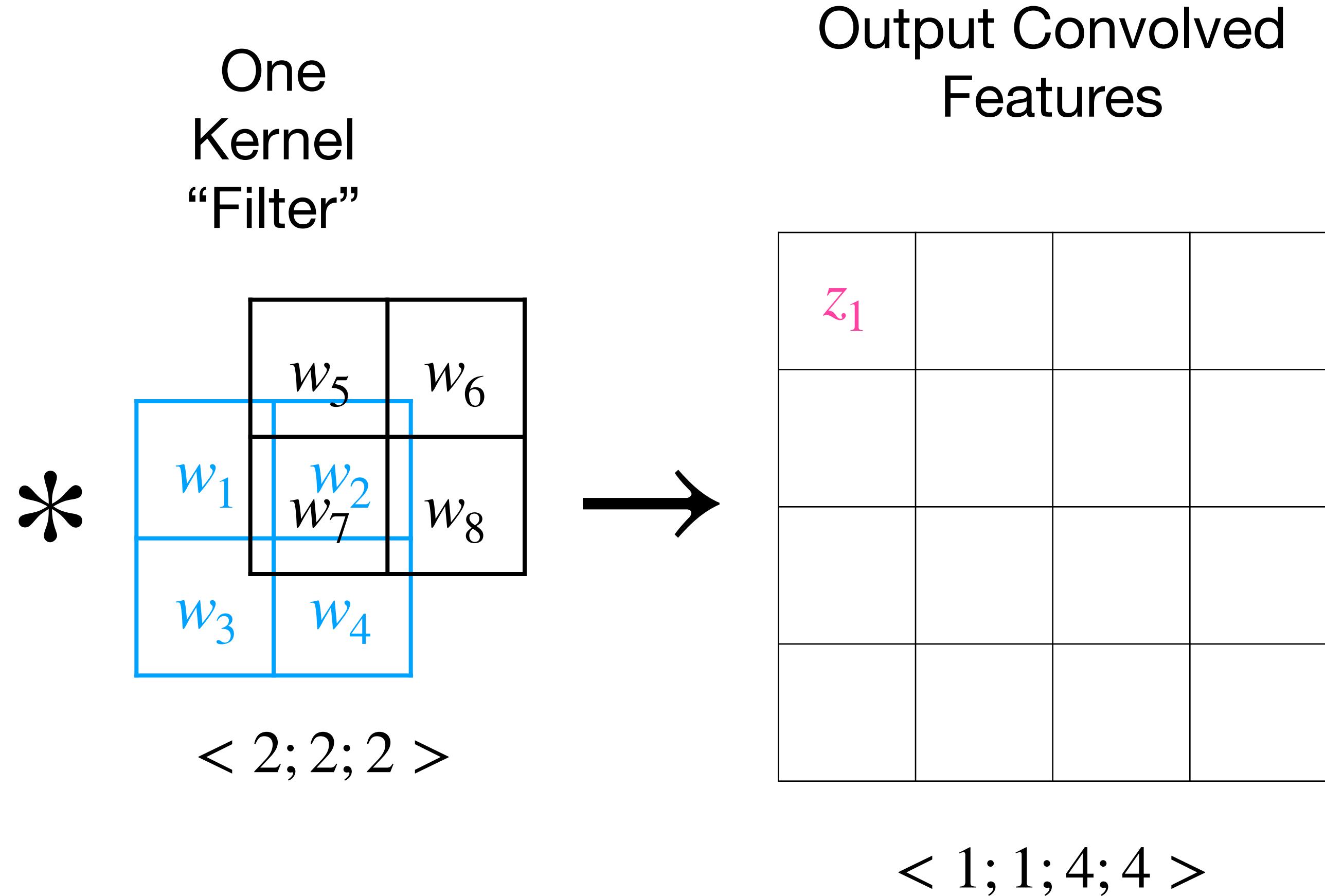
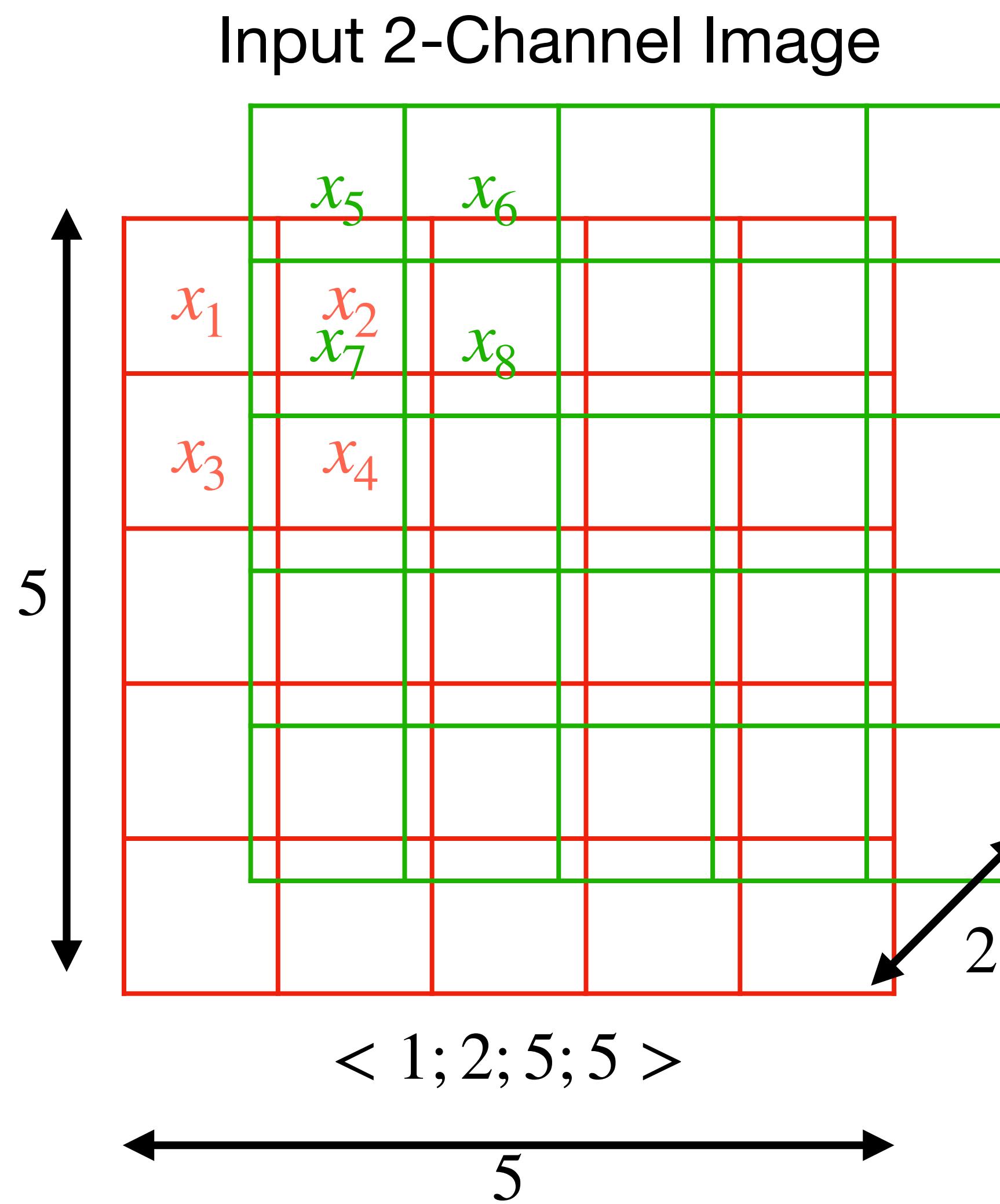
Output Convolved Features

$z_1$		

Bias !

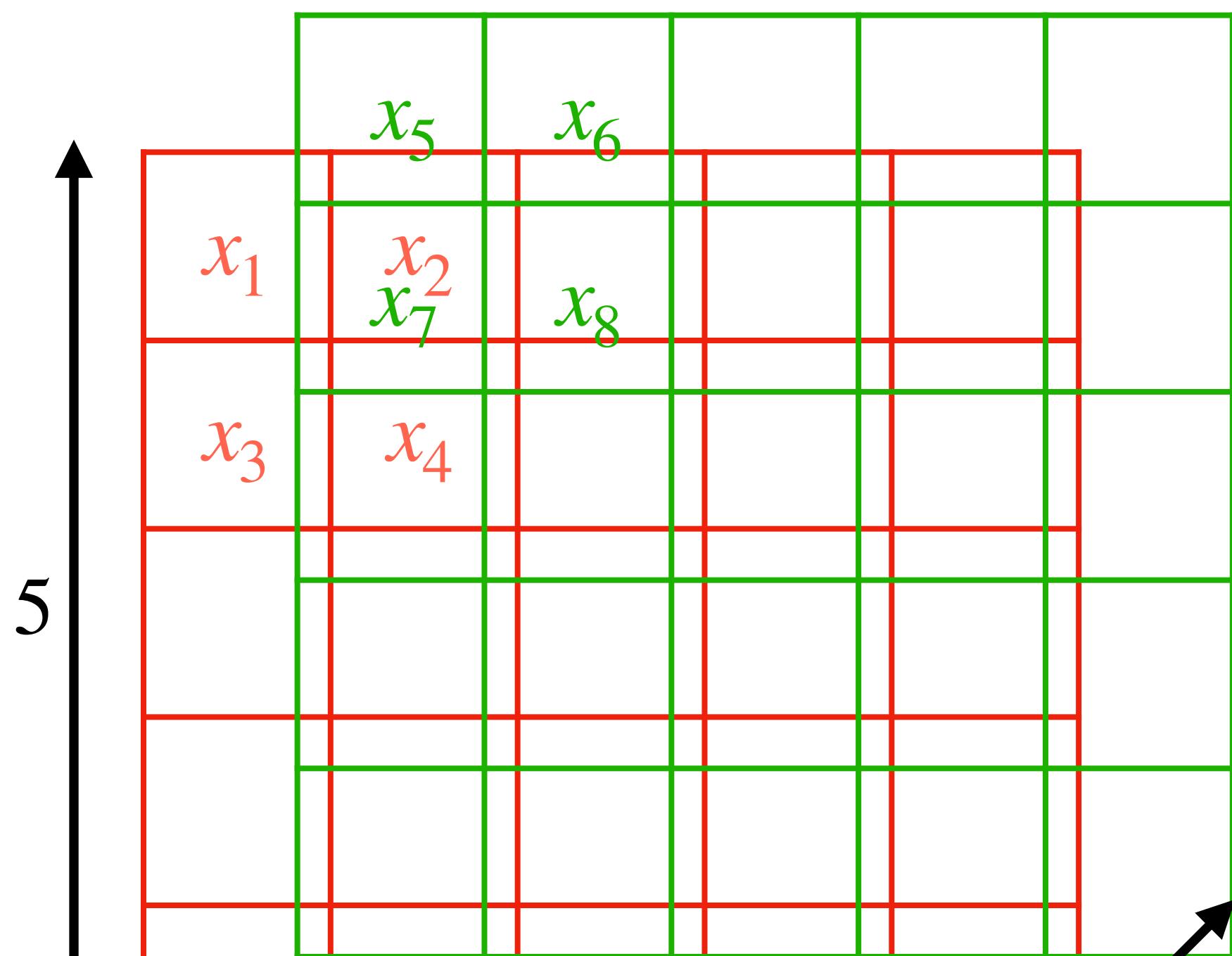
$$z_1 = w_1x_1 + w_2x_2 + \dots + w_9x_9 + b$$

# Convolutional Layer Example

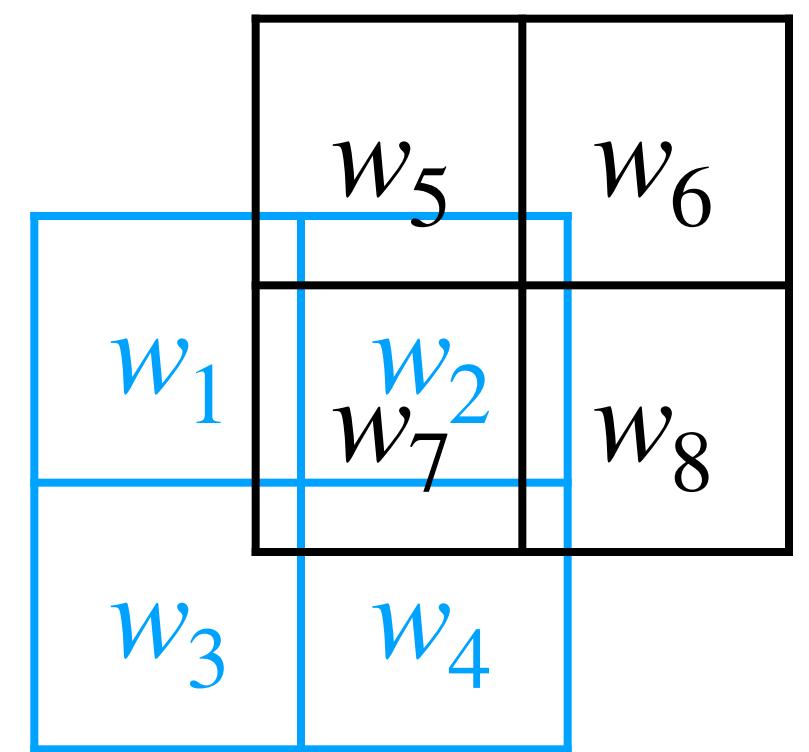
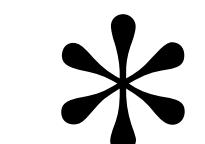


# Convolutional Layer Example

Input 2-Channel Image

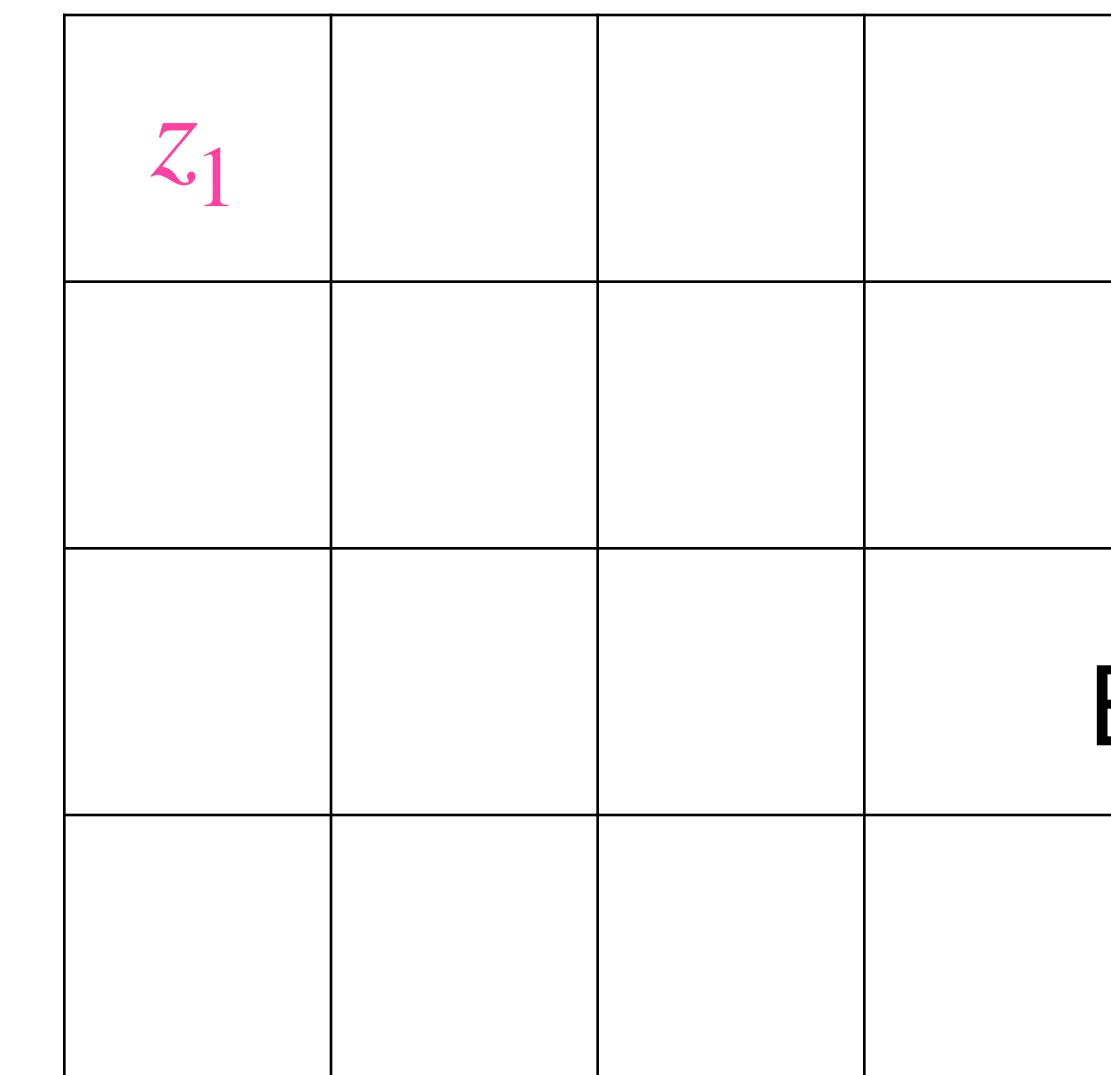


One  
Kernel  
“Filter”



$< 2; 2; 2 >$

Output Convolved  
Features

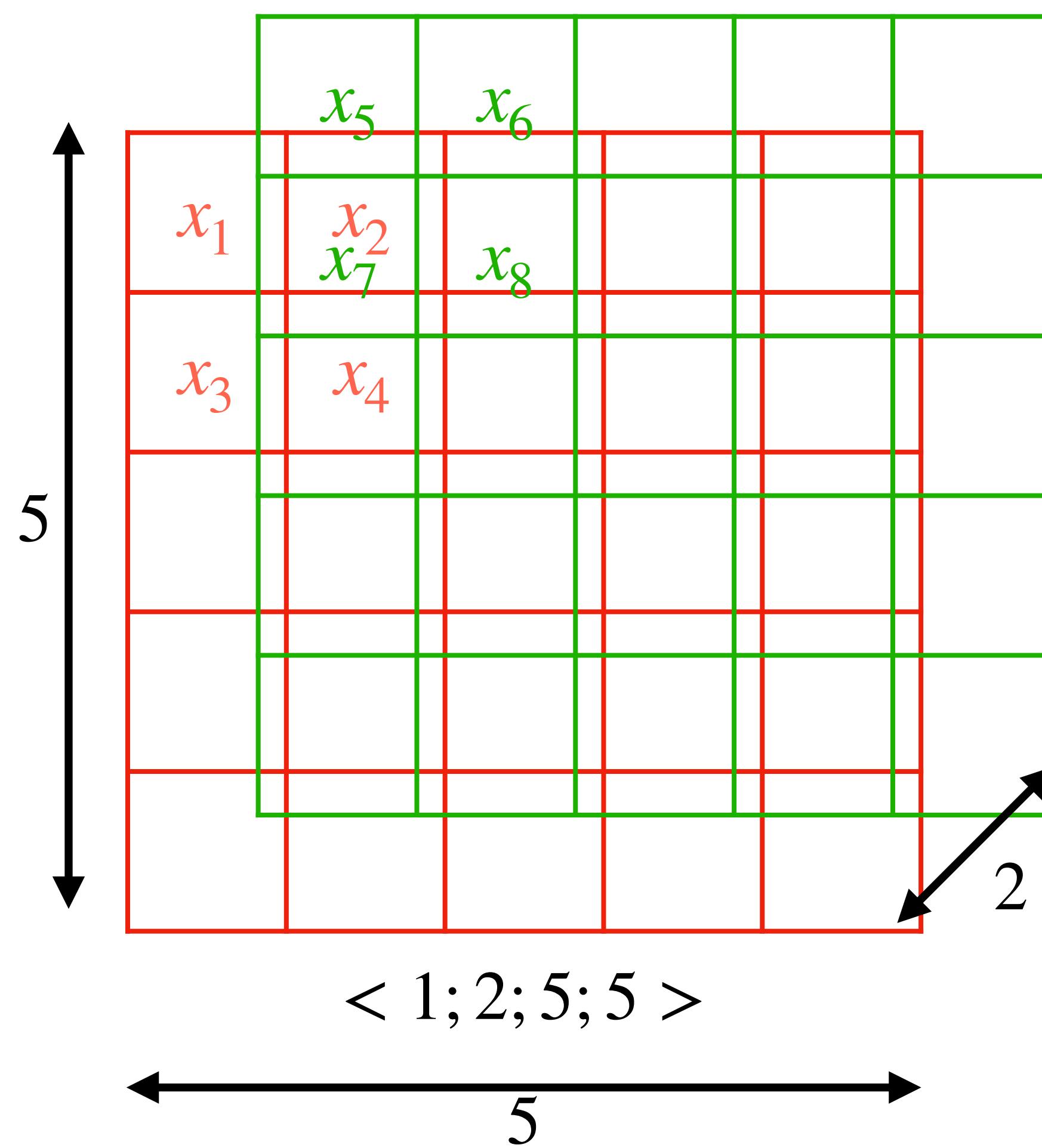


$$z_1 = w_1 x_1 + w_2 x_2 + \dots + w_7 x_7 + w_8 x_8 + b$$

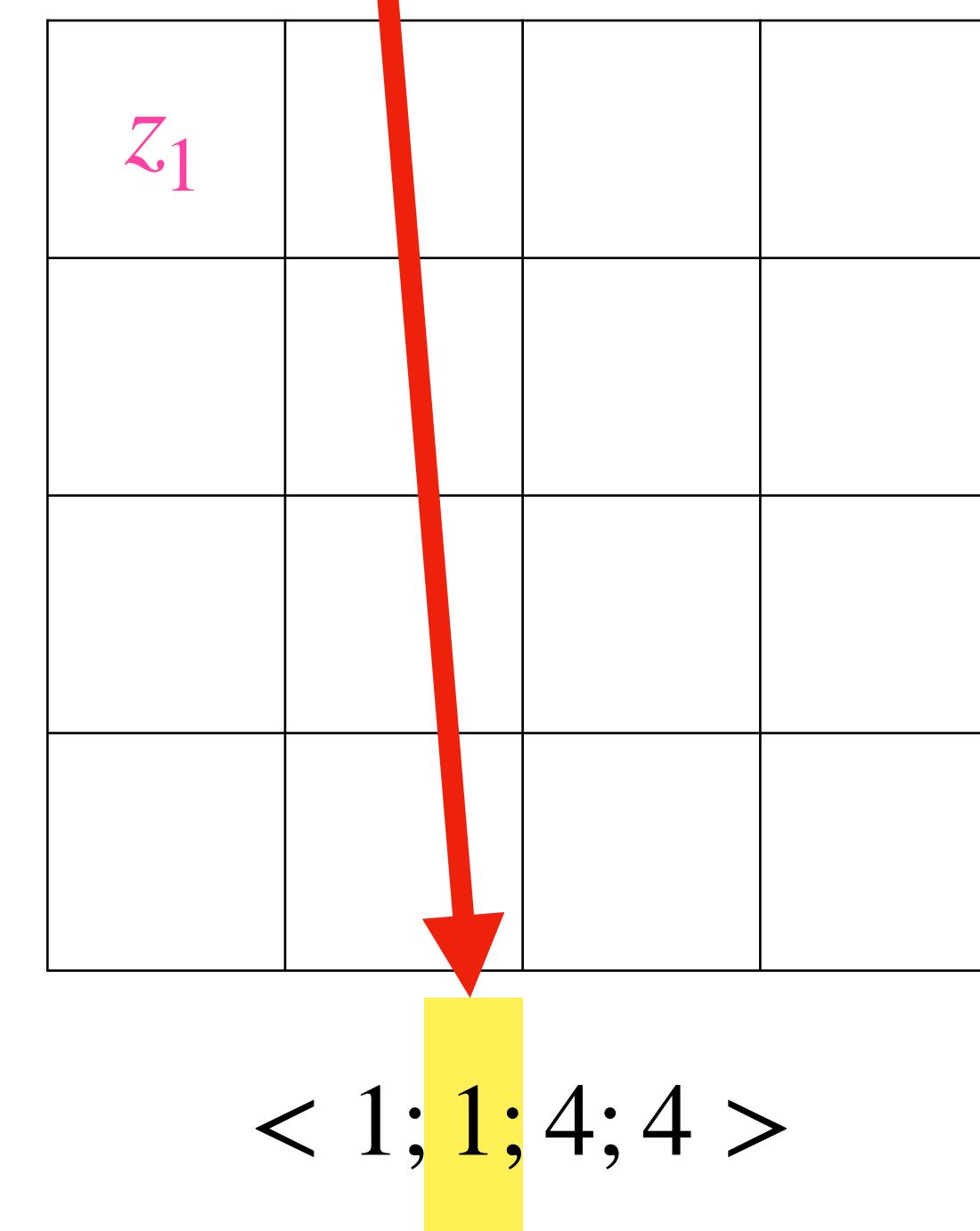
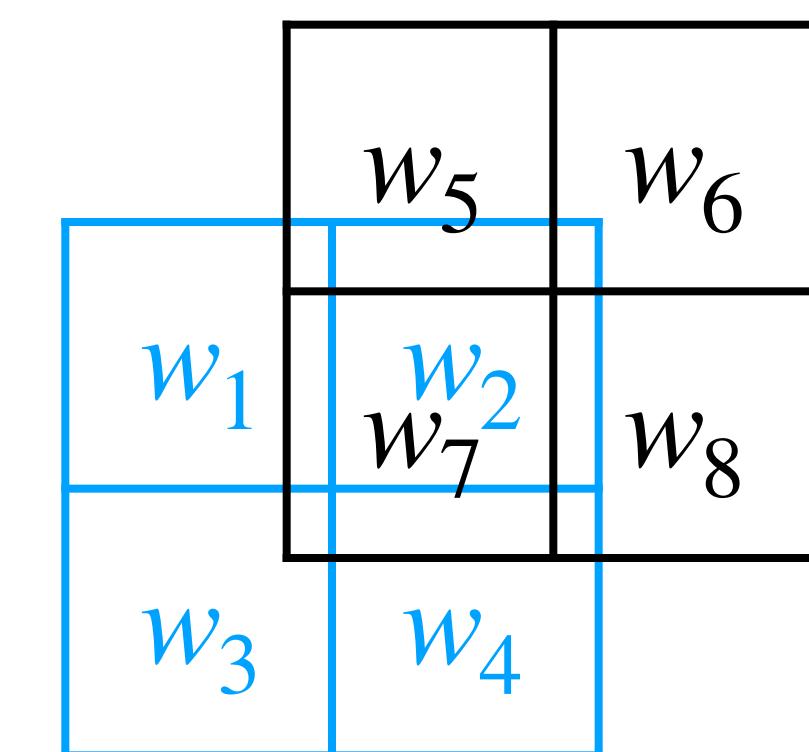
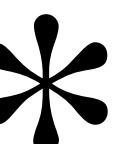
# Convolutional Layer

## Example

Input 2-Channel Image



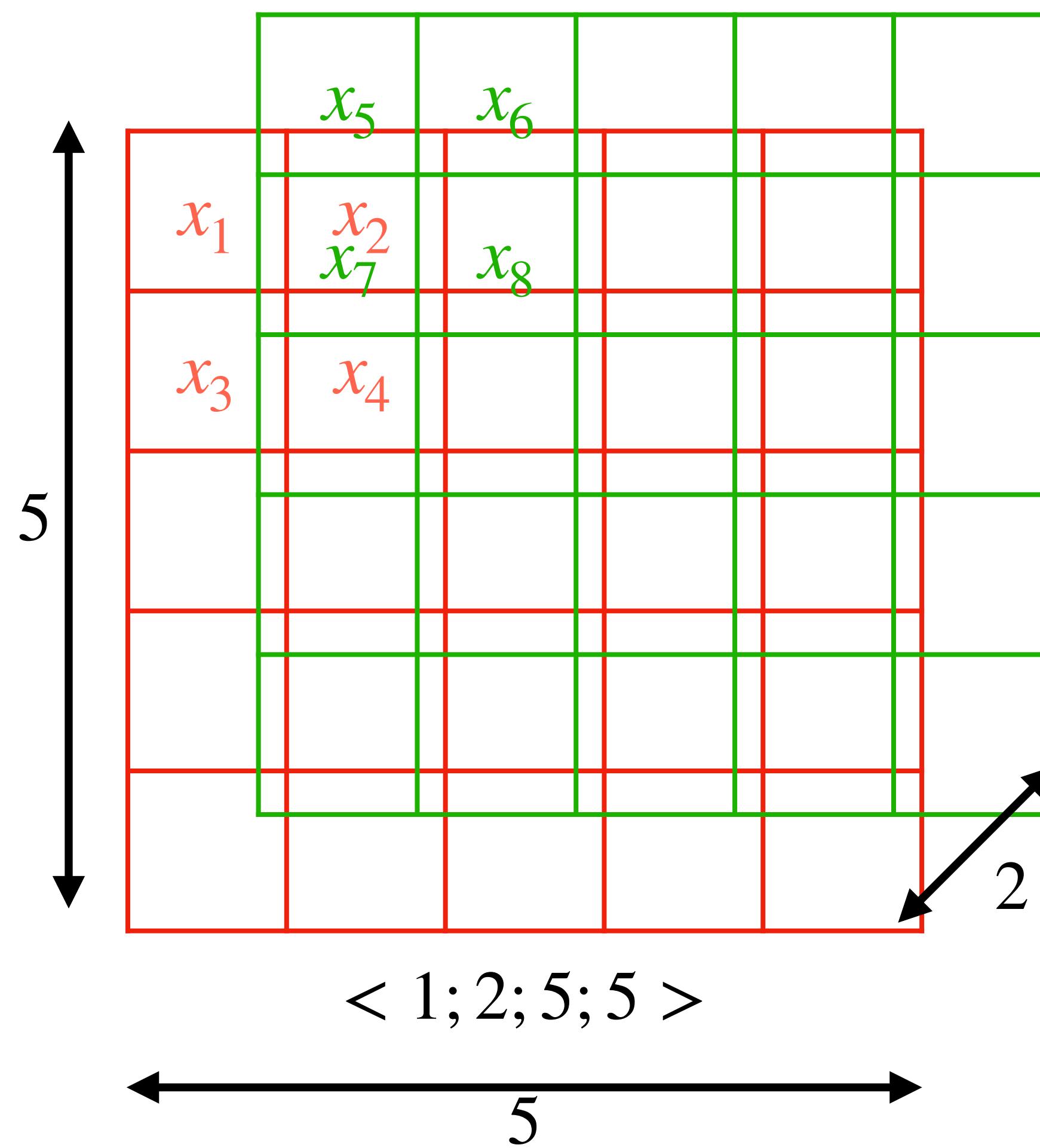
One  
Kernel  
“Filter”



One filter

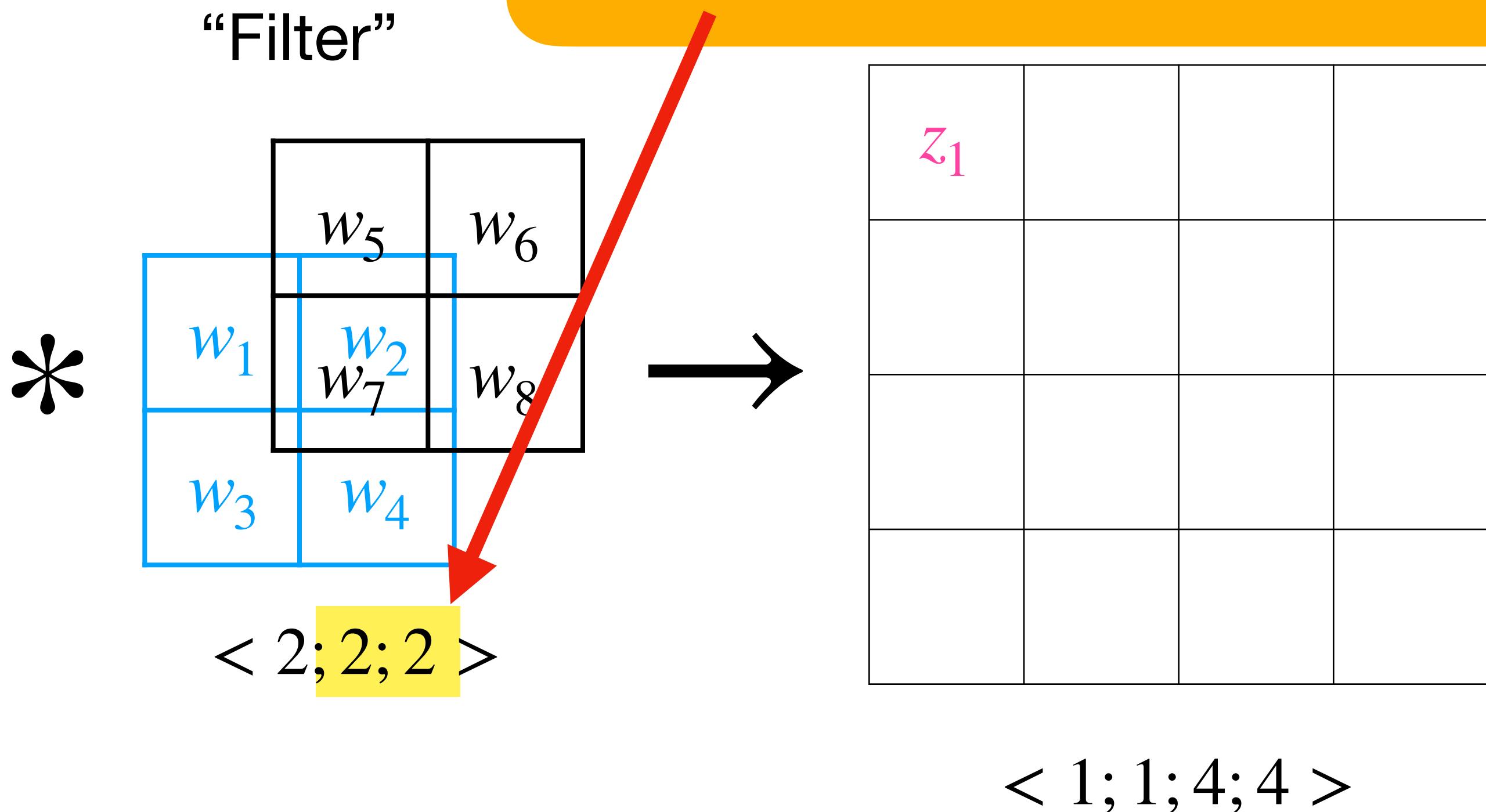
# Convolutional Layer Example

Input 2-Channel Image



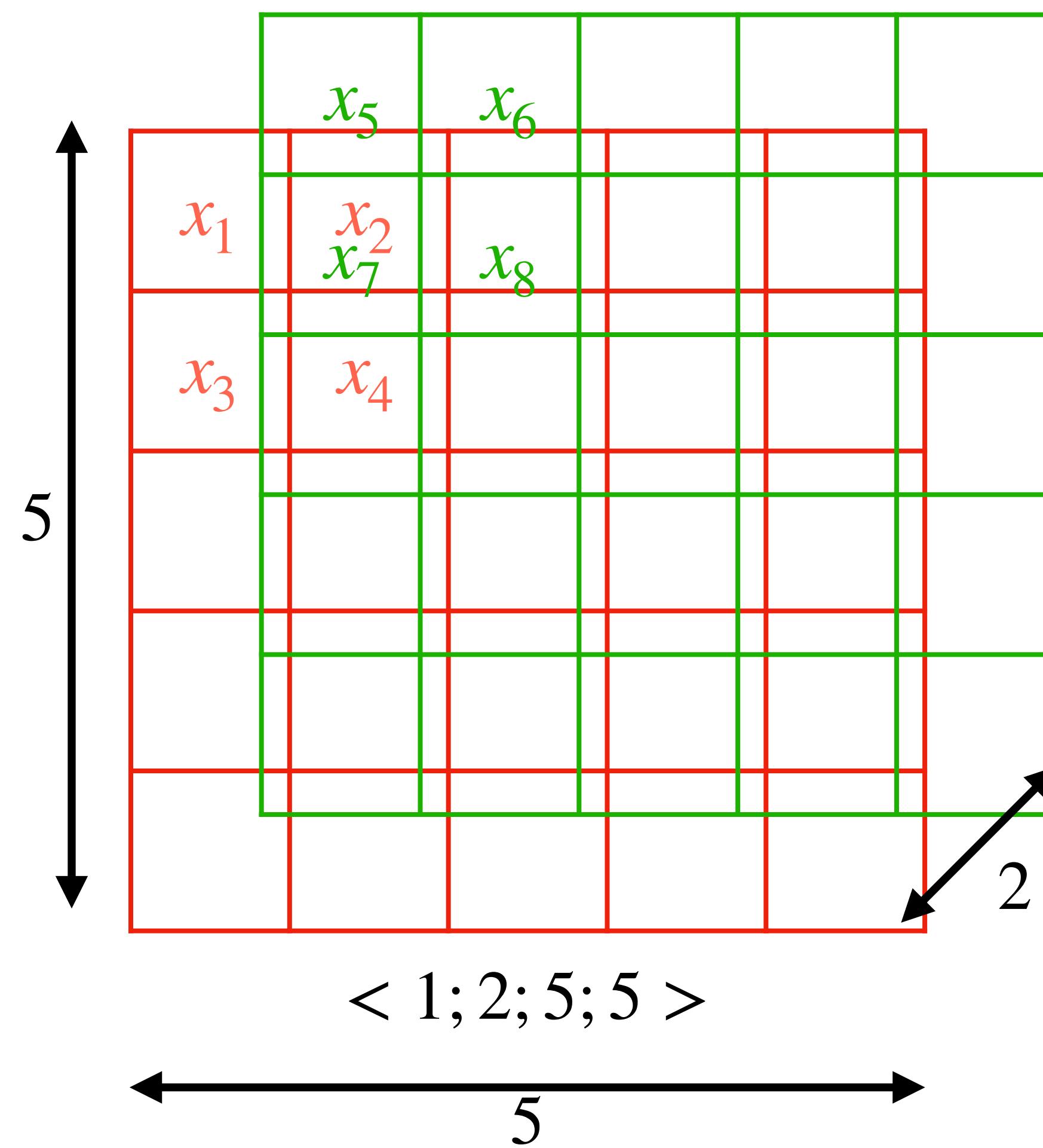
One Kernel  
“Filter”

One filter  
with kernel size  $2 \times 2$

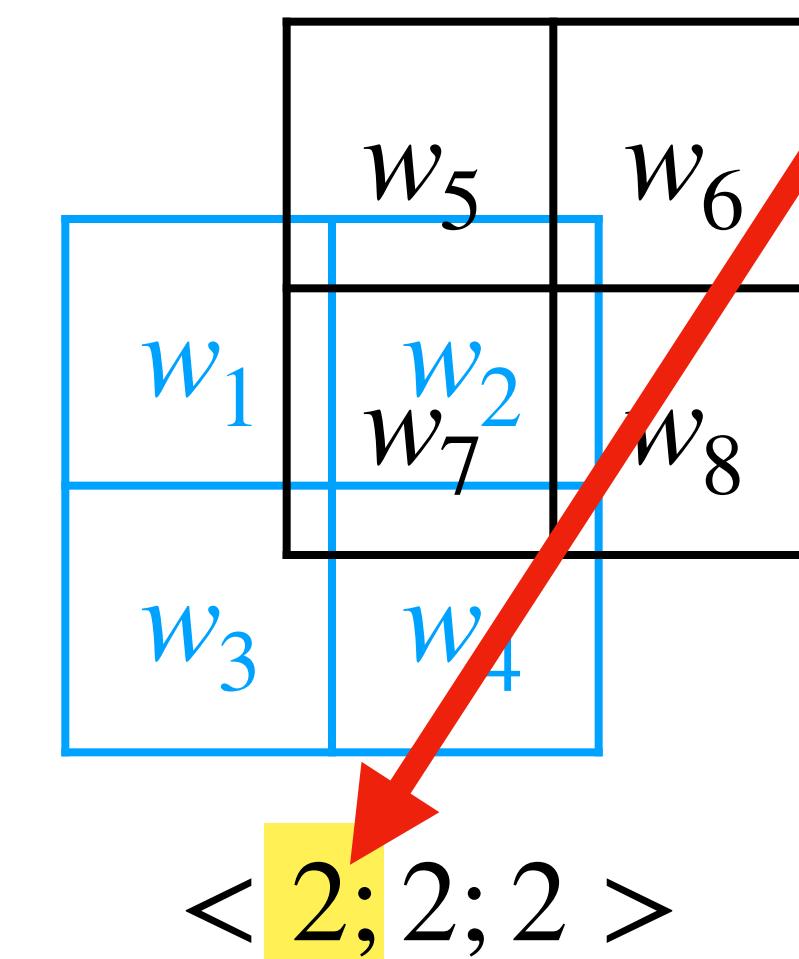
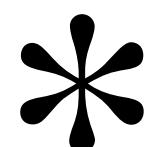


# Convolutional Layer Example

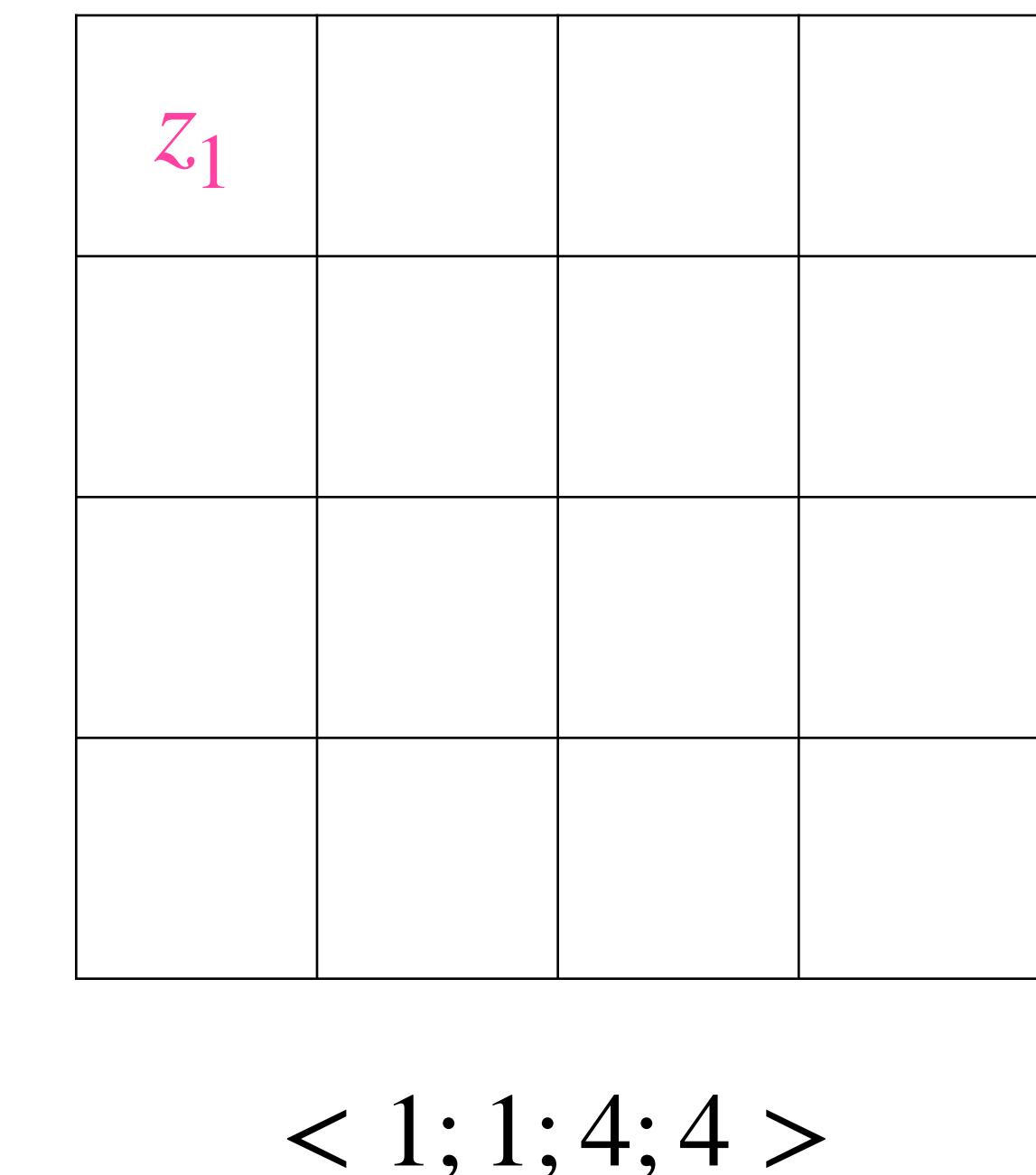
Input 2-Channel Image



One  
Kernel  
“Filter”

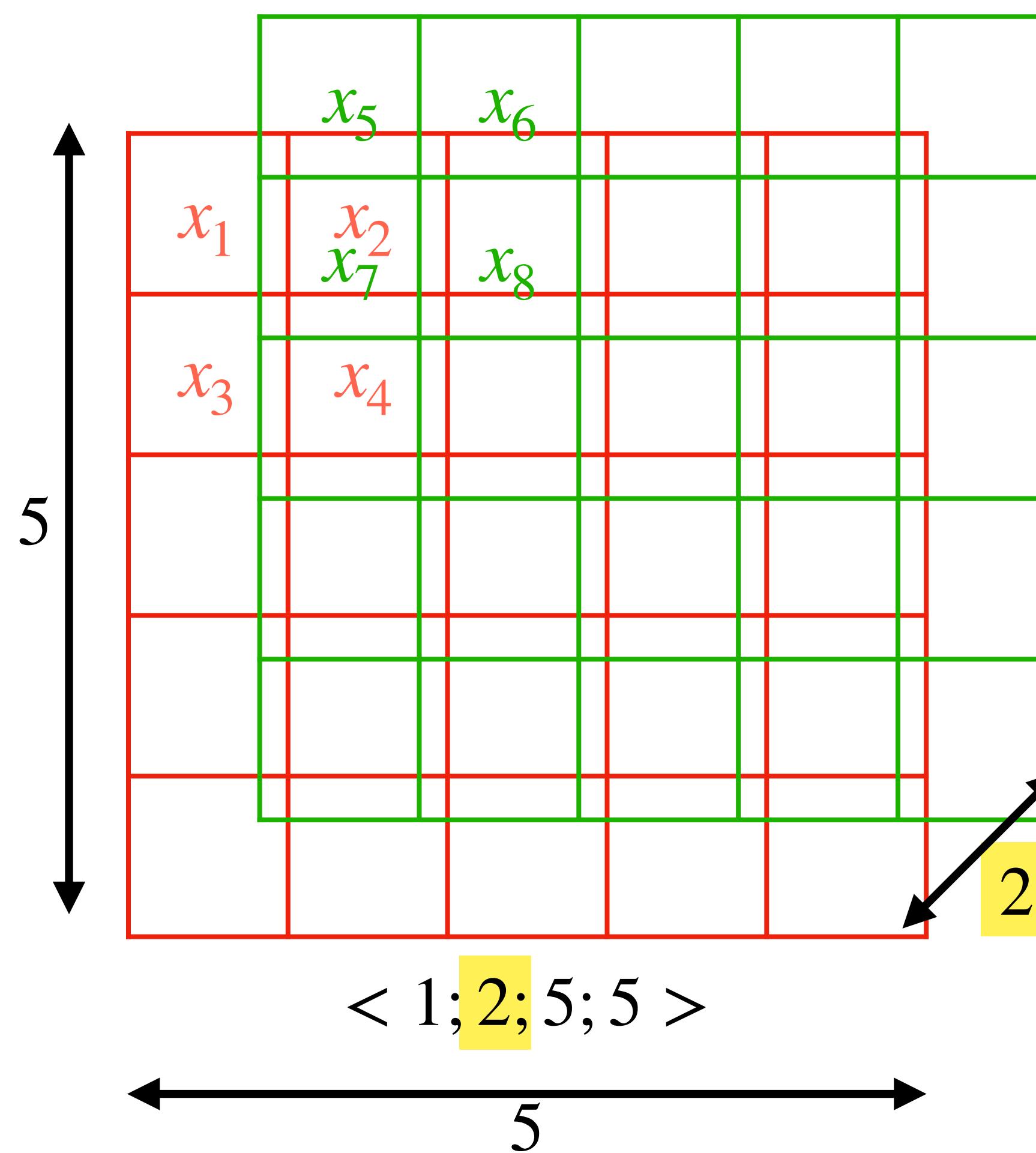


One filter  
with kernel size  $2 \times 2$   
must have the same number  
of channels



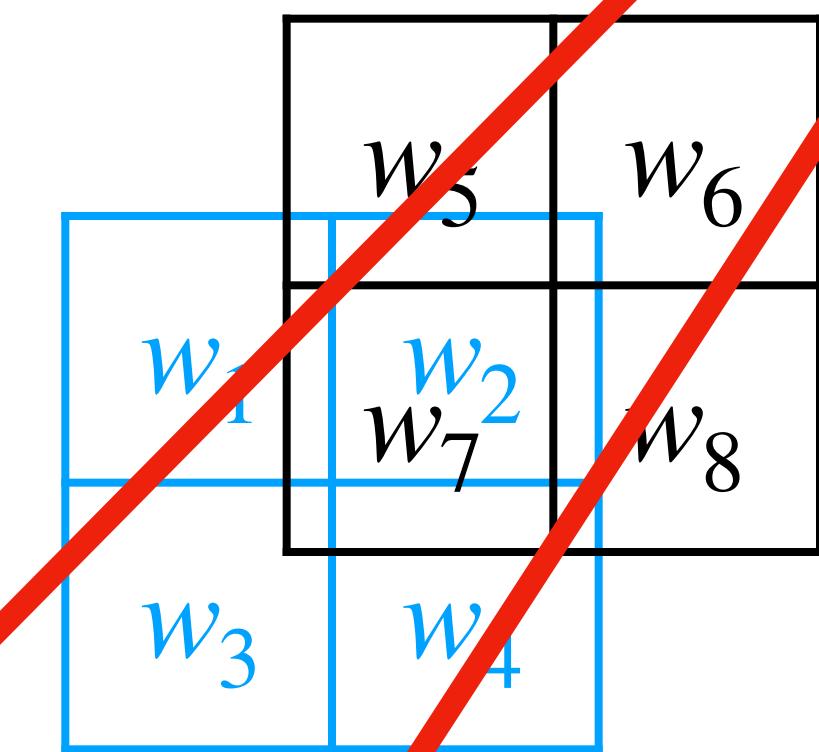
# Convolutional Layer Example

Input 2-Channel Image



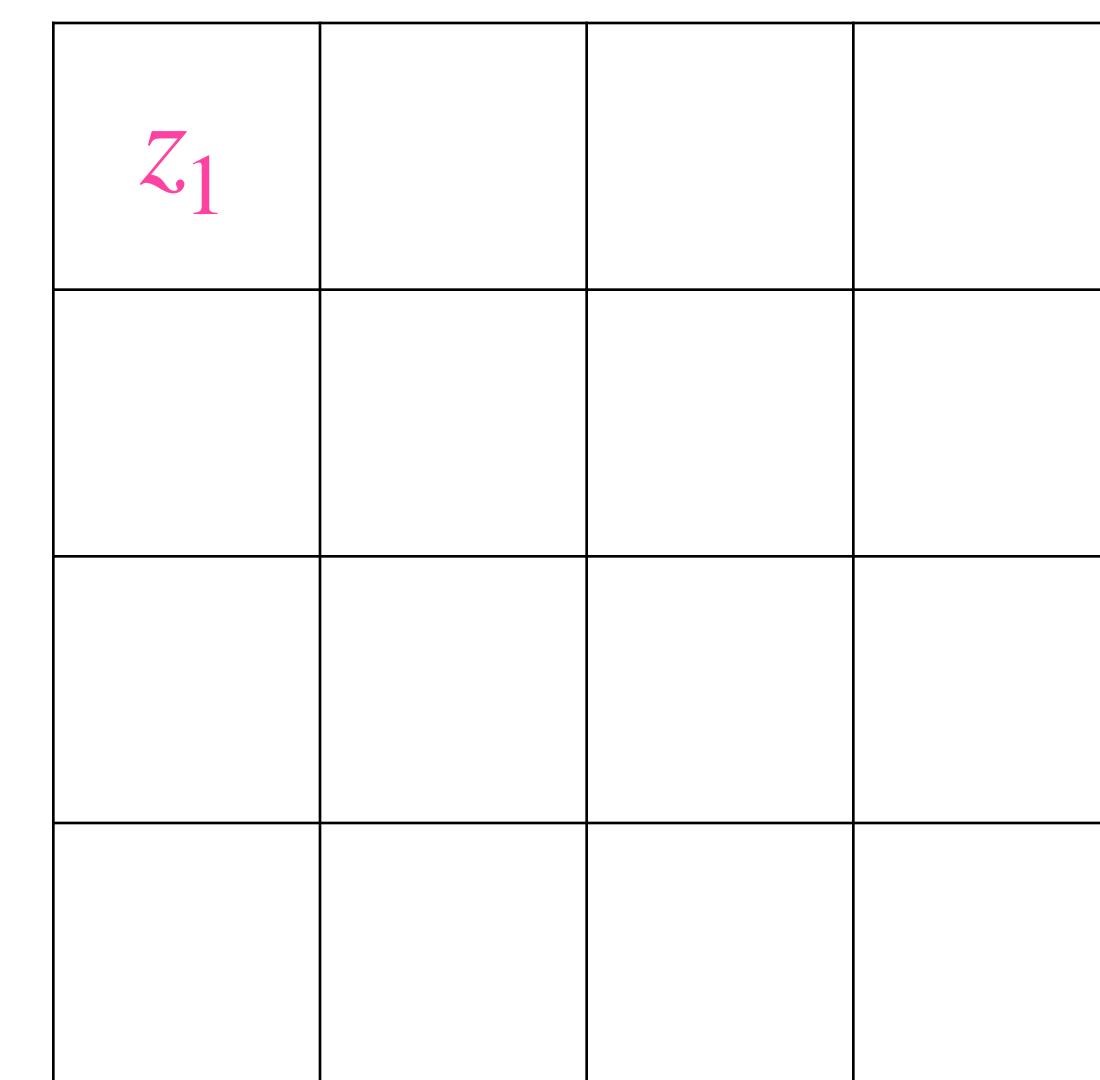
One Kernel  
“Filter”

\*



$< 2; 2; 2 >$

One filter  
with kernel size  $2 \times 2$   
must have the same number  
of channels  
as the input image



# Convolutional Layer

- Convolution is a linear operator !
  - We need non-linear activations to capture complex patterns

# Convolutional Layer

- Convolution is a linear operator !
  - We need non-linear activations to capture complex patterns
- **Parameters = Kernel matrices (tensors)**
  - One for each output channel (filter)

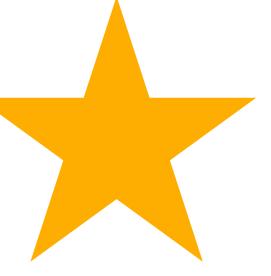
# Convolutional Layer

- Convolution is a linear operator !
  - We need non-linear activations to capture complex patterns
- **Parameters = Kernel matrices (tensors)**
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- Total number of parameters does not depend on input spatial resolution
  - Can be applied to images of **any size**



# Convolutional Layer

- Convolution is a linear operator !
  - We need non-linear activations to capture complex patterns
- **Parameters = Kernel matrices (tensors)**
  - One for each output channel (filter)
- Total number of parameters does not depend on input spatial resolution
  - Can be applied to images of **any size**
- **Translation invariant**



# Convolutional Layer

## Number of Parameters

- Number of input channels -  $ch_{in}$
- Number of output channels (filters) -  $ch_{out}$
- Kernel size -  $(k_h \times k_w)$

$$p = ch_{out}(k_h k_w | ch_{in} + 1)$$

Number of output channels

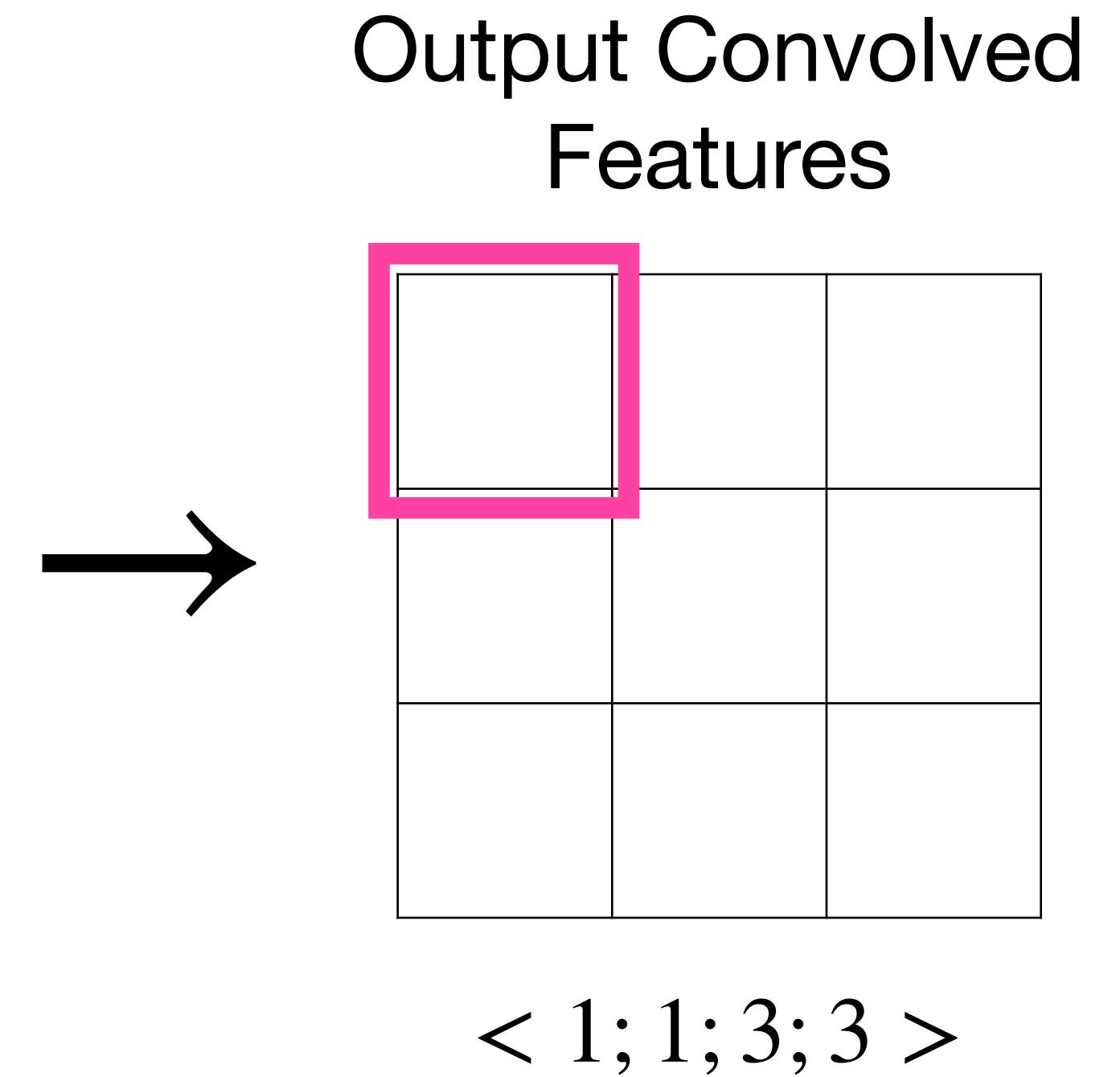
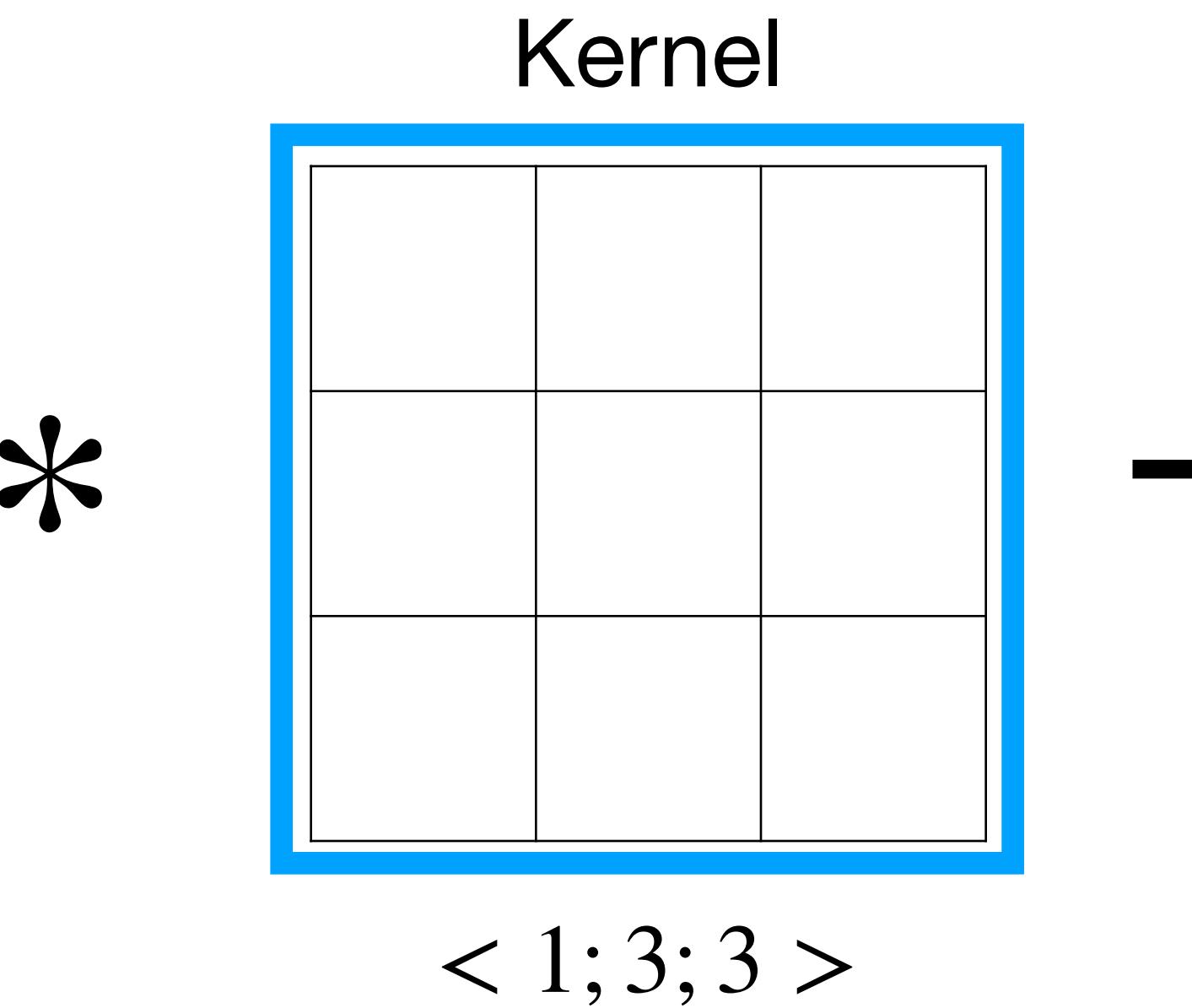
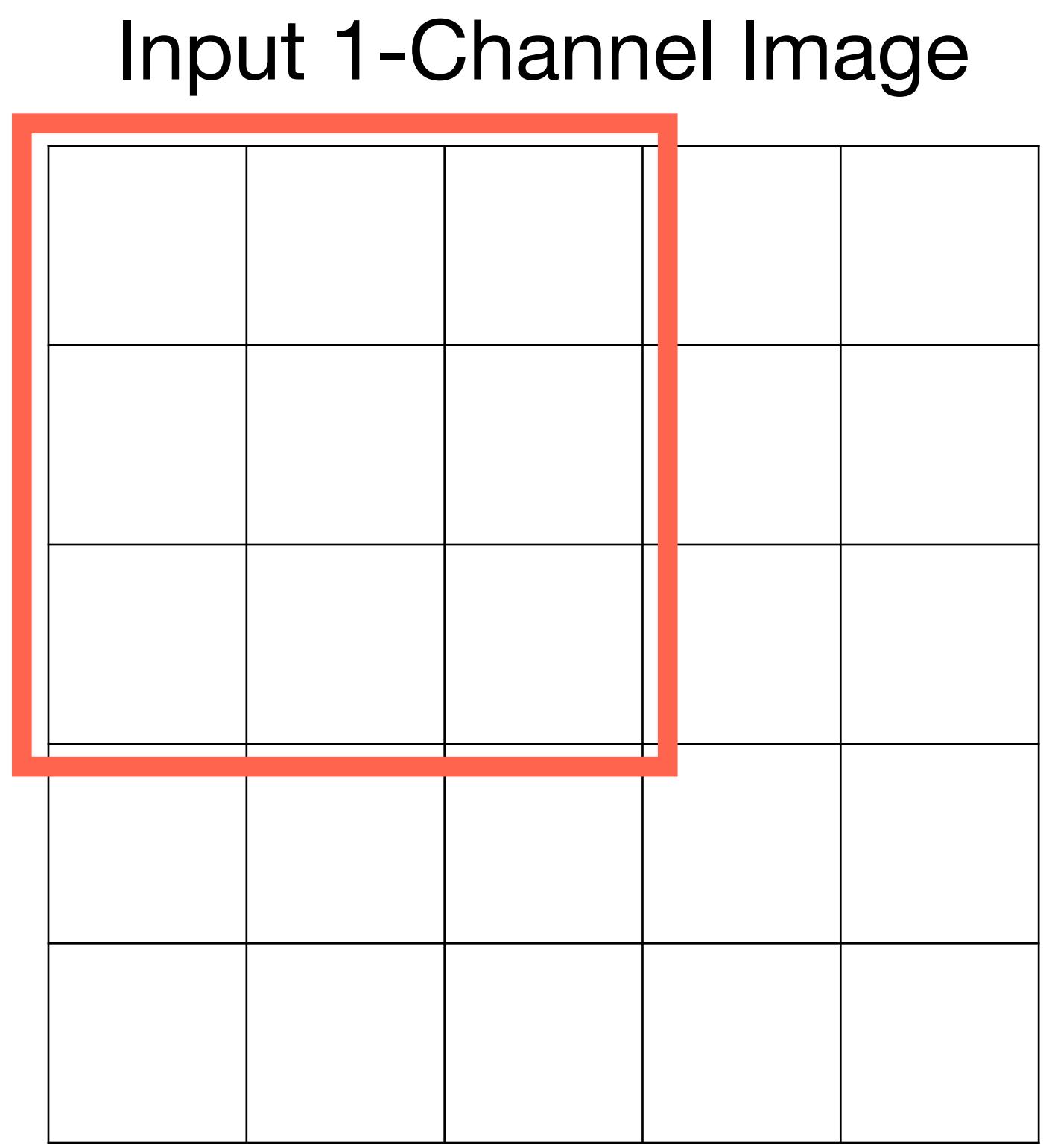
Kernel size

Number of input channels

Bias

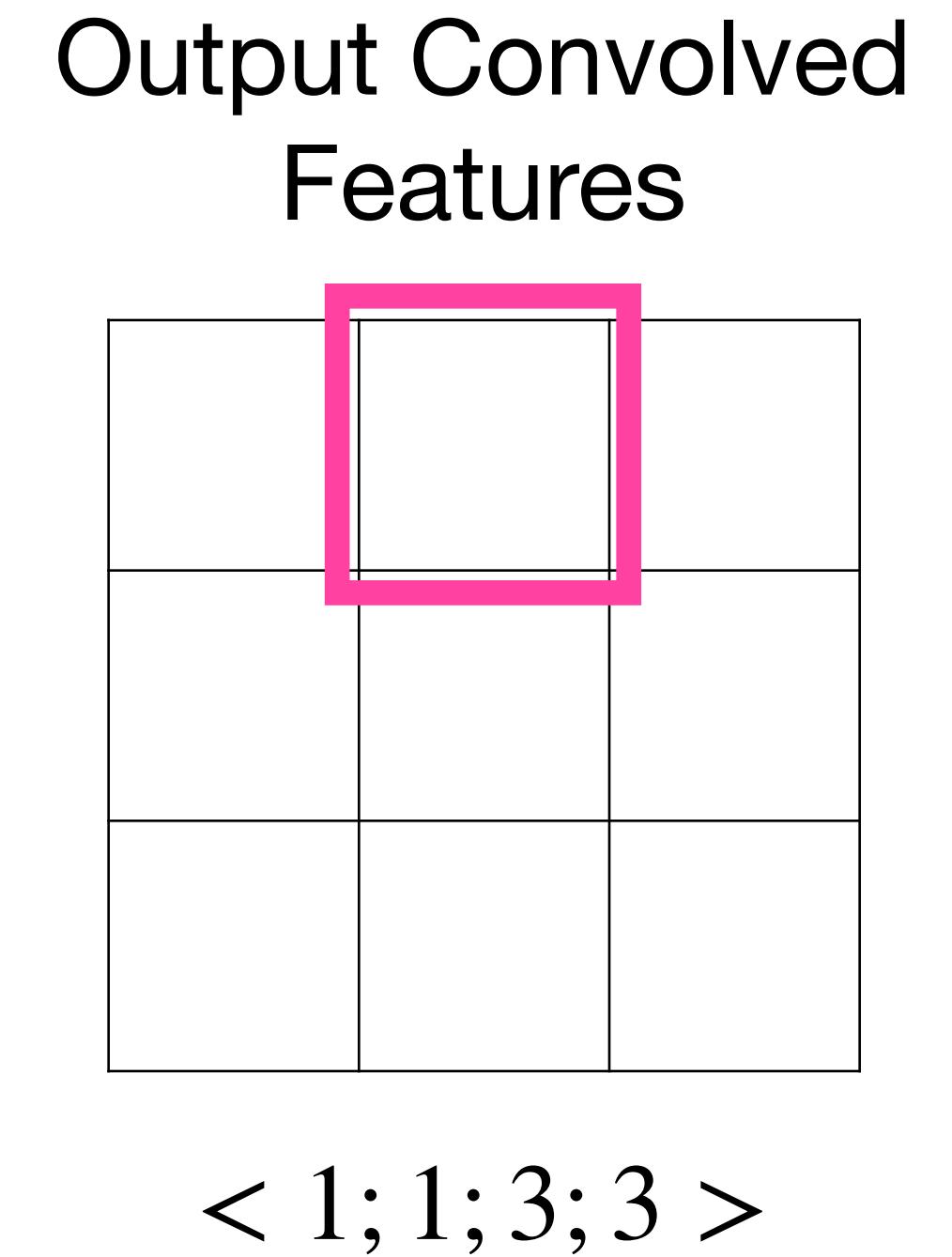
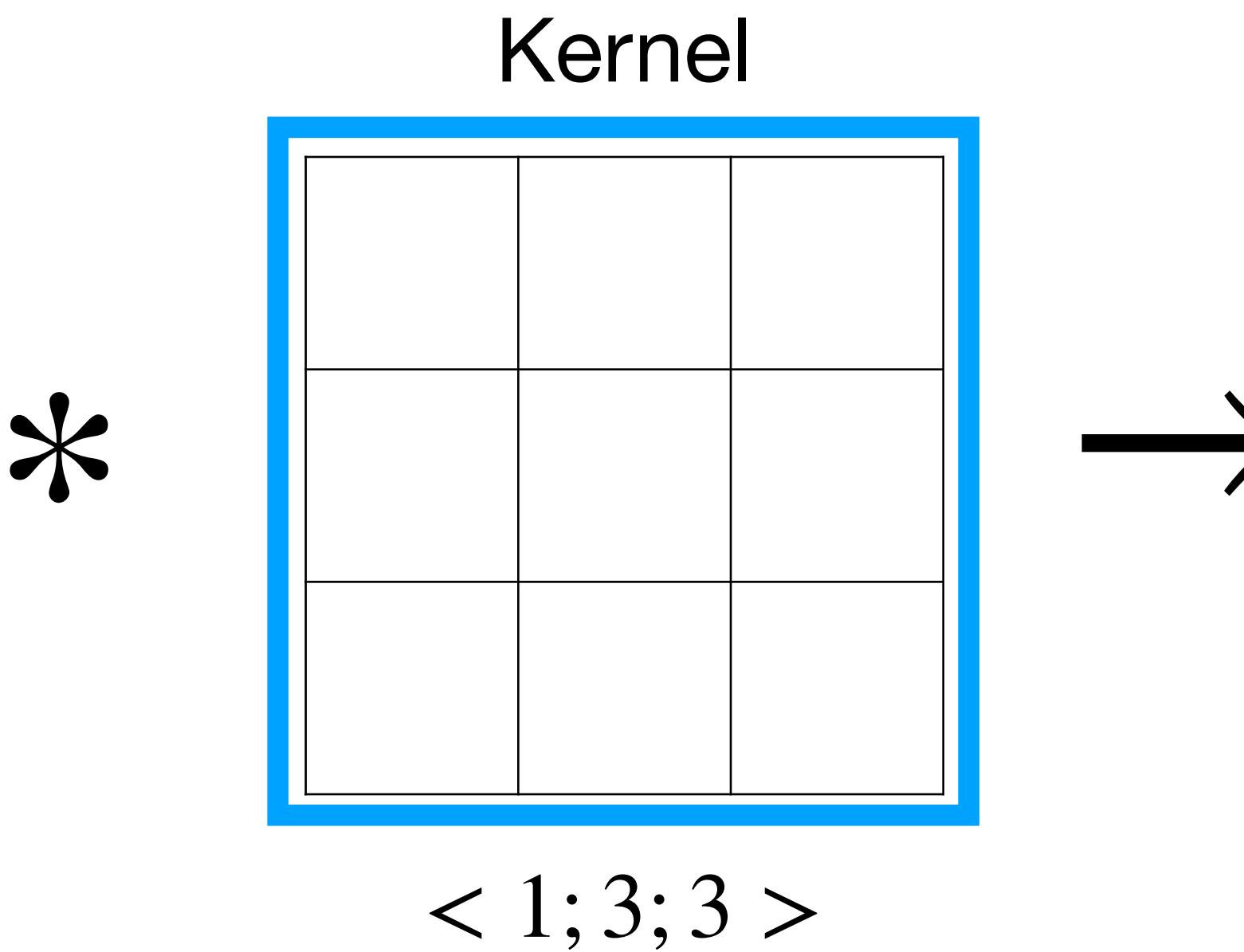
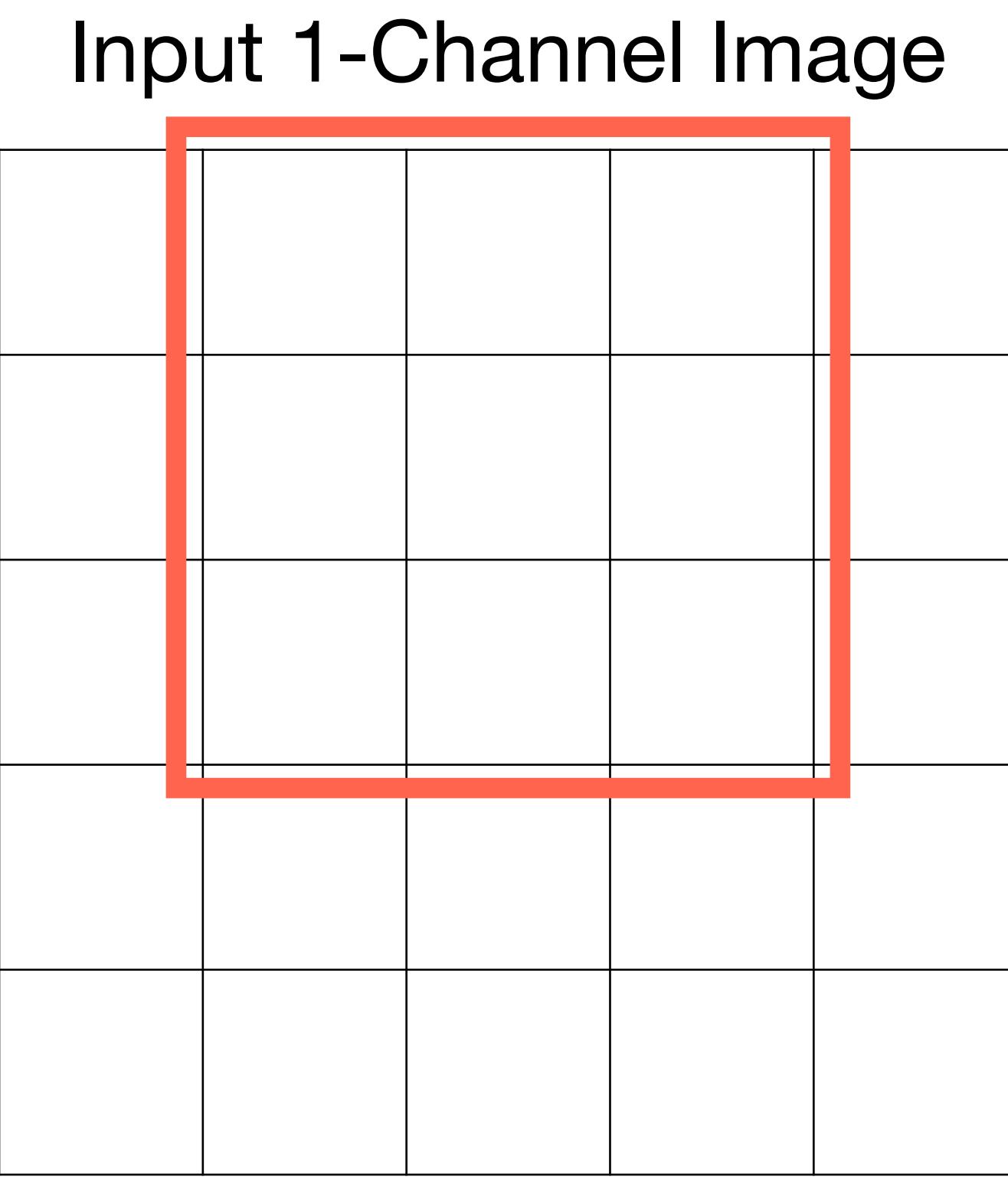
# Convolutional Layer

## Input & Output Shapes



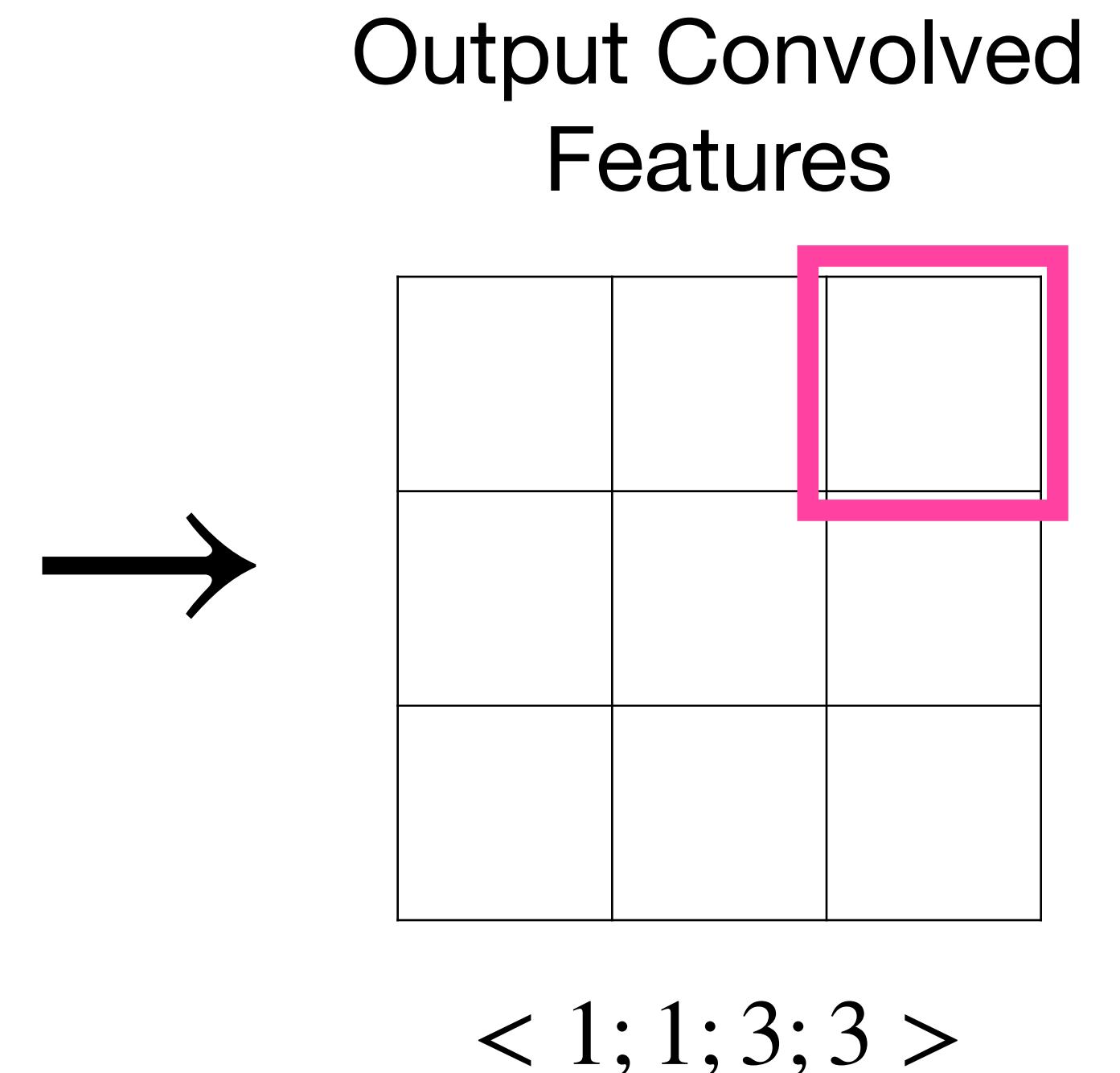
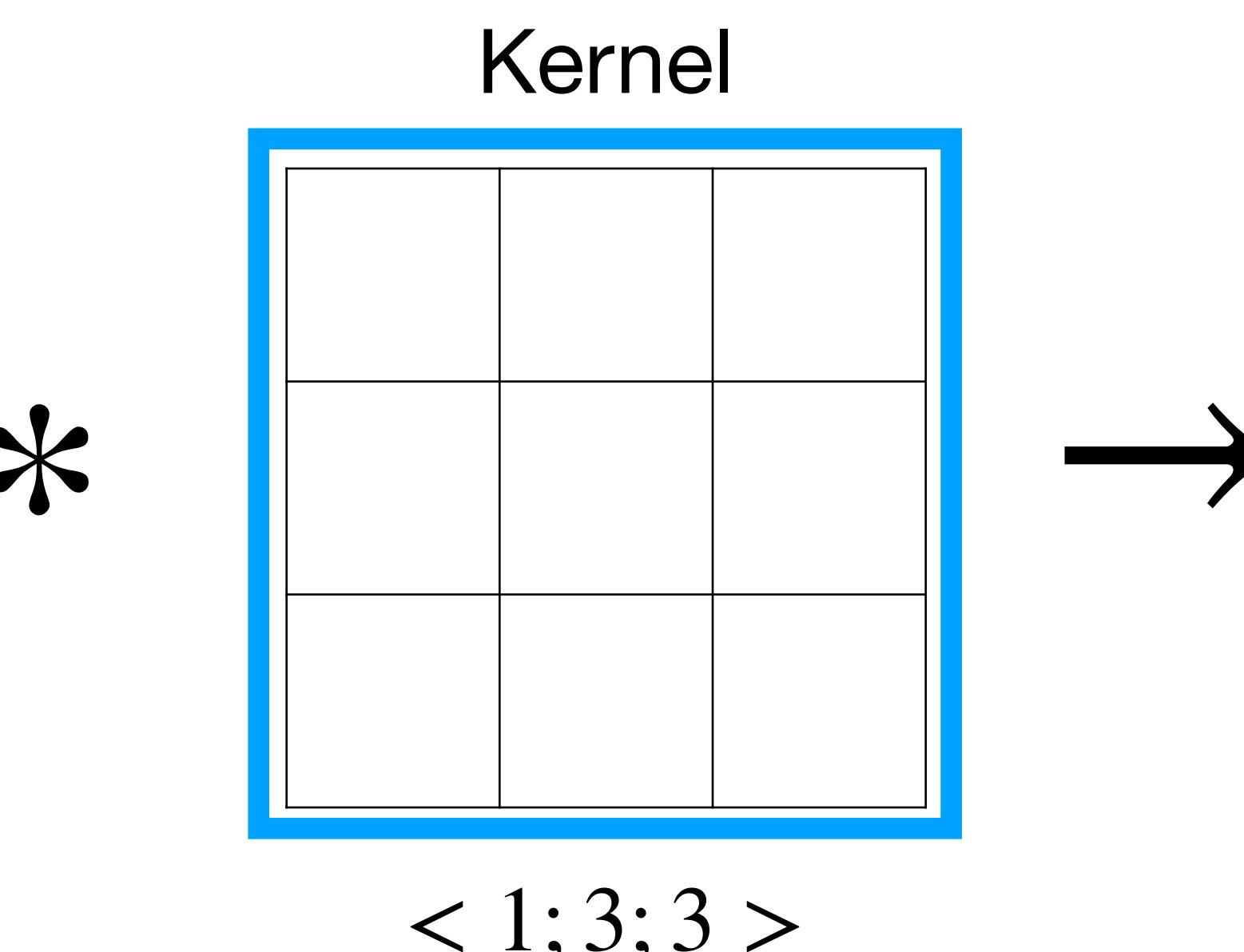
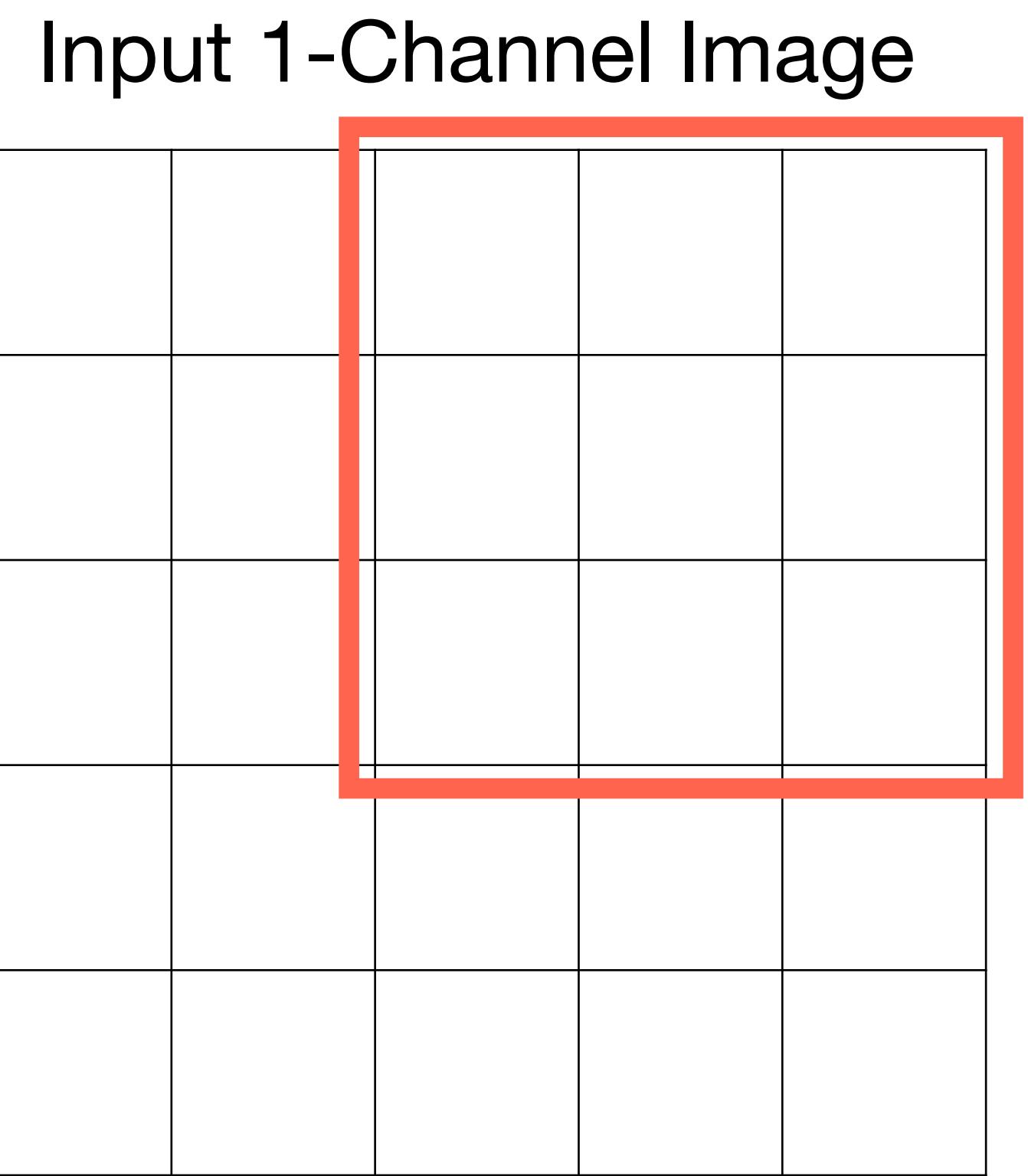
# Convolutional Layer

## Input & Output Shapes



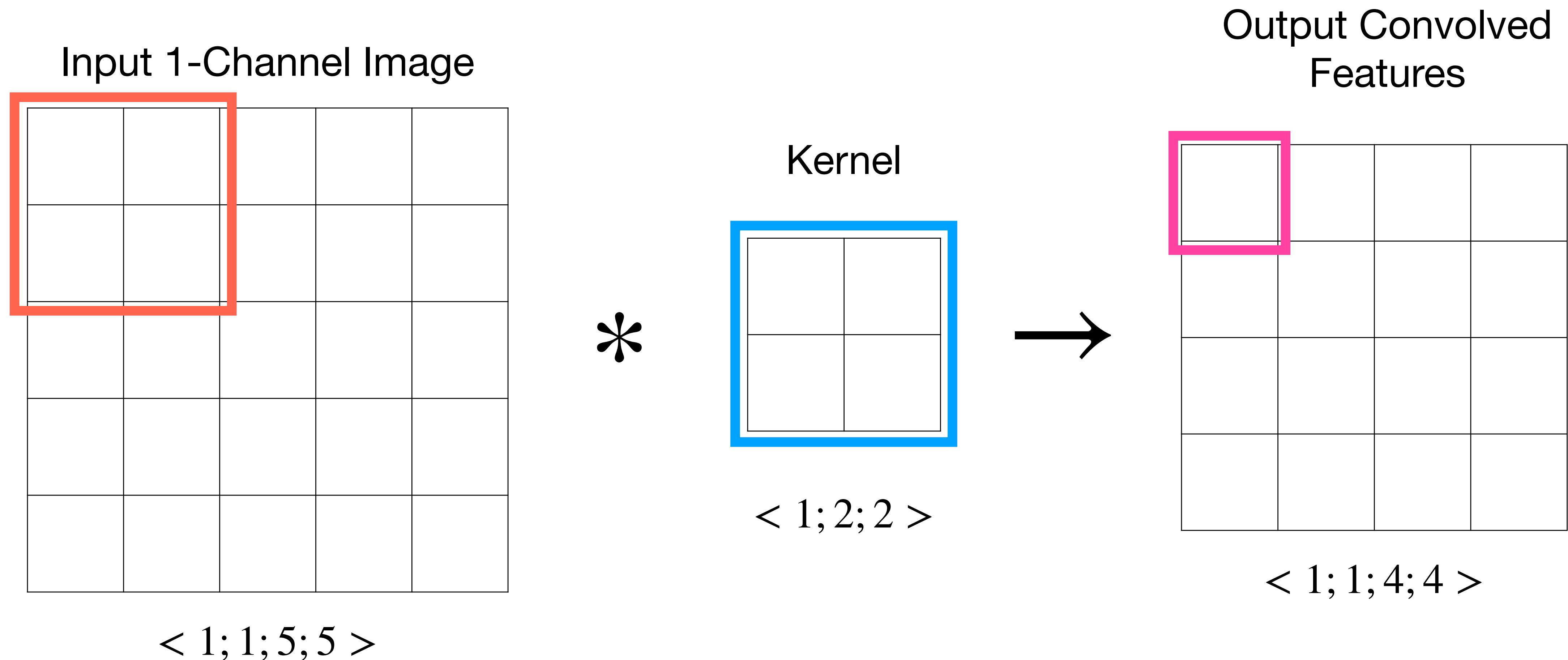
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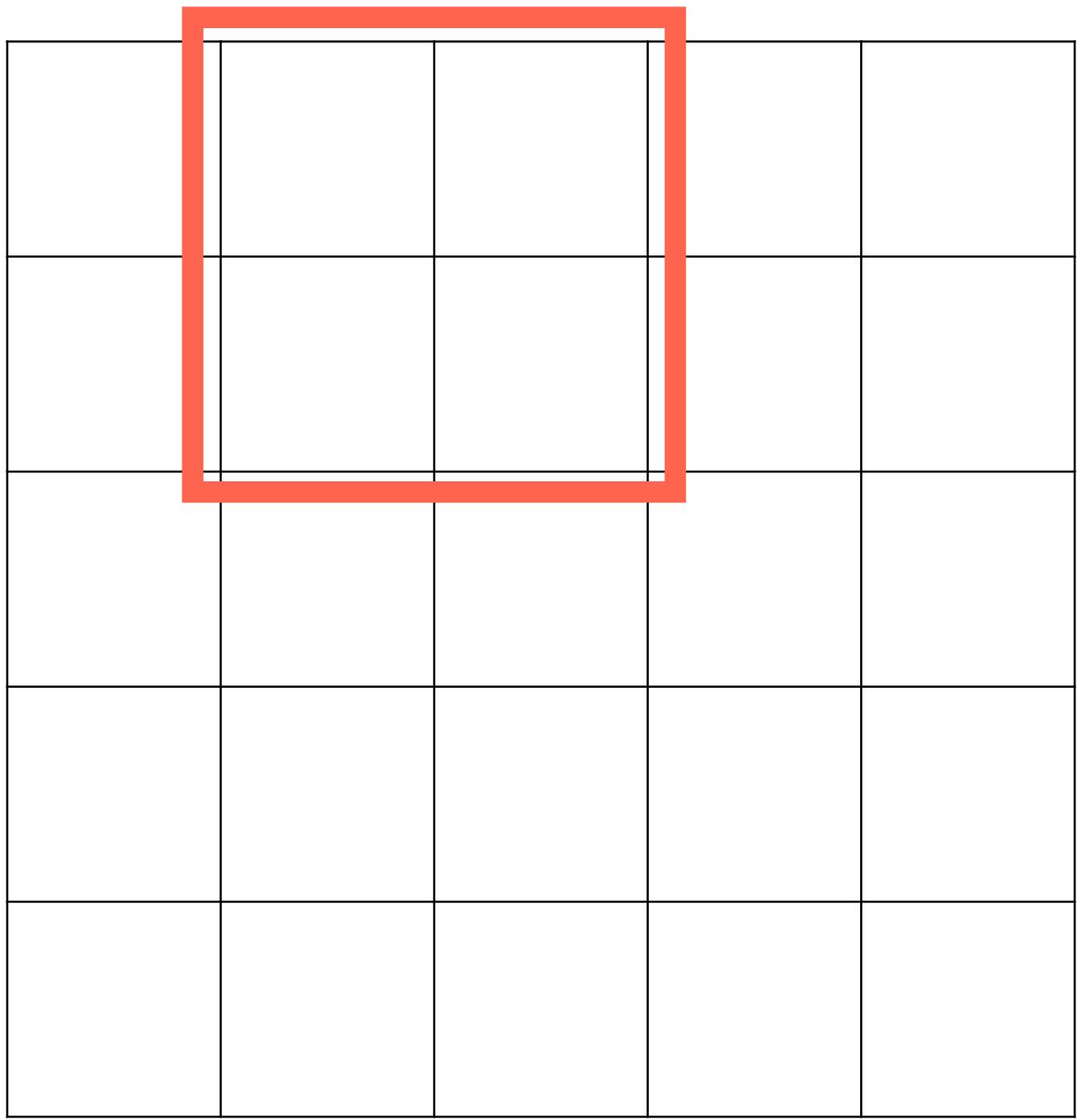
## Input & Output Shapes



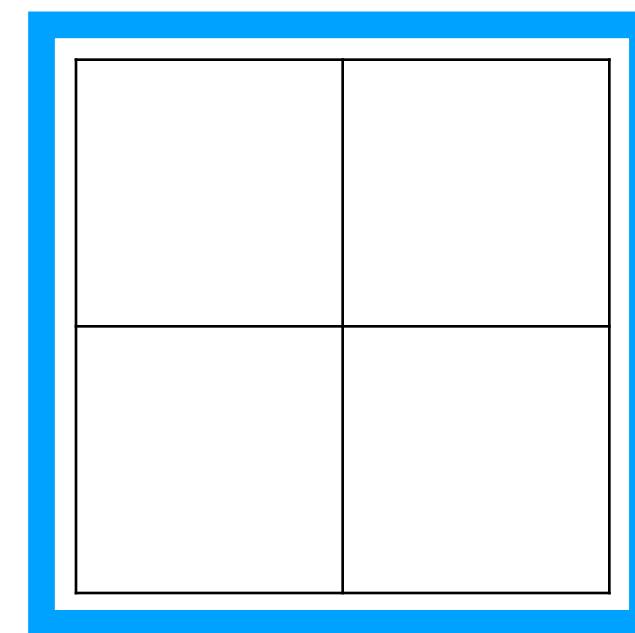
# Convolutional Layer

## Input & Output Shapes

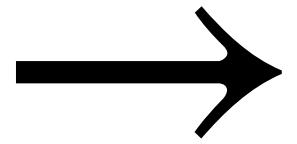
Input 1-Channel Image



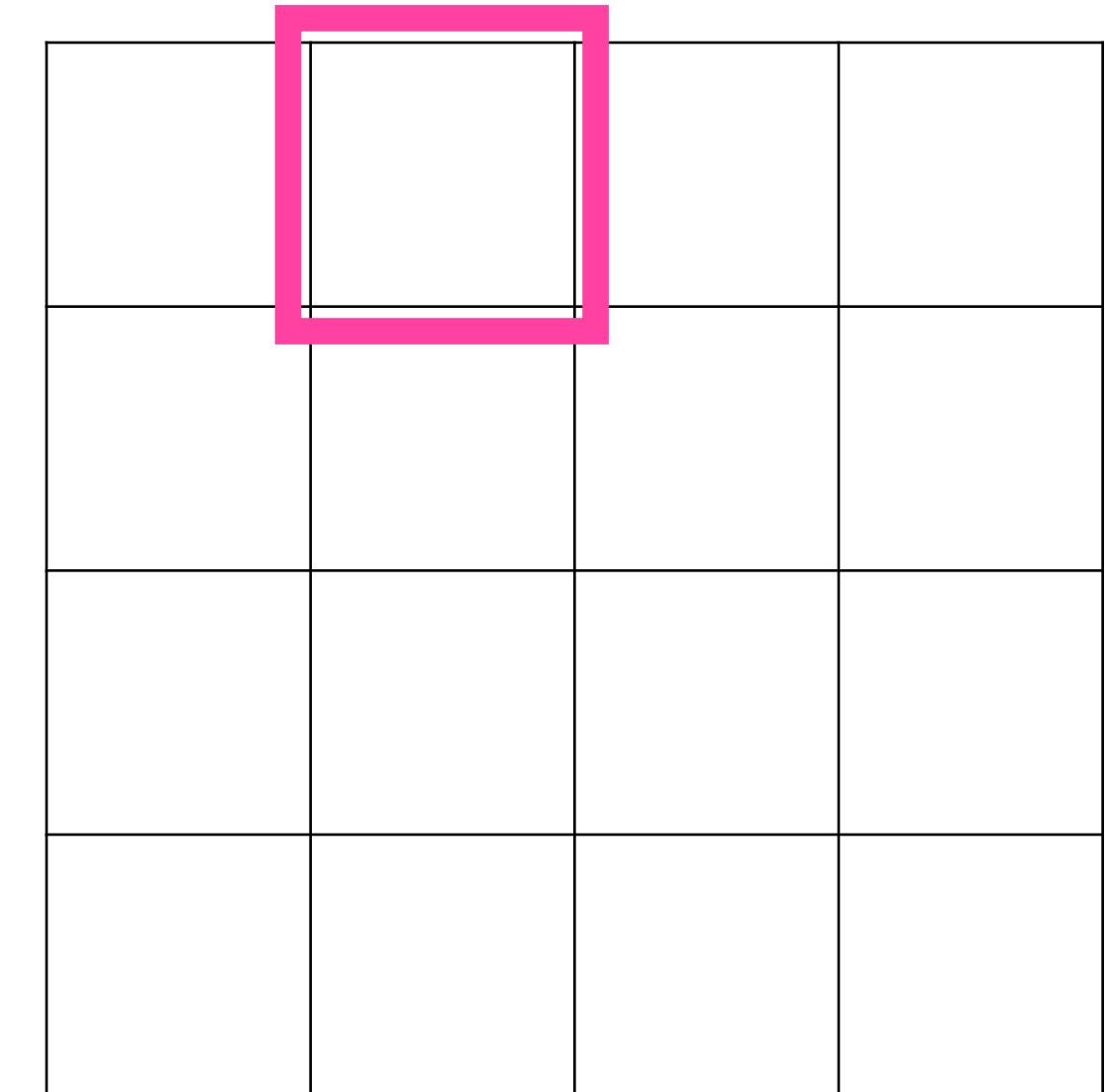
Kernel



$< 1; 2; 2 >$



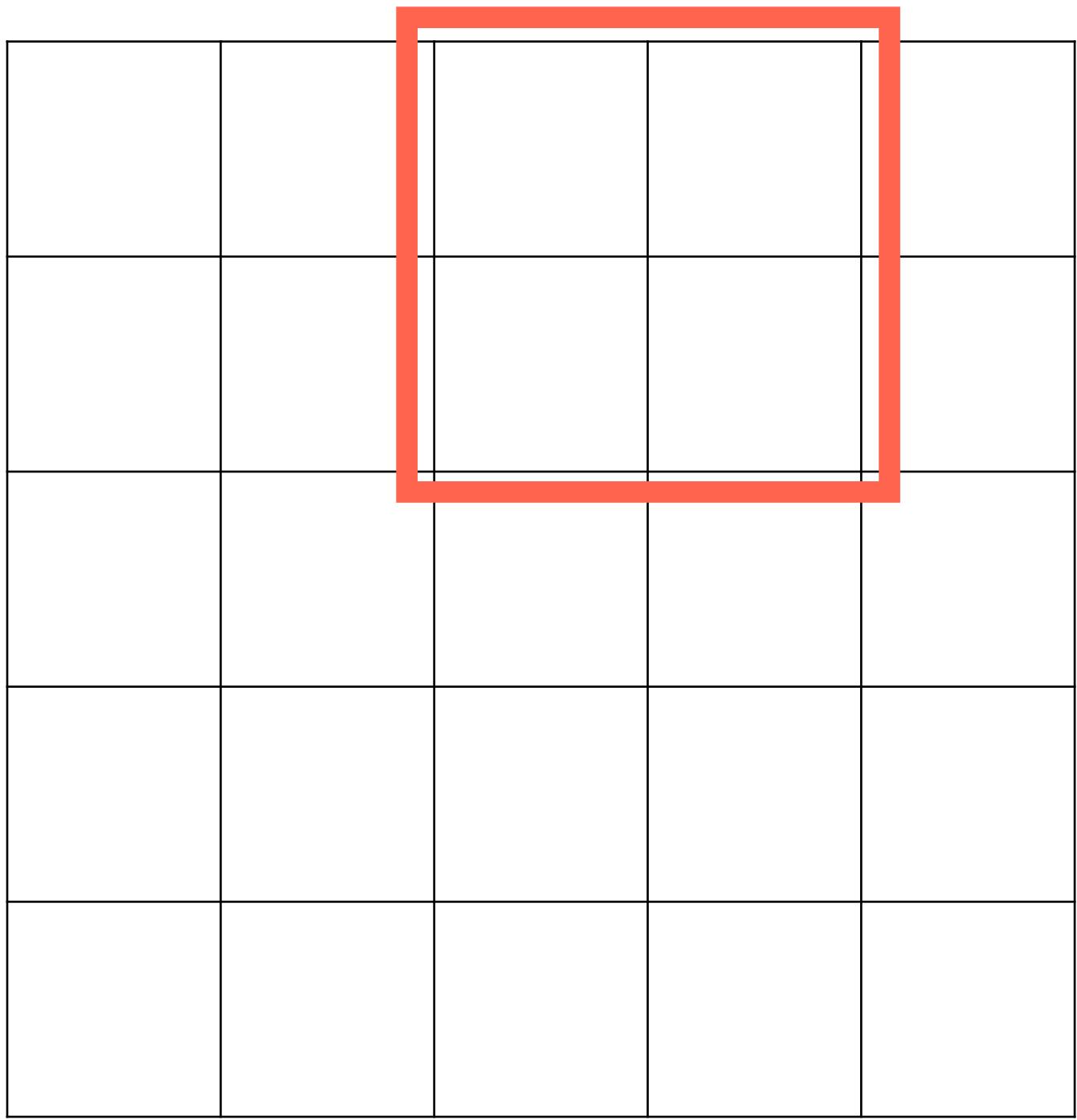
Output Convolved Features



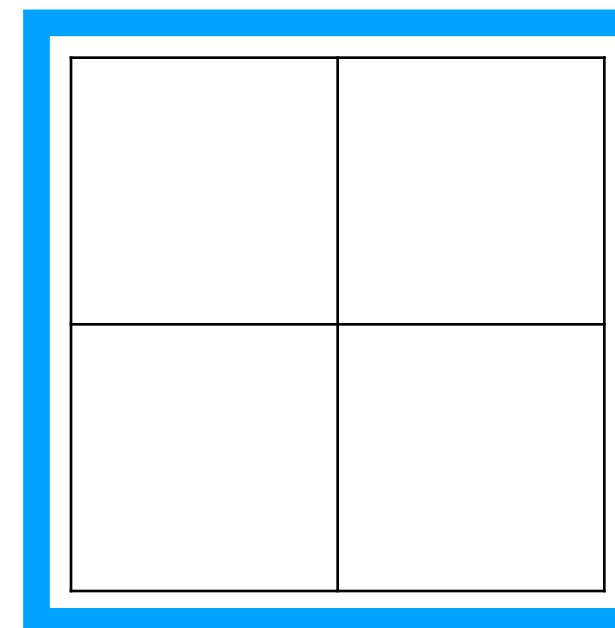
# Convolutional Layer

## Input & Output Shapes

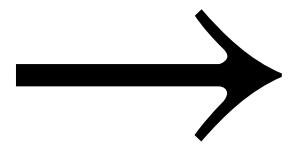
Input 1-Channel Image



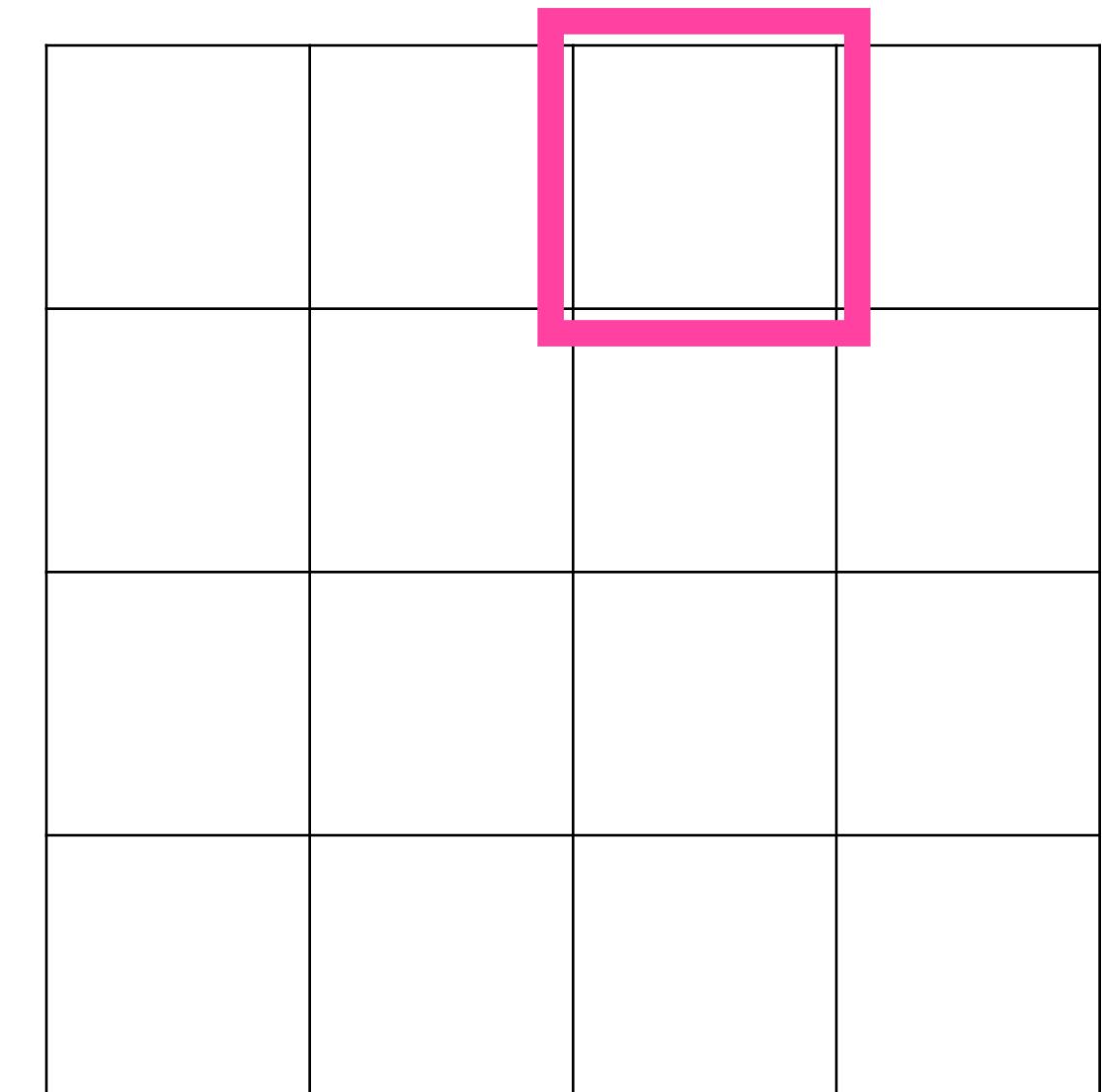
Kernel



$< 1; 2; 2 >$

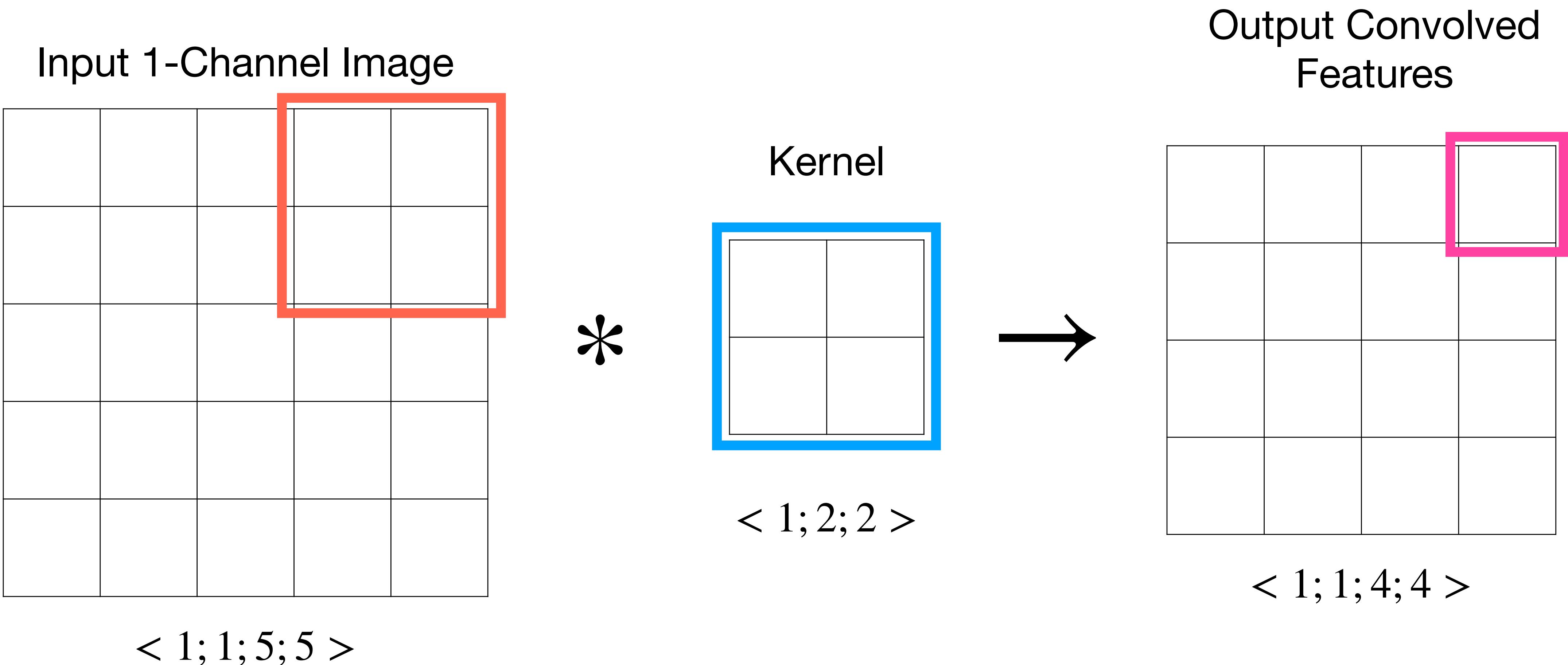


Output Convolved Features



# Convolutional Layer

## Input & Output Shapes



# Convolutional Layer

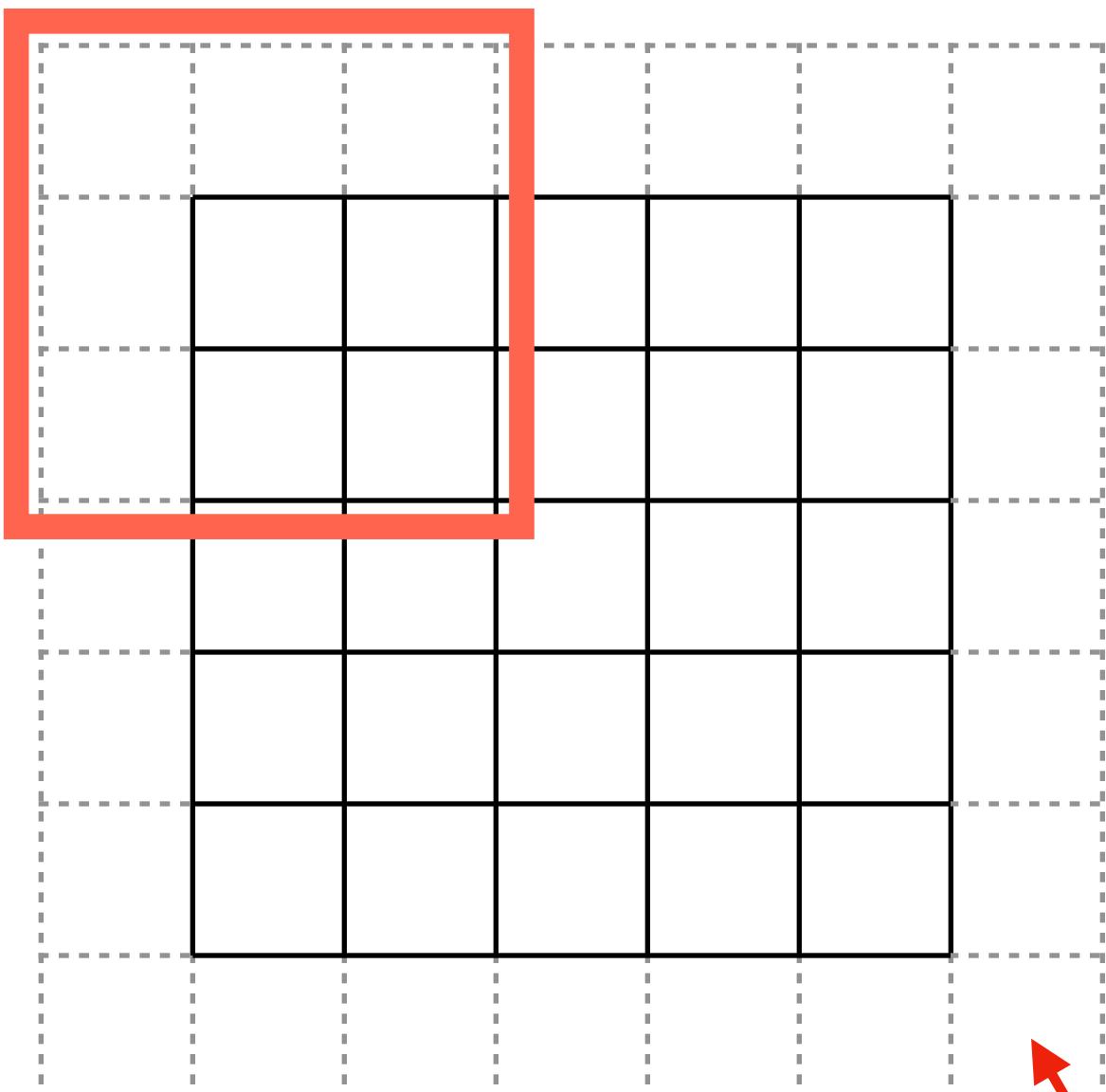
## Padding

- Add extra information to make convolution **preserve the original shape**
  - Zeros
  - Mirror
  - Constant
  - Copy edge pixels
  - ...

# Convolutional Layer

## Input & Output Shapes - Padding

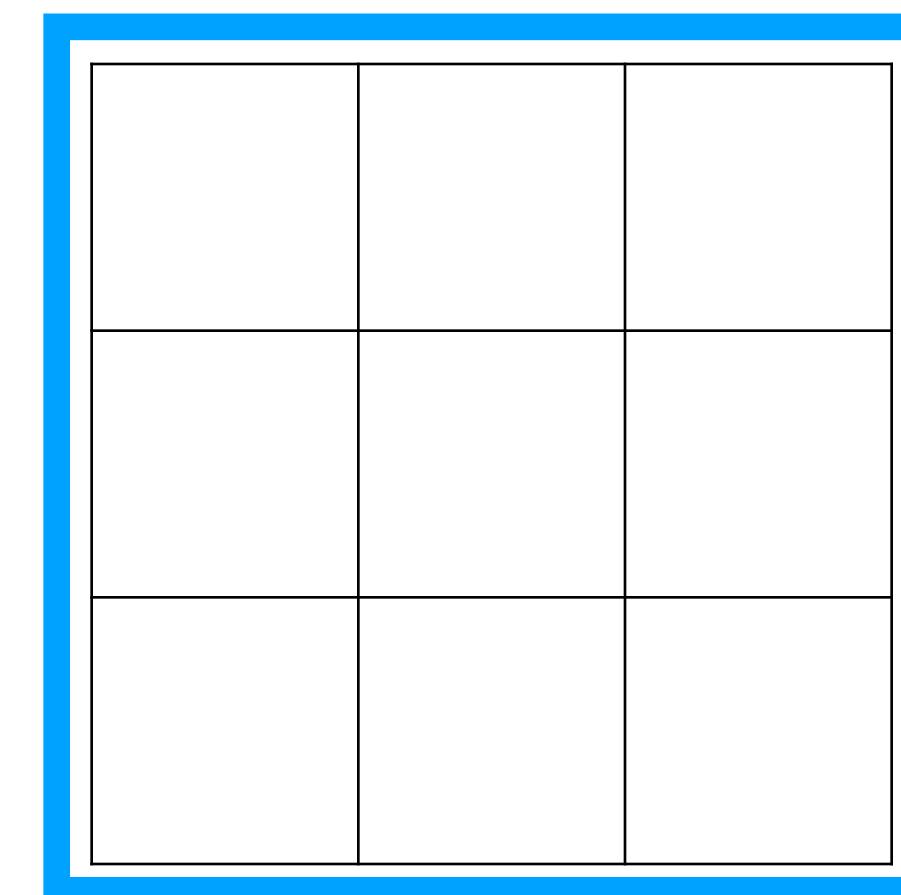
Input 1-Channel Image



$< 1; 1; 5; 5 >$

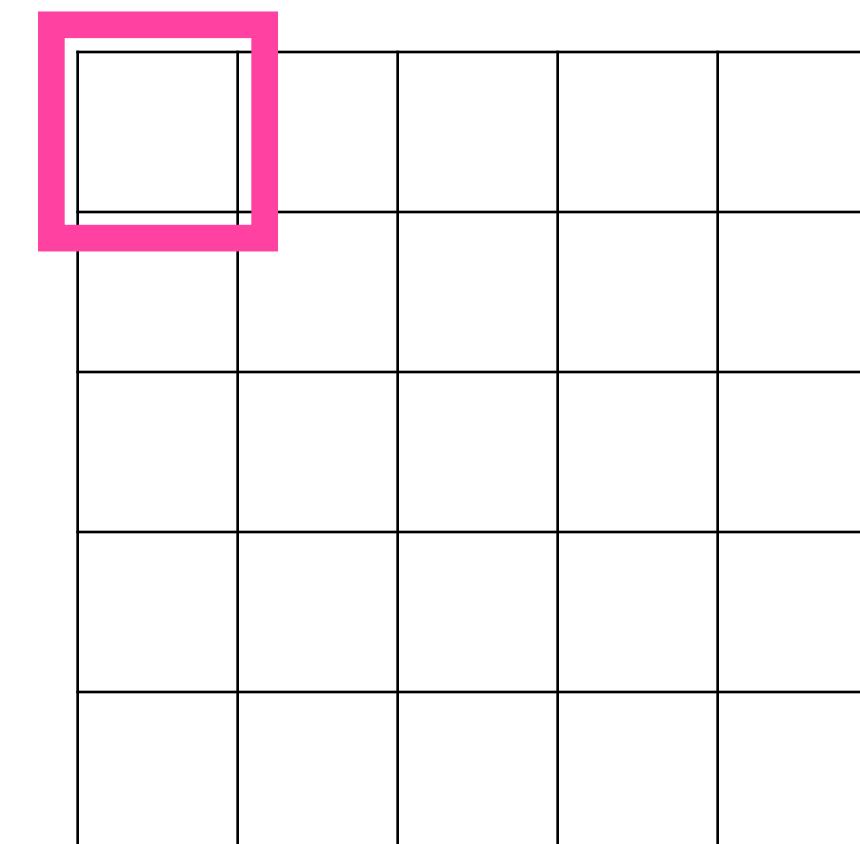
1 pixel extra padding

Kernel



$< 1; 3; 3 >$

Output Convolved Features

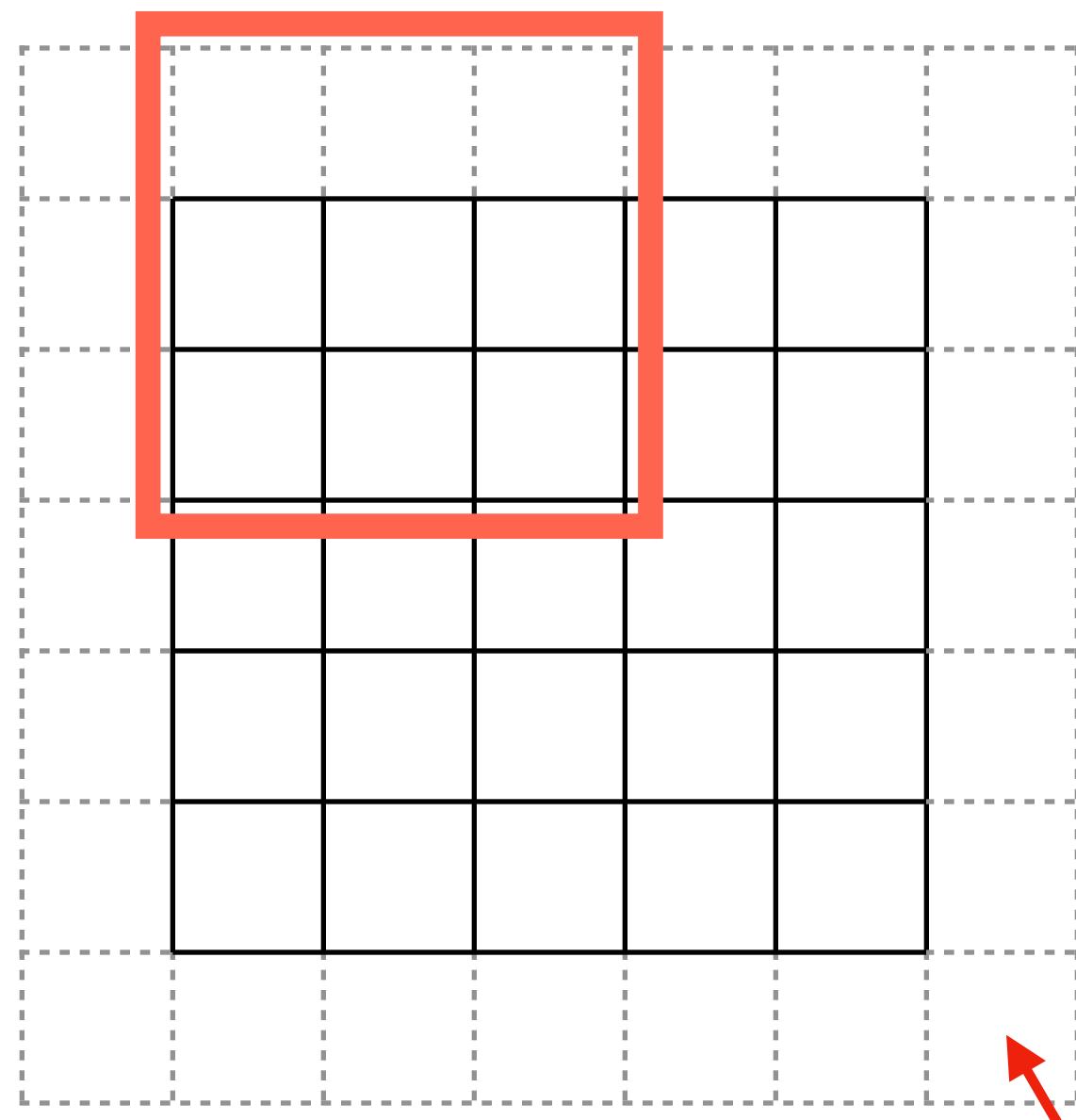


$< 1; 1; 5; 5 >$

# Convolutional Layer

## Input & Output Shapes - Padding

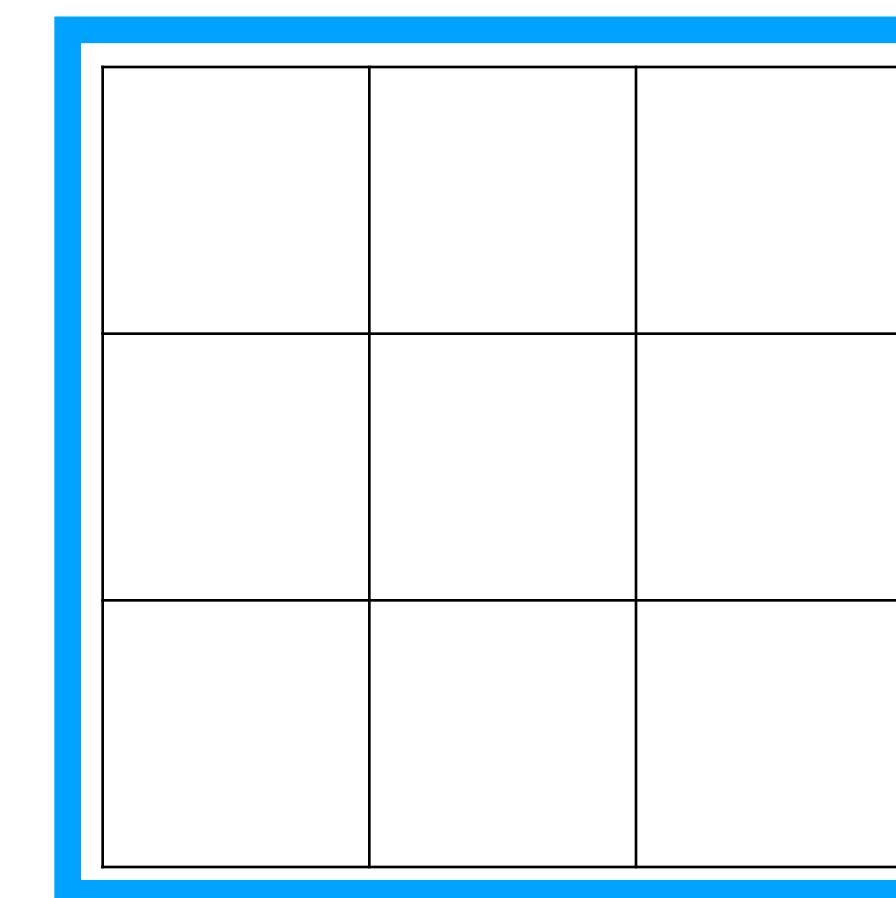
Input 1-Channel Image



$< 1; 1; 5; 5 >$

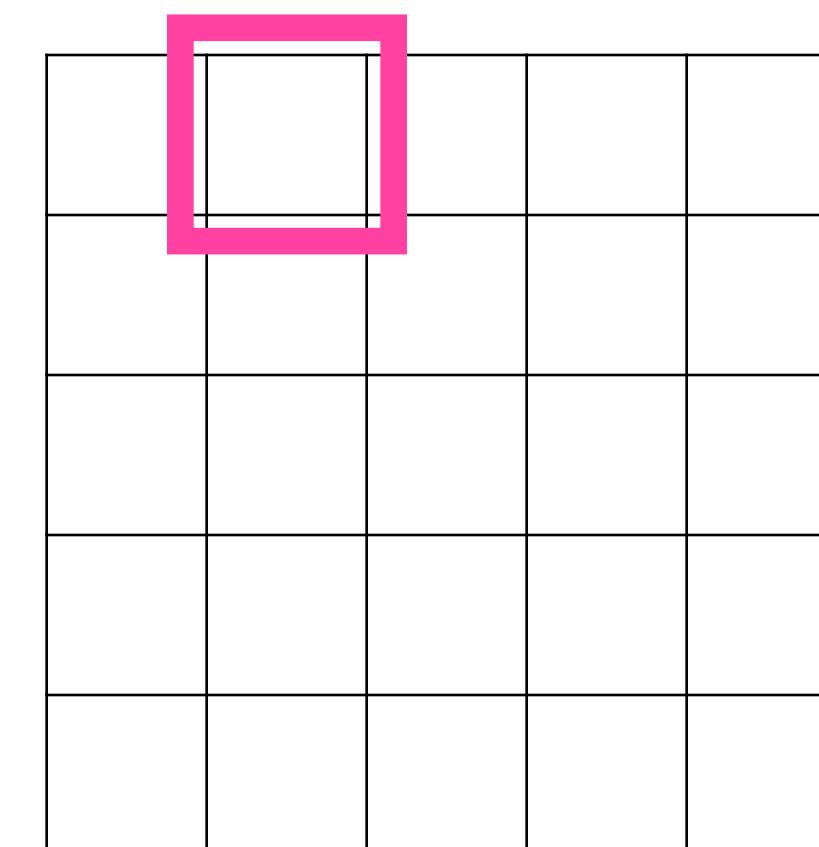
1 pixel extra padding

Kernel



$< 1; 3; 3 >$

Output Convolved  
Features

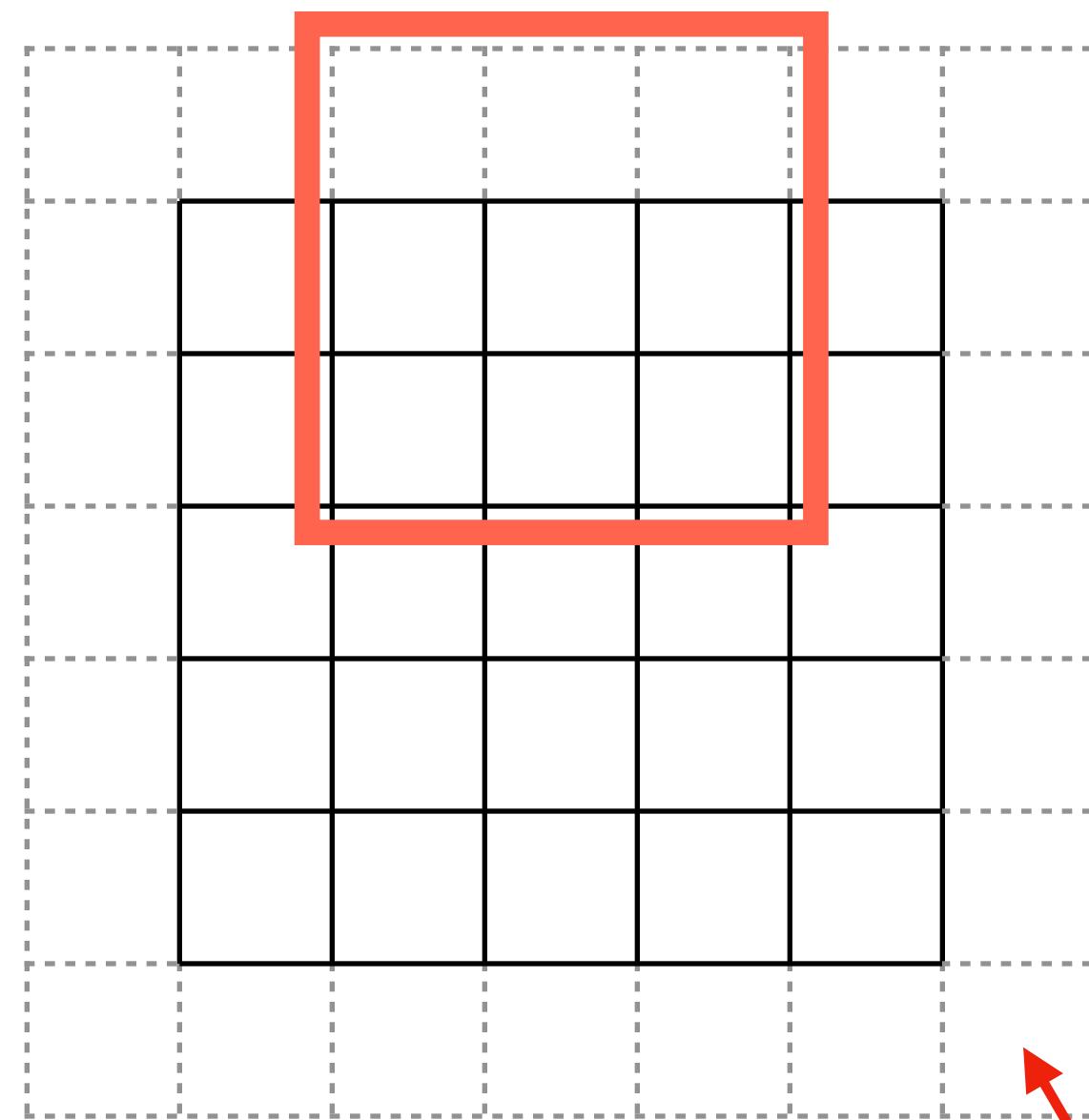


$< 1; 1; 5; 5 >$

# Convolutional Layer

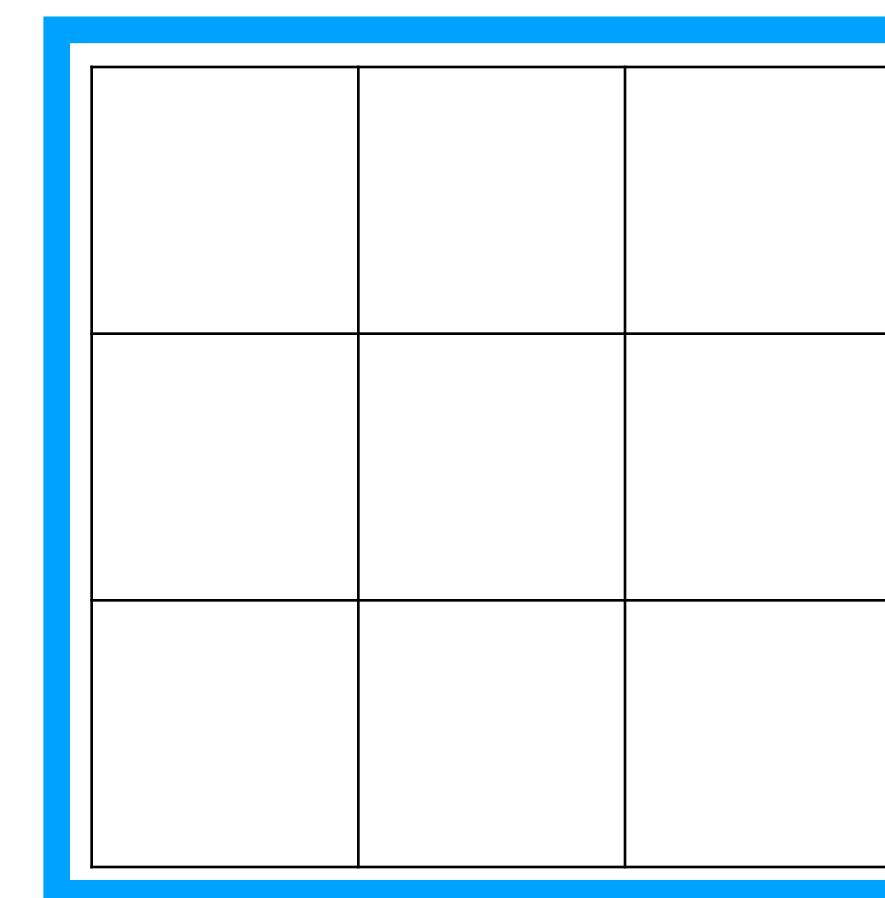
## Input & Output Shapes - Padding

Input 1-Channel Image

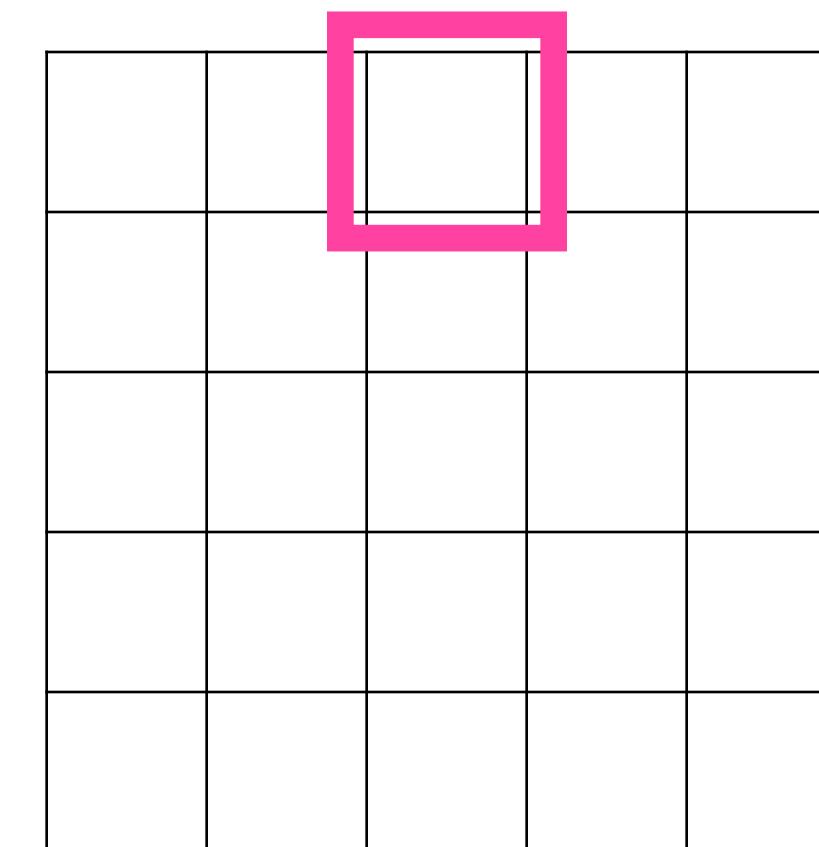


\*

Kernel



Output Convolved  
Features



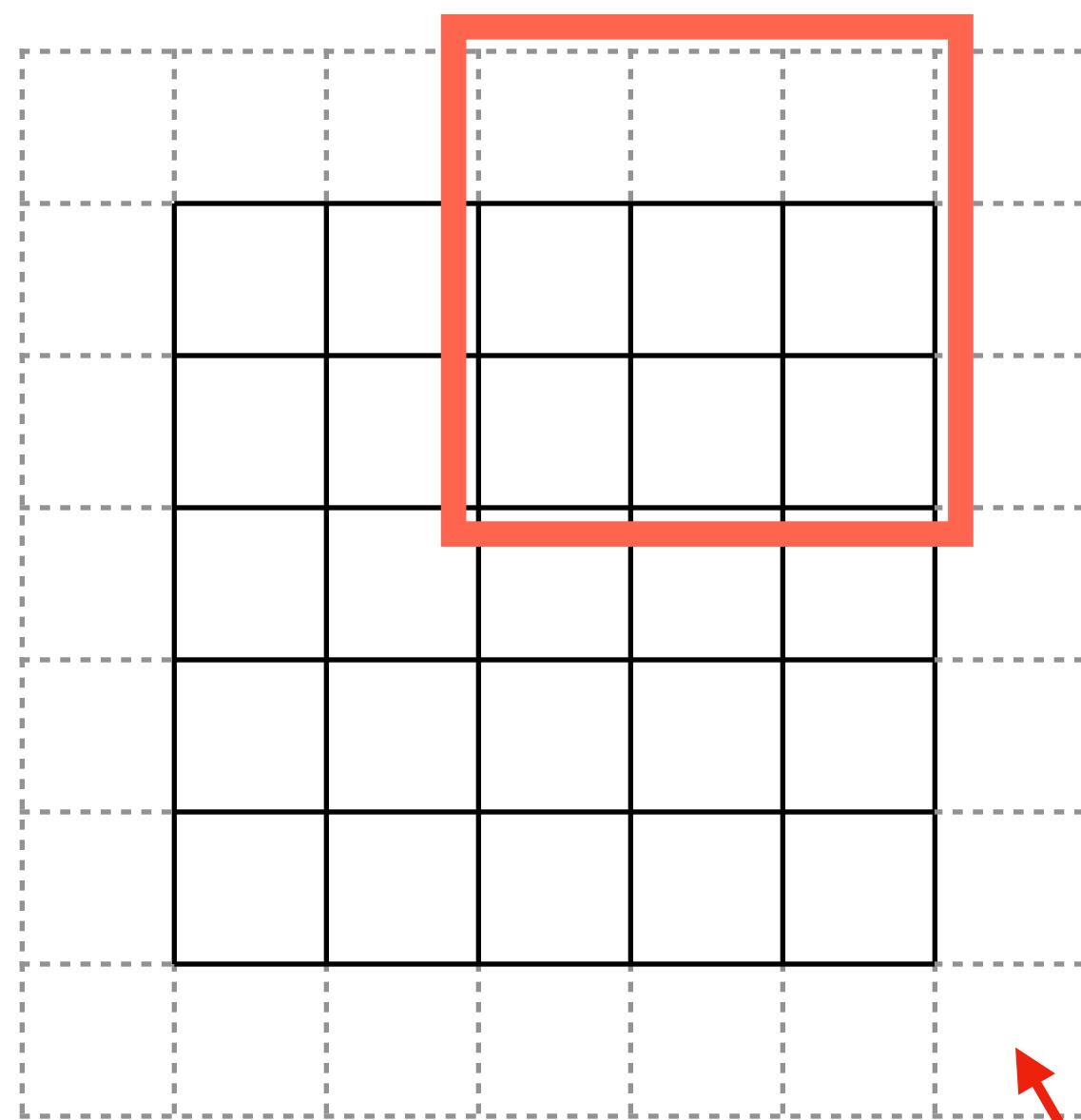
$< 1; 1; 5; 5 >$

1 pixel extra padding

# Convolutional Layer

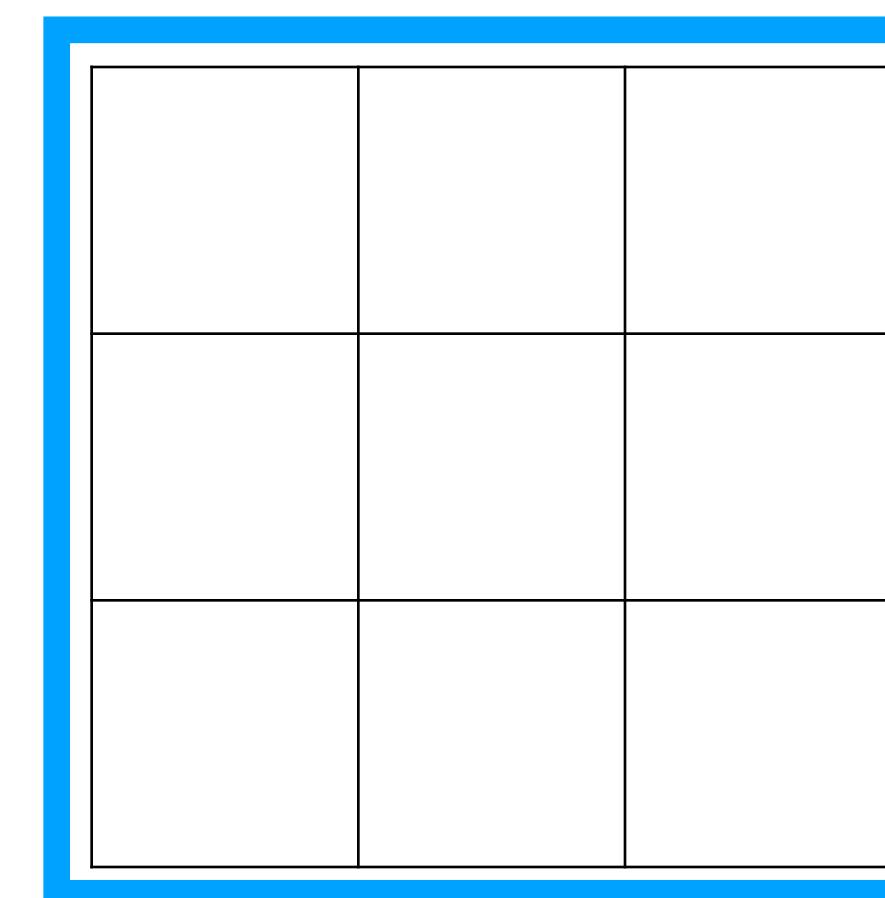
## Input & Output Shapes - Padding

Input 1-Channel Image

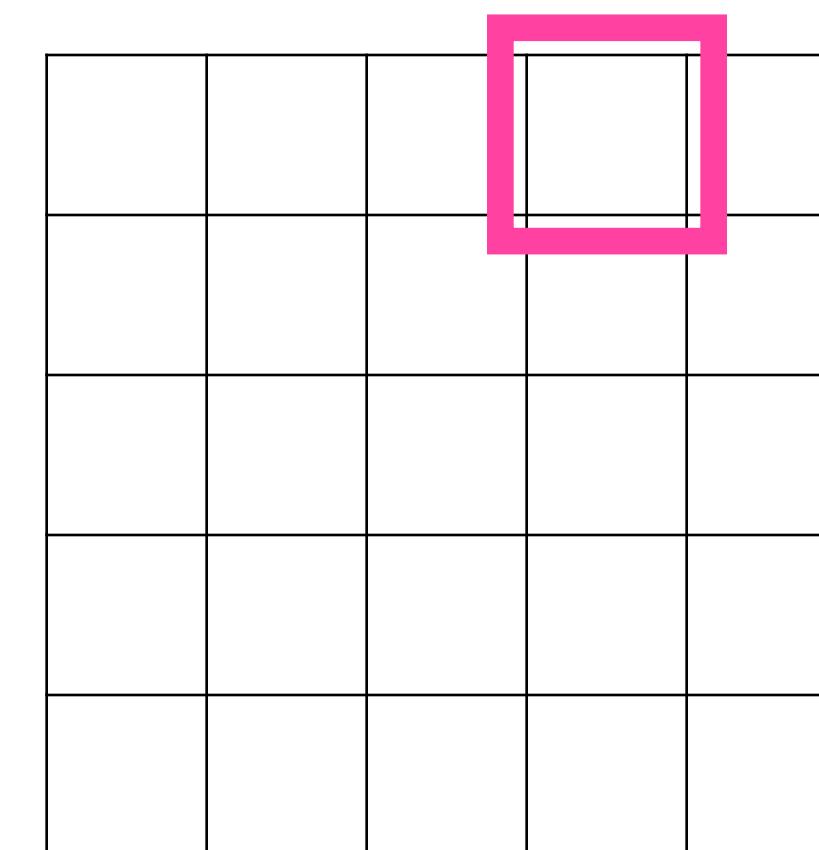


\*

Kernel



Output Convolved  
Features



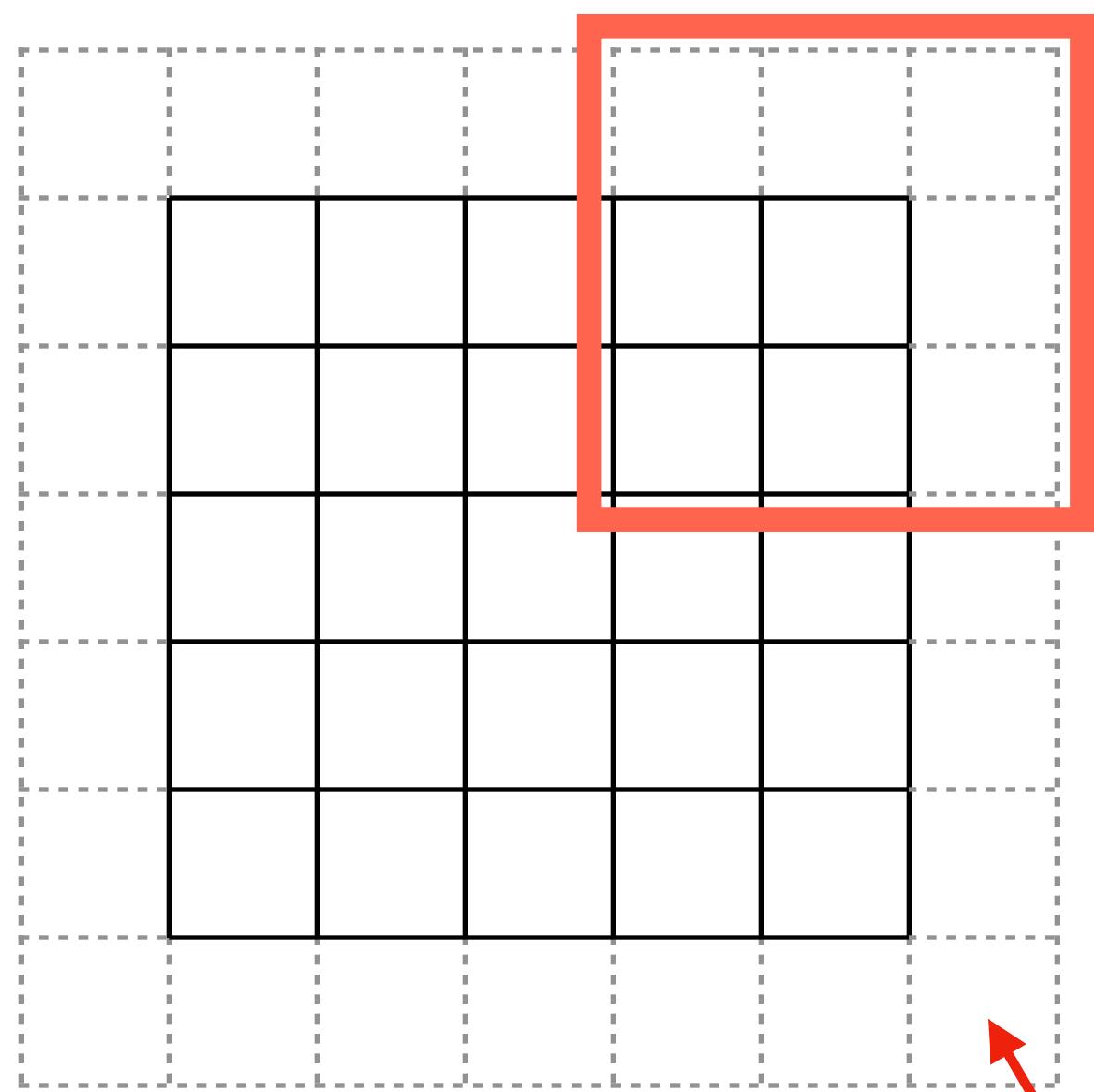
$< 1; 1; 5; 5 >$

1 pixel extra padding

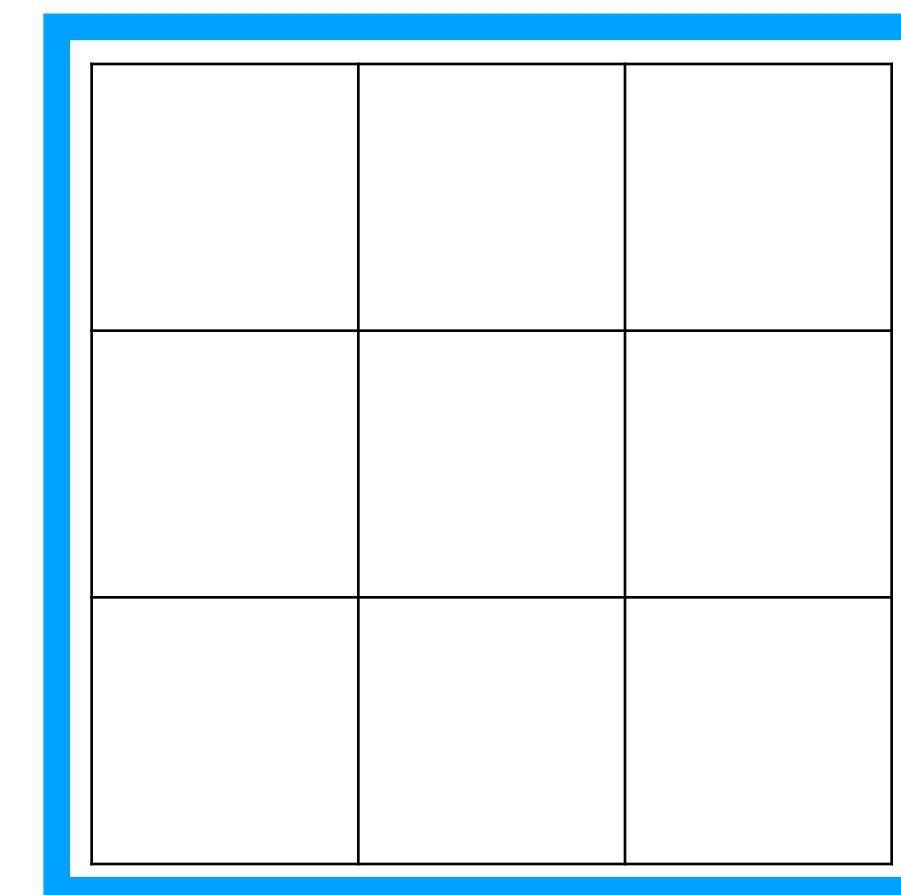
# Convolutional Layer

## Input & Output Shapes - Padding

Input 1-Channel Image

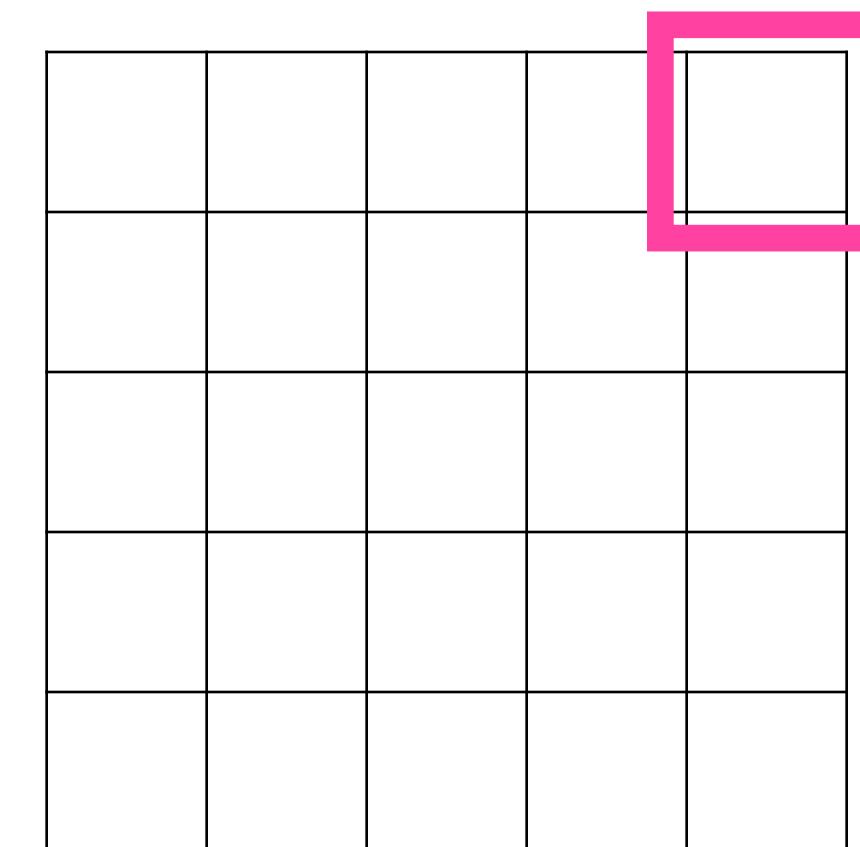


Kernel



$< 1; 3; 3 >$

Output Convolved  
Features



# Convolutional Layer

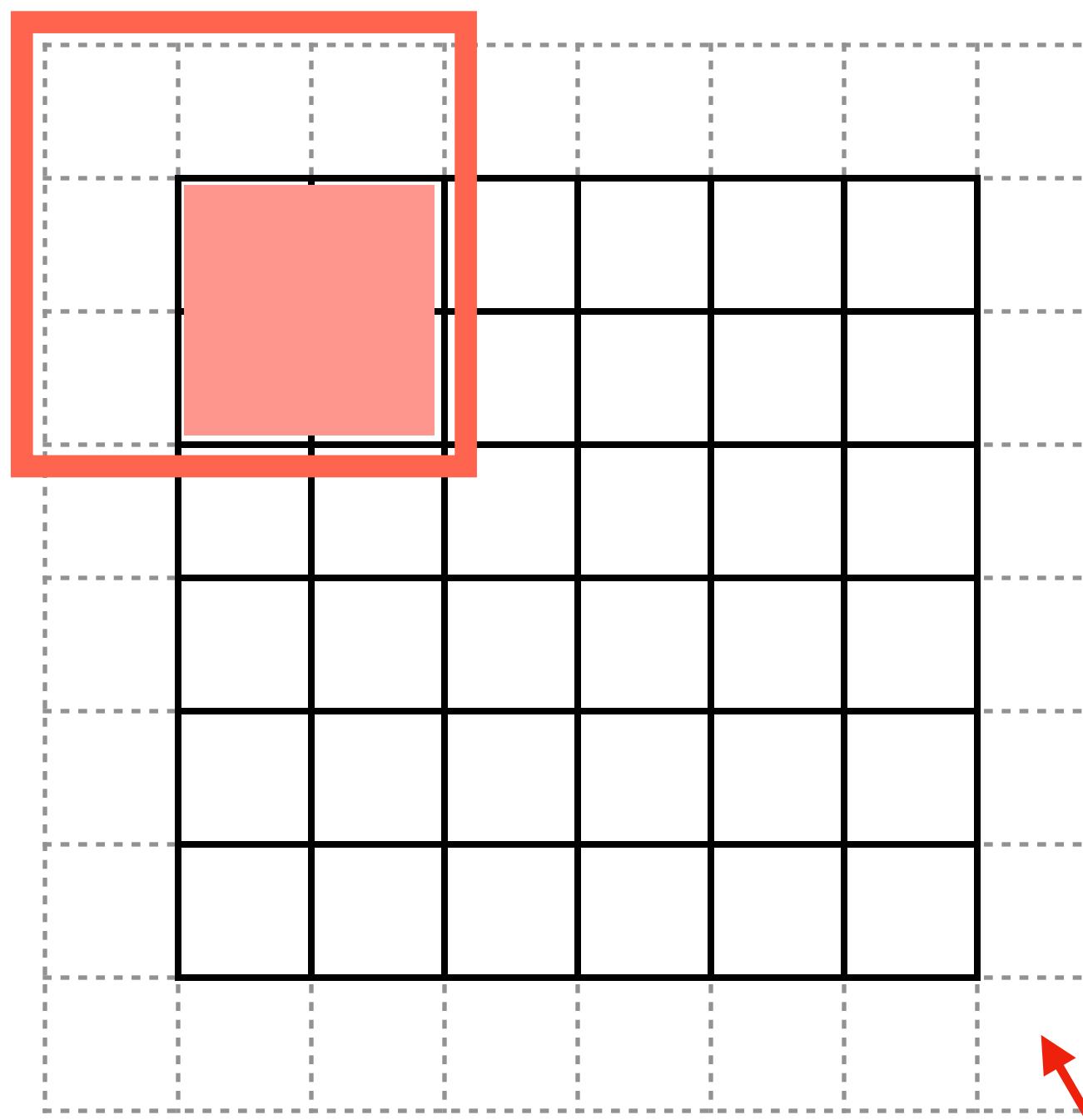
## Stride

- Convolution will produce tensors with **reduced spatial resolution**
- Alternative to pooling layers
  - “Smart” downsampling

# Convolutional Layer

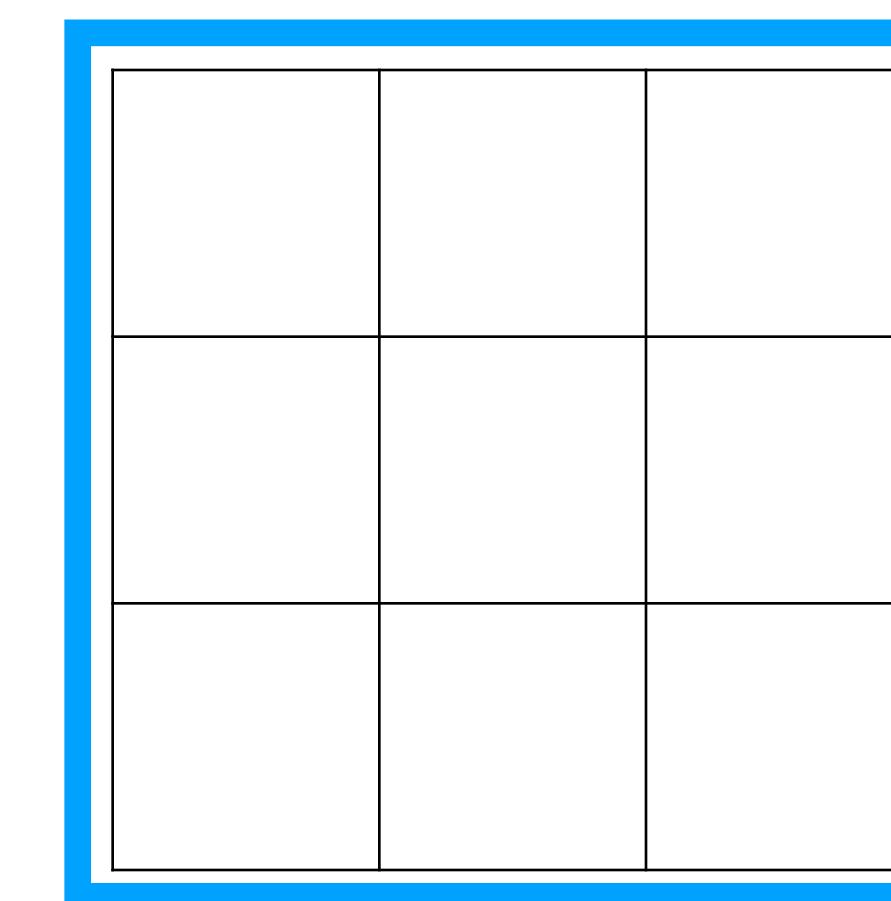
## Input & Output Shapes - Stride & Padding

Input 1-Channel Image



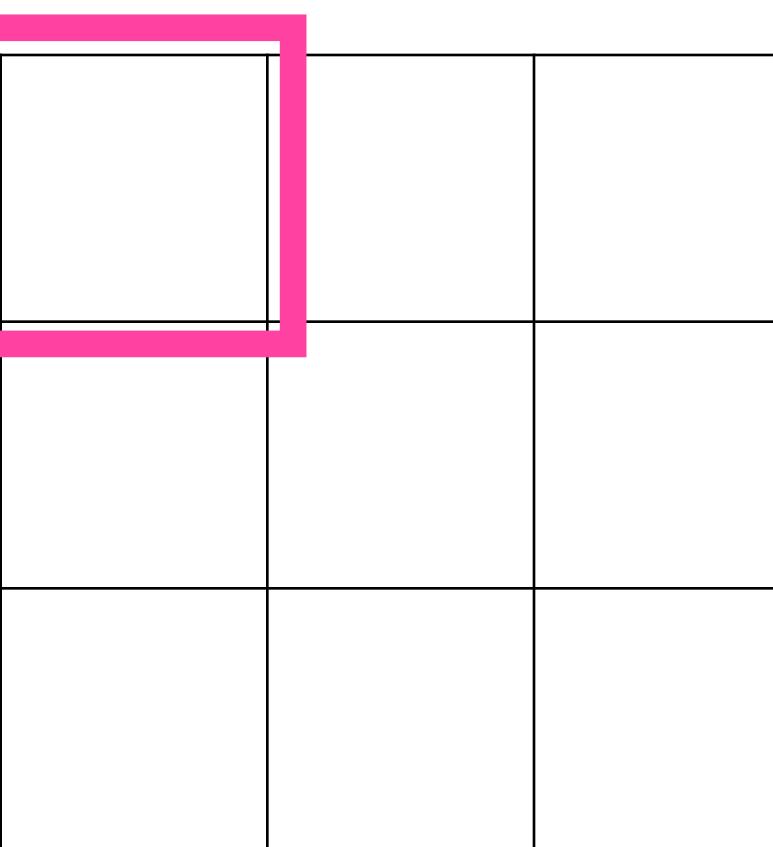
\*

Kernel



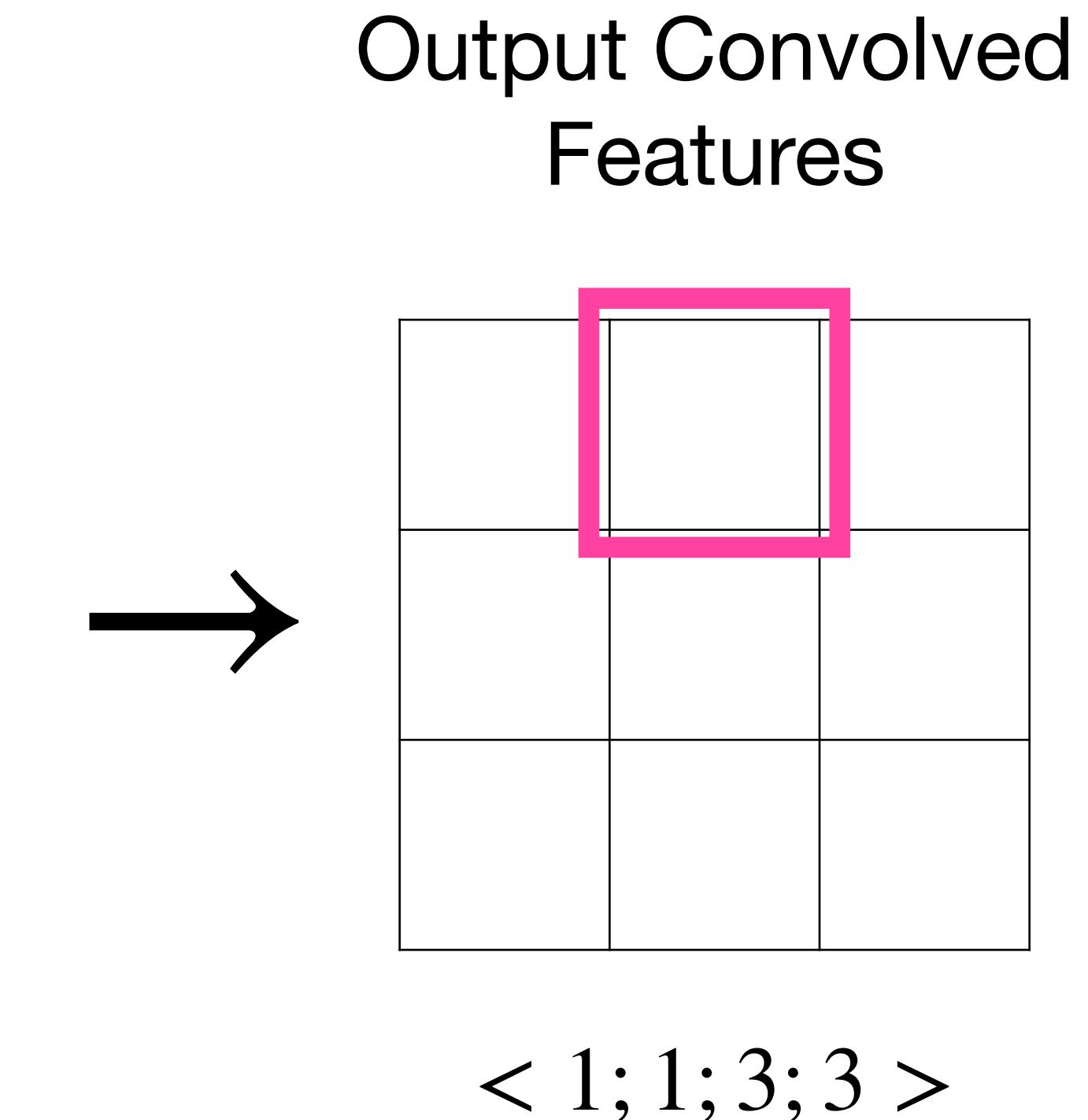
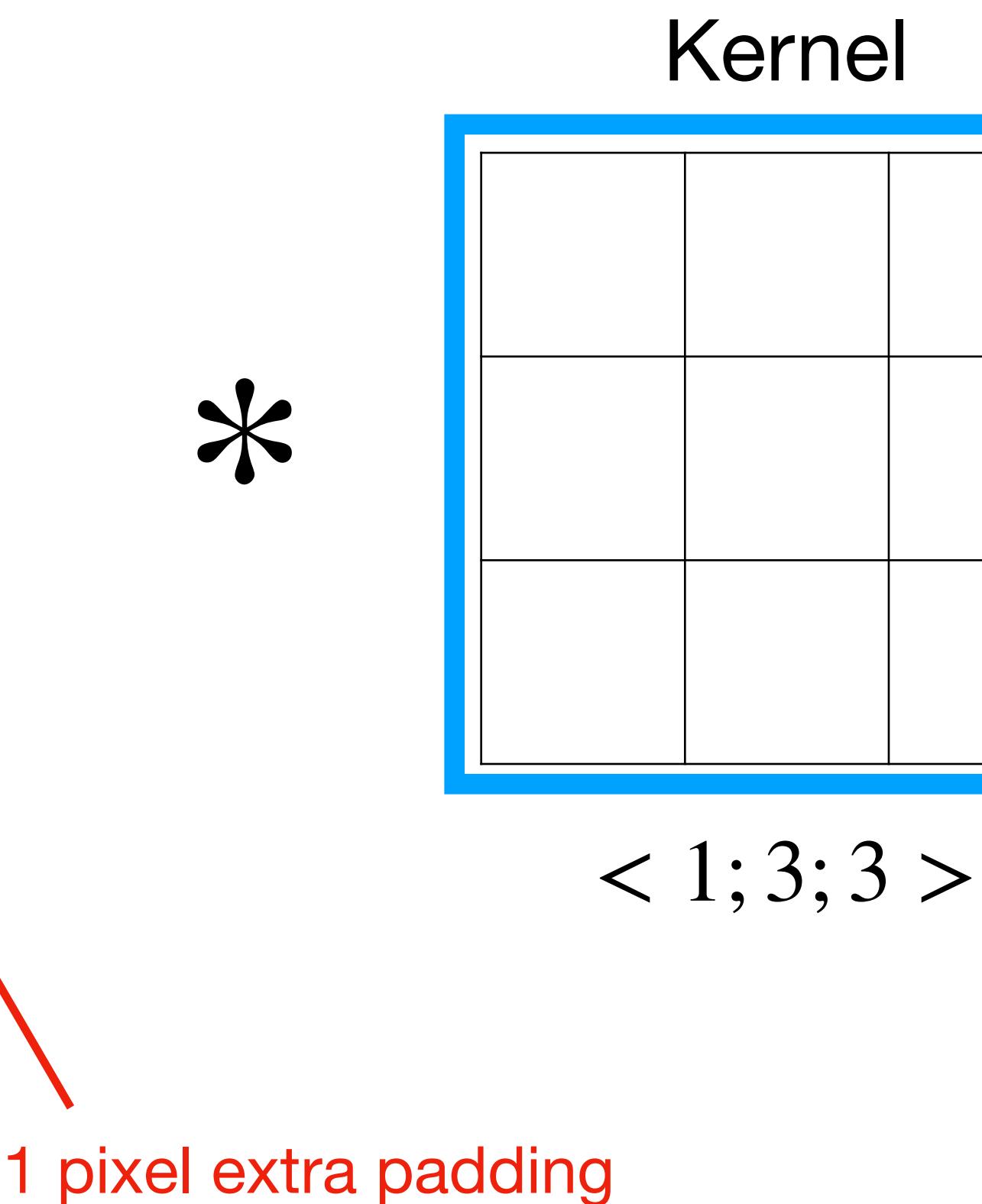
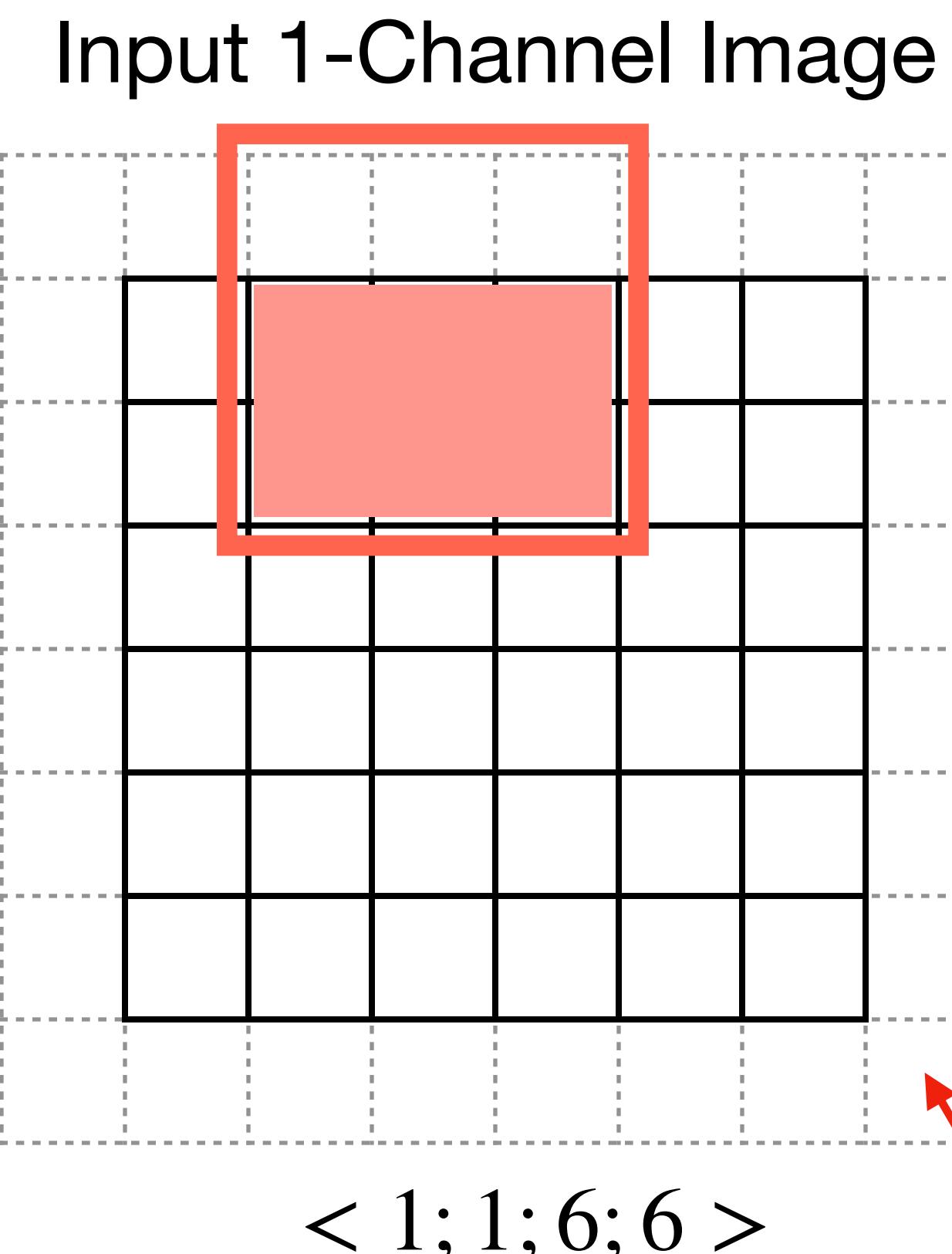
1 pixel extra padding

Output Convolved Features



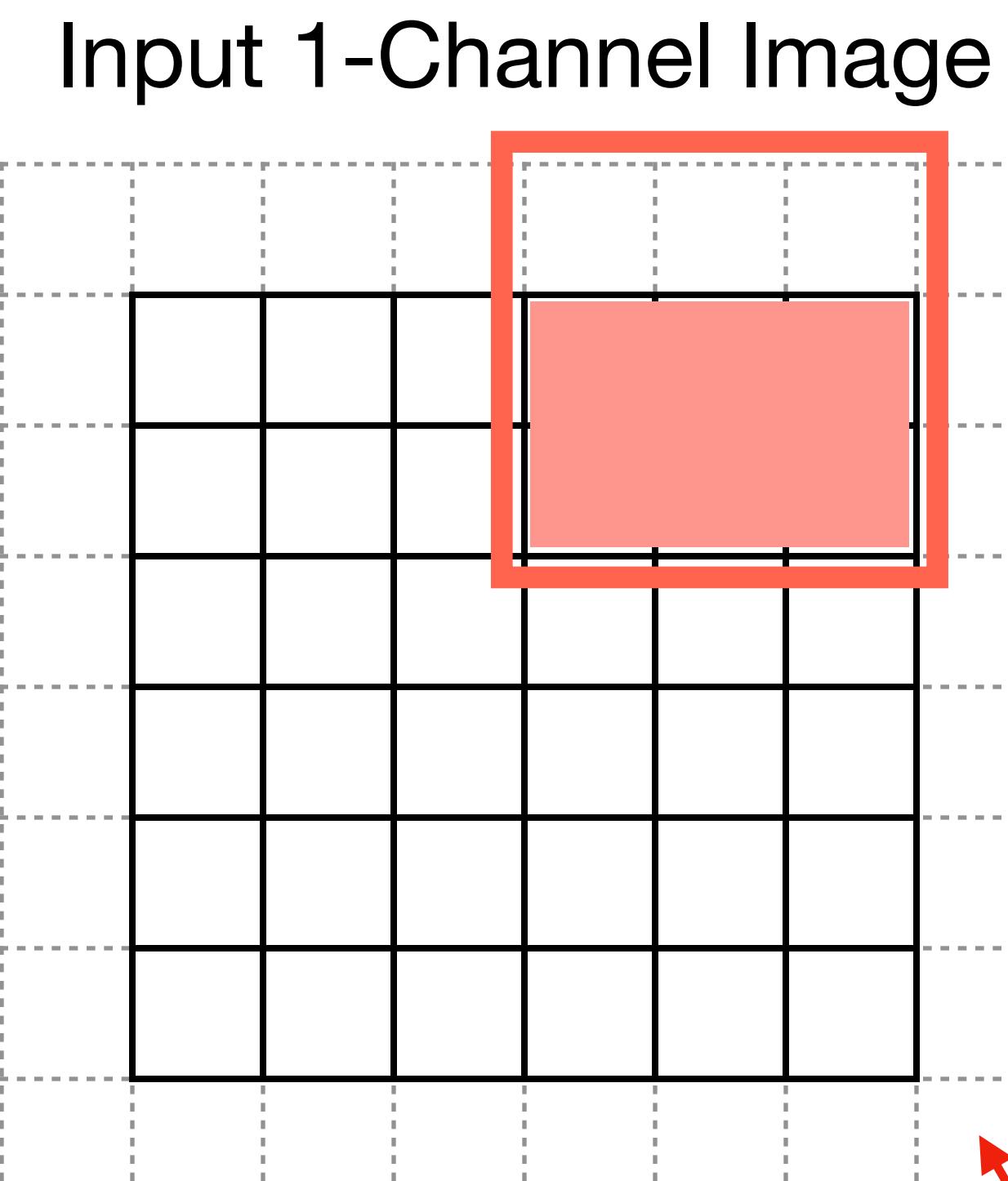
# Convolutional Layer

## Input & Output Shapes - Stride & Padding

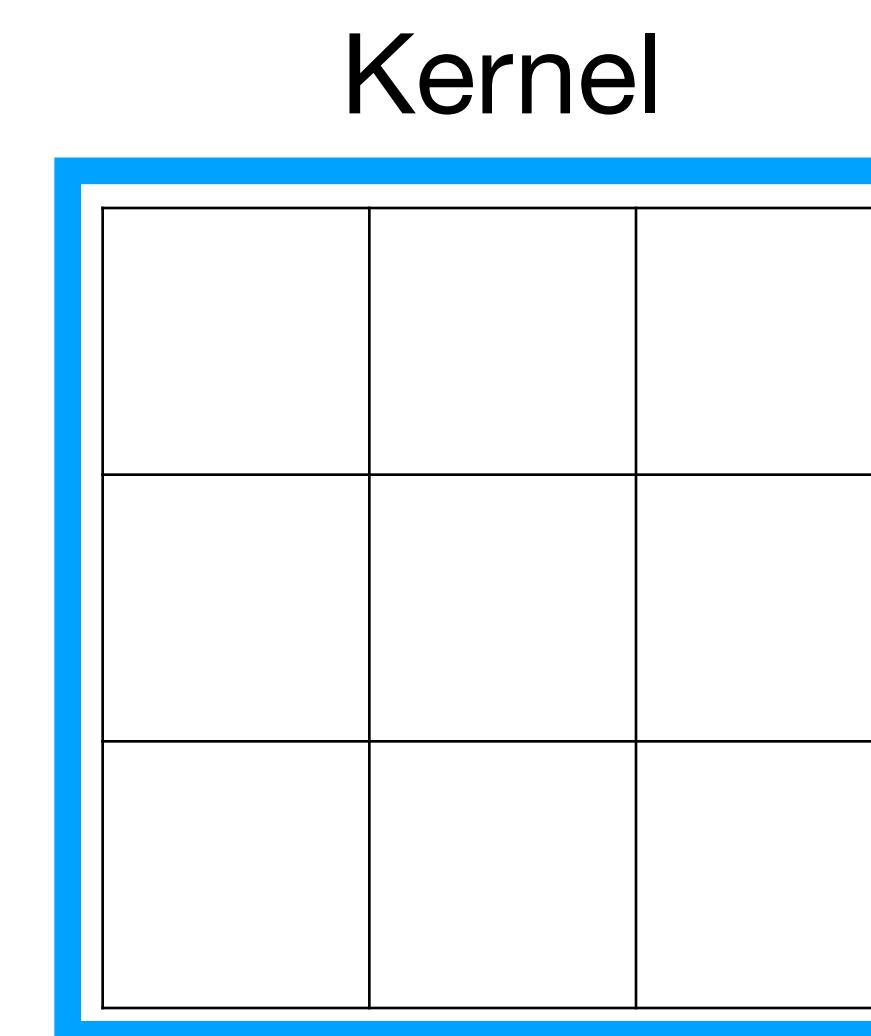


# Convolutional Layer

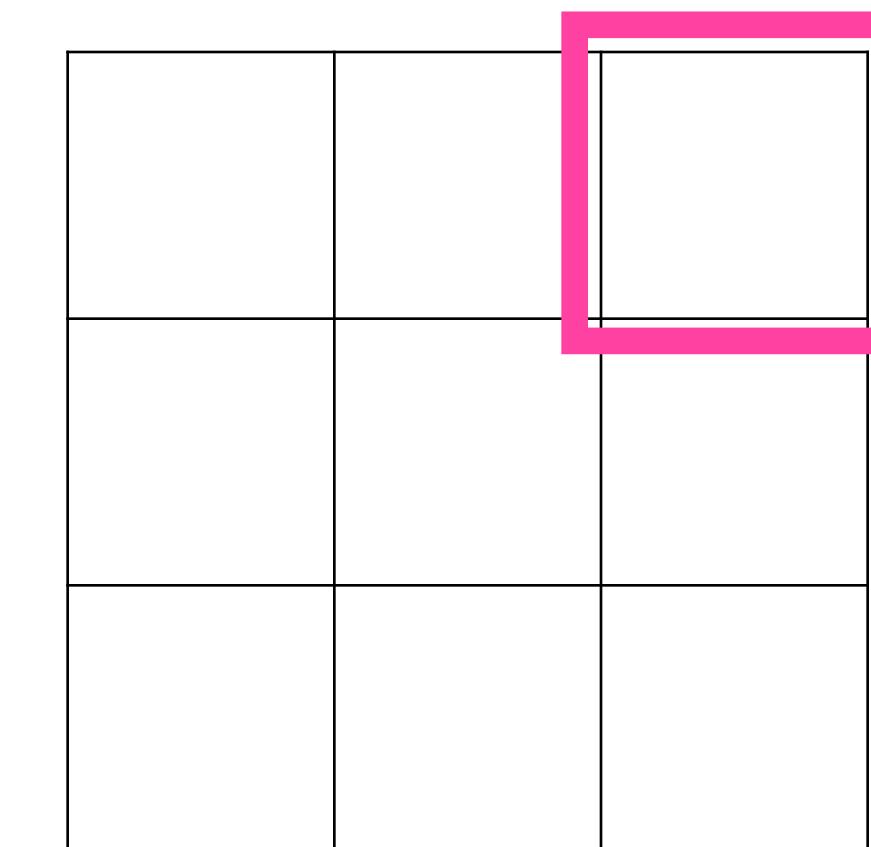
## Input & Output Shapes - Stride & Padding



1 pixel extra padding



Output Convolved  
Features





# Convolutional Layer

## Input & Output Shapes

- Input spatial resolution -  $h_{in}, w_{in}$
- Kernel size -  $k_h \times k_w$
- Stride -  $s_h, s_w$
- Padding -  $p_h, p_w$
- Output spatial resolution -  $h_{out}, w_{out}$

$$h_{out} = \left\lfloor \frac{h_{in} + 2p_h - (k_h - 1) - 1}{s_h} + 1 \right\rfloor$$

$$w_{out} = \left\lfloor \frac{w_{in} + 2p_w - (k_w - 1) - 1}{s_w} + 1 \right\rfloor$$

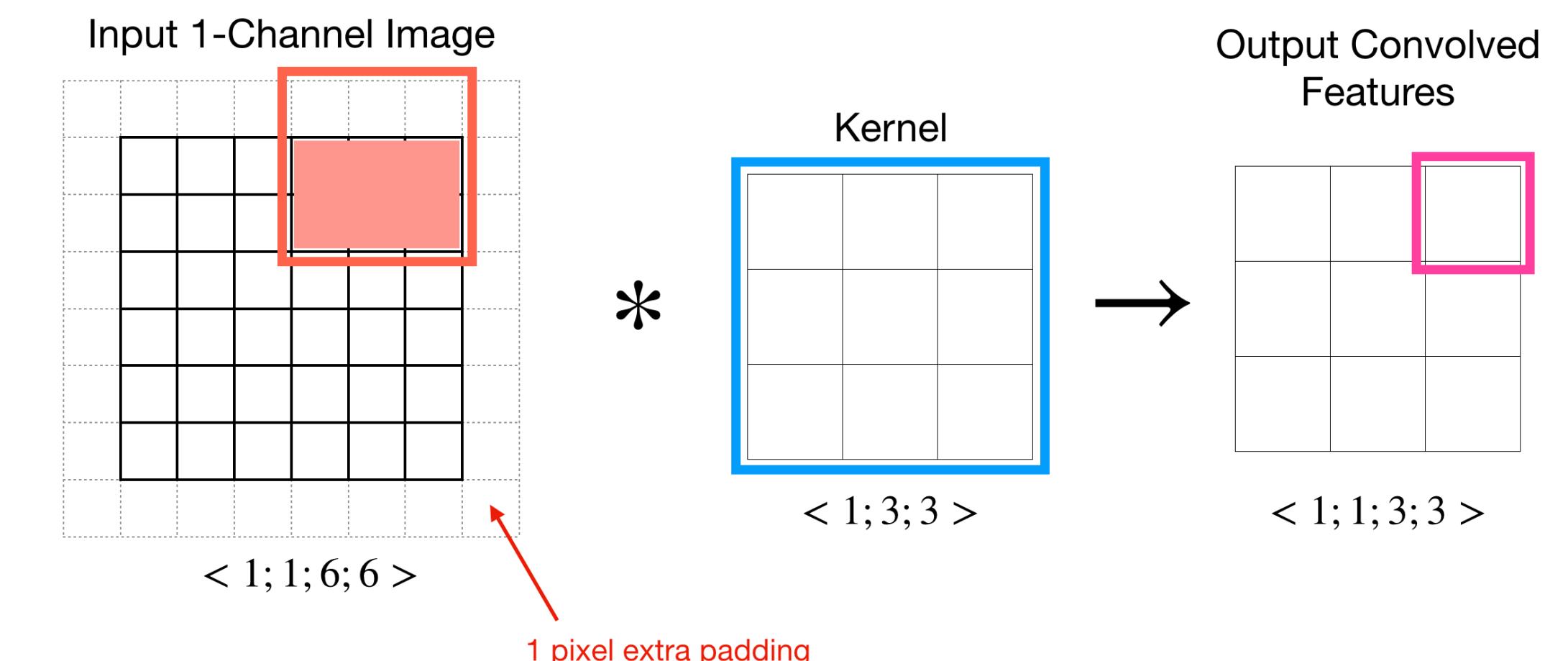
# Convolutional Layer

## Input & Output Shapes

- Input spatial resolution -  $h_{in}, w_{in}$
- Kernel size -  $k_h \times k_w$
- Stride -  $s_h, s_w$
- Padding -  $p_h, p_w$
- Output spatial resolution -  $h_{out}, w_{out}$

$$h_{out} = \left\lfloor \frac{6 + 2 - (3 - 1) - 1}{2} + 1 \right\rfloor = 3$$

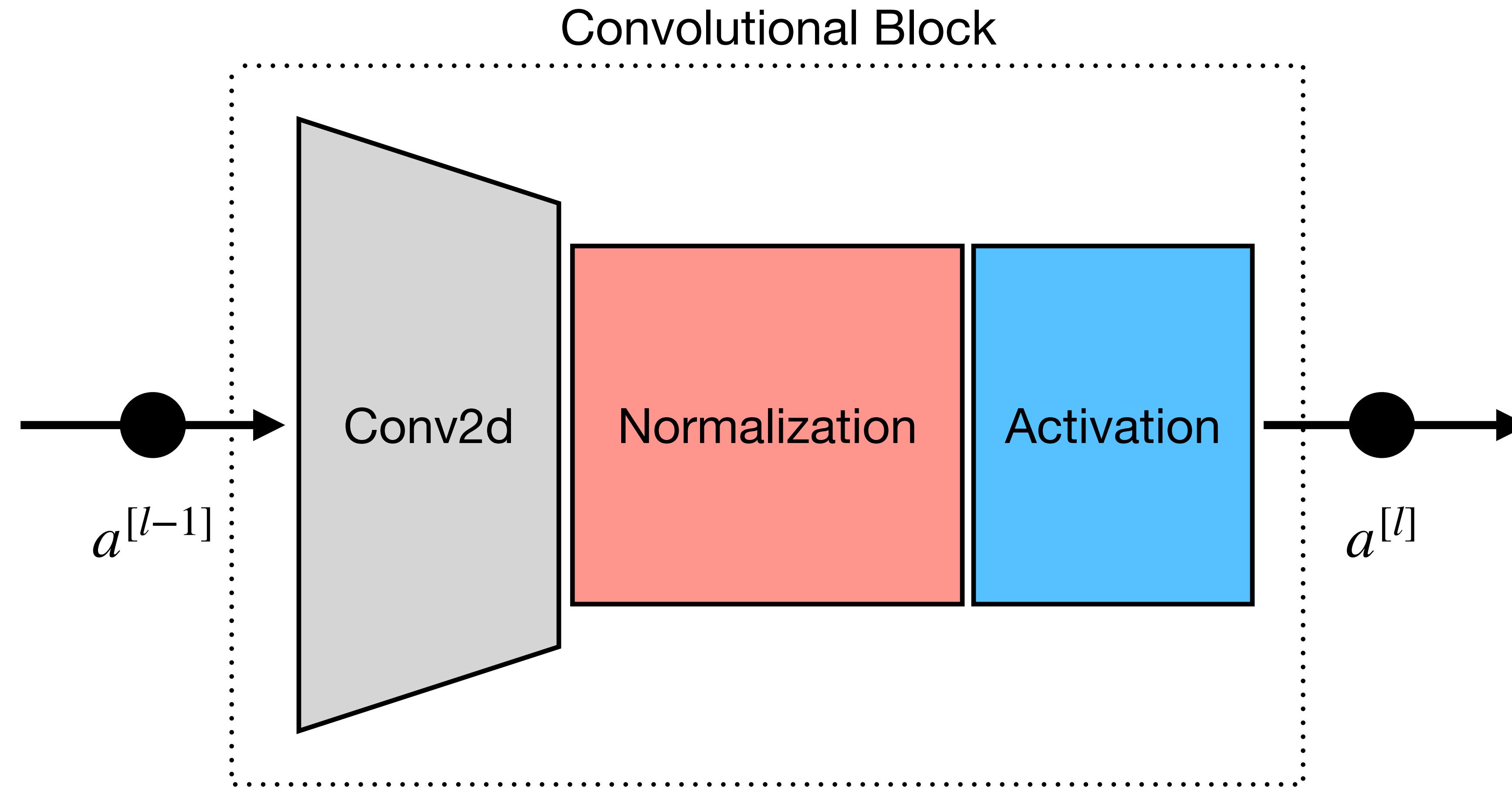
$$h_{out} = \left\lfloor \frac{h_{in} + 2p_h - (k_h - 1) - 1}{s_h} + 1 \right\rfloor$$
$$w_{out} = \left\lfloor \frac{w_{in} + 2p_w - (k_w - 1) - 1}{s_w} + 1 \right\rfloor$$





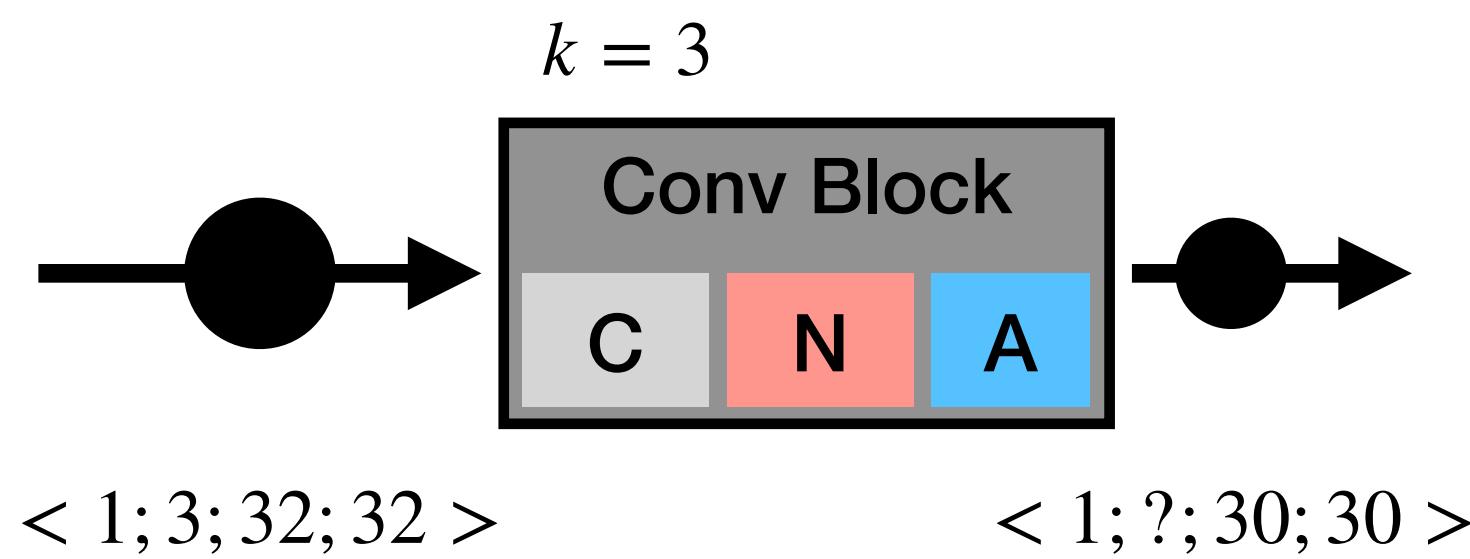
# Multiple Convolutional Layers

## Convolutional Block

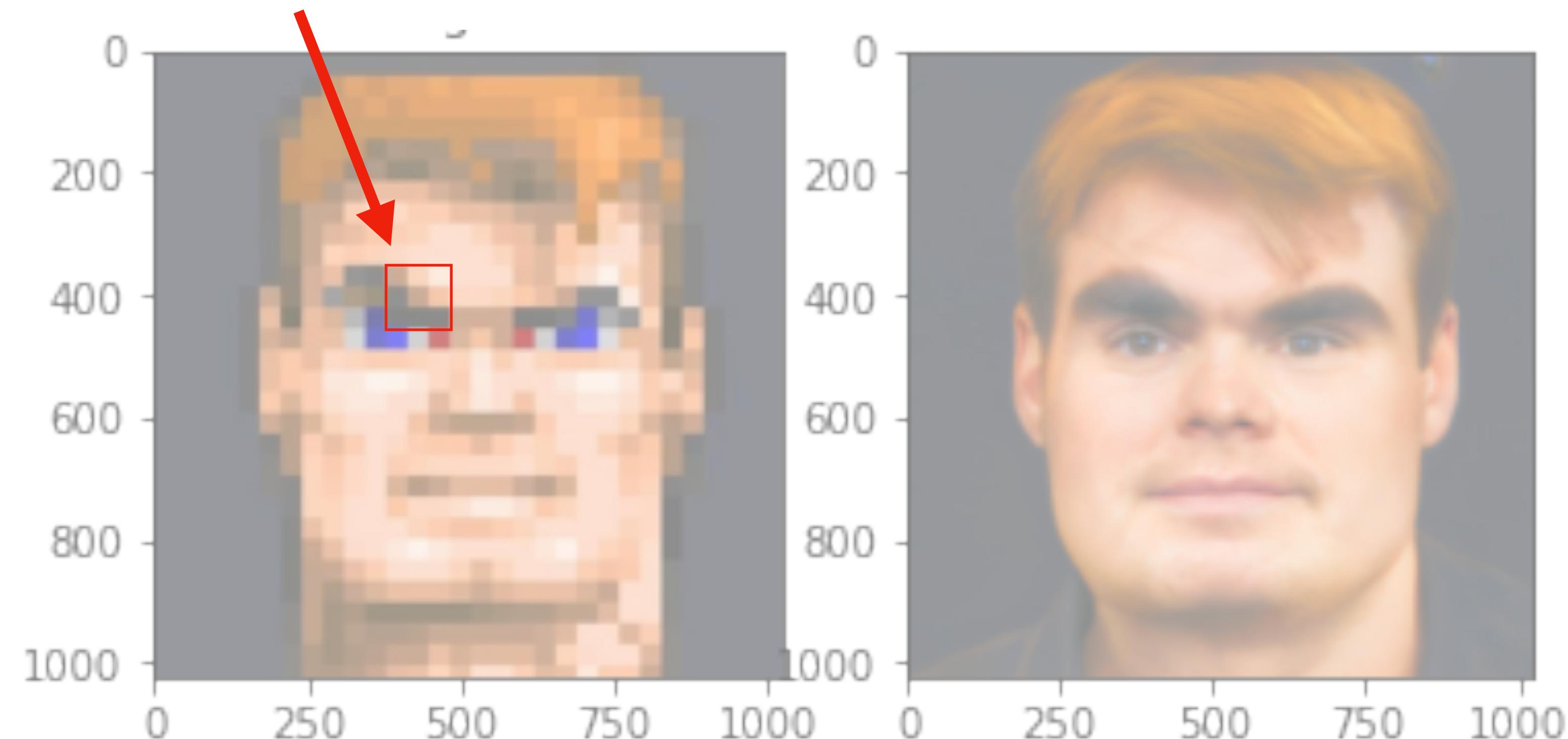


# Convolutional Layers

## Receptive Field

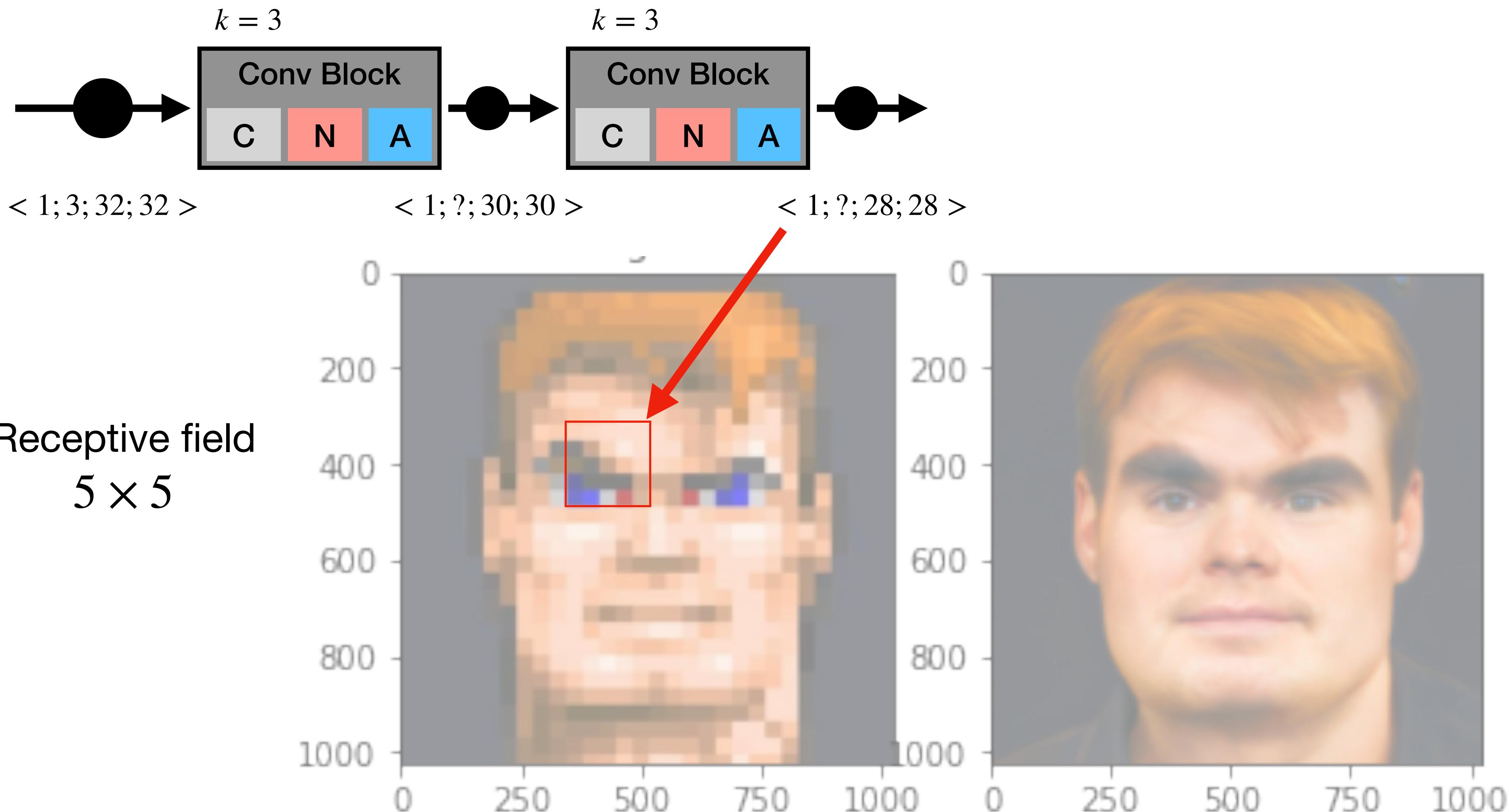


Receptive field  
 $3 \times 3$



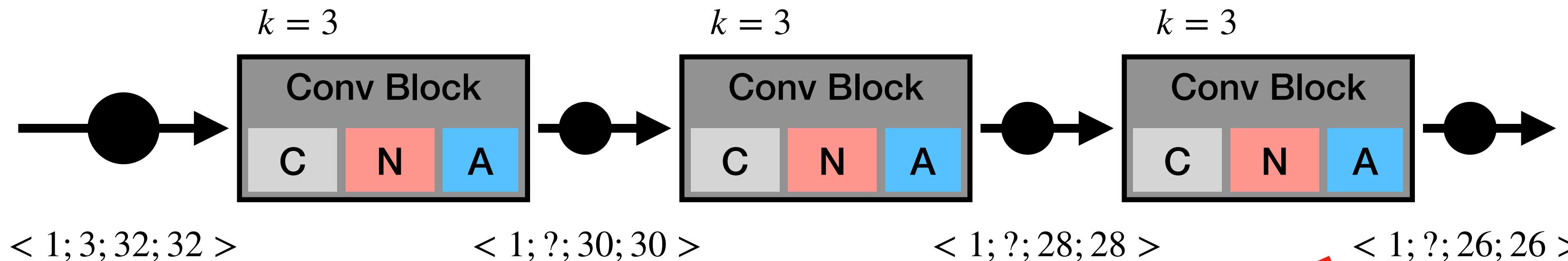
# Convolutional Layers

## Receptive Field

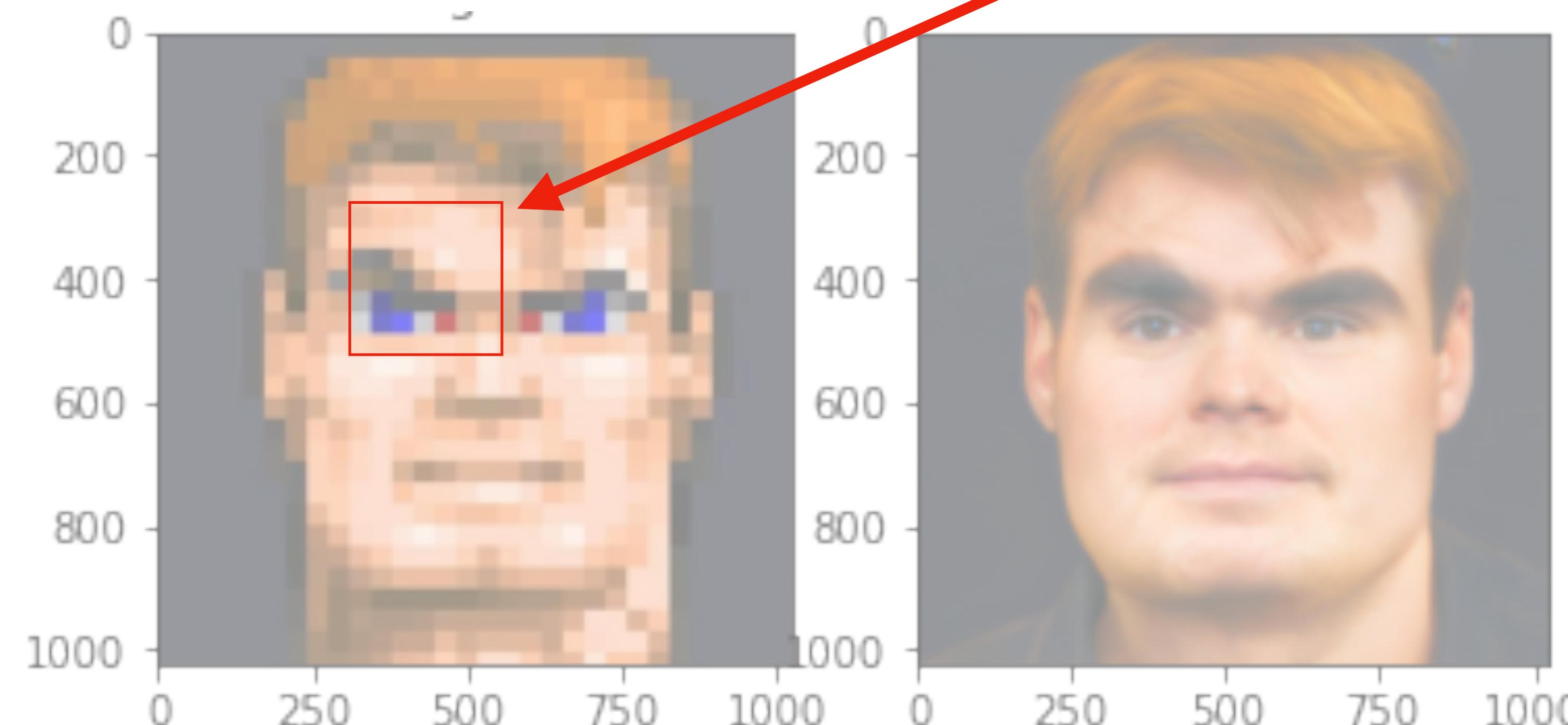


# Convolutional Layers

## Receptive Field



Receptive field  
 $7 \times 7$



# Pooling Layers

- Convolution lowers spatial dimensions of activation maps (without padding)
- Convolution operates on a **small local neighborhood**
  - Multiple blocks are necessary for the receptive field to grow
- **Pooling - allows the receptive field to grow faster**

# Pooling Layers

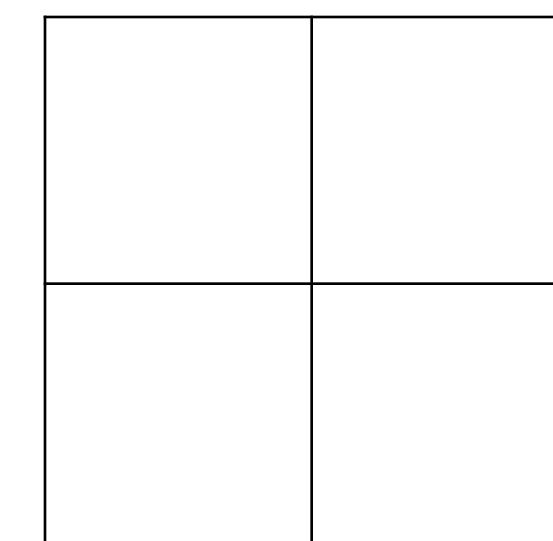
Input  $n$ -Channel Image

1	4	8	9	5	7
3	6	2	5	2	3
7	8	2	3	6	6
5	6	1	4	4	7

$< 1; n; 6; 6 >$

Pool  
operation

Kernel



$< 2; 2 >$

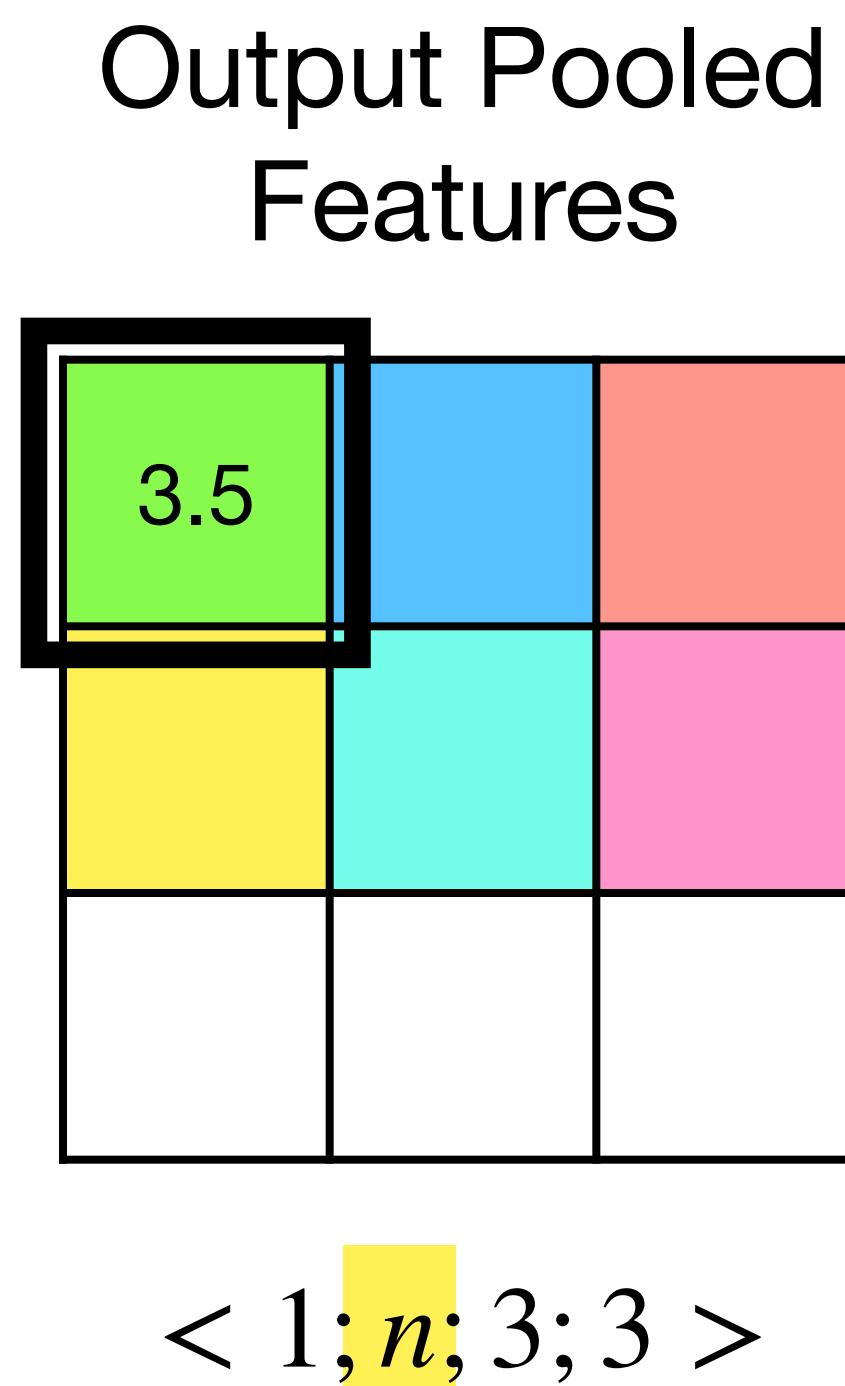
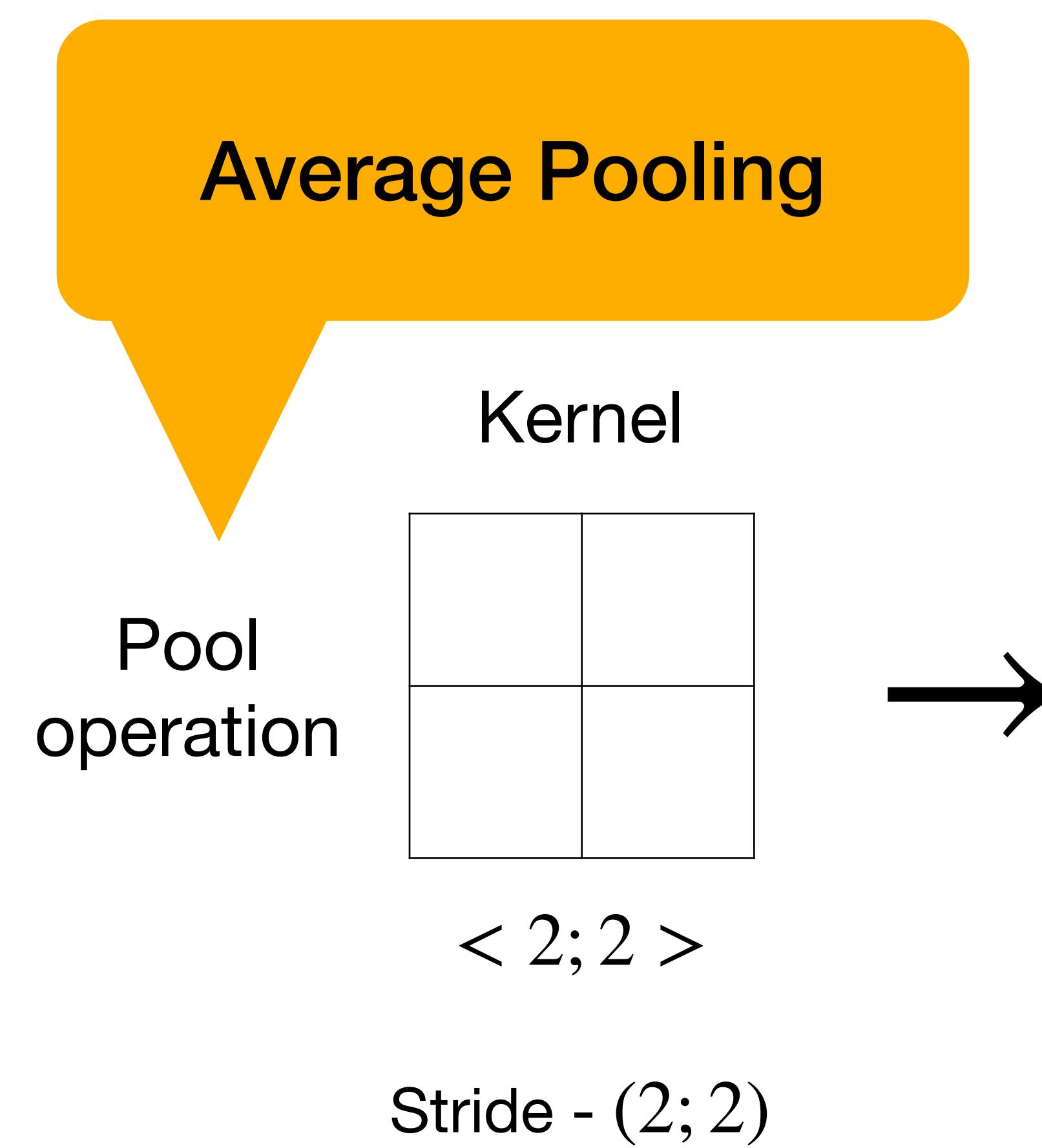
Stride -  $(2; 2)$

Output Pooled  
Features

1	2	3
4	5	6
7	8	9

$< 1; n; 3; 3 >$

# Pooling Layers

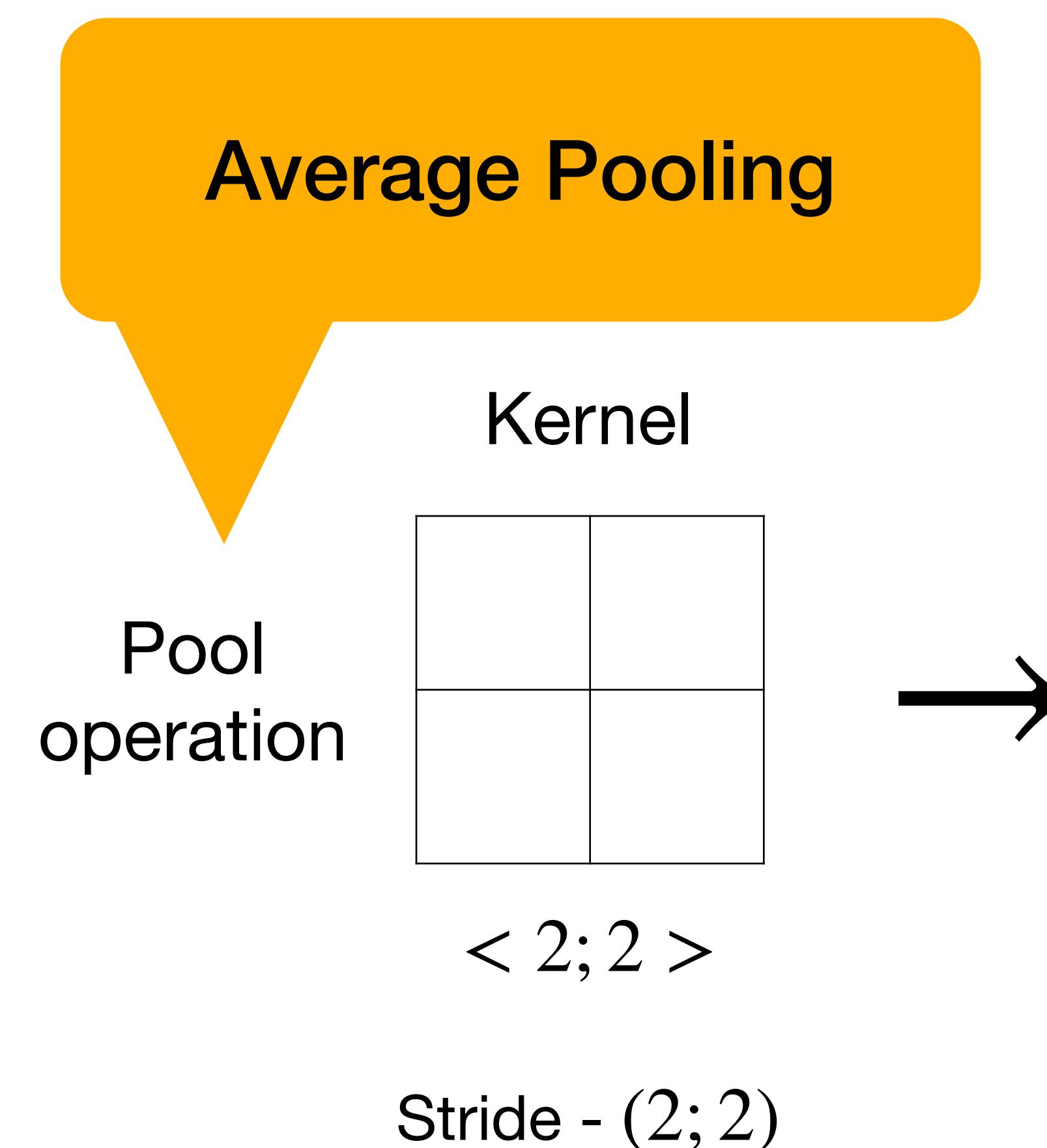


# Pooling Layers

Input  $n$ -Channel Image

1	4	8	9	5	7
3	6	2	5	2	3
7	8	2	3	6	6
5	6	1	4	4	7

$< 1; n; 6; 6 >$



Output Pooled Features

3.5	6	

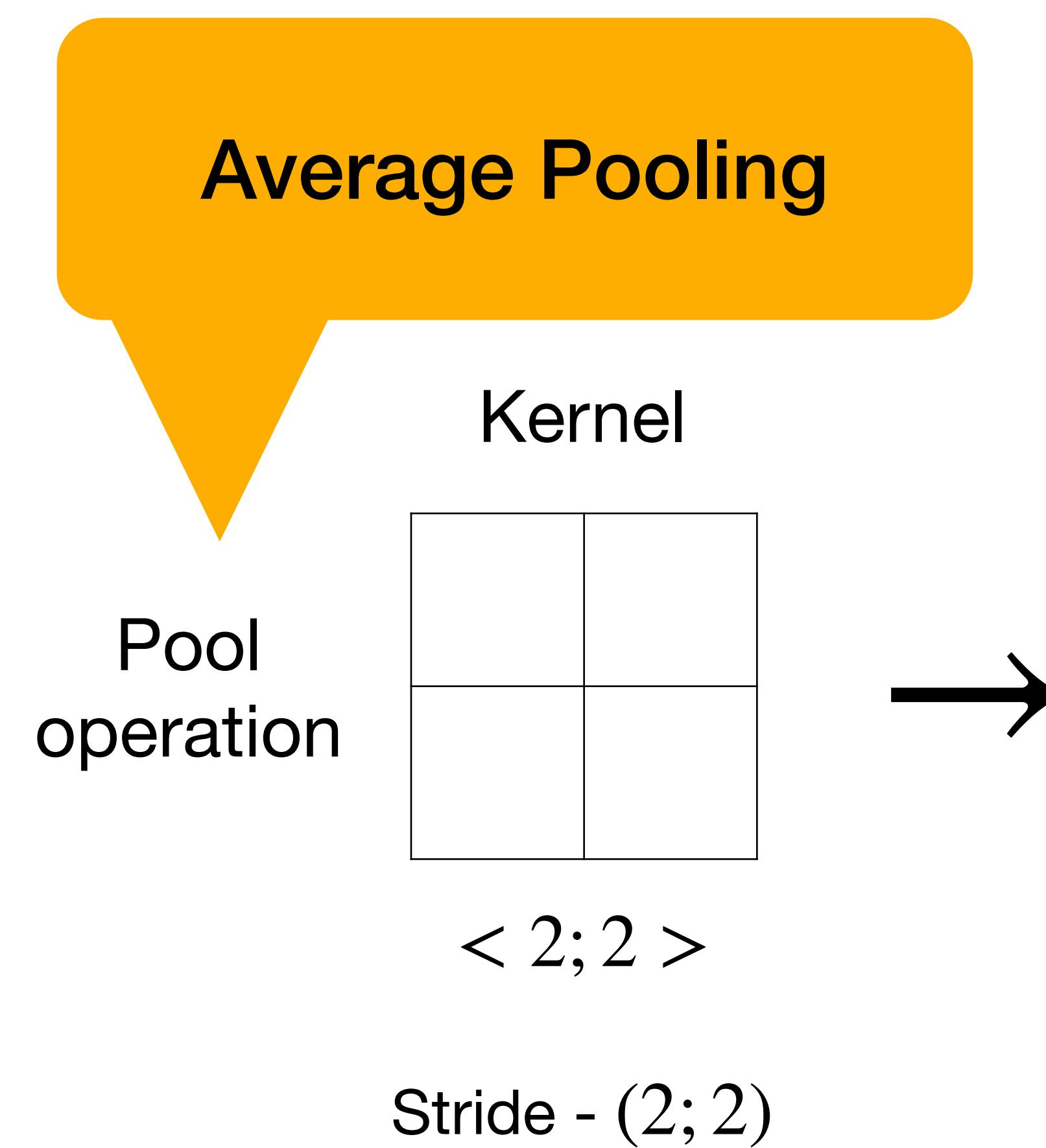
$< 1; n; 3; 3 >$

# Pooling Layers

Input  $n$ -Channel Image

1	4	8	9	5	7
3	6	2	5	2	3
7	8	2	3	6	6
5	6	1	4	4	7

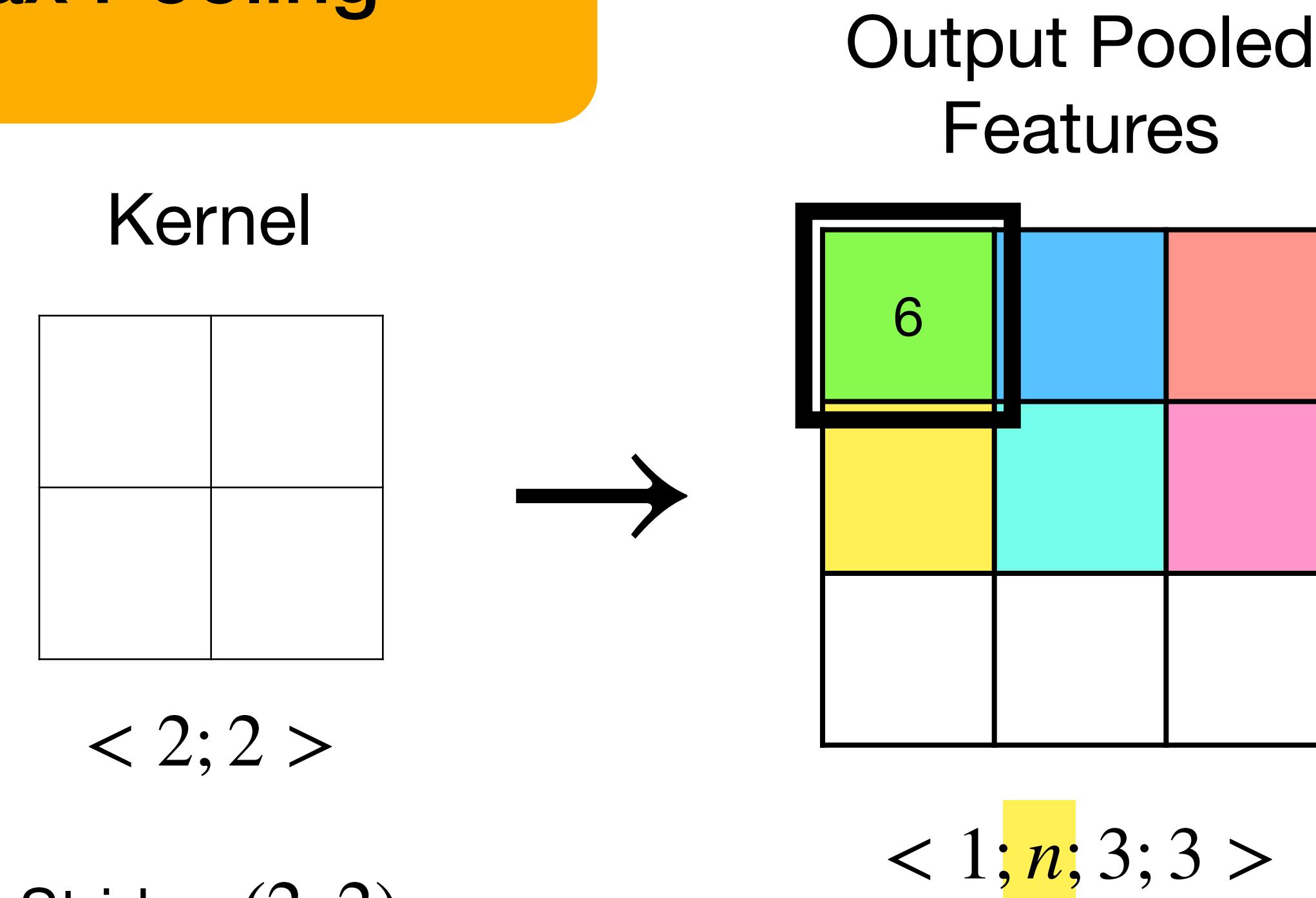
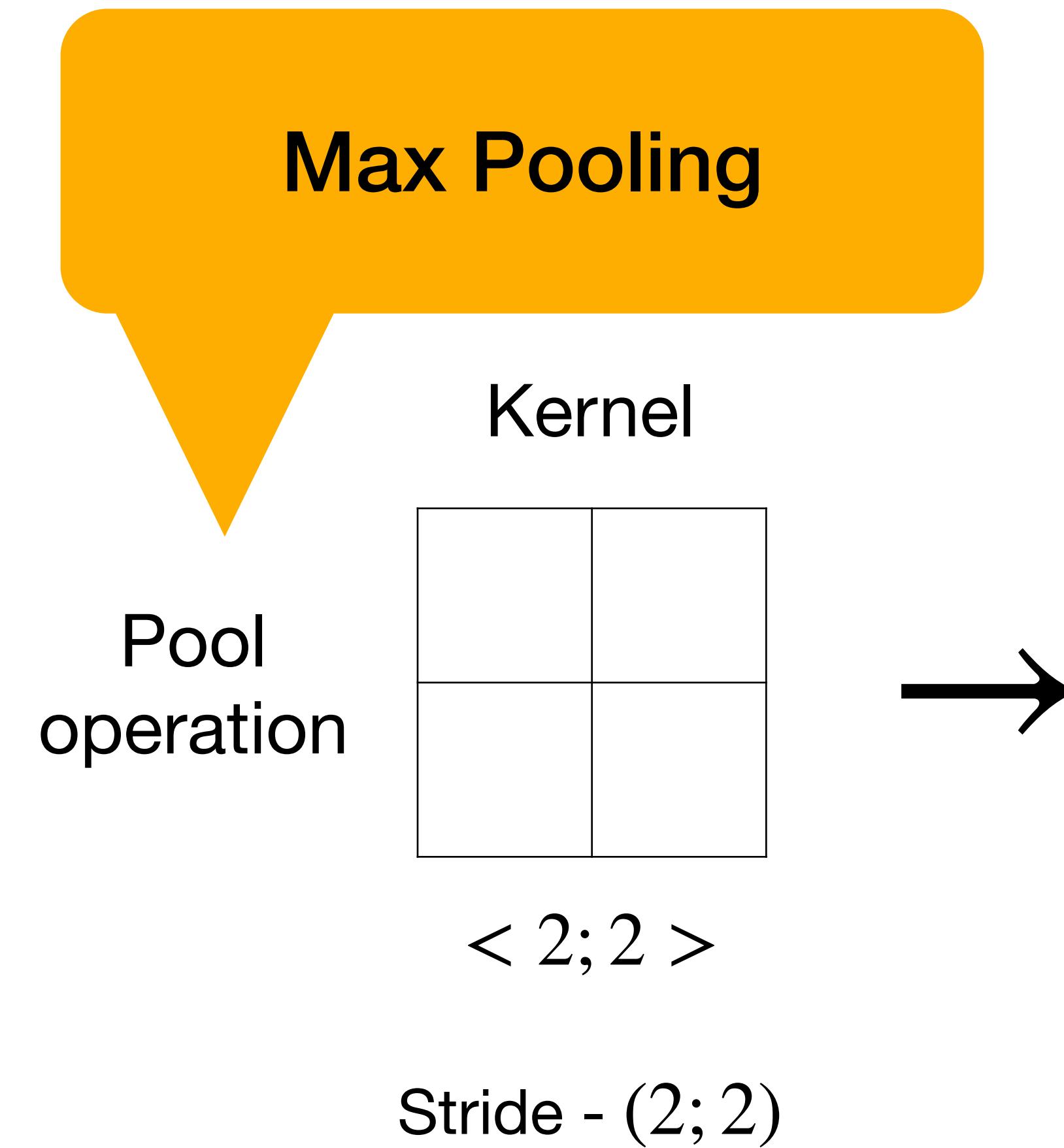
$< 1; n; 6; 6 >$



Output Pooled Features


$< 1; n; 3; 3 >$

# Pooling Layers

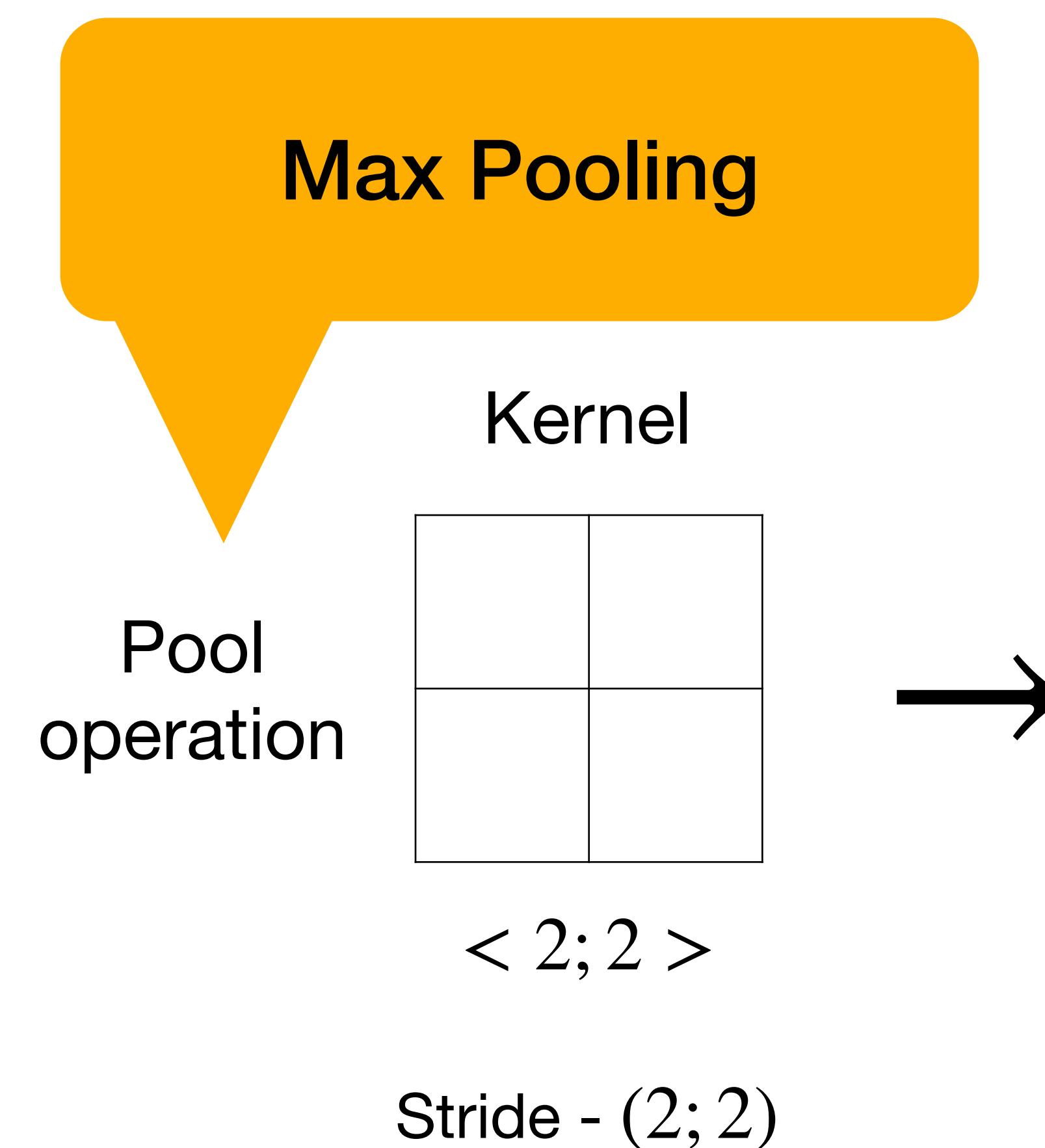


# Pooling Layers

Input  $n$ -Channel Image

1	4	8	9	5	7
3	6	2	5	2	3
7	8	2	3	6	6
5	6	1	4	4	7

$< 1; n; 6; 6 >$



Output Pooled Features

6	9	

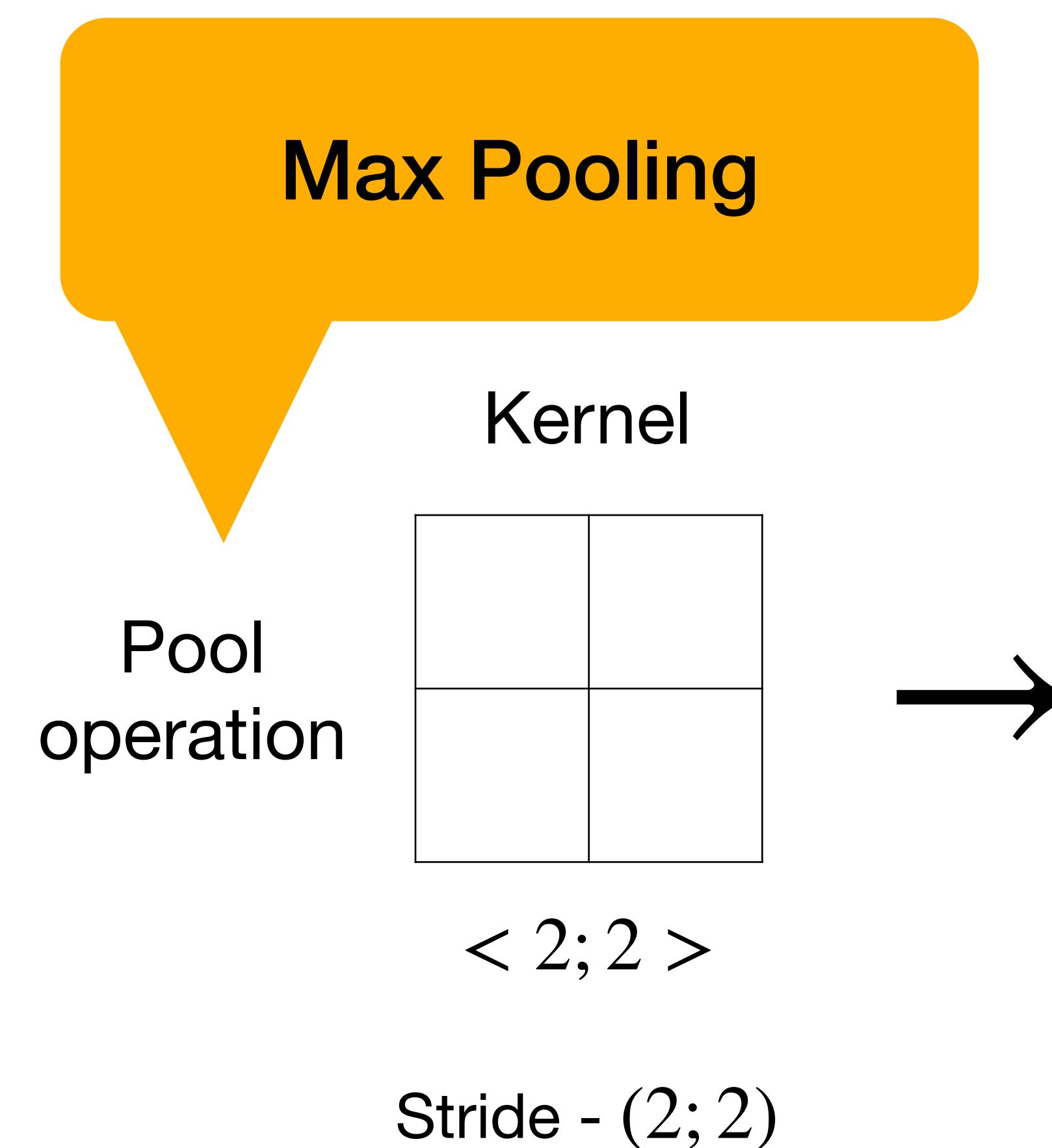
$< 1; n; 3; 3 >$

# Pooling Layers

Input  $n$ -Channel Image

1	4	8	9	5	7
3	6	2	5	2	3
7	8	2	3	6	6
5	6	1	4	4	7

$< 1; n; 6; 6 >$



Output Pooled Features

6	9	7

$< 1; n; 3; 3 >$

# Pooling Layers

## Global Average Pooling (Adaptive Pooling)

Input  $n$ -Channel Image

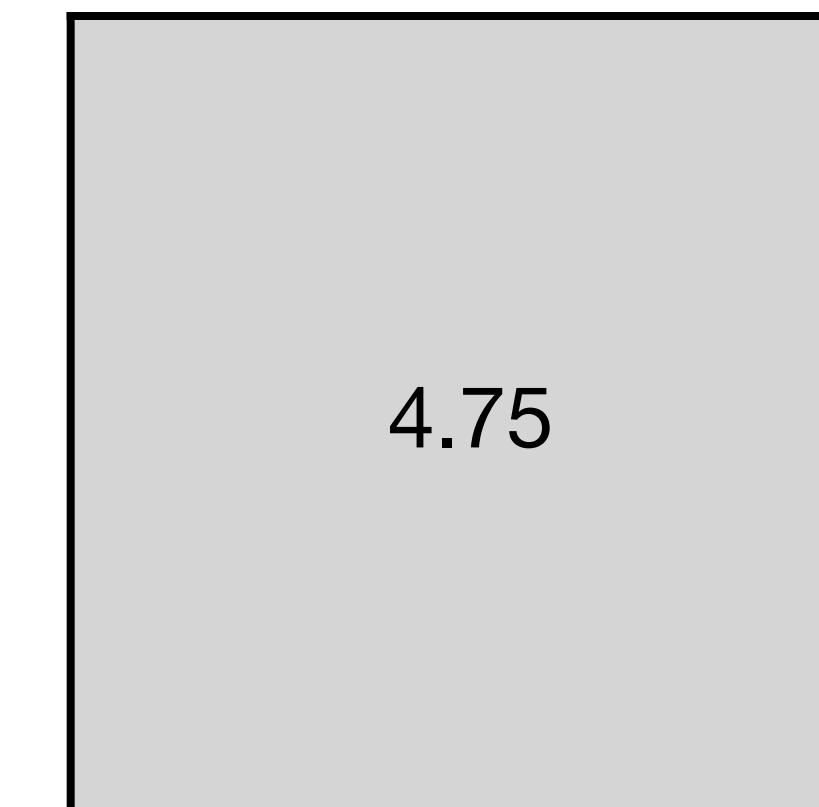
1	4	8	9	5	7
3	6	2	5	2	3
7	8	2	3	6	6
5	6	1	4	4	7

$< 1; n; 6; 6 >$

Pool  
operation



Output Pooled  
Features



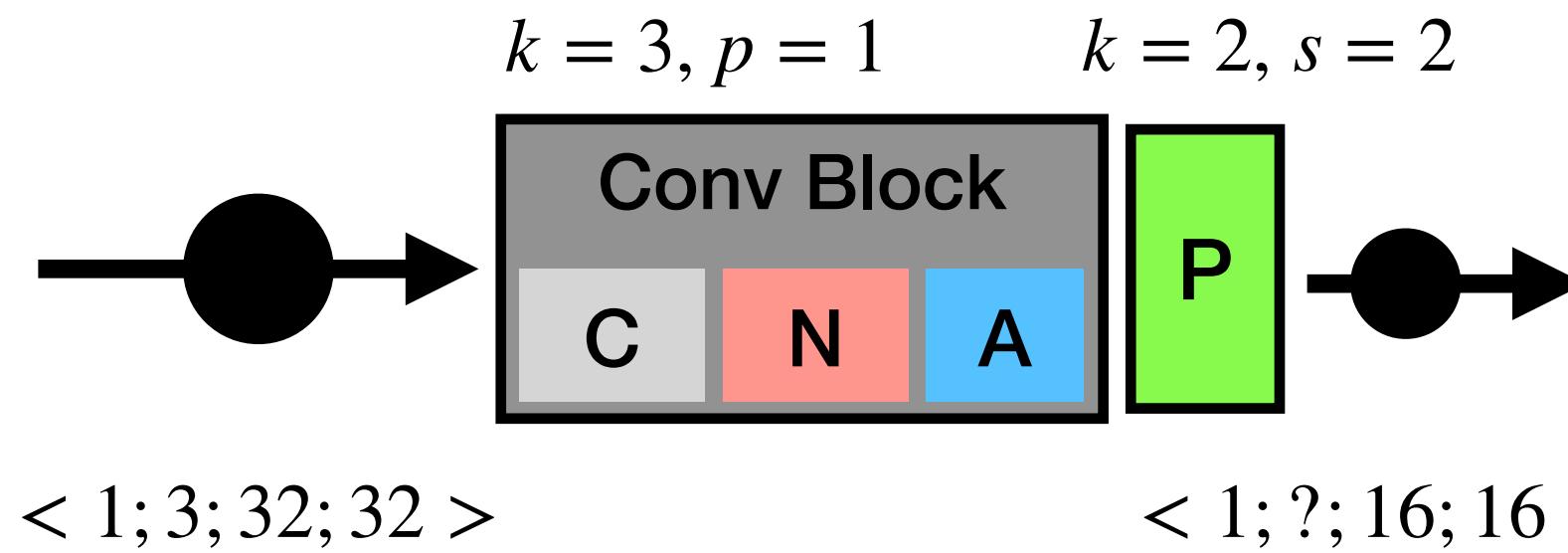
$< 1; n; 1; 1 >$



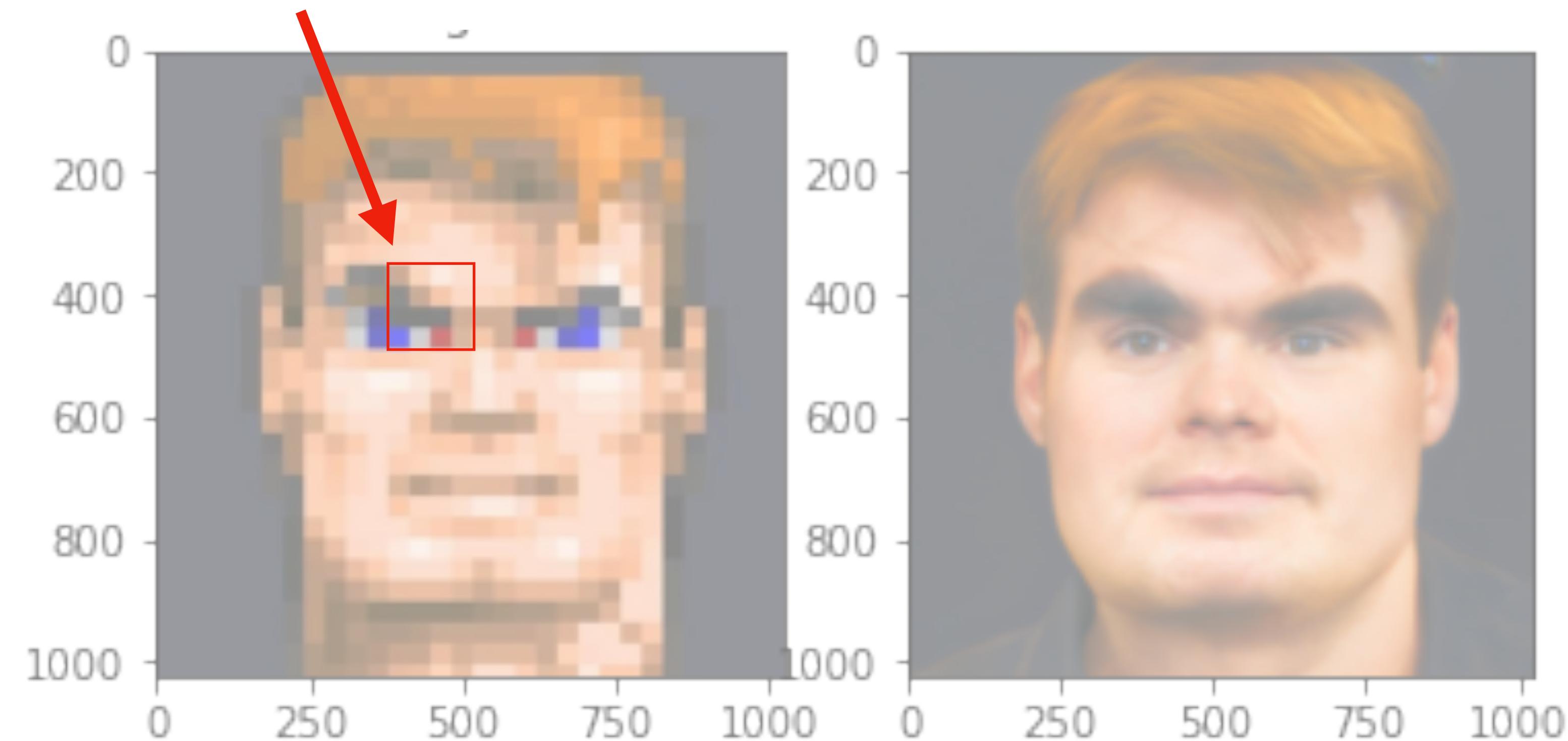
We can directly decide the output shape  
and interpolate each channel

# Convolutional Layers

## Receptive Field

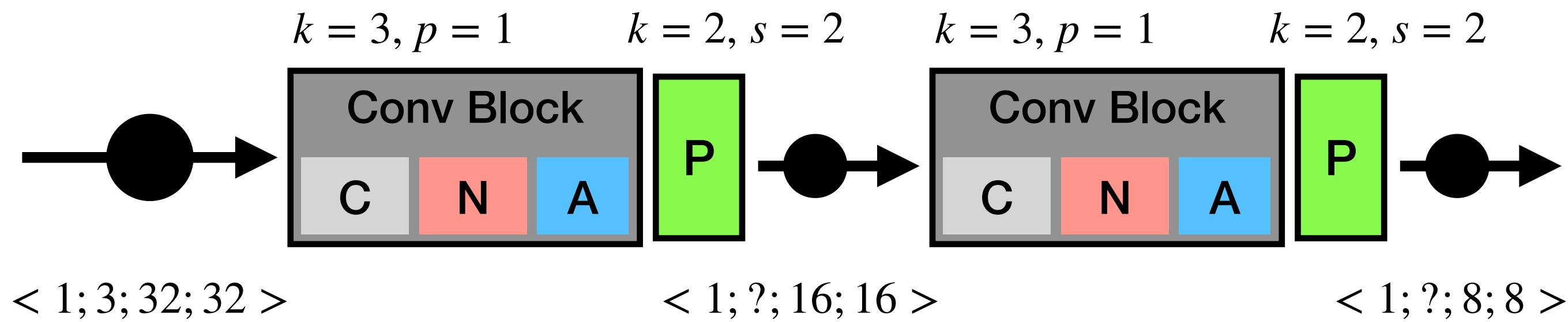


Receptive field  
 $4 \times 4$

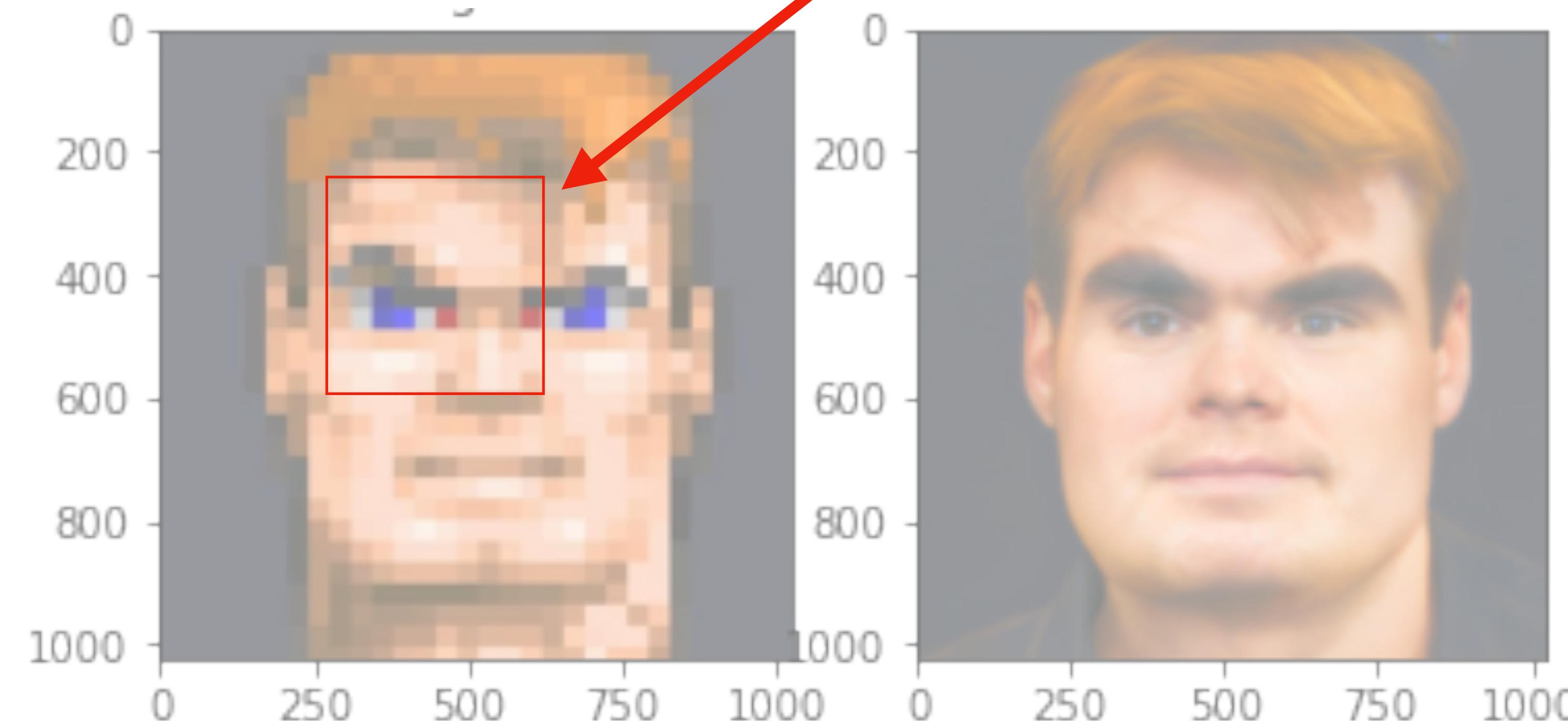


# Convolutional Layers

## Receptive Field

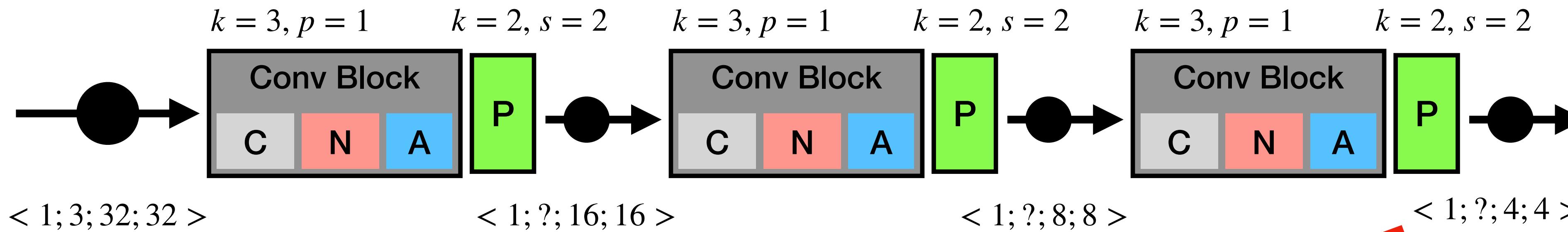


Receptive field  
 $10 \times 10$

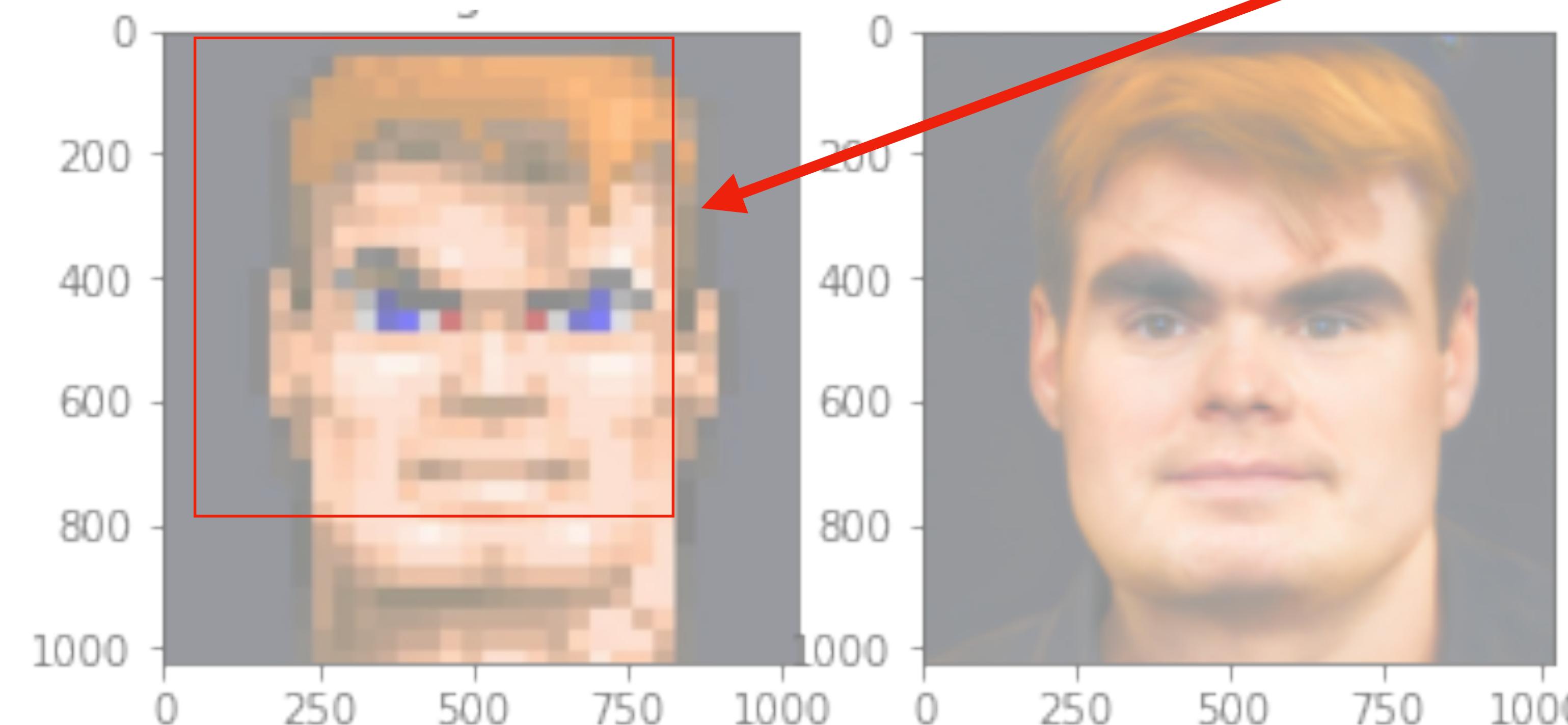


# Convolutional Layers

## Receptive Field



Receptive field  
 $22 \times 22$





# Convolutional Layers

## Receptive Field - How to Compute ?

- Go backward, ignore padding

$$h_{out} = \left\lfloor \frac{h_{in} + 2p_h - (k_h - 1) - 1}{s_h} + 1 \right\rfloor$$

$$h_{out} = \left\lfloor \frac{h_{in} - (k_h - 1) - 1}{s_h} + 1 \right\rfloor$$

$$s_h(h_{out} - 1) = h_{in} - (k_h - 1) - 1$$

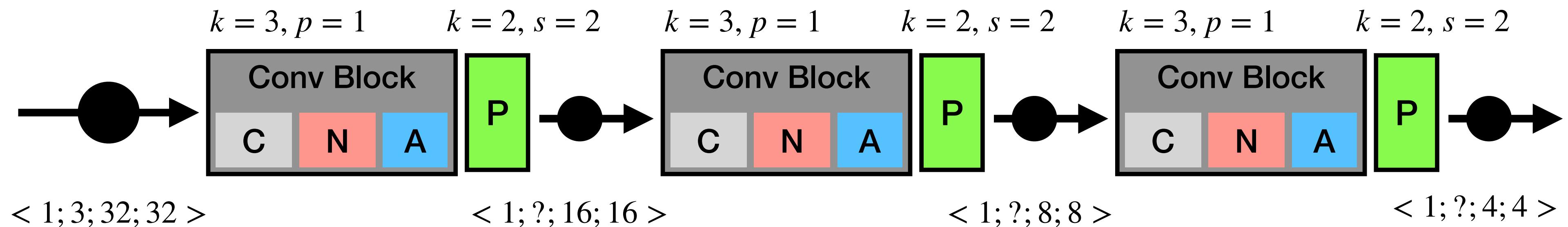
$$s_h(h_{out} - 1) + 1 + (k_h - 1) = h_{in}$$

$$s_h(h_{out} - 1) + k_h = h_{in}$$

# Convolutional Layers

## Receptive Field

$$s_h(h_{out} - 1) + k_h = h_{in}$$



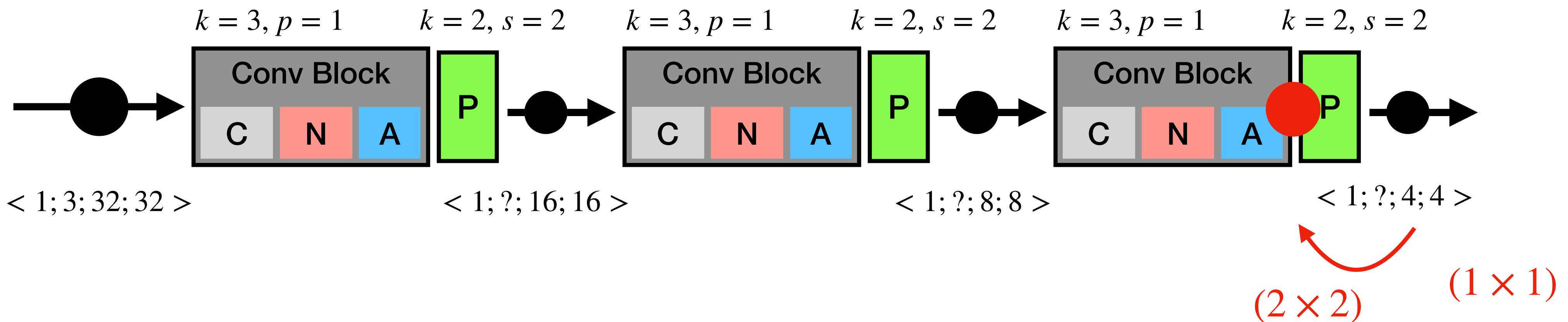
$(1 \times 1)$

Start with  $1 \times 1$   
we are looking for the reception  
field of each element of the  
final activation map

# Convolutional Layers

## Receptive Field

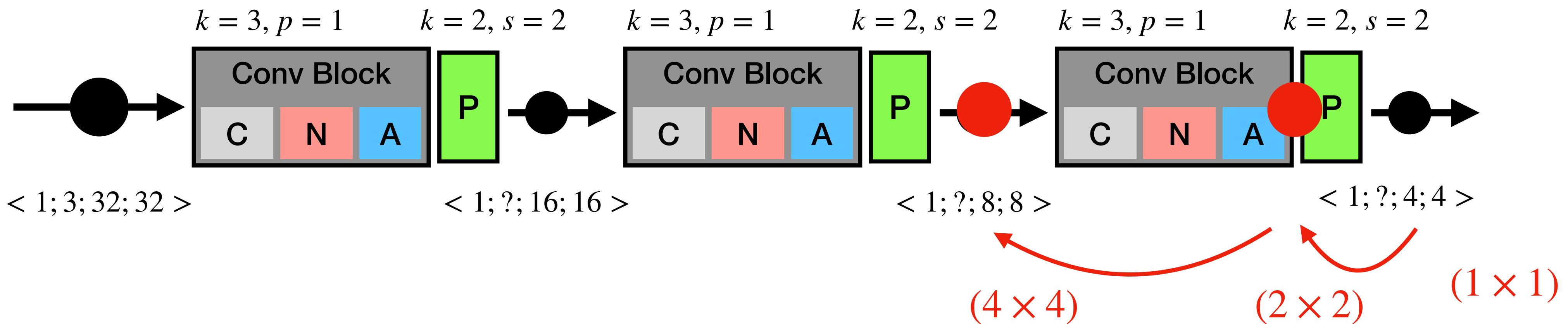
$$s_h(h_{out} - 1) + k_h = h_{in}$$



# Convolutional Layers

## Receptive Field

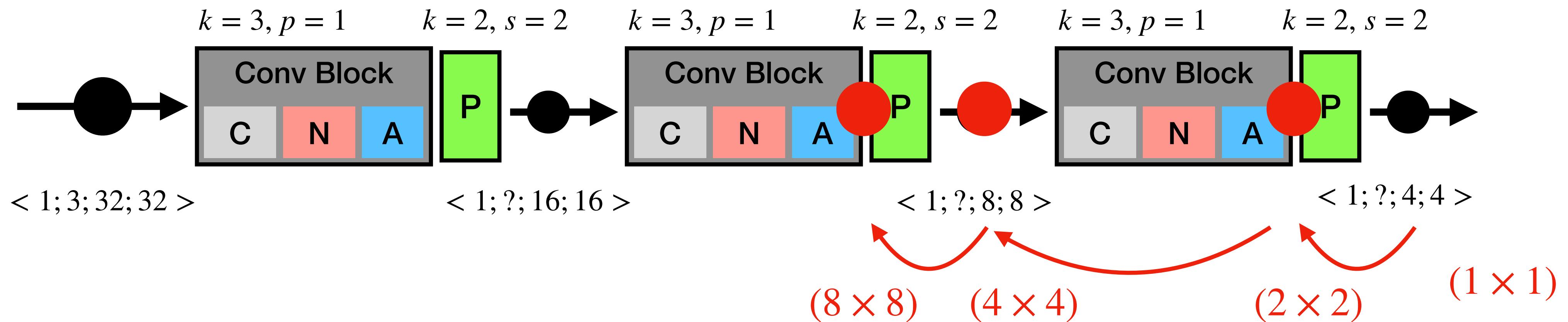
$$s_h(h_{out} - 1) + k_h = h_{in}$$



# Convolutional Layers

## Receptive Field

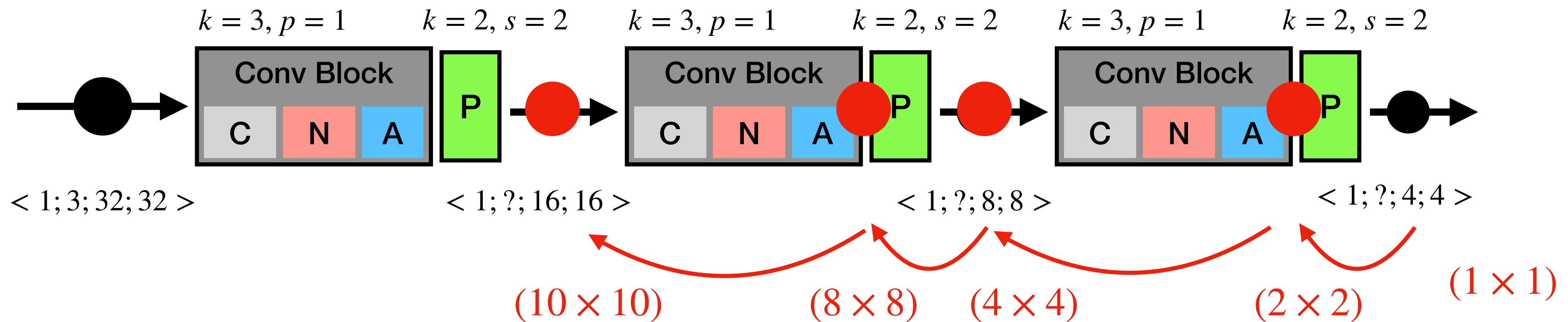
$$s_h(h_{out} - 1) + k_h = h_{in}$$



# Convolutional Layers

## Receptive Field

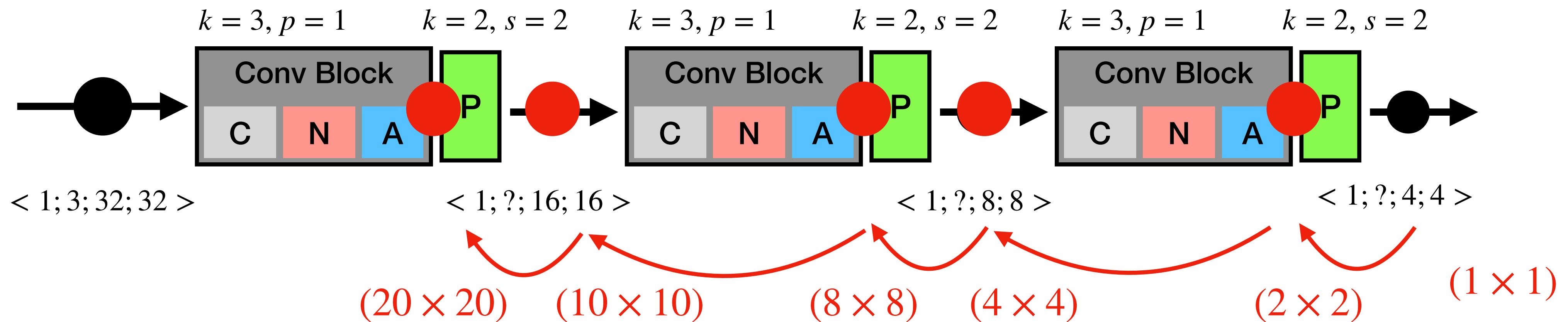
$$s_h(h_{out} - 1) + k_h = h_{in}$$



# Convolutional Layers

## Receptive Field

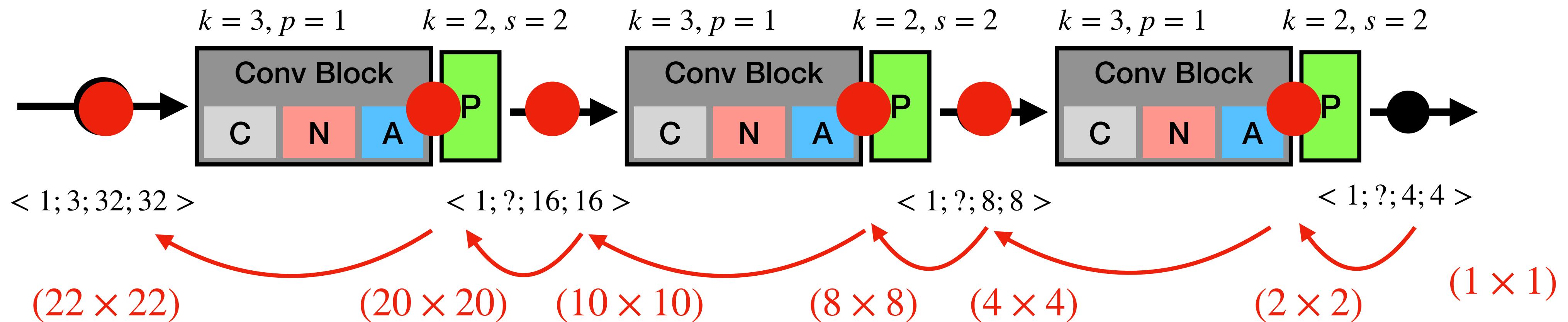
$$s_h(h_{out} - 1) + k_h = h_{in}$$



# Convolutional Layers

## Receptive Field

$$s_h(h_{out} - 1) + k_h = h_{in}$$



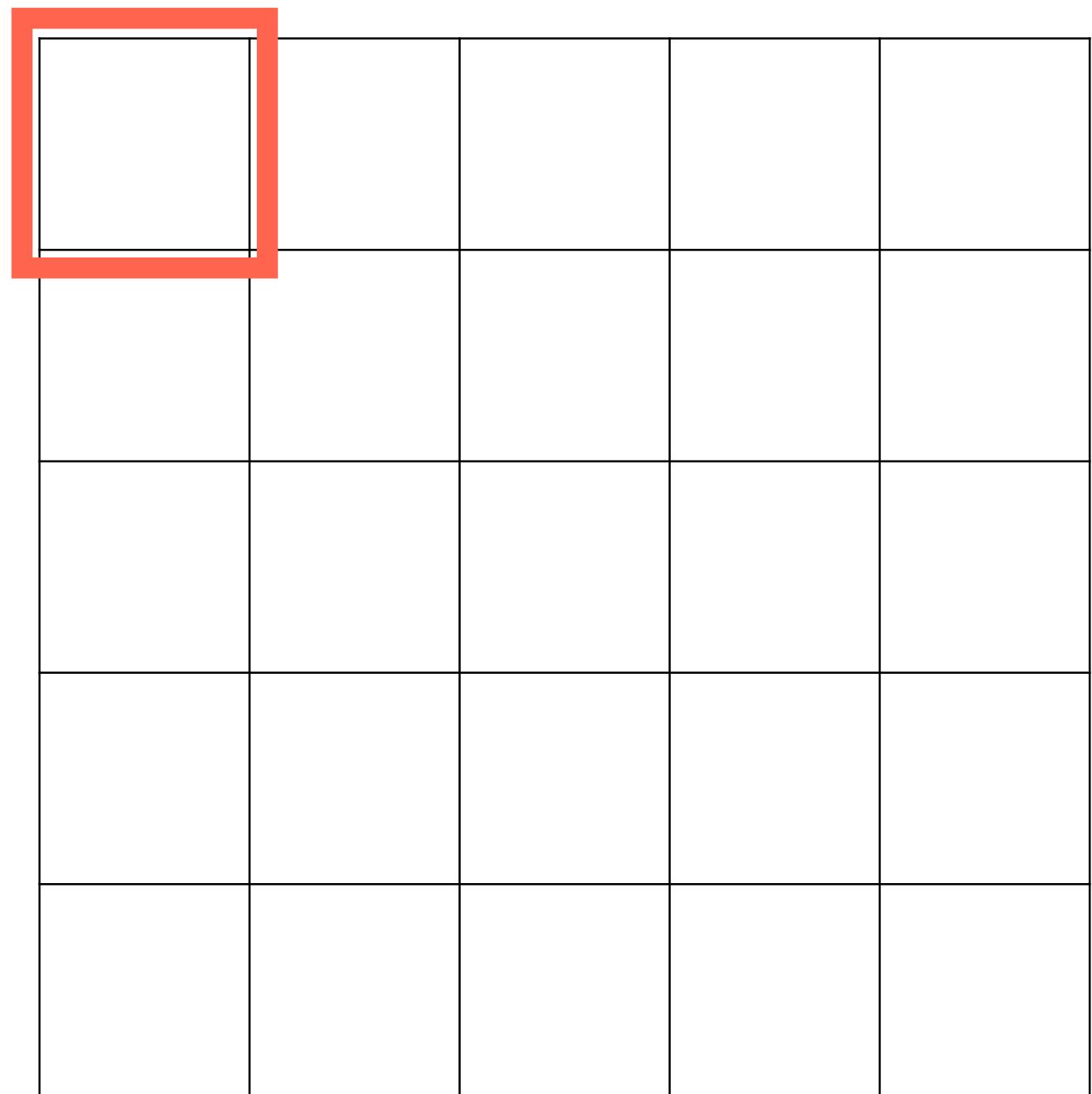
# Convolutional Layer

## 1x1 Convolution

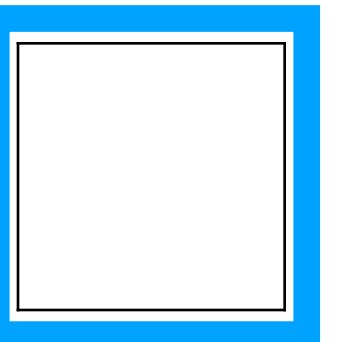


Output Convolved  
Features

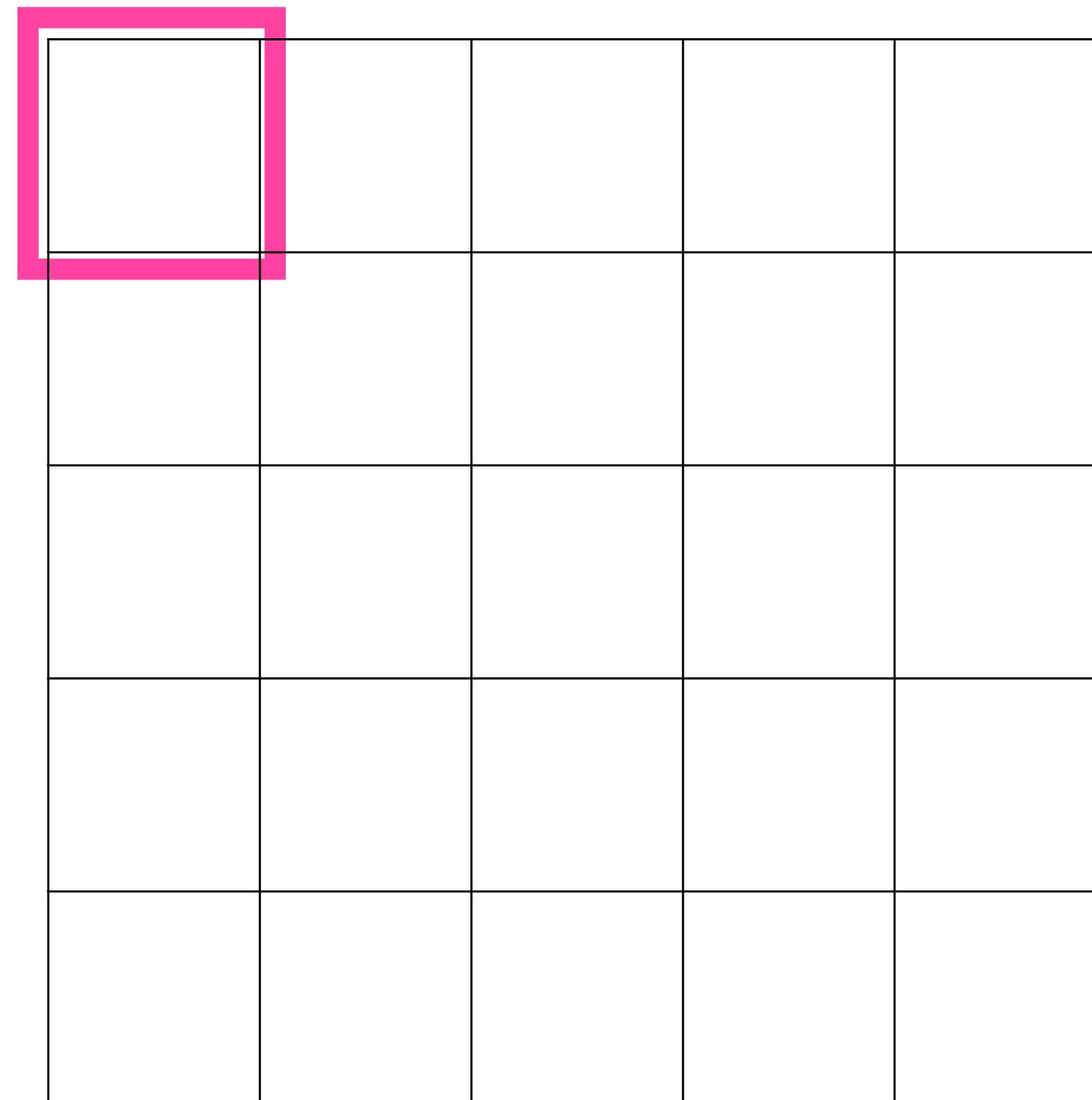
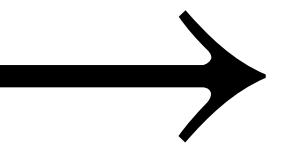
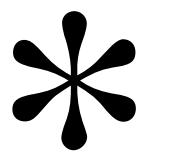
Input  $n$ -Channel Image



Kernel



$< n; 1; 1 >$



$< 1; n; 5; 5 >$

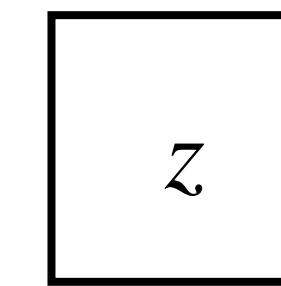
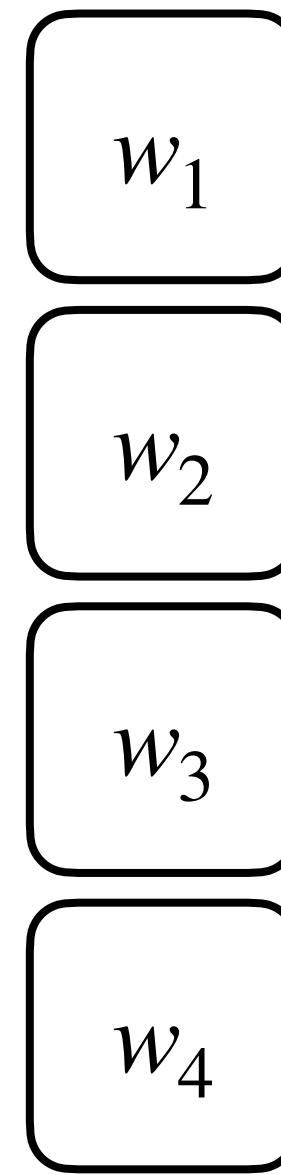
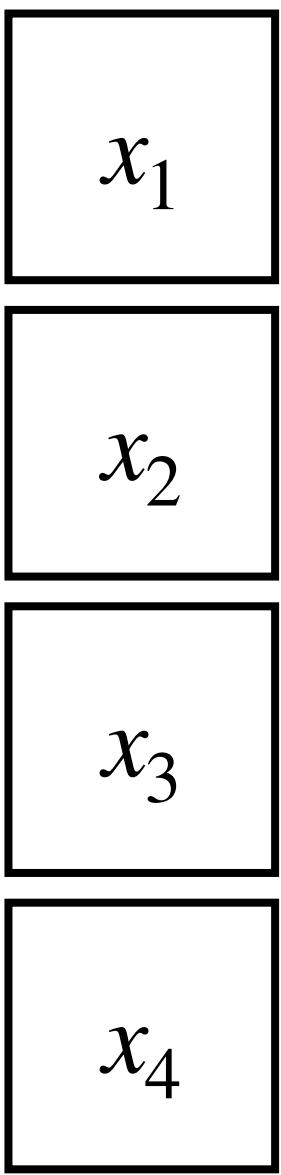
$< 1; 1; 5; 5 >$

# Convolutional Layer

1x1 Convolution

Single Artificial Neuron

$$z = w_1x_1 + w_2x_2 + w_3x_3 + w_4x_4 + b$$

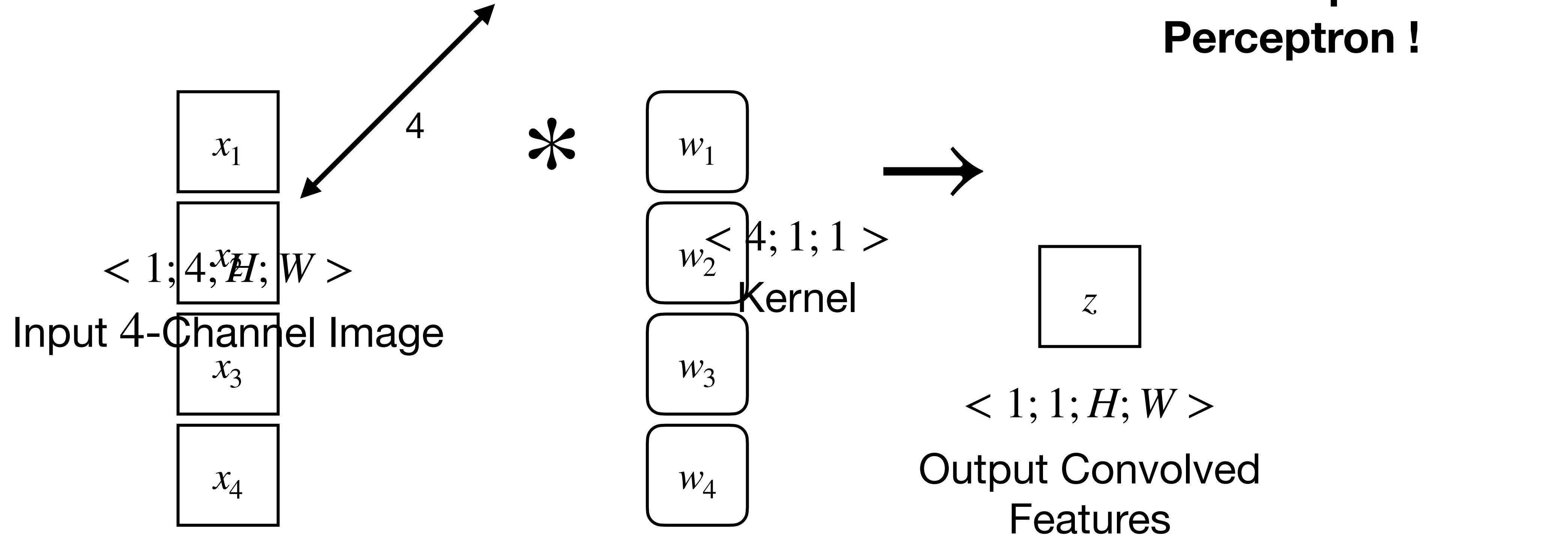


# Convolutional Layer

## 1x1 Convolution

1x1 Convolution

$$z = w_1x_1 + w_2x_2 + w_3x_3 + w_4x_4 + b$$



# Convolutional Layer

## 1x1 Convolution

- Is equivalent to “Linear” for flat vectors
  - Computed channel-wise for each element of activation map
  - Can be applied to input of any size
- Can be used for dimensionality reduction

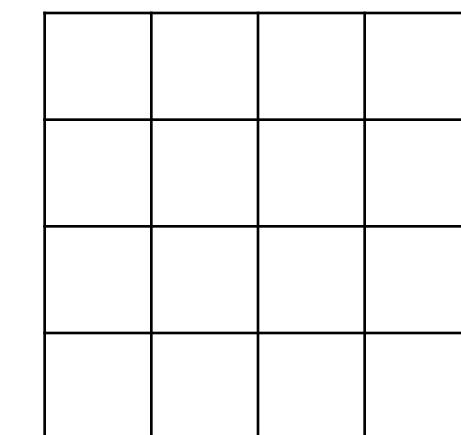
# Implementation of Convolution

$\langle B; C; H; W \rangle$

## 1. Unfold operation

- Extracts  $(k_h \times k_w)$  patches from input according to stride and padding rules

$k = 3$



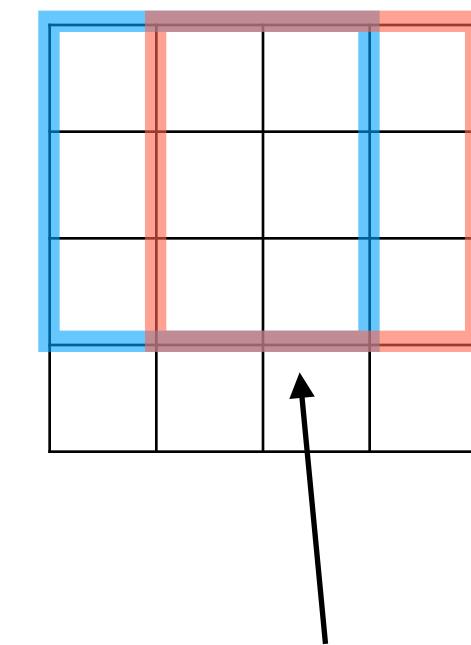
# Implementation of Convolution

$\langle B; C; H; W \rangle$

## 1. Unfold operation

- Extracts  $(k_h \times k_w)$  patches from input according to stride and padding rules

$k = 3$



$n_x = 2$

Two possible positions of kernel  
in horizontal direction

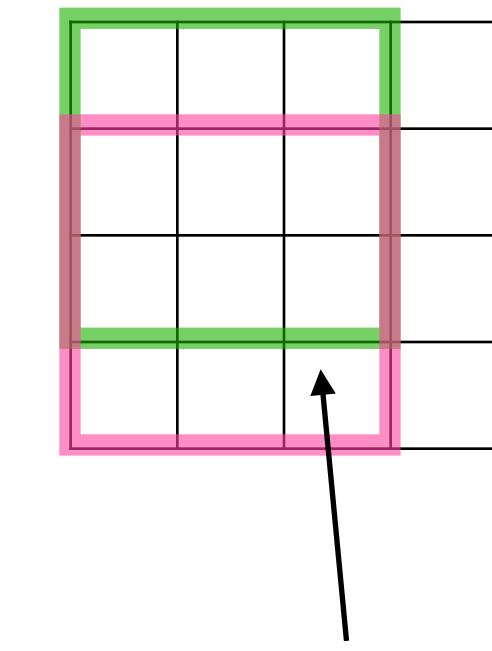
# Implementation of Convolution

$\langle B; C; H; W \rangle$

## 1. Unfold operation

- Extracts  $(k_h \times k_w)$  patches from input according to stride and padding rules

$k = 3$



$n_y = 2$

Two possible positions of kernel  
in vertical direction

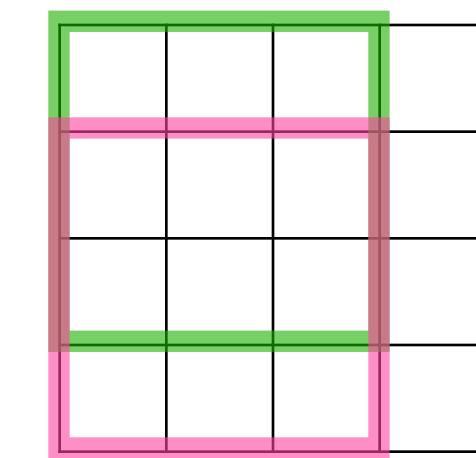
# Implementation of Convolution

$\langle B; C; H; W \rangle$

## 1. Unfold operation

- Extracts  $(k_h \times k_w)$  patches from input according to stride and padding rules

$k = 3$



Total number of patches  
(output activation map size)

$$l = n_x \cdot n_y$$

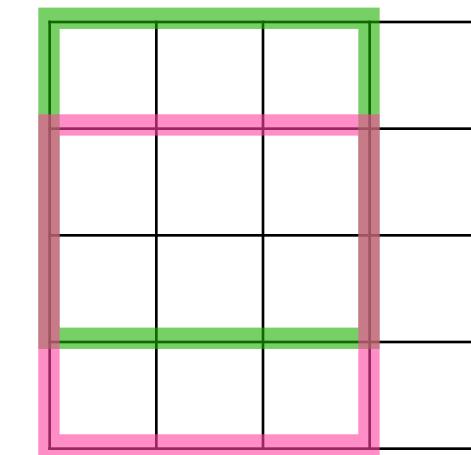
# Implementation of Convolution

$\langle B; C; H; W \rangle$

## 1. Unfold operation

- Extracts  $(k_h \times k_w)$  patches from input according to stride and padding rules
- Each patch with  $C$  features (channel depth) is flattened into a simple feature vector

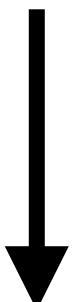
$k = 3$



Total number of patches  
(output activation map size)

$$l = n_x \cdot n_y$$

$$n_f = C \cdot k_h \cdot k_w$$



$\langle B; n_f; l \rangle$

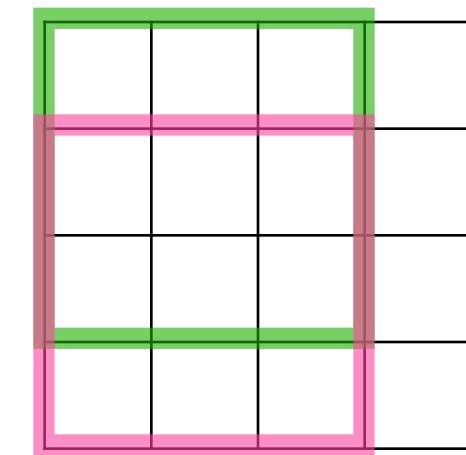
# Implementation of Convolution

$$< B; \ C; \ H; \ W >$$

## 1. Unfold operation

- Extracts  $(k_h \times k_w)$  patches from input according to stride and padding rules
  - Each patch with  $C$  features (channel depth) is flattened into a simple feature vector
  - Transpose  $2^{nd}$  and  $3^{rd}$  dimension

$$k = 3$$



Total number of patches  
(output activation map size)

$$l = n_x \cdot n_y$$

$$n_f = C \cdot k_h \cdot k_w$$



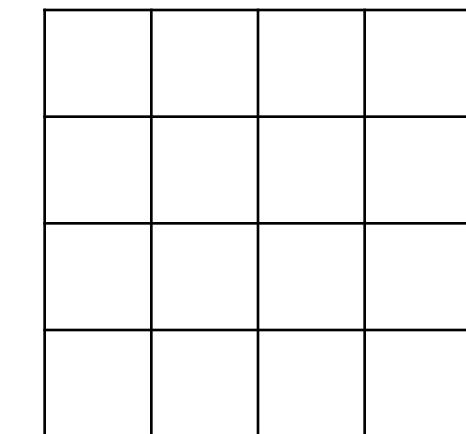
# Implementation of Convolution

$\langle B; C; H; W \rangle$

## 2. Matrix multiplication

- Output channels -  $C_{out}$
- Weights matrix  $W$  has shape  $\langle n_f; C_{out} \rangle$

$k = 3$



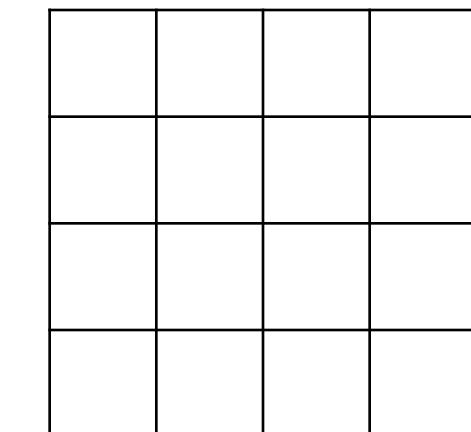
$$\langle B; l; n_f \rangle \times \langle n_f; C_{out} \rangle = \boxed{\langle B; l; C_{out} \rangle}$$

# Implementation of Convolution

$\langle B; C; H; W \rangle$

3. Transpose 2<sup>nd</sup> and 3<sup>rd</sup> dimension again

$k = 3$



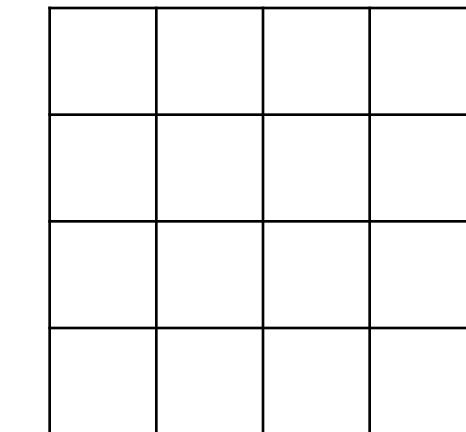
$\langle B; l; C_{out} \rangle \longrightarrow \langle B; C_{out}; l \rangle$

# Implementation of Convolution

$\langle B; C; H; W \rangle$

4. Reshape into  $(n_y \times n_x)$

$k = 3$



$\langle B; C_{out}; l \rangle \rightarrow \langle B; C_{out}; n_y; n_x \rangle$

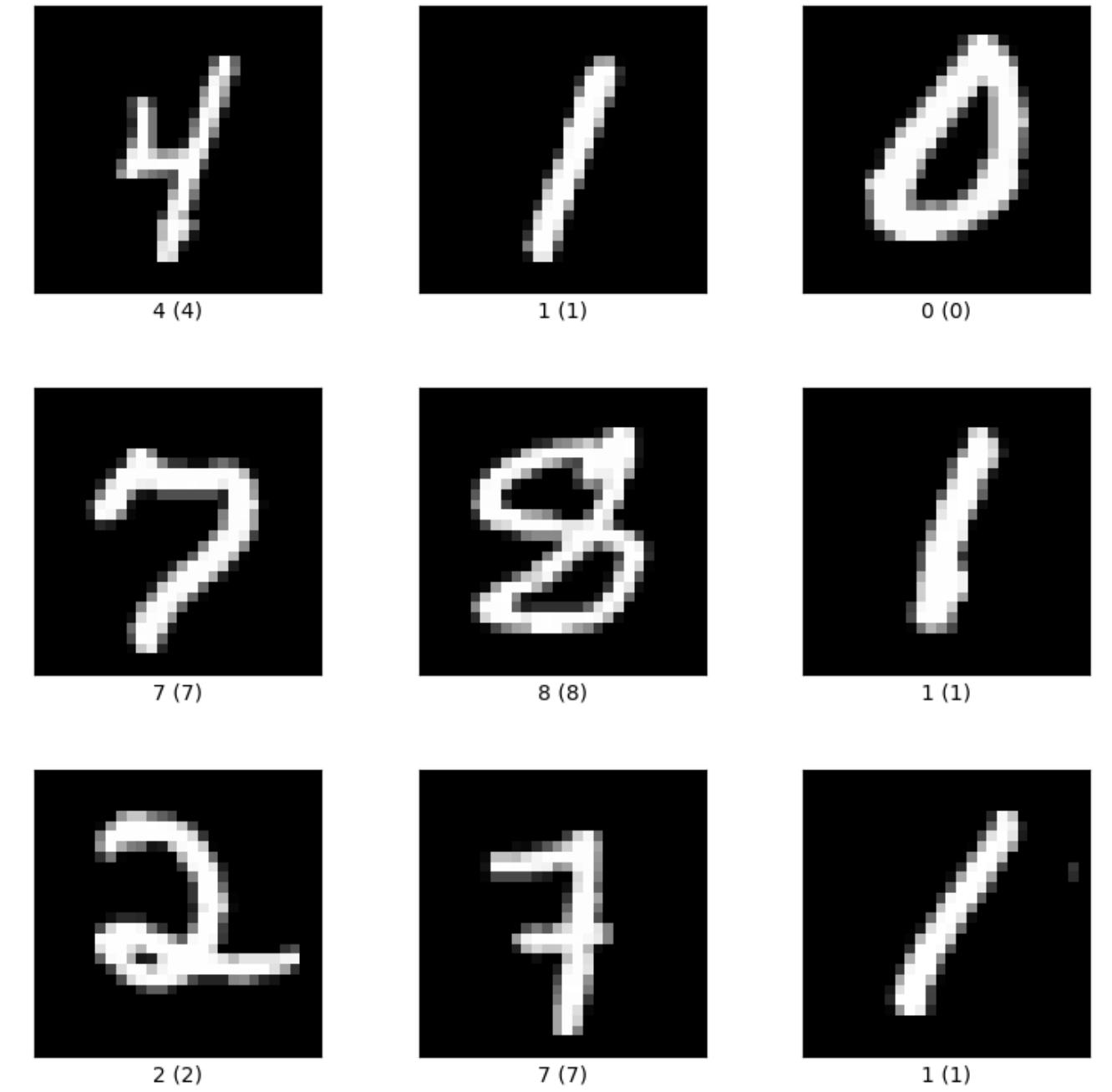
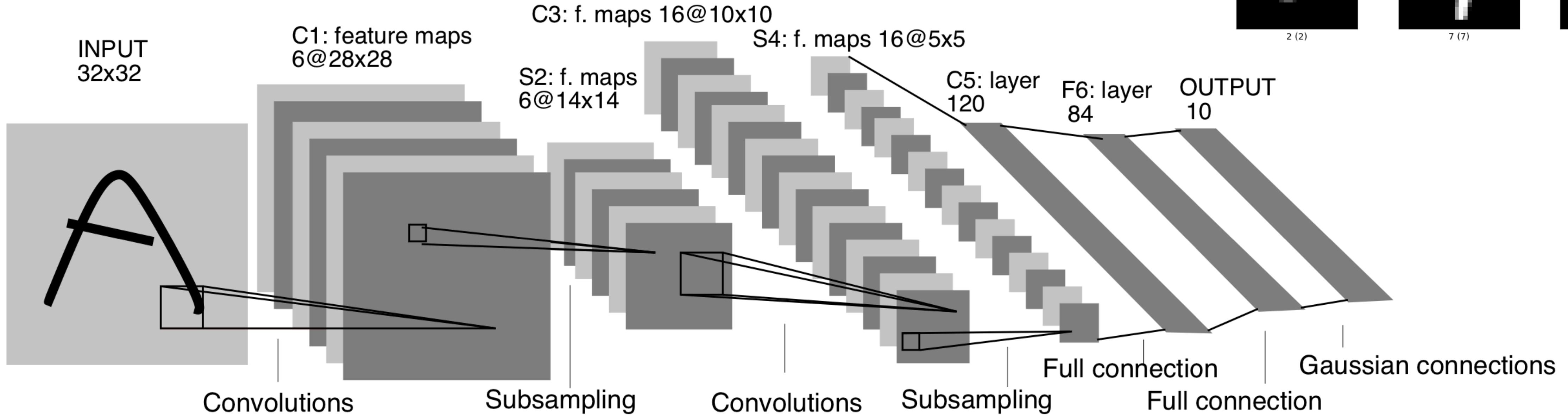
# Convolutional Architectures

- 1989 - LeNet-1
- 2012 - AlexNet
- 2014 - Inception
- 2015 - ResNet
- 2016 - DenseNet
- 2019 - EfficientNet
- 2022 - ConvNext
- ...

# LeNet-5

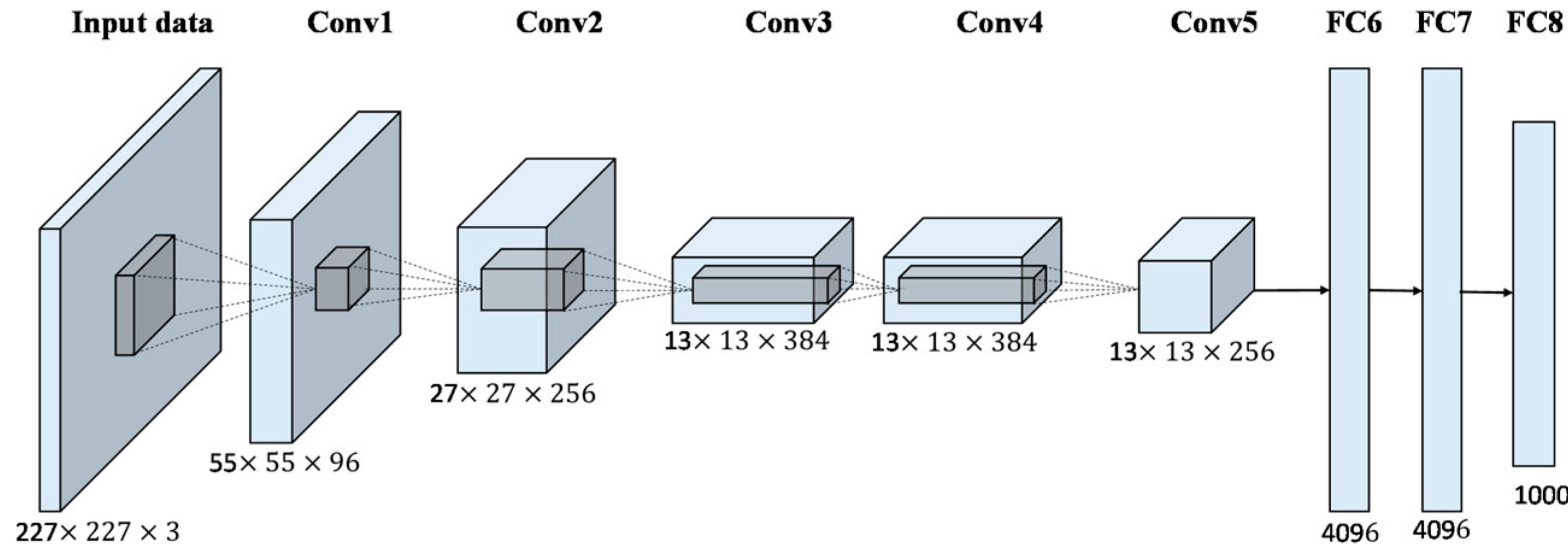
1998

Sigmoid Activation  
2x (Conv + Pooling)  
3x Fully Connected ( $120 \rightarrow 84 \rightarrow 10$ )



# AlexNet

## 2012

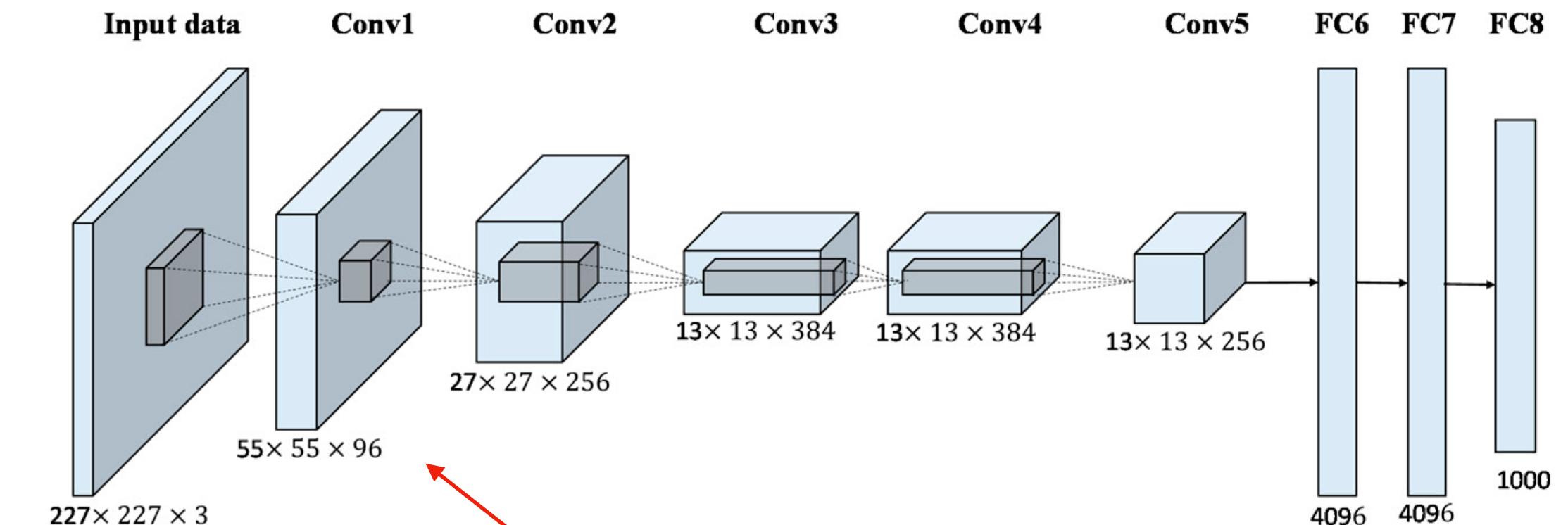


Krizhevsky, Alex, Ilya Sutskever, and Geoffrey E. Hinton. "Imagenet classification with deep convolutional neural networks." *Advances in neural information processing systems* 25 (2012).

# AlexNet

## 2012

- ImageNet Classification - 1000 classes
- Deep Network by today's standard
- **Trained on 2 GeForce GTX 580 GPUs (3GB each)**
- ReLU activation
- DropOut
- Normalization
- **60 million parameters**



96 filters learned in Conv1



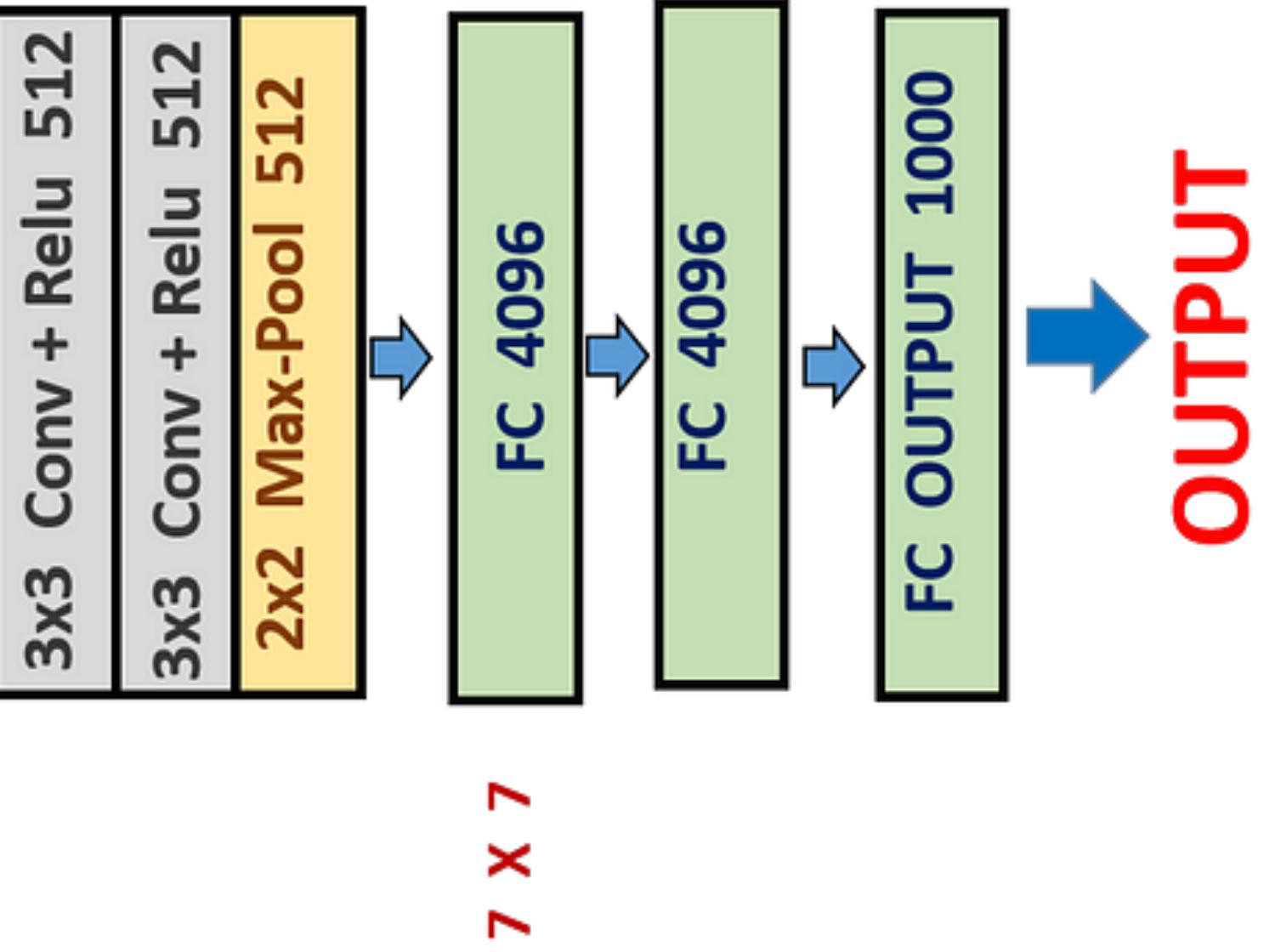
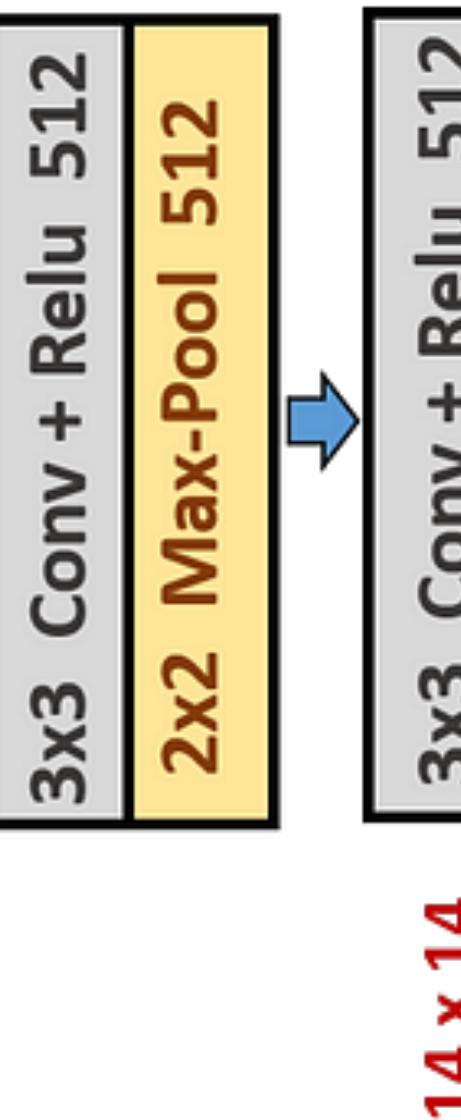
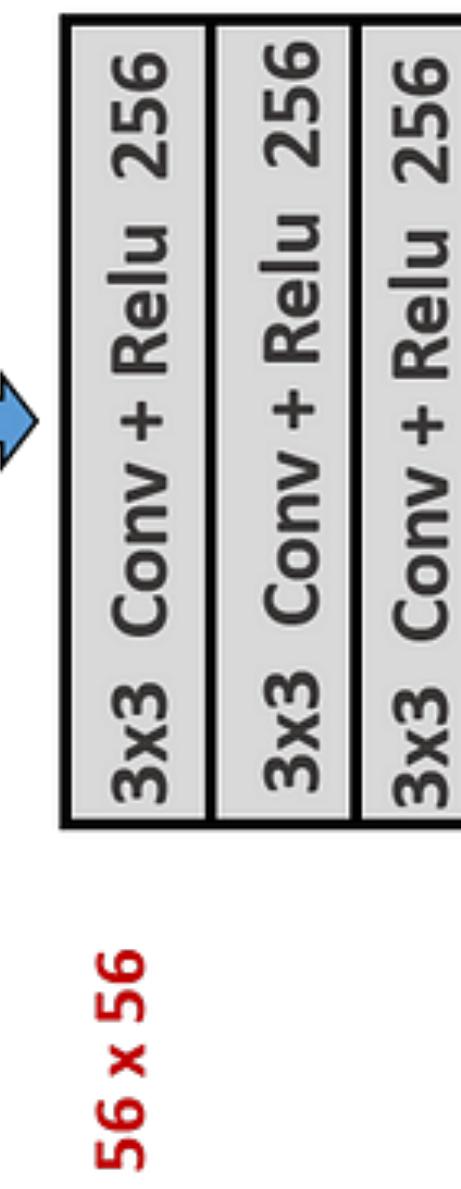
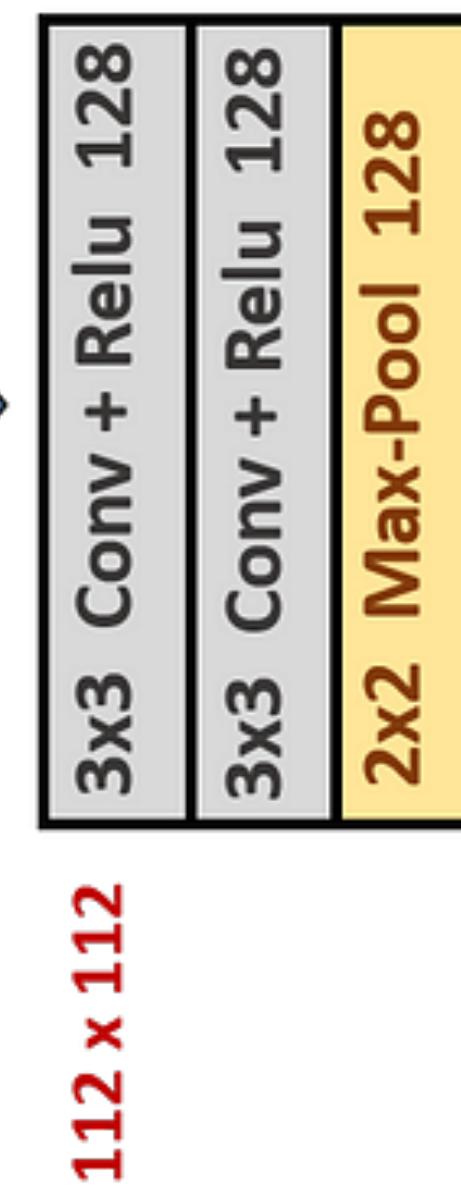
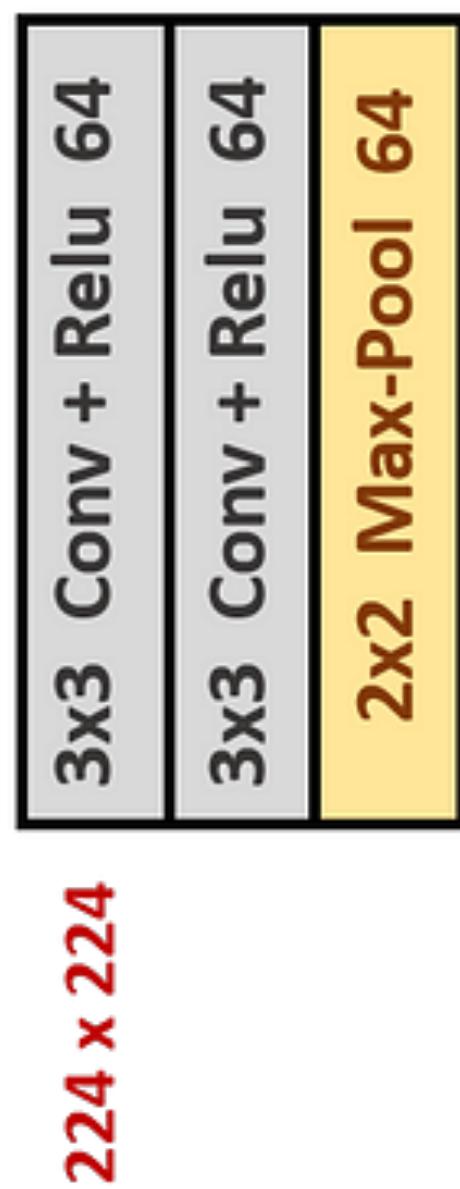
# VGGNet

## 2014

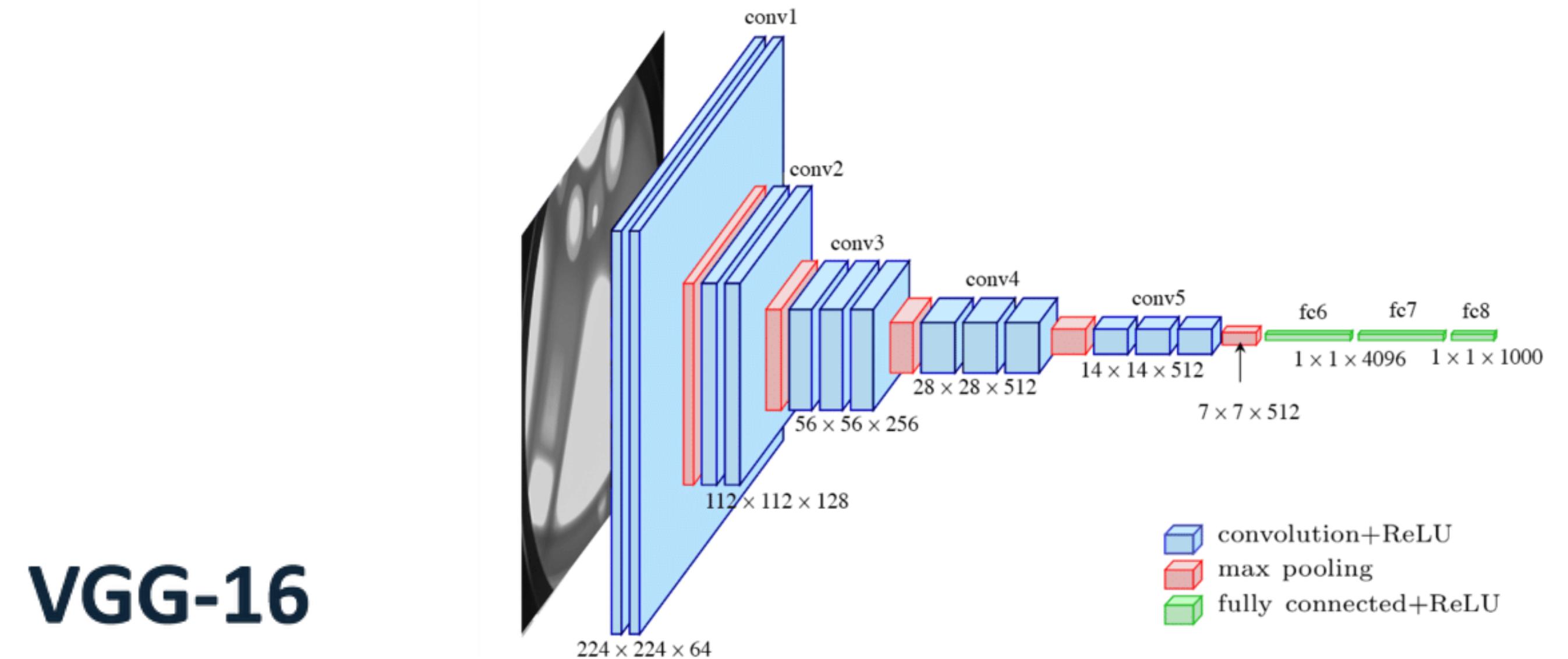
**138 millions of parameters**

**INPUT**

**224 x 224**



**OUTPUT**



Simonyan, Karen, and Andrew Zisserman. "Very deep convolutional networks for large-scale image recognition." *arXiv preprint arXiv:1409.1556* (2014).

Images - <https://medium.com/@siddheshb008/vgg-net-architecture-explained-71179310050f>

# Inception

## 2014

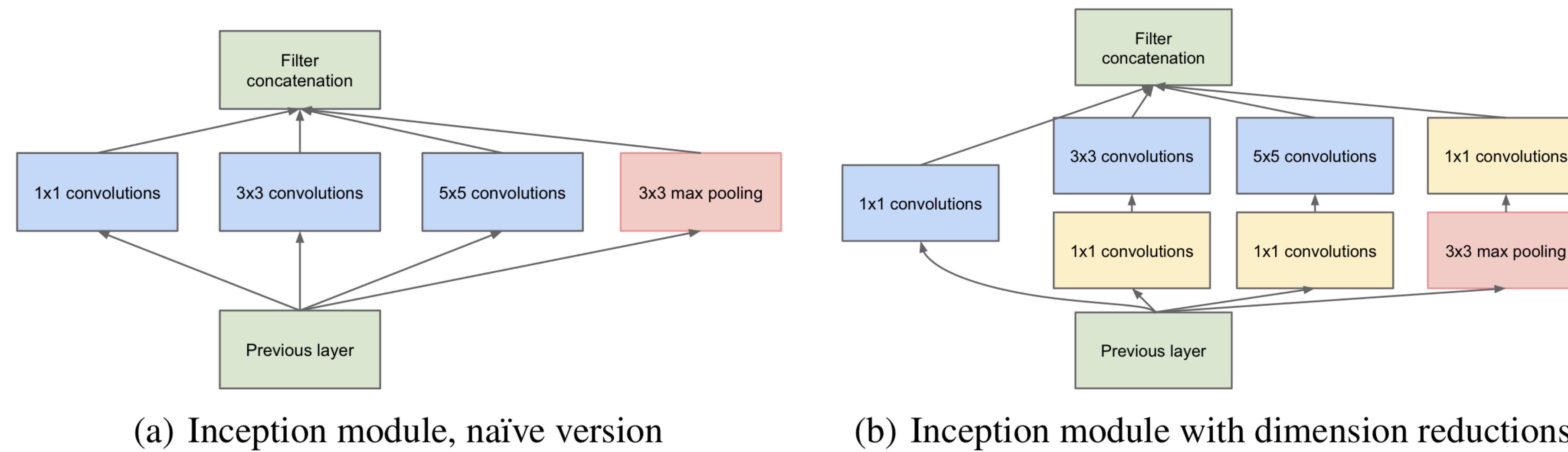


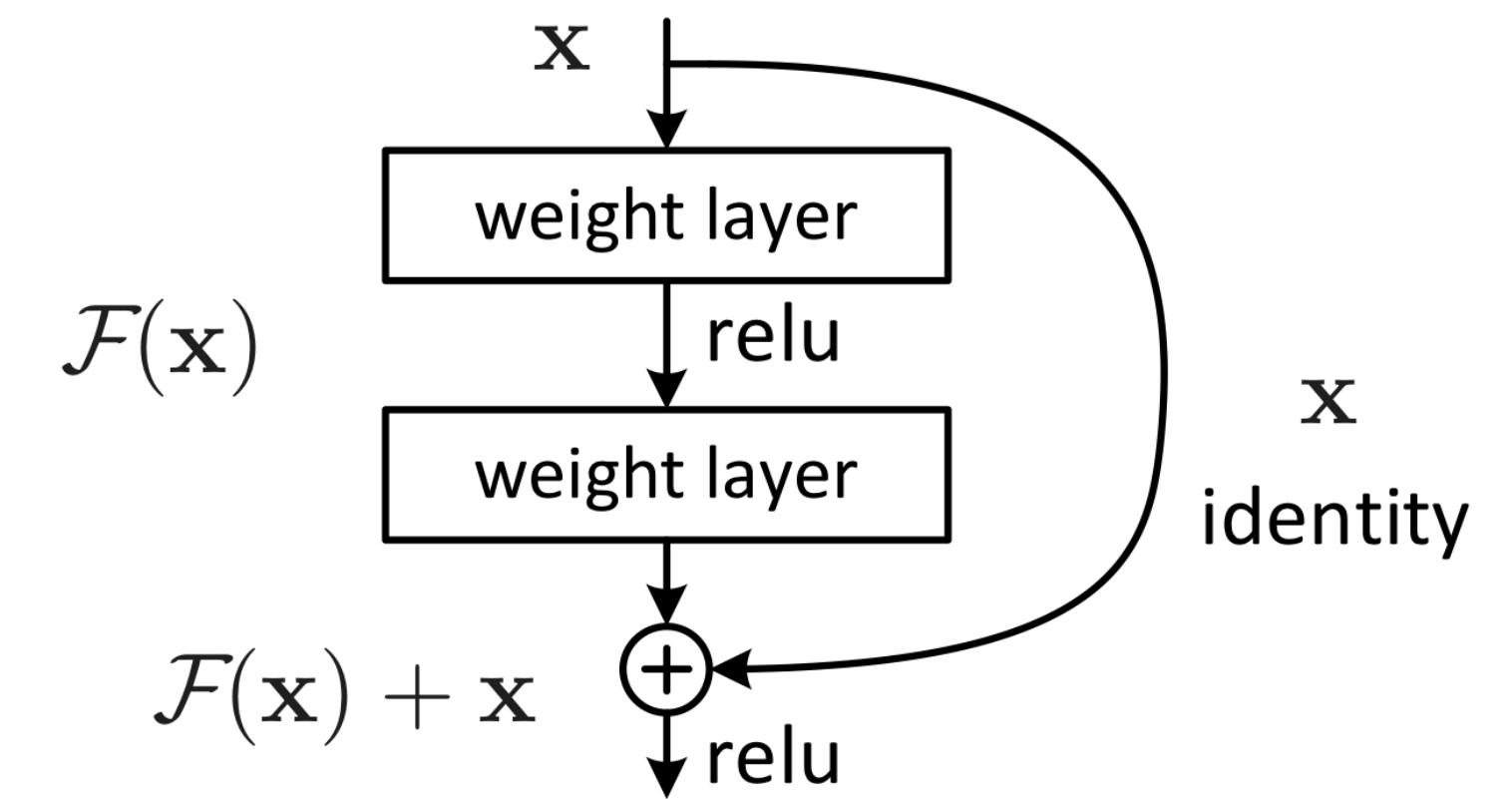
Figure 2: Inception module



# ResNet

## 2015

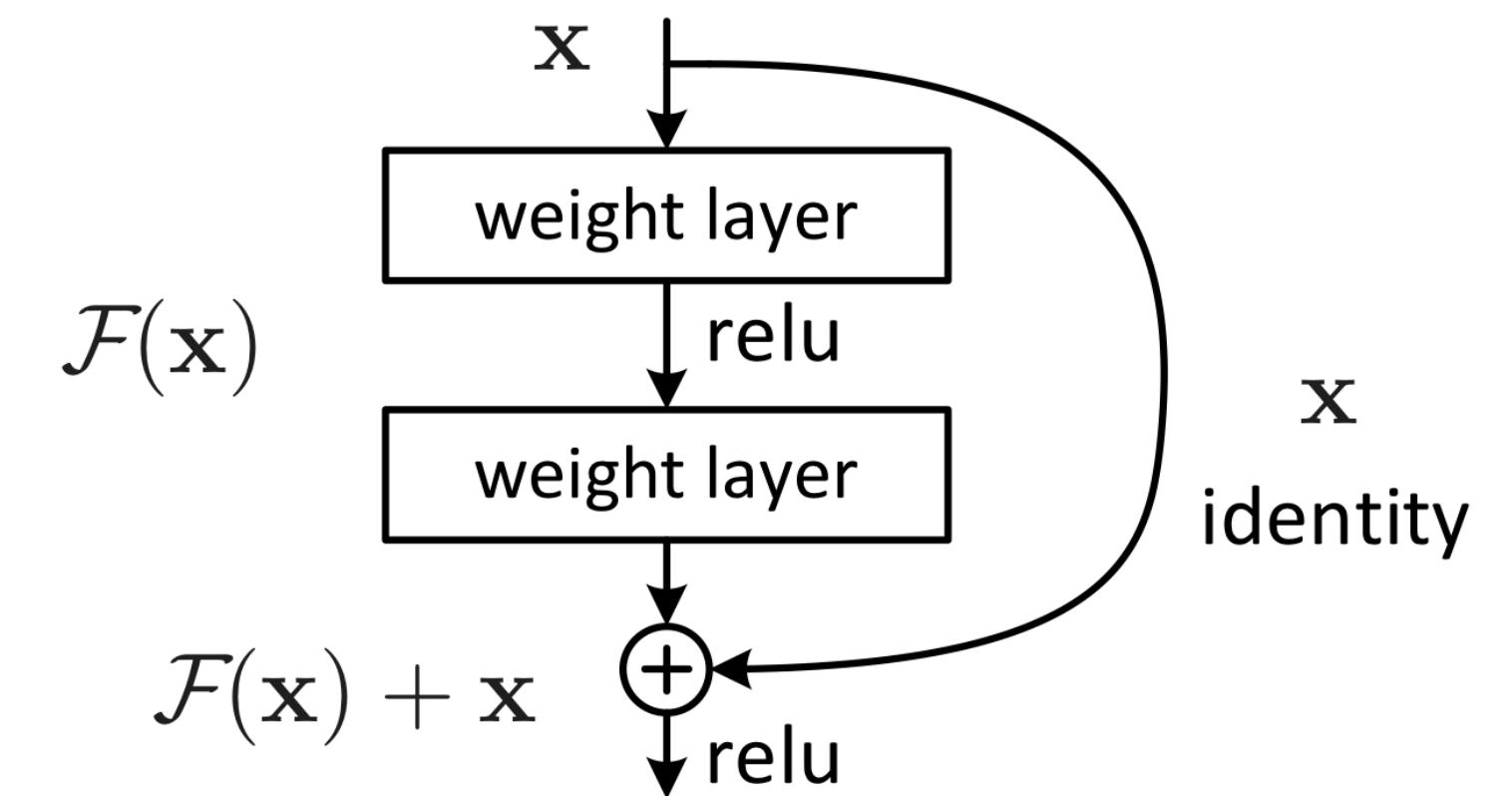
- Even with ReLU, normalization, initialization, ...
  - Deep networks are difficult to train
- Proposed **residual block**
  - Skip connection improves gradient propagation



# ResNet

## 2015

- Even with ReLU, normalization, initialization, ...
  - Deep networks are difficult to train
- Proposed **residual block**
  - Skip connection improves gradient propagation



Convolutional Block:

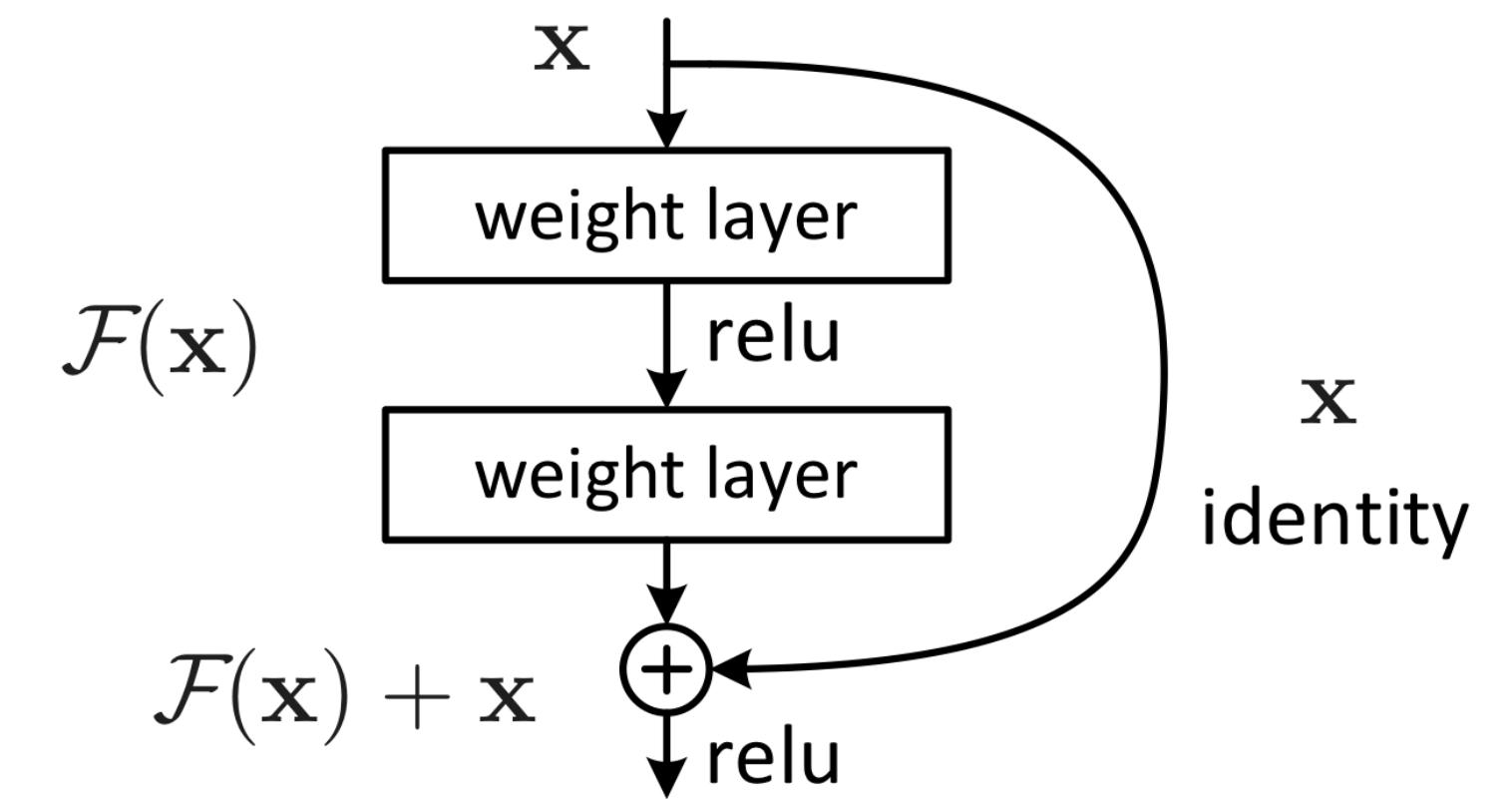
*“What representation of  $x$  can be useful ?”*



# ResNet

## 2015

- Even with ReLU, normalization, initialization, ...
  - Deep networks are difficult to train
- Proposed **residual block**
  - Skip connection improves gradient propagation



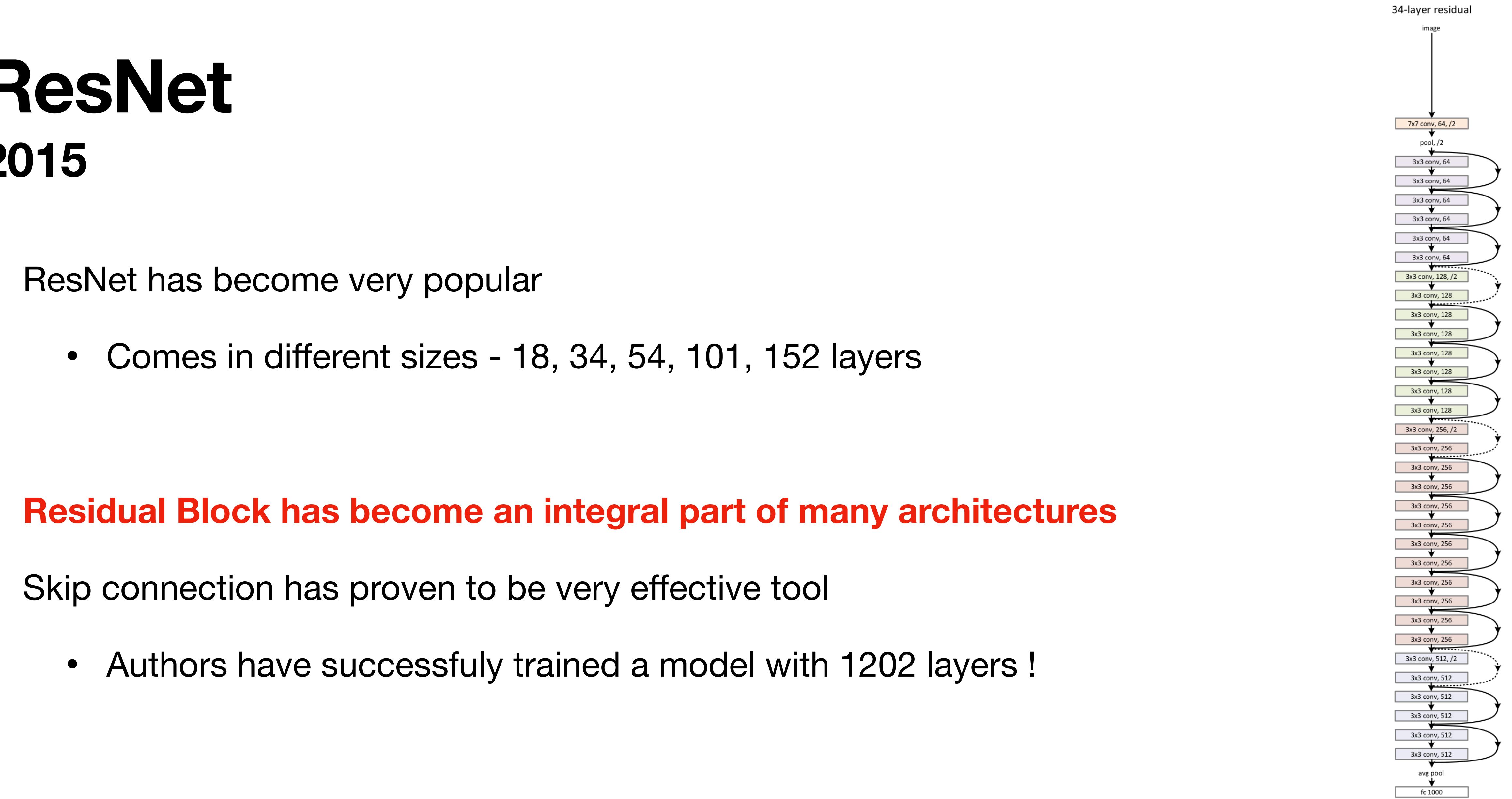
Residual Block:

*“How much does  $x$  need to be changed to be useful ?”*

# ResNet

## 2015

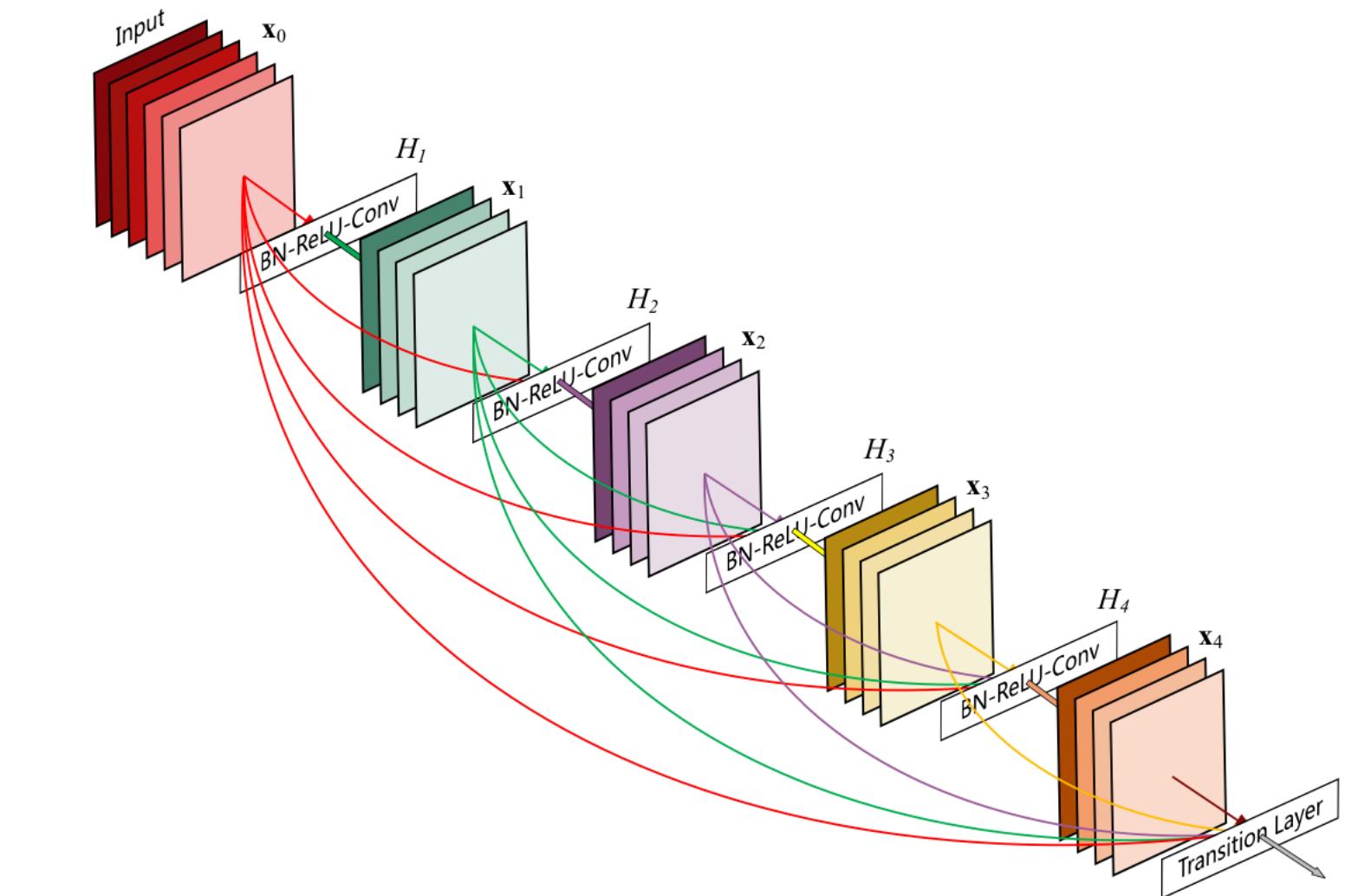
- ResNet has become very popular
  - Comes in different sizes - 18, 34, 54, 101, 152 layers
- **Residual Block has become an integral part of many architectures**
- Skip connection has proven to be very effective tool
  - Authors have successfully trained a model with 1202 layers !



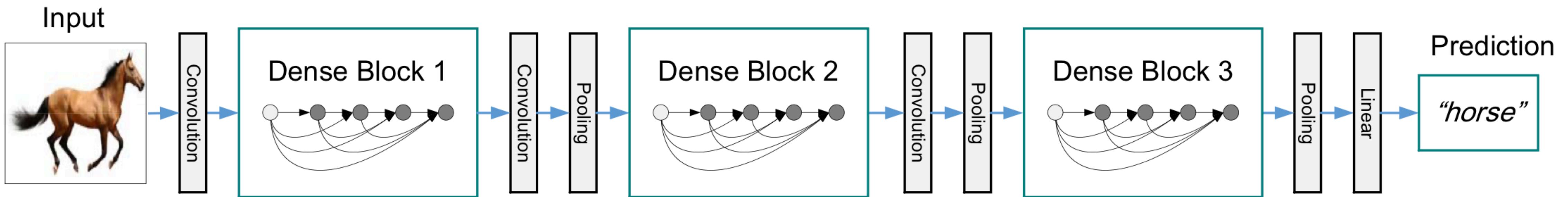
# DenseNet

## 2016

- Direct connections between any two layers with the same feature-map size



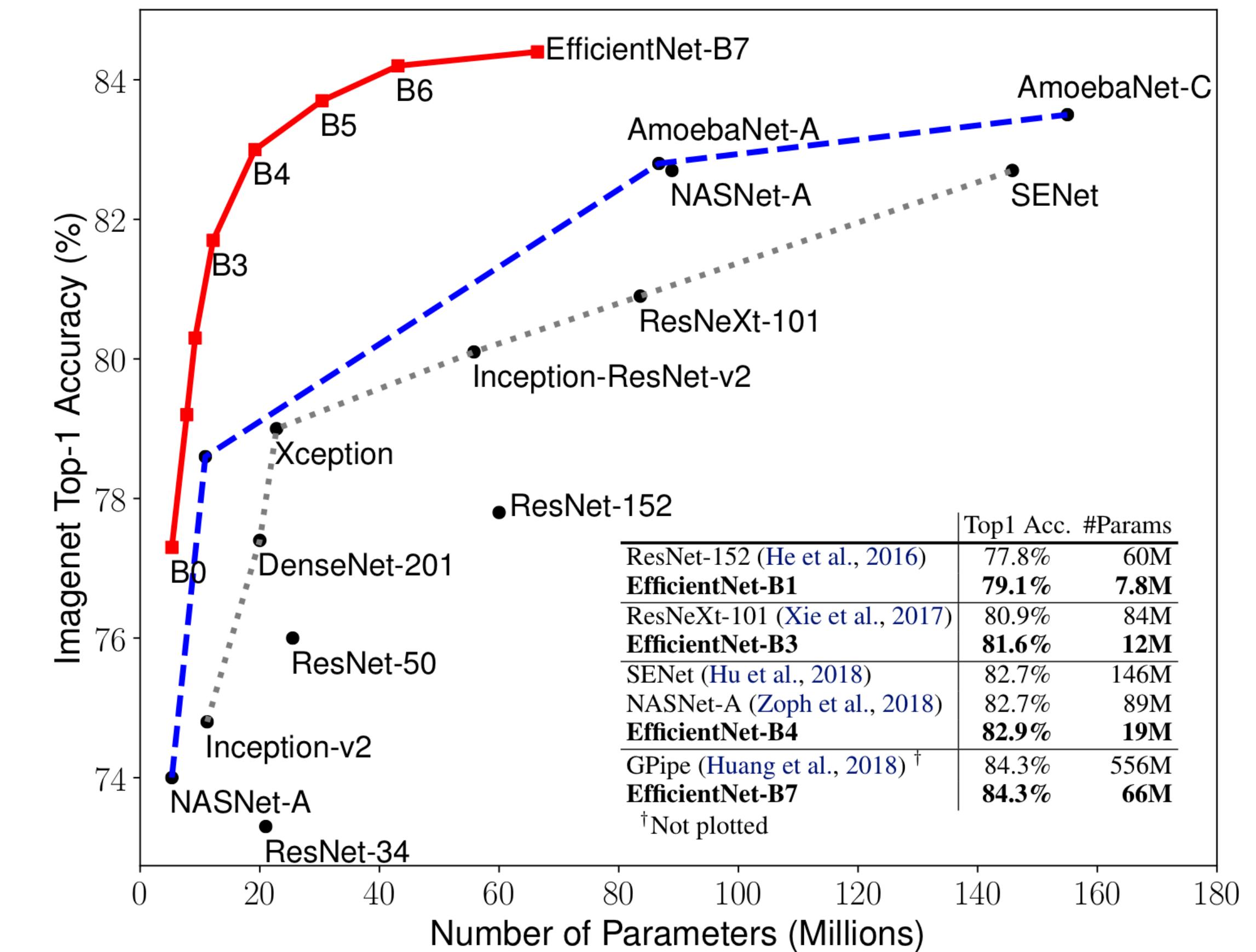
**Figure 1:** A 5-layer dense block with a growth rate of  $k = 4$ . Each layer takes all preceding feature-maps as input.



# EfficientNet

2019

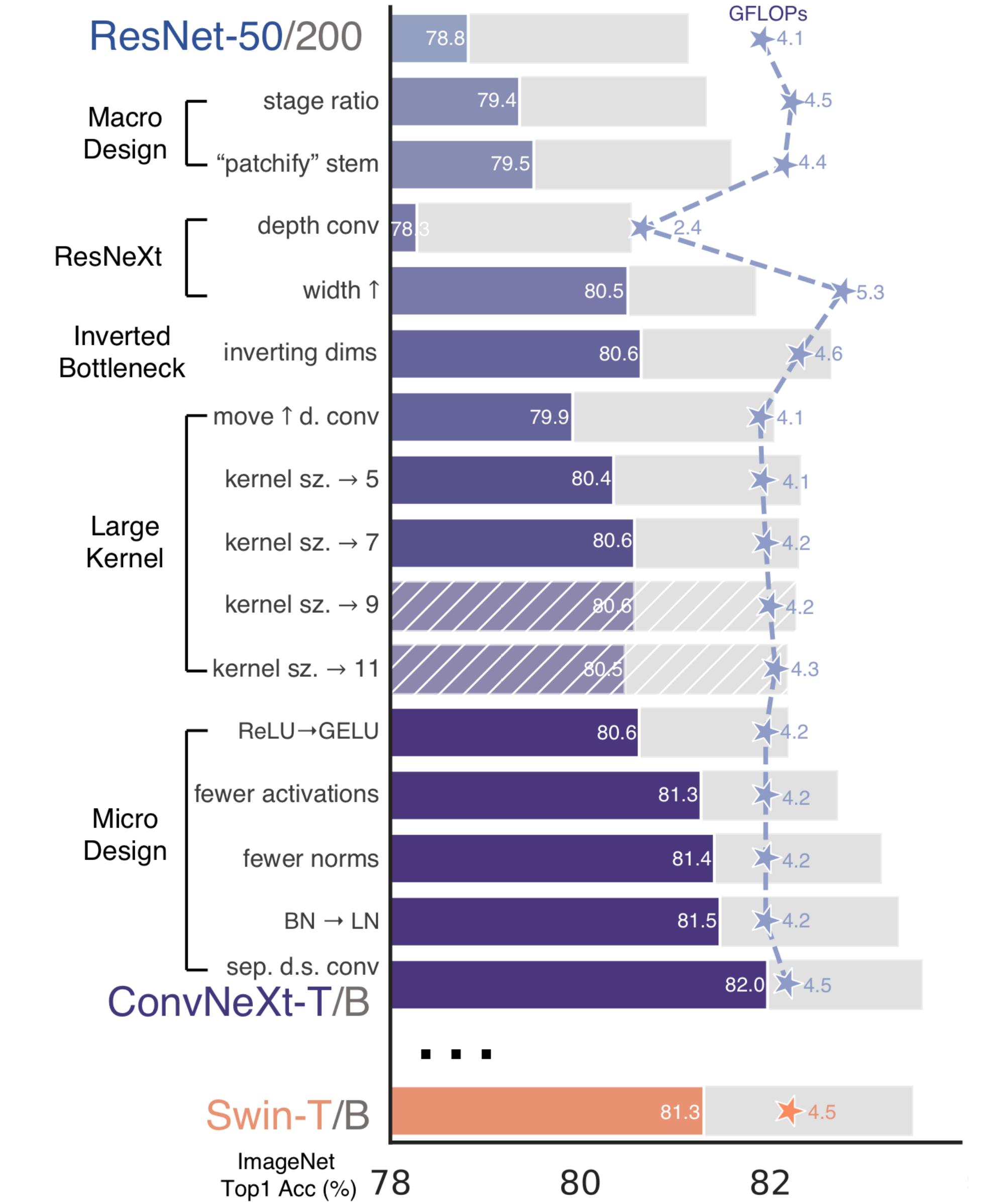
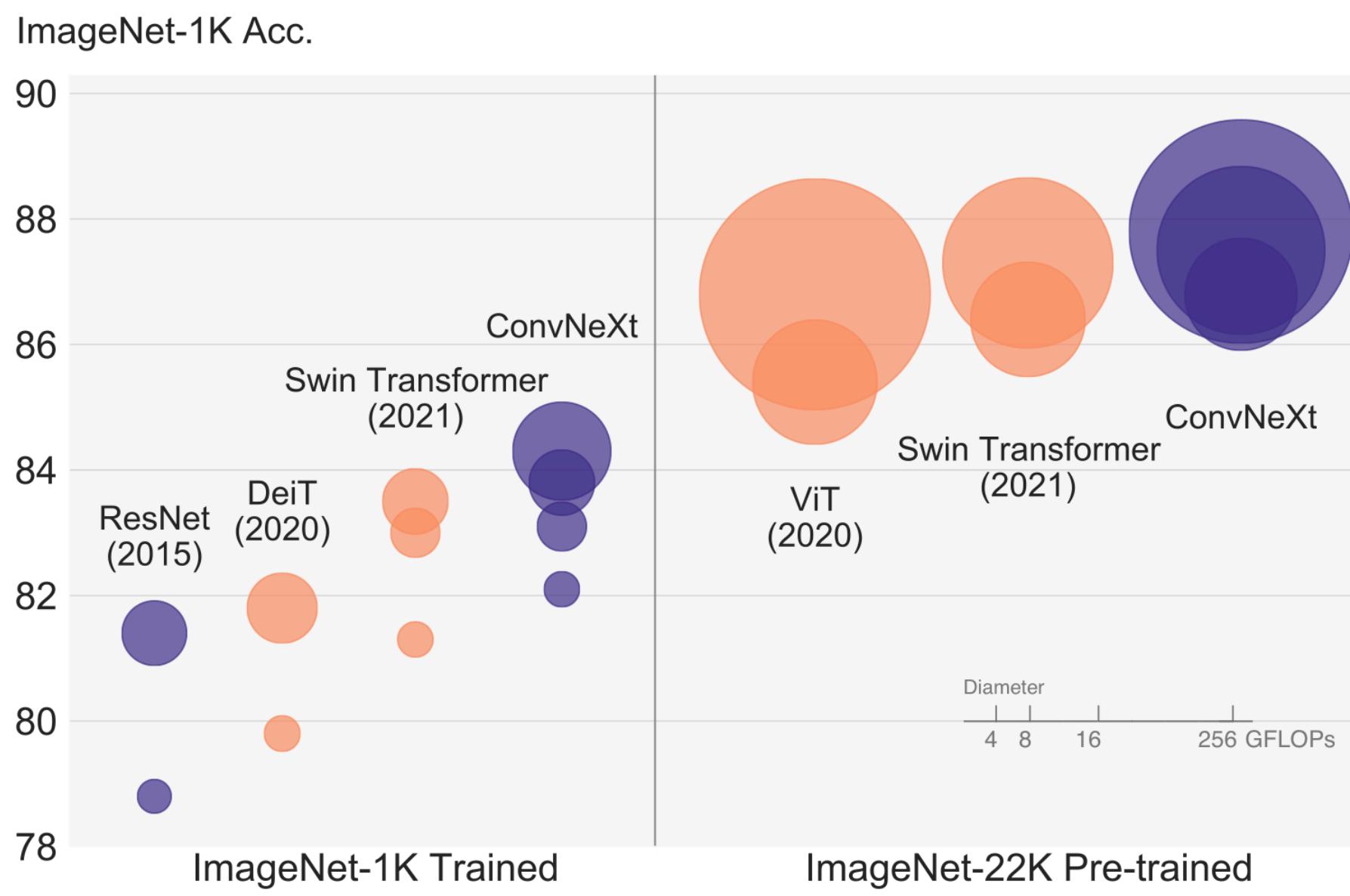
- Systematic study of how to scale up convolutional models properly
- Demonstrated on scaling up **MobileNets and ResNets**



# ConvNeXt

## 2022

- Modernized ResNet toward the design of Vision Transformer

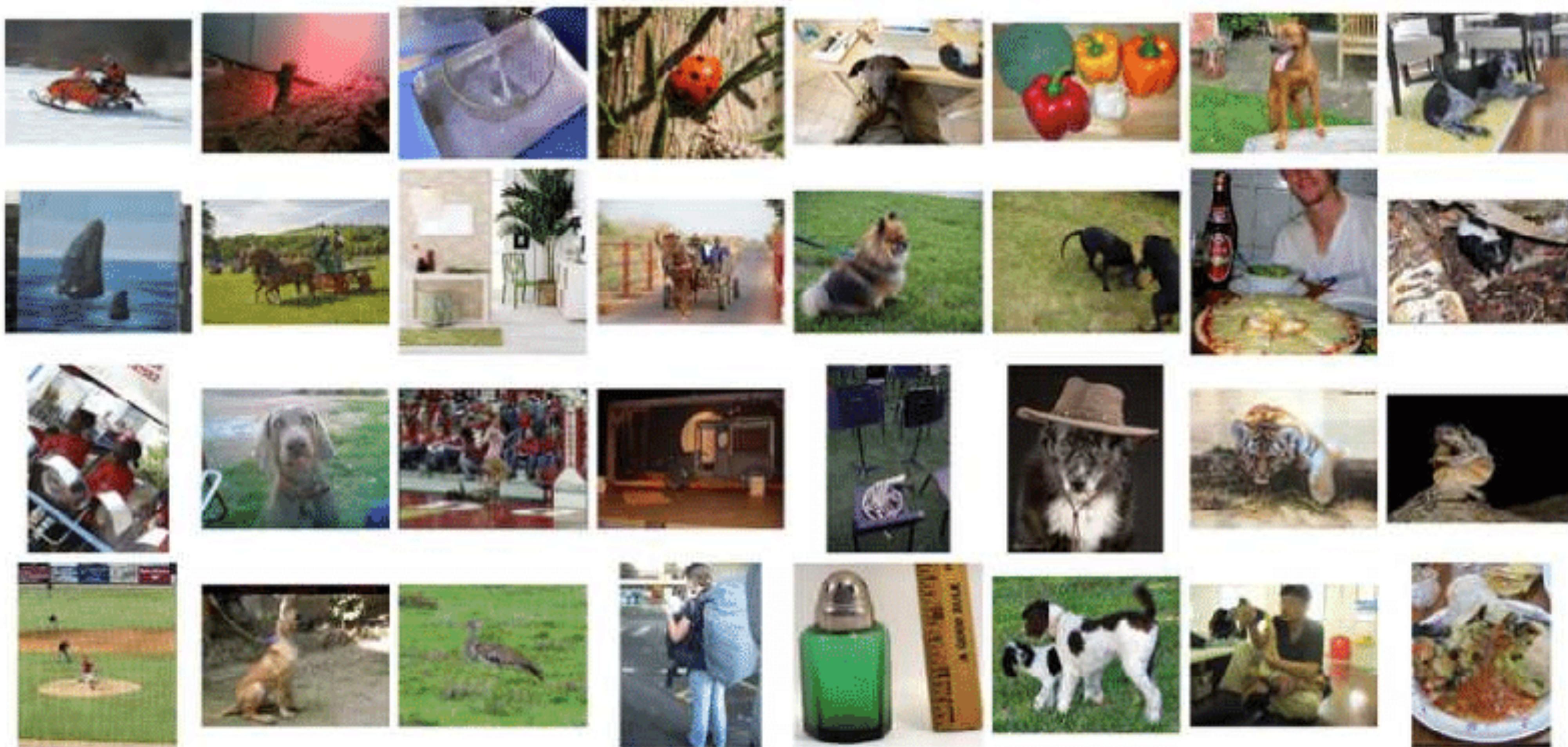


# Using a Pre-Trained Model

- Decide which architecture is most suitable for you
  - Can you use big model ?
  - Do you need fast inference ?
  - Do you have CPU/GPU/memory limitations ?

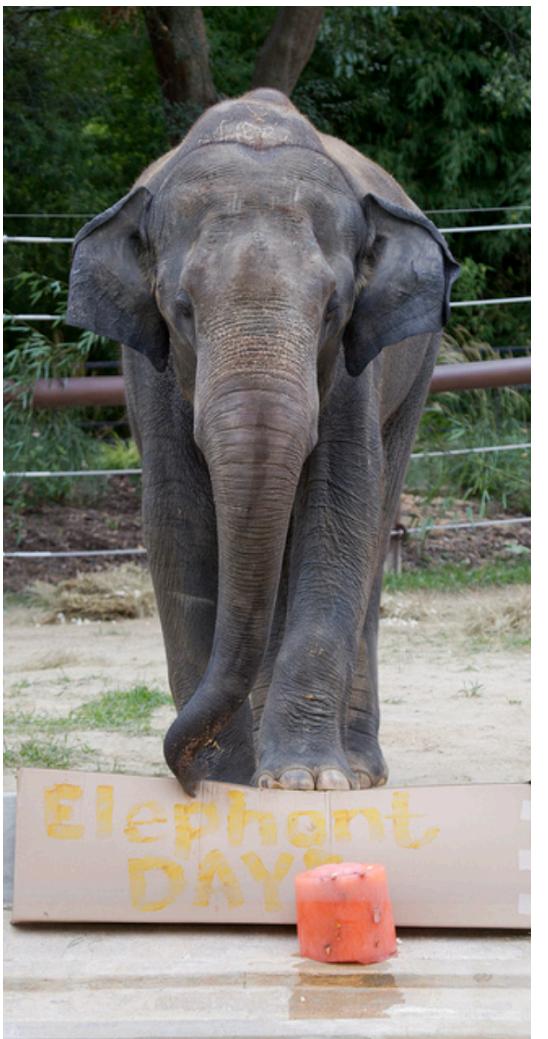
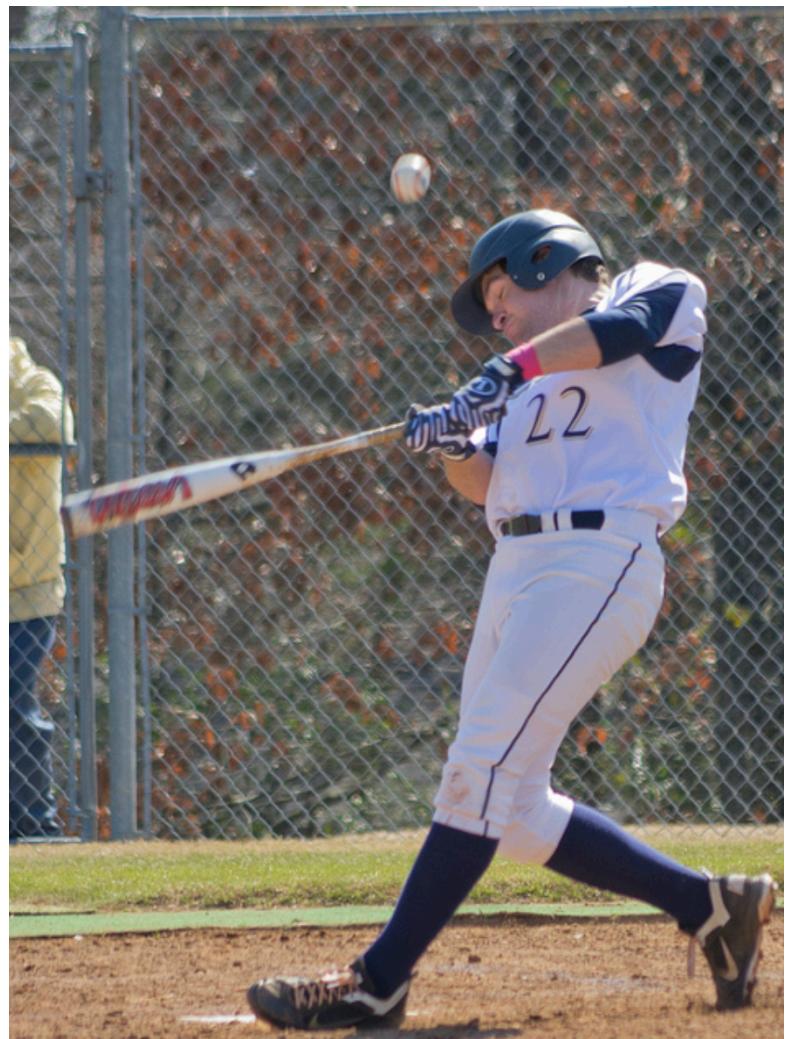
# Using a Pre-Trained Model

## Training Dataset - ImageNet



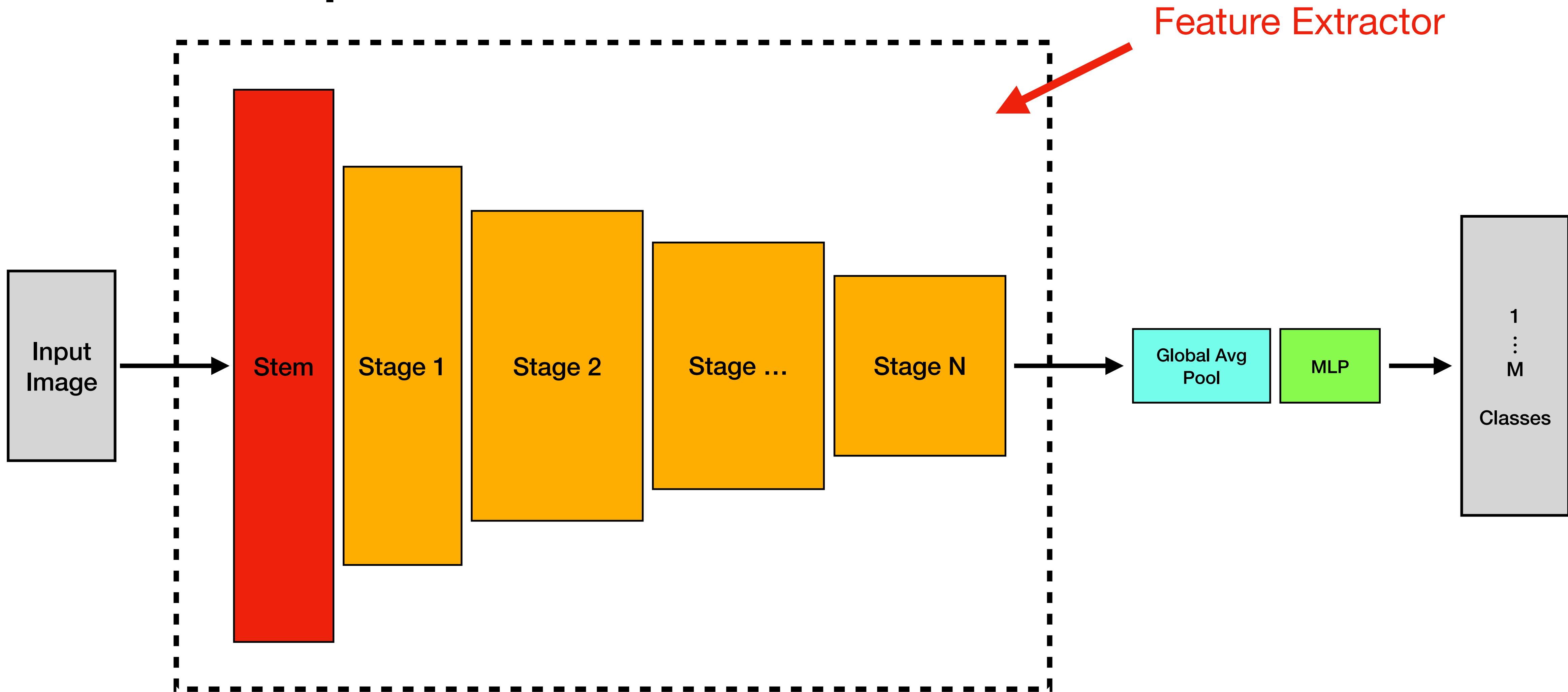
# Using a Pre-Trained Model

## Training Dataset - COCO - Common Objects in Context



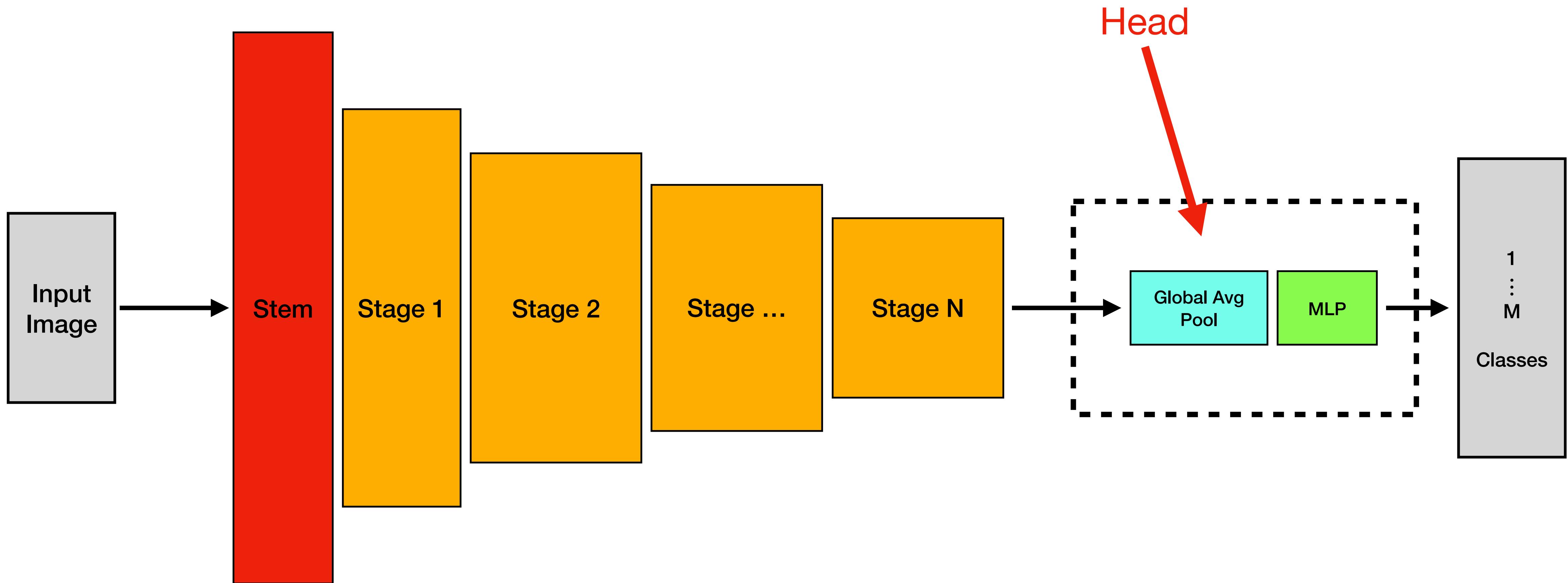
# Using a Pre-Trained Model

## Generic Components



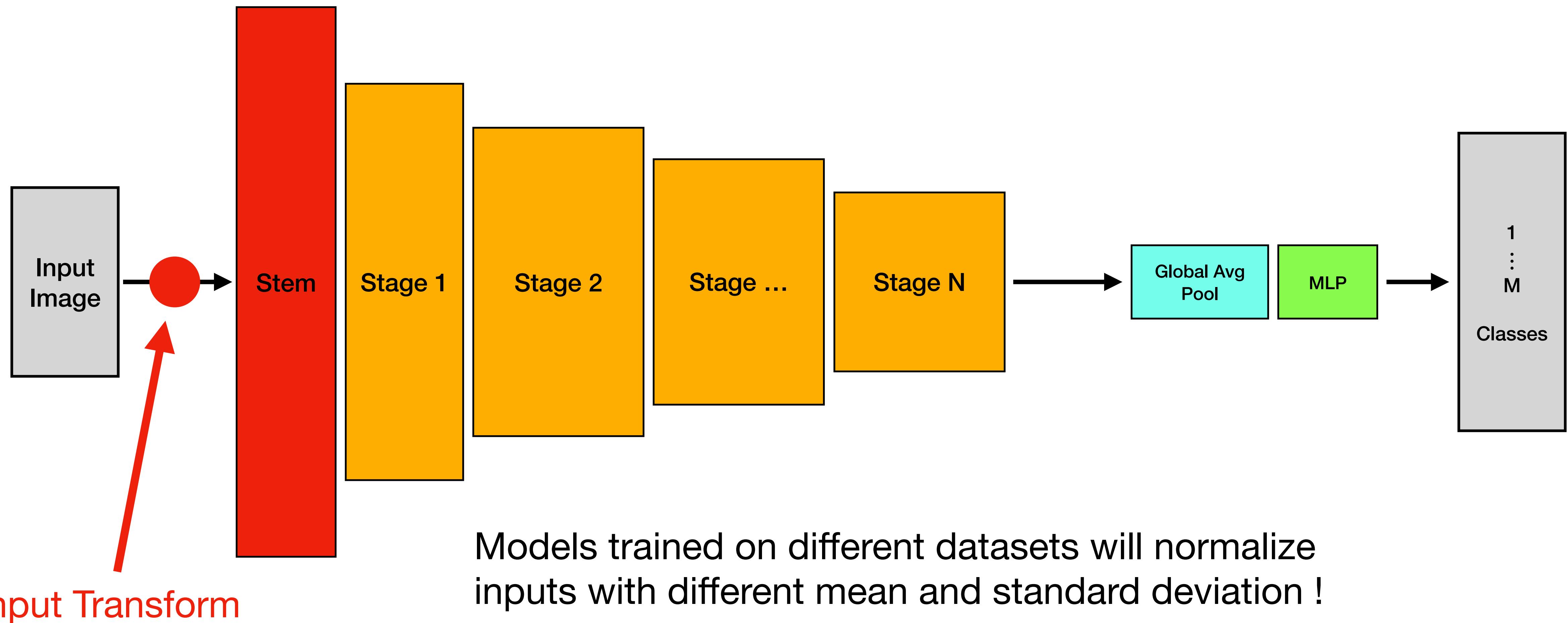
# Using a Pre-Trained Model

## Generic Components



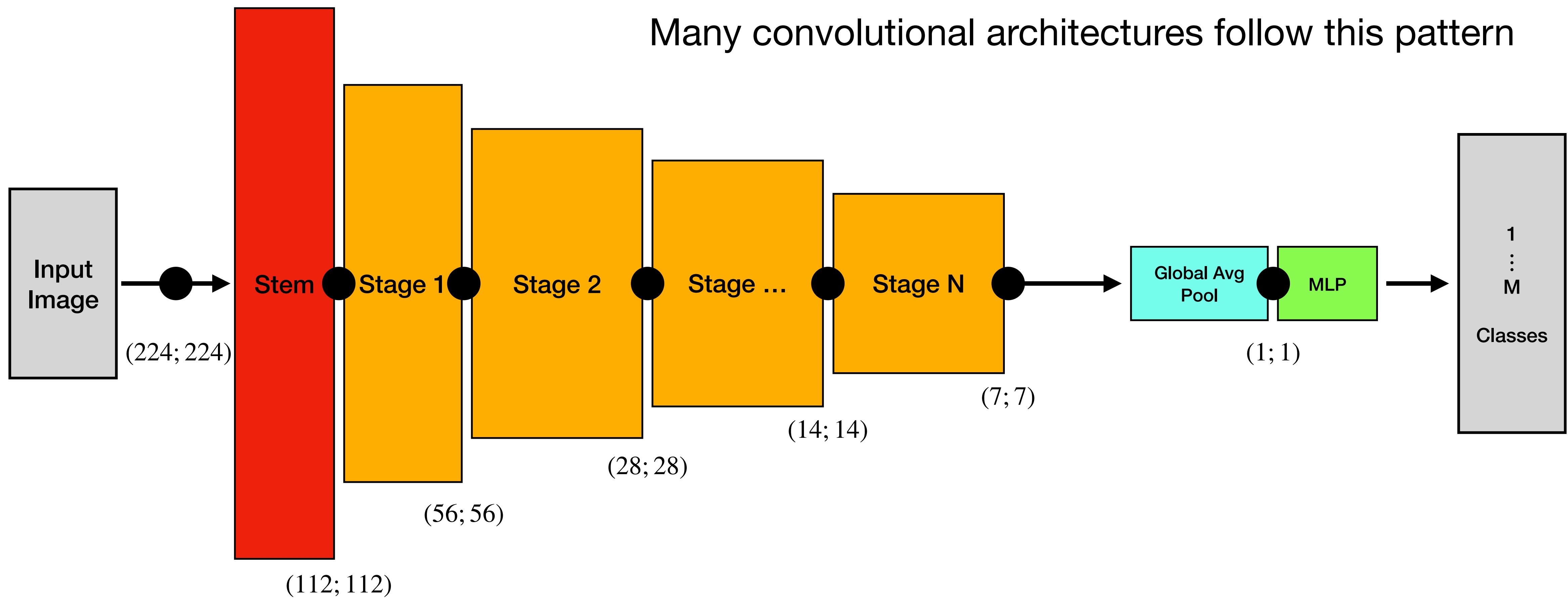
# Using a Pre-Trained Model

## Generic Components



# Using a Pre-Trained Model

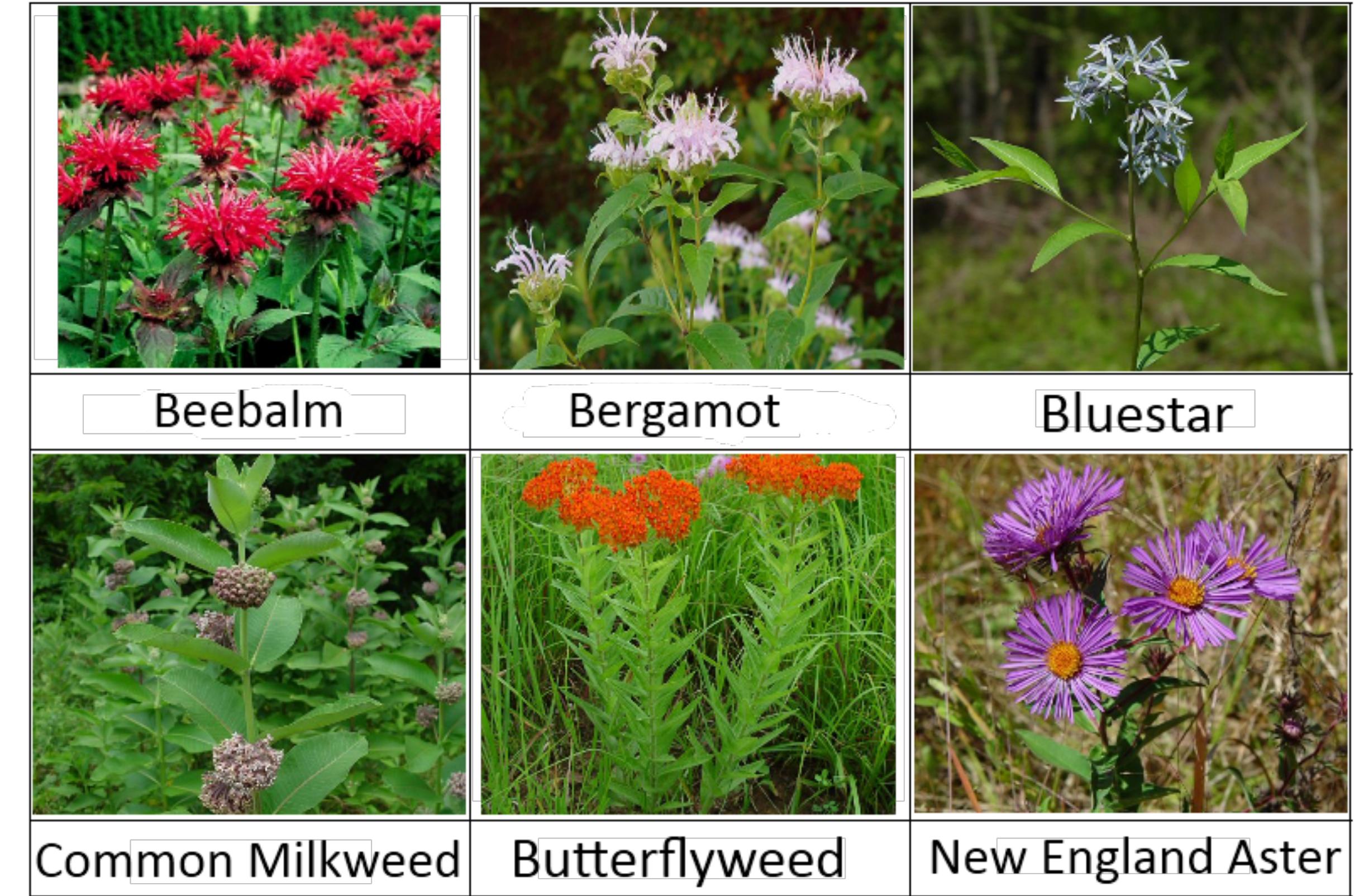
## Generic Components - Spatial Dimensions



# Using a Pre-Trained Model

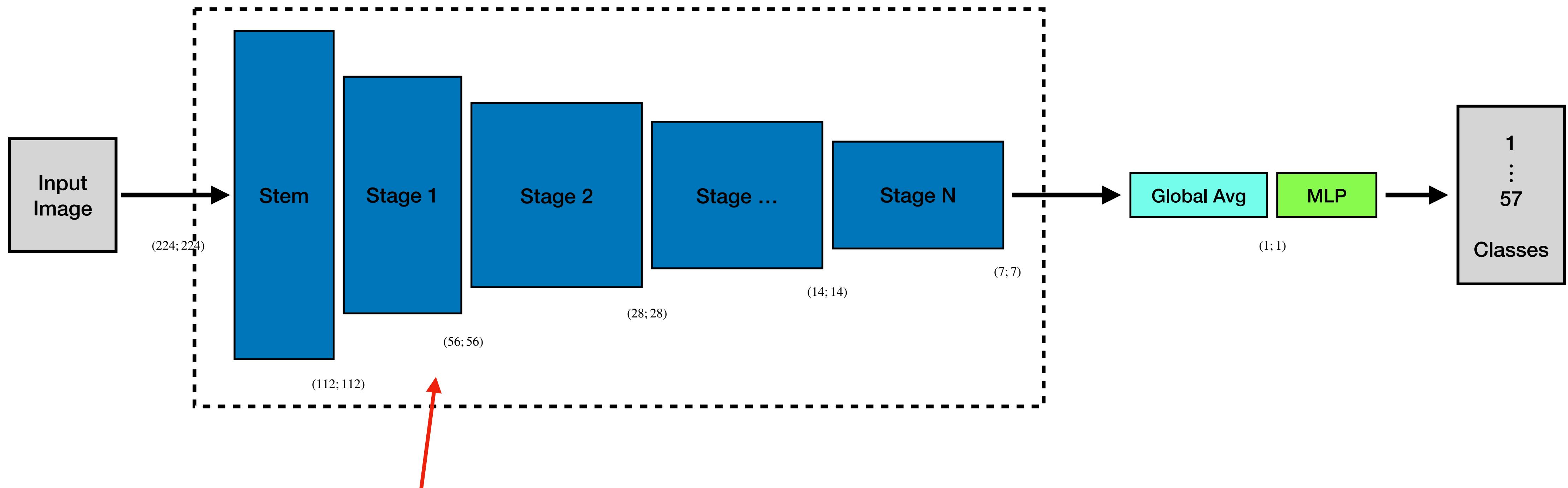
## Transfer Learning - Identify Plant Species

- Goal
  - Identify plant species
- Dataset
  - 57 classes
  - Few thousands images



# Using a Pre-Trained Model

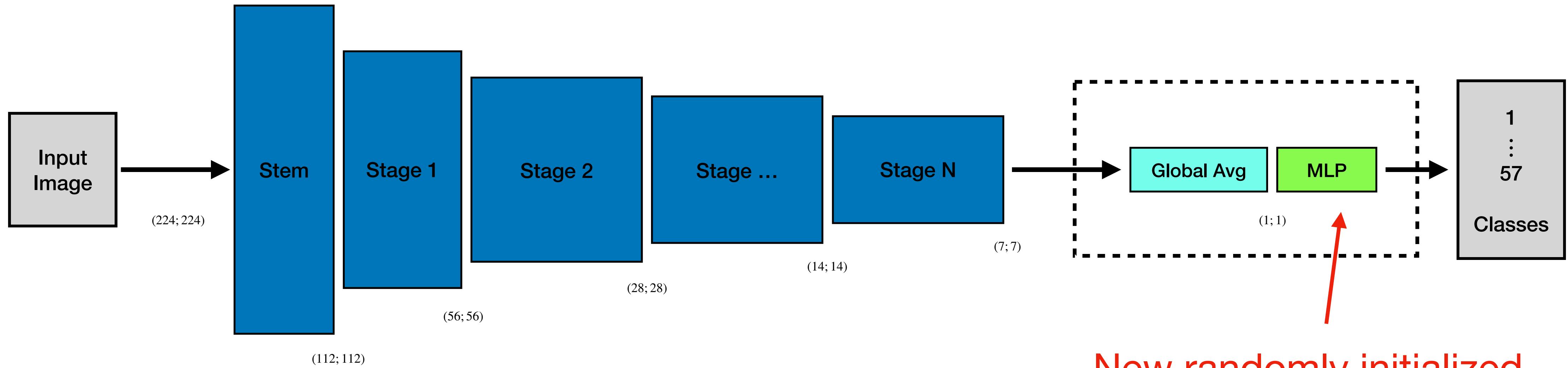
## Transfer Learning - Identify Plant Species



Keep these layers frozen

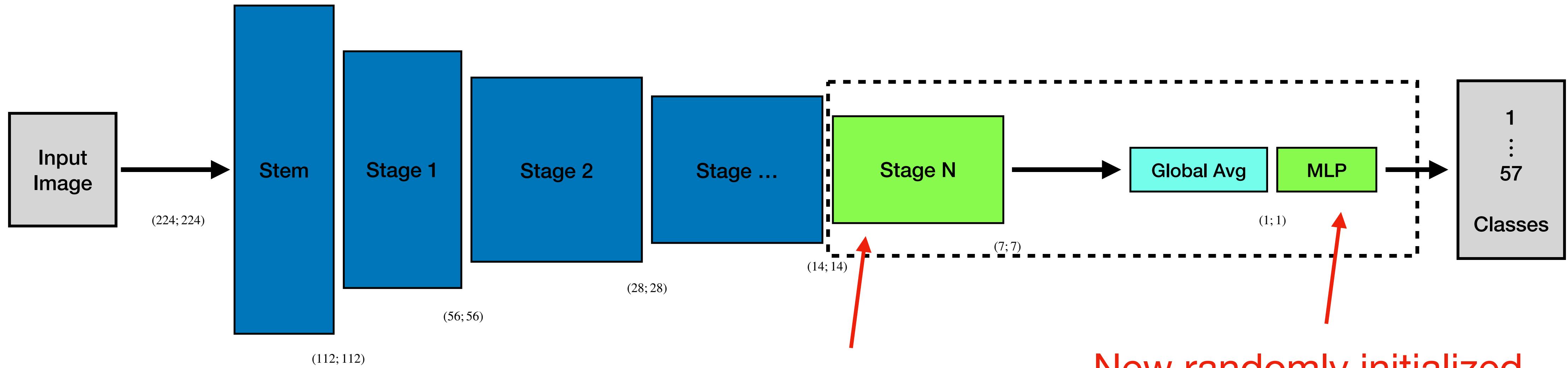
# Using a Pre-Trained Model

## Transfer Learning - Identify Plant Species



# Using a Pre-Trained Model

## Transfer Learning - Identify Plant Species



If we have enough data,  
we can fine-tune more layers

New randomly initialized  
MLP with 57 output units

# Augmentations

- **Always consider** the possibility of augmenting your data
  - Make **training set** bigger, add more variability
  - Cover the edge cases
- **Synthetic data !**
  - We will talk about them later



# Augmentations

## Convolution

- Is translation-invariant
- **Is not** rotation-invariant
- **Is not** scale-invariant

Original Image



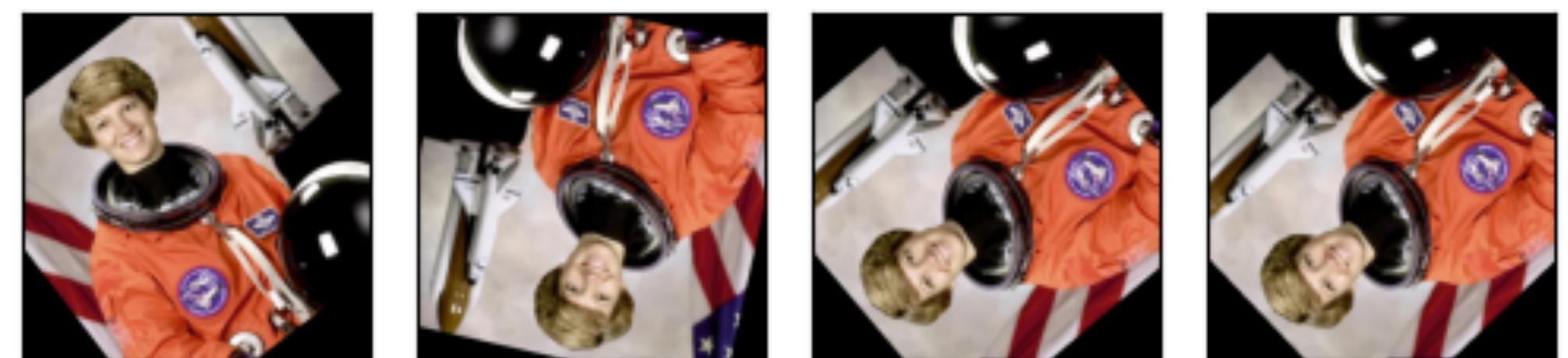
Random Crop



Original Image



Random Rotation



Original Image



Random Resized Crop





# Augmentations

- Perspective
- Image deformations

Original Image



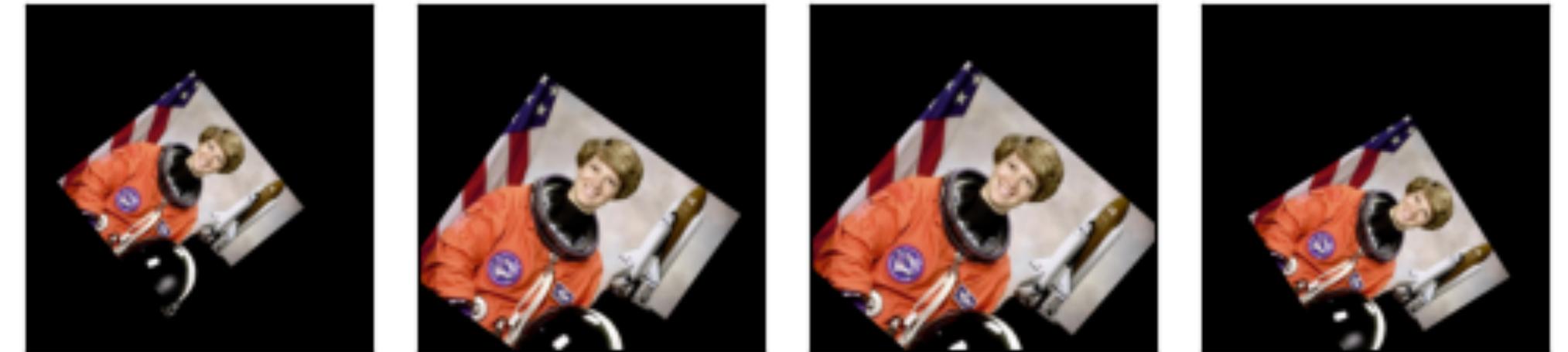
Random Perspective



Original Image



Random Affine Transformation



Original Image



Random Elastic Transformation





# Augmentations

- Color
- Contrast
- Saturation
- Brightness
- Color channel bit-depth

Original Image



Color Jitter



Original Image



Invert Colors



Original Image



Bit-depth reduction





# Augmentations

## Corrupt Information

- Noise
- Blur
- Cut-out
- Compression





# Augmentations

## Histogram Matching

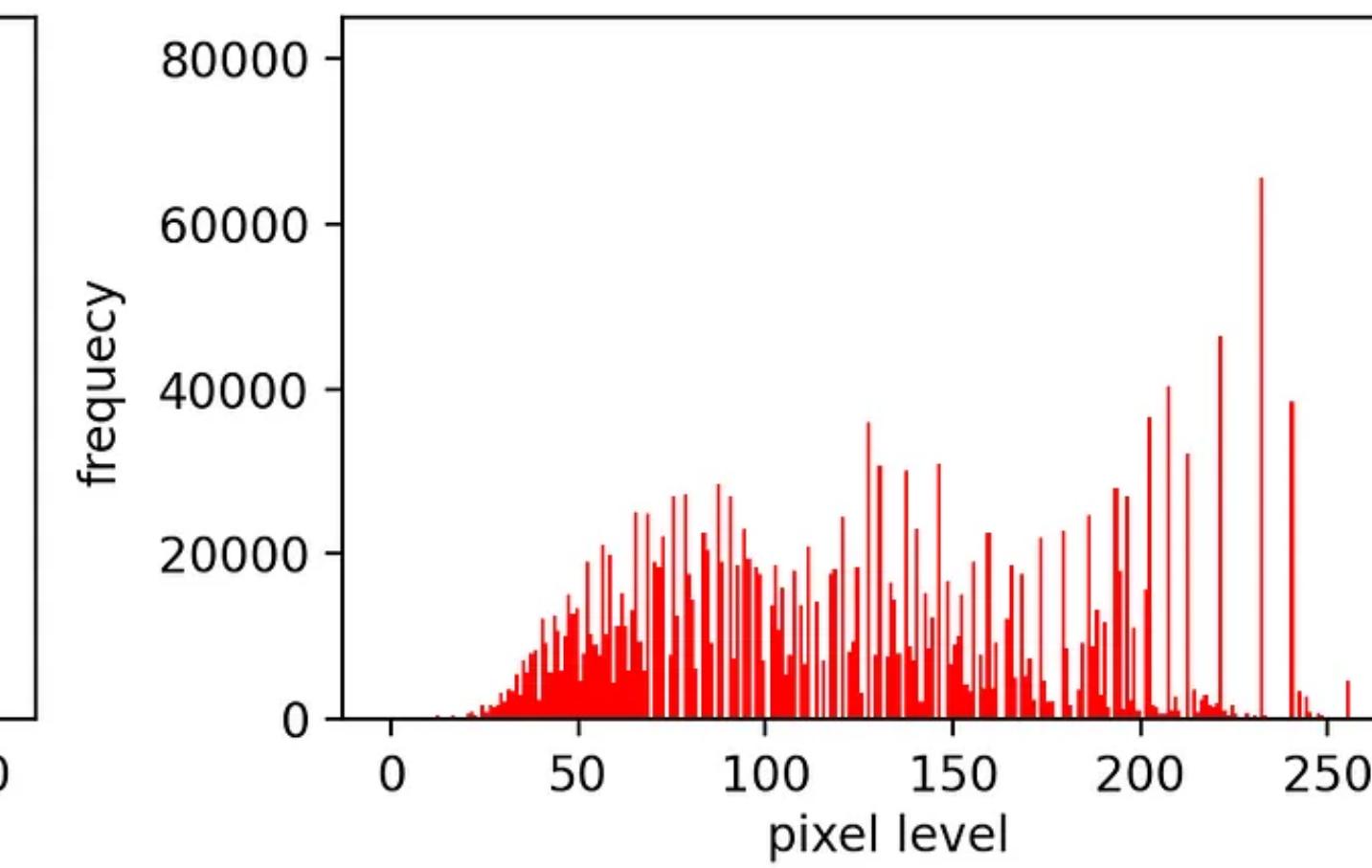
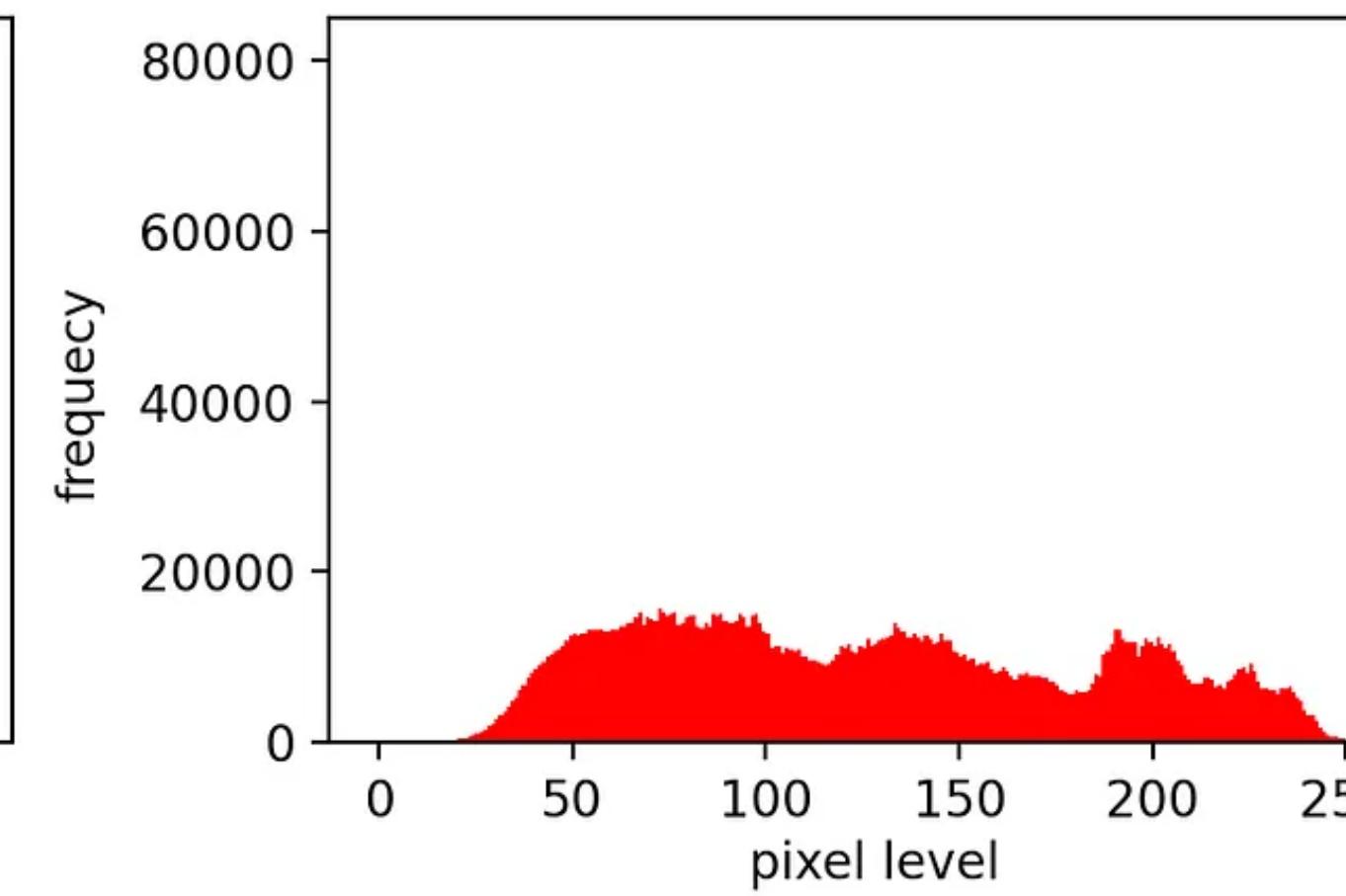
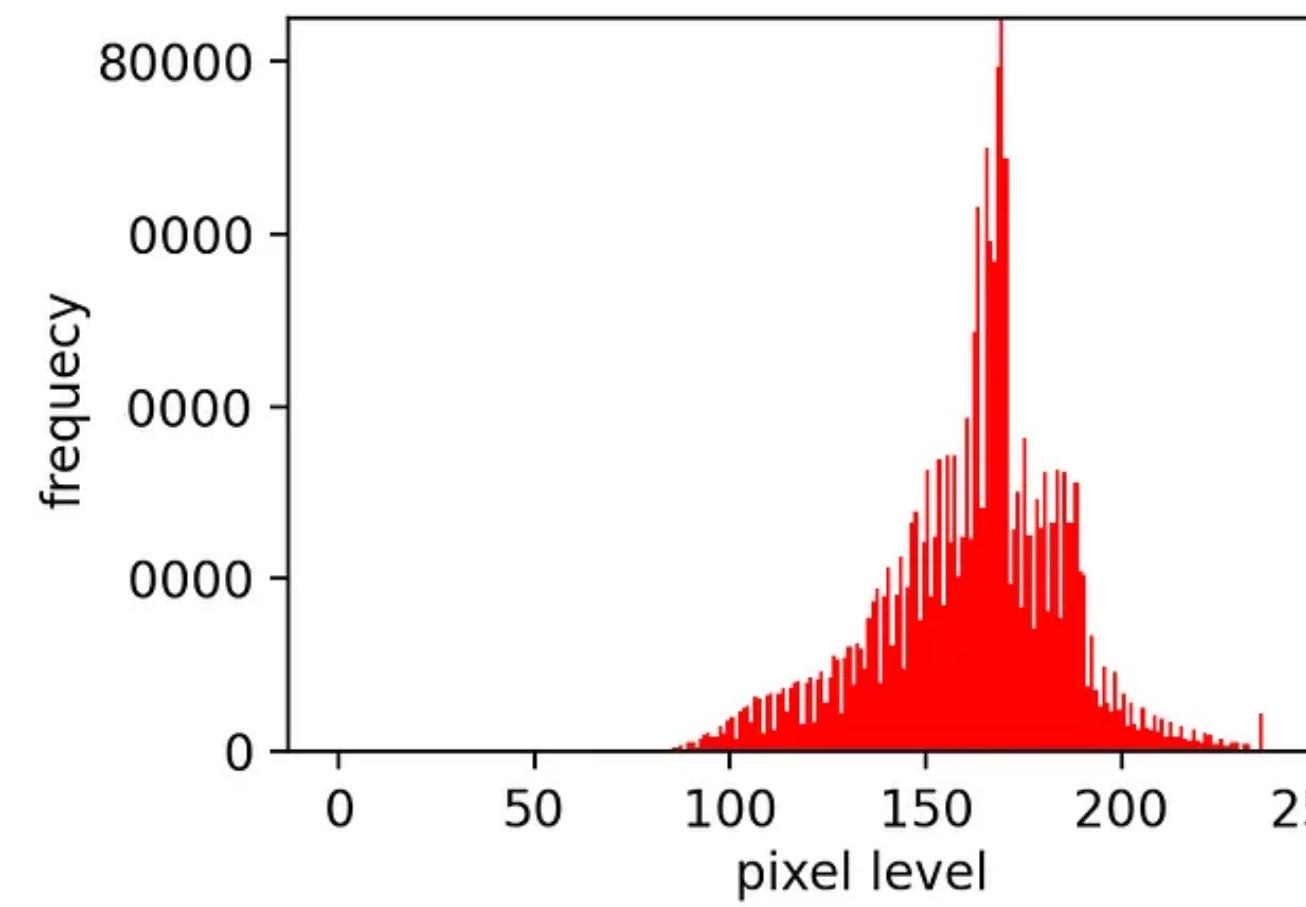
Original



Reference



Result



# 3 Siamese Networks

# Siamese Networks

- Image similarity
- Written signature verification
- Face recognition / verification
- ...

# Image Similarity



Image - <https://ioannotator.com/image-similarity-search>

# Image Similarity

Query Image  
Label: angular\_leaf\_spot



Similar Image # 1  
Label: angular\_leaf\_spot



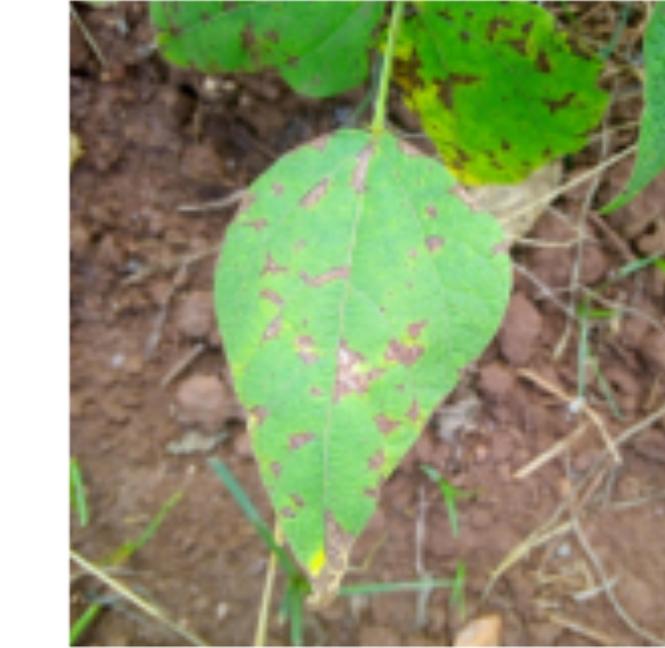
Similar Image # 2  
Label: angular\_leaf\_spot



Similar Image # 3  
Label: angular\_leaf\_spot



Similar Image # 4  
Label: angular\_leaf\_spot

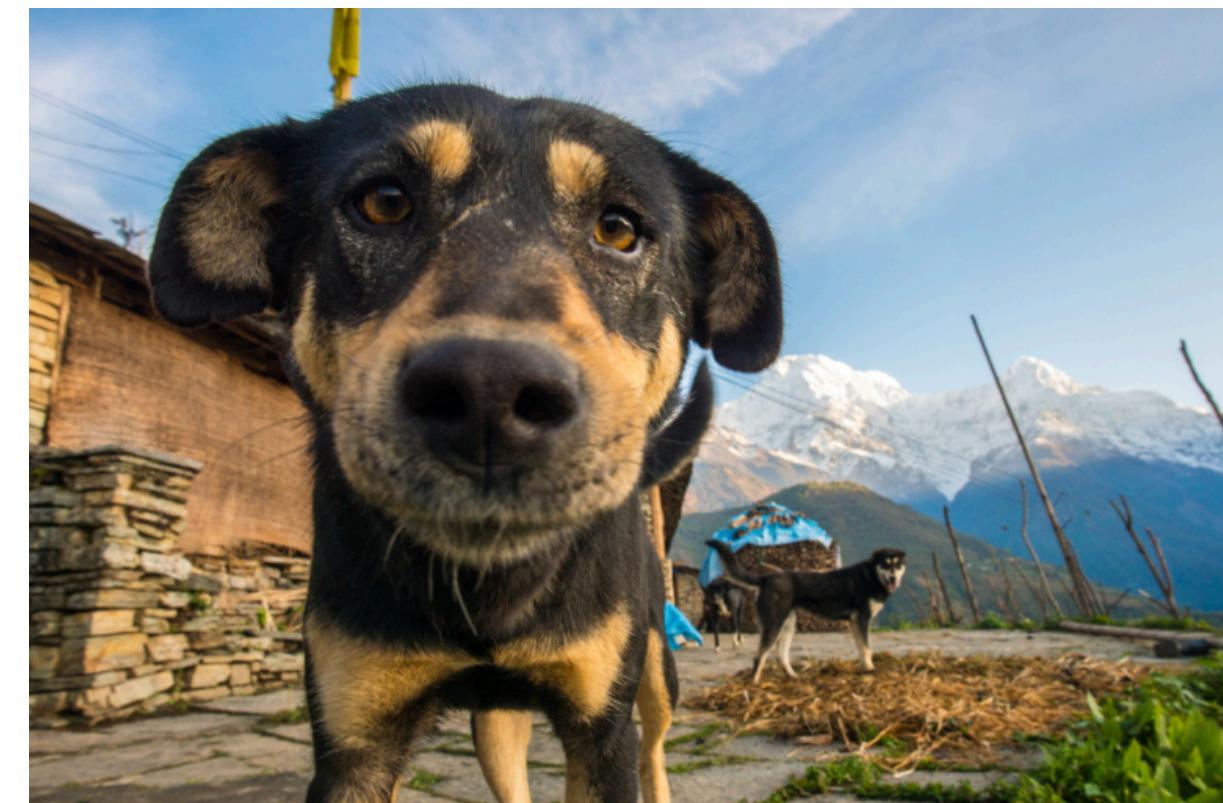


Similar Image # 5  
Label: angular\_leaf\_spot



# Image Similarity

Which Images Are Similar ?



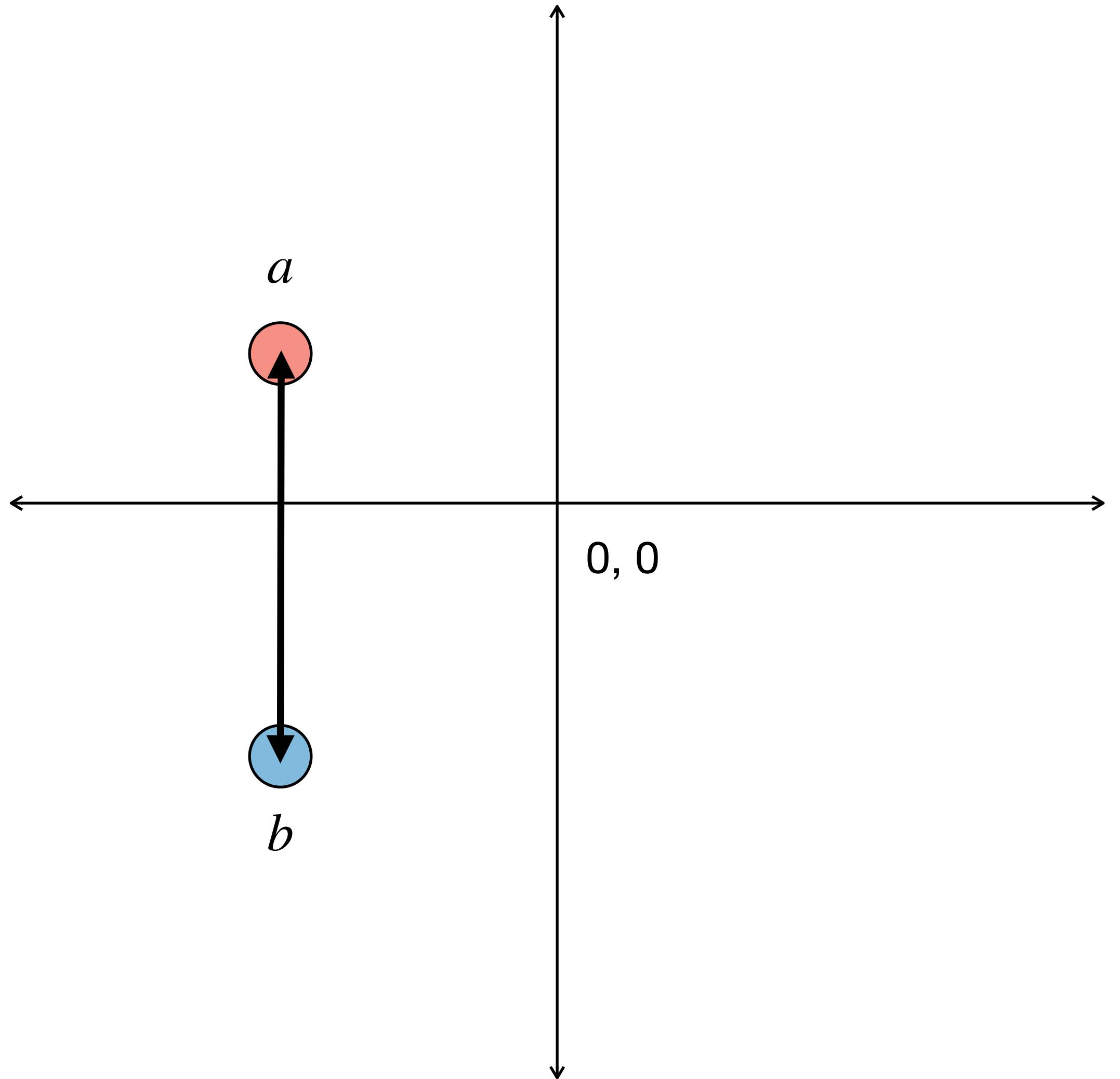
# Image Similarity

## Vector Similarity Metrics

- **Euclidean distance**

In multi-dimensional space all vectors are often “far away” from each other

$$d(a, b) = \|a - b\|$$





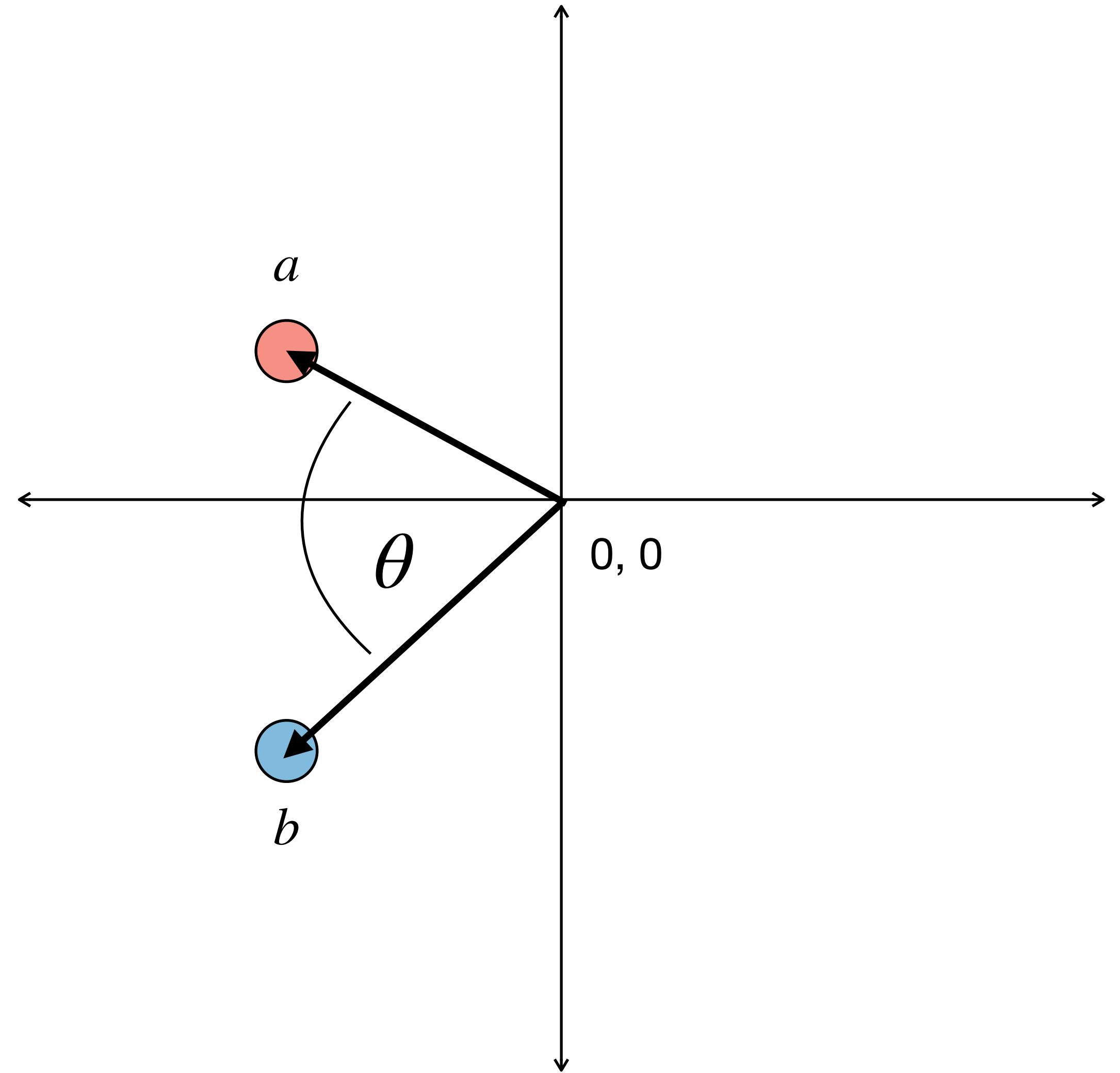
# Image Similarity

## Vector Similarity Metrics

- Euclidean distance
- **Cosine similarity**

$$S_C(a, b) = \cos \theta = \frac{a \cdot b}{\|a\| \|b\|}$$

$$D_C(a, b) = 1 - S_C(a, b)$$





# Image Similarity

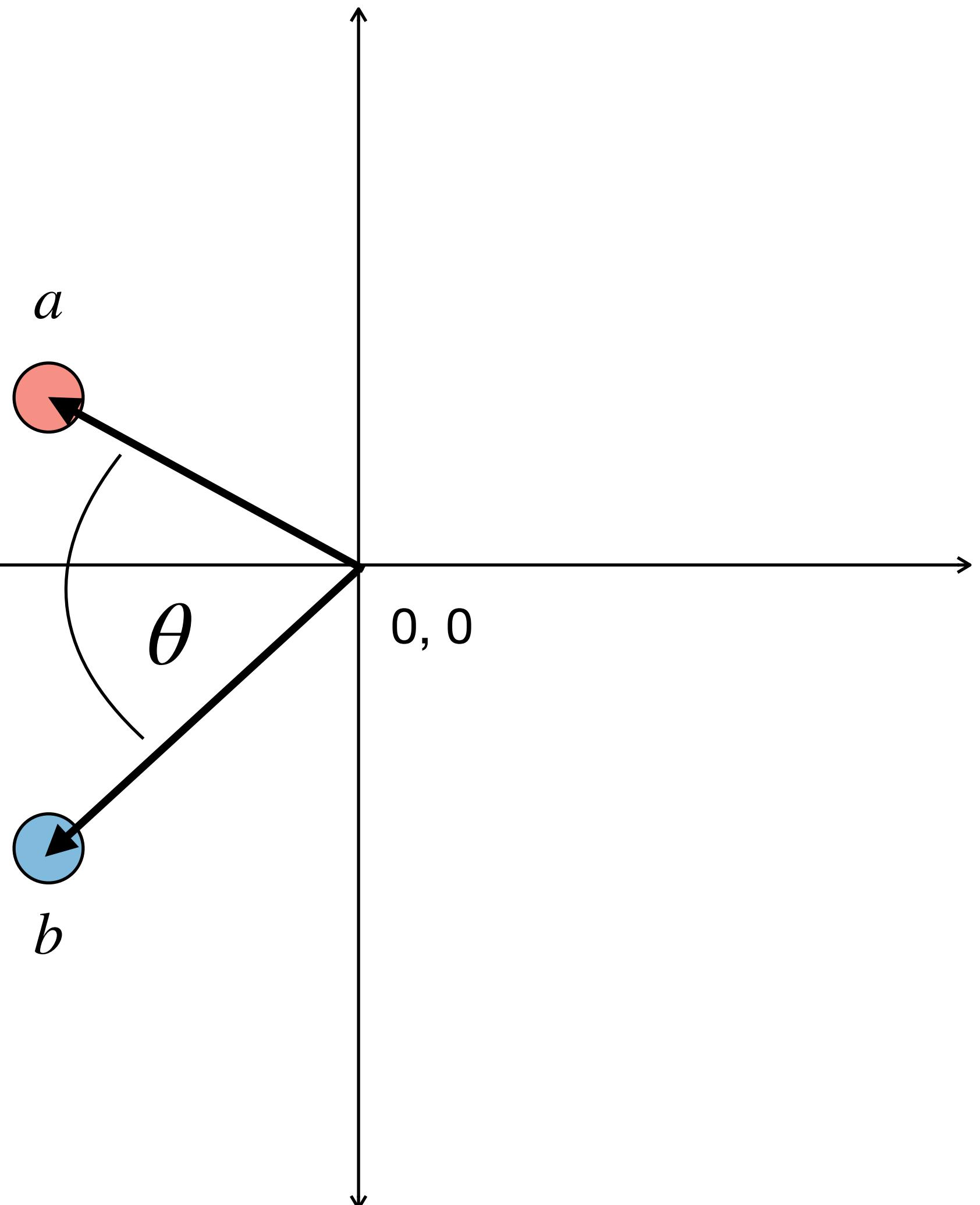
## Vector Similarity Metrics

- Euclidean distance
- **Cosine similarity**

For normalized vectors

$$S_C(a, b) = a \cdot b$$

$$S_C(a, b) = \cos \theta = \frac{a \cdot b}{\|a\| \|b\|}$$

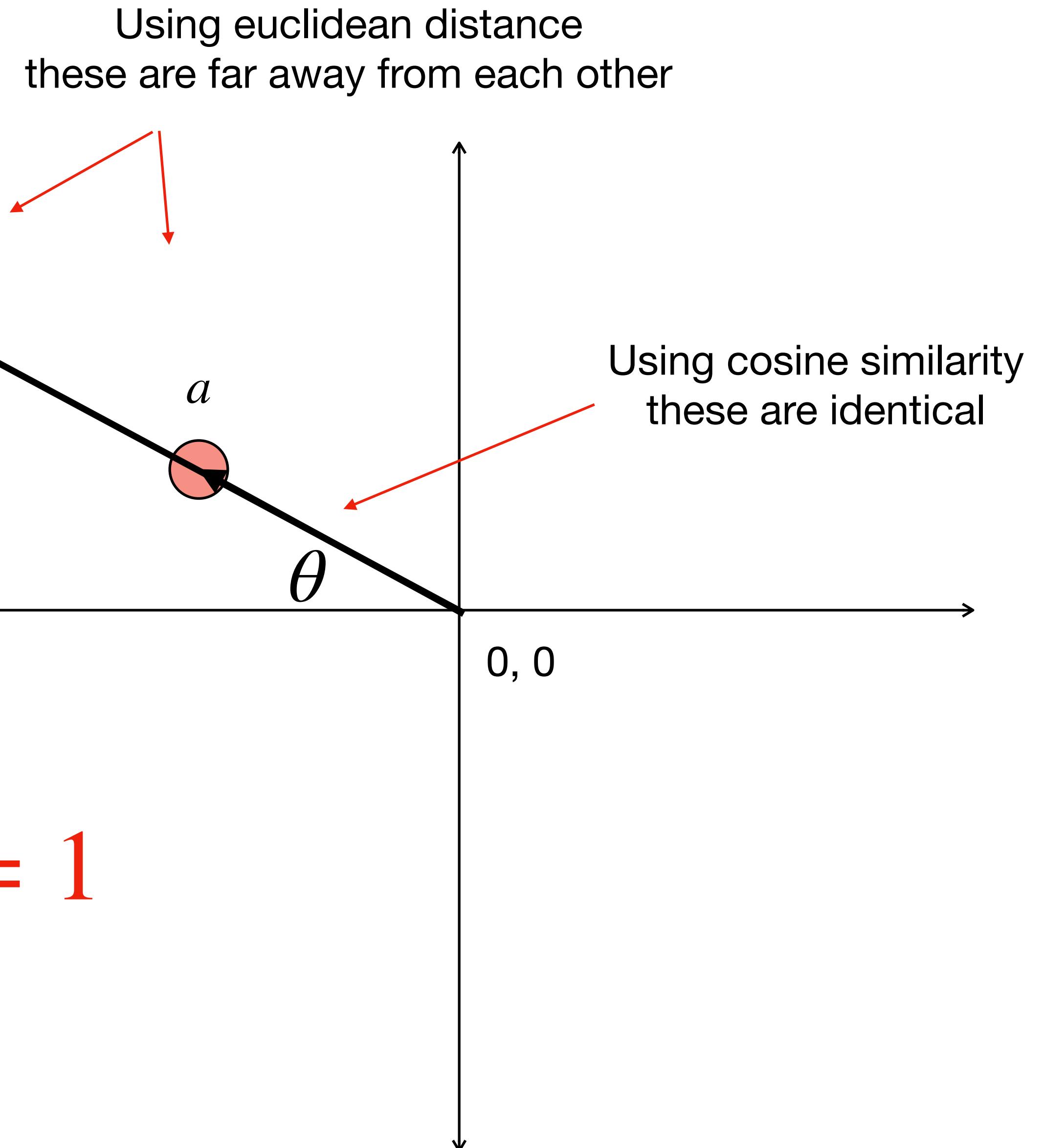


# Image Similarity

## Vector Similarity Metrics

- Euclidean distance
- **Cosine similarity**

$$S_C(a, b) = \cos \theta = \frac{a \cdot b}{\|a\| \|b\|} = 1$$

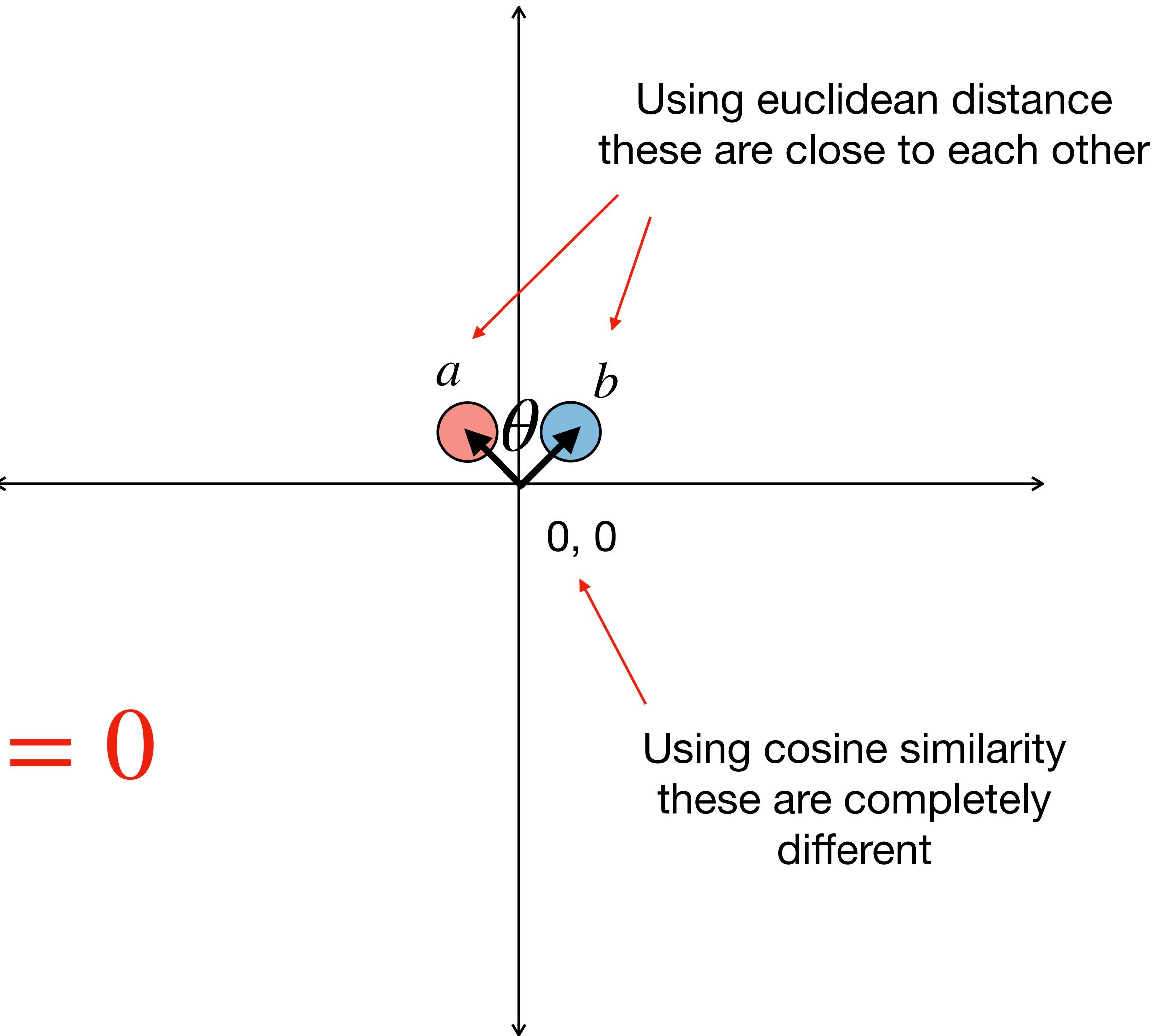


# Image Similarity

## Vector Similarity Metrics

- Euclidean distance
- **Cosine similarity**

$$S_C(a, b) = \cos \theta = \frac{a \cdot b}{\|a\| \|b\|} = 0$$





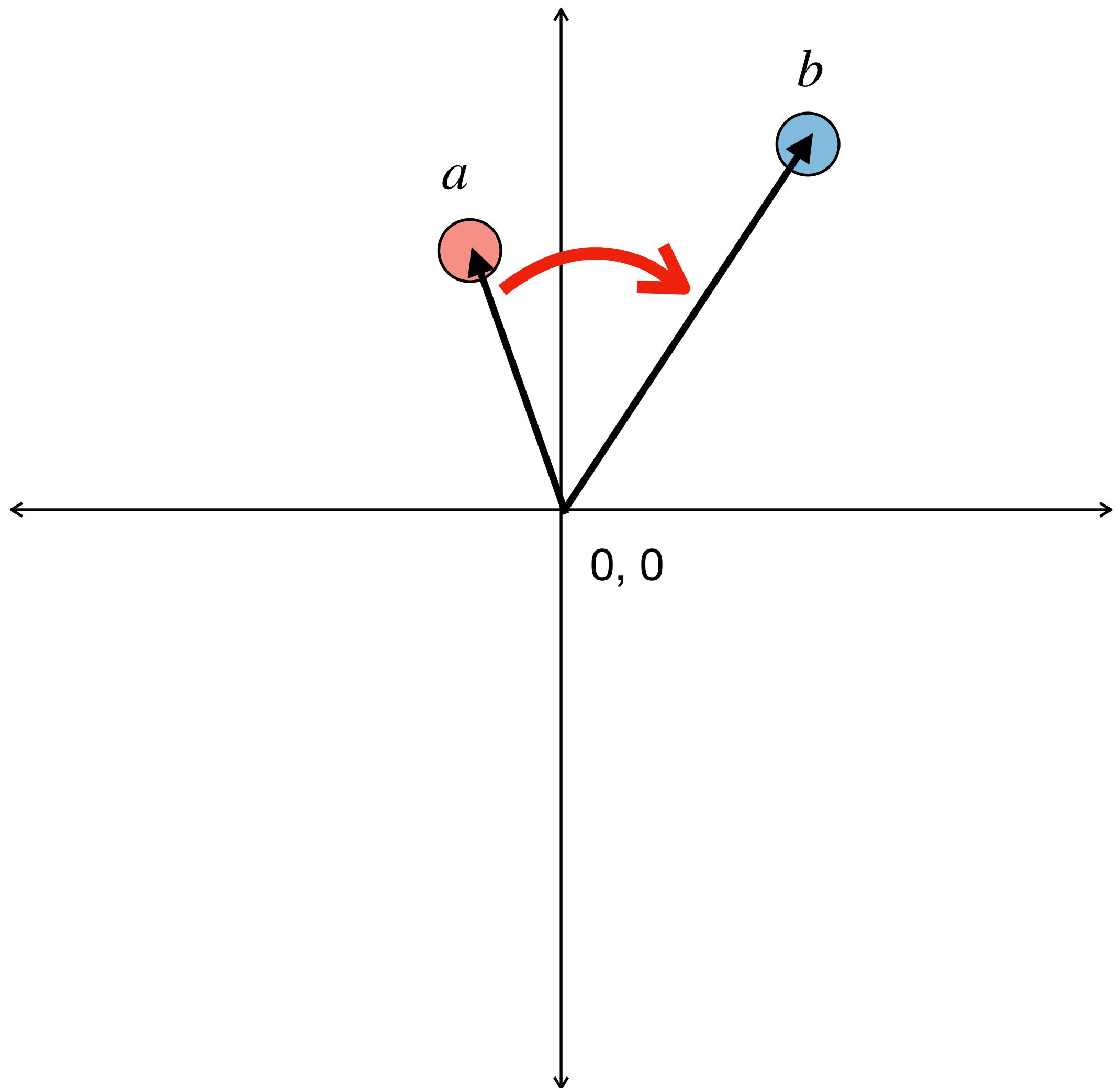
# Image Similarity

## Cosine Similarity Loss

- If we want two vectors to be **similar**

$$\mathcal{L}(a, b) = 1 - S_C(a, b)$$

$$= 1 - \frac{a \cdot b}{\|a\| \|b\|}$$

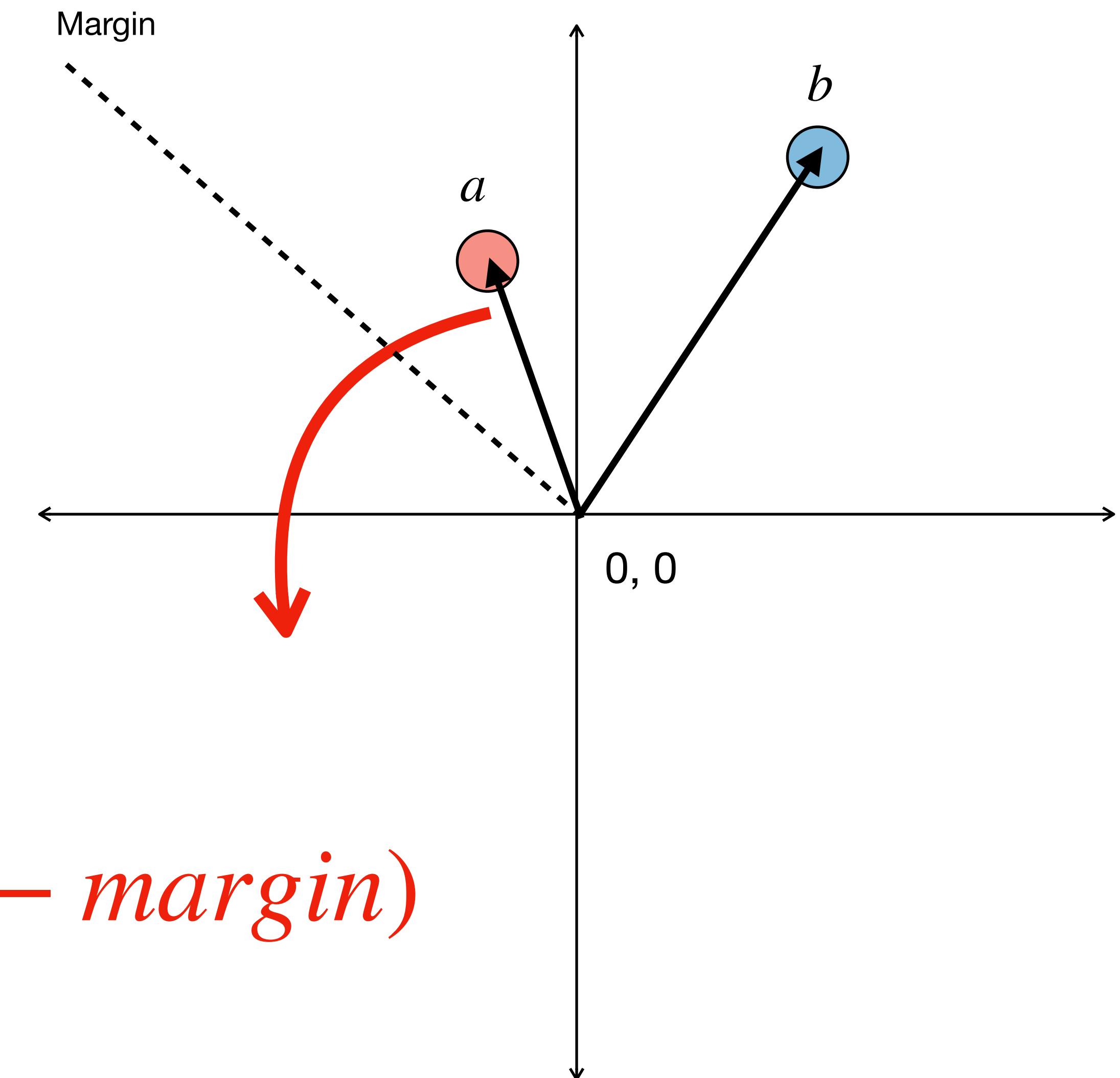




# Image Similarity

## Cosine Similarity Loss

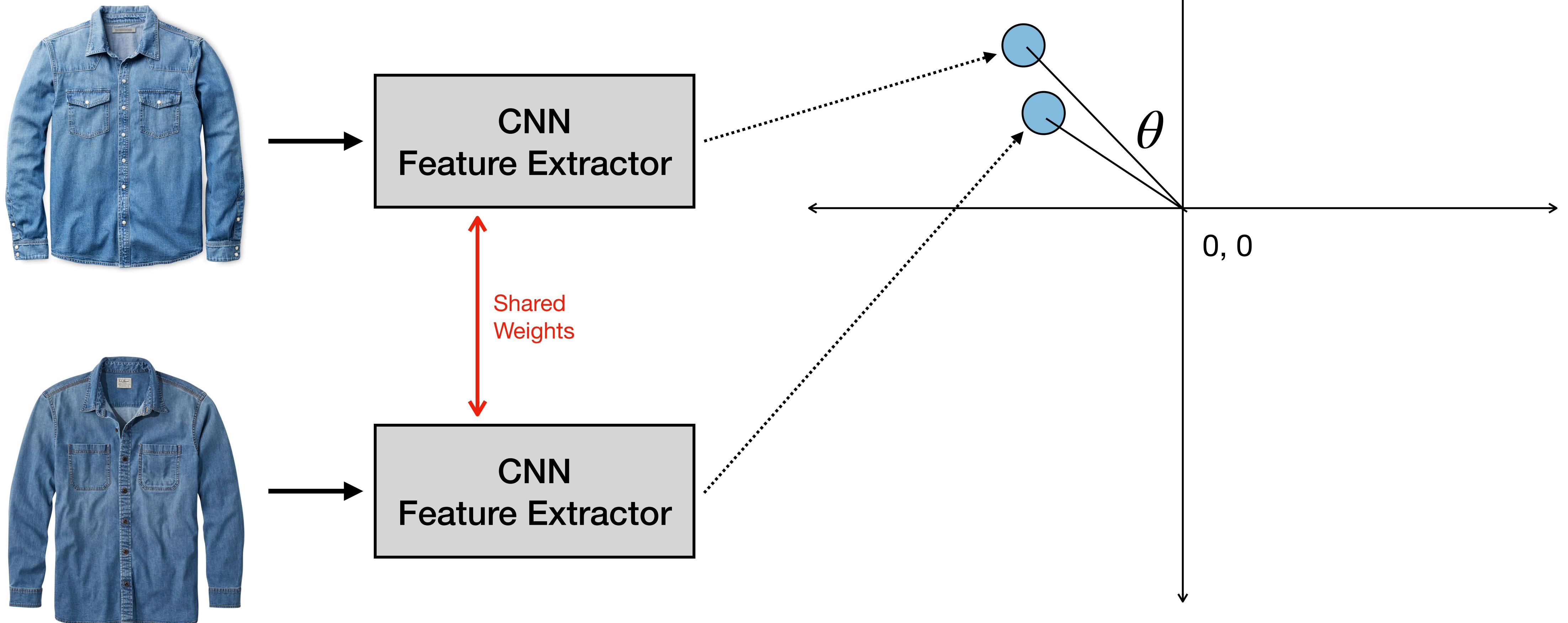
- If we want two vectors to be **different**



$$\mathcal{L}(a, b) = \max(0, S_C(a, b) - \text{margin})$$

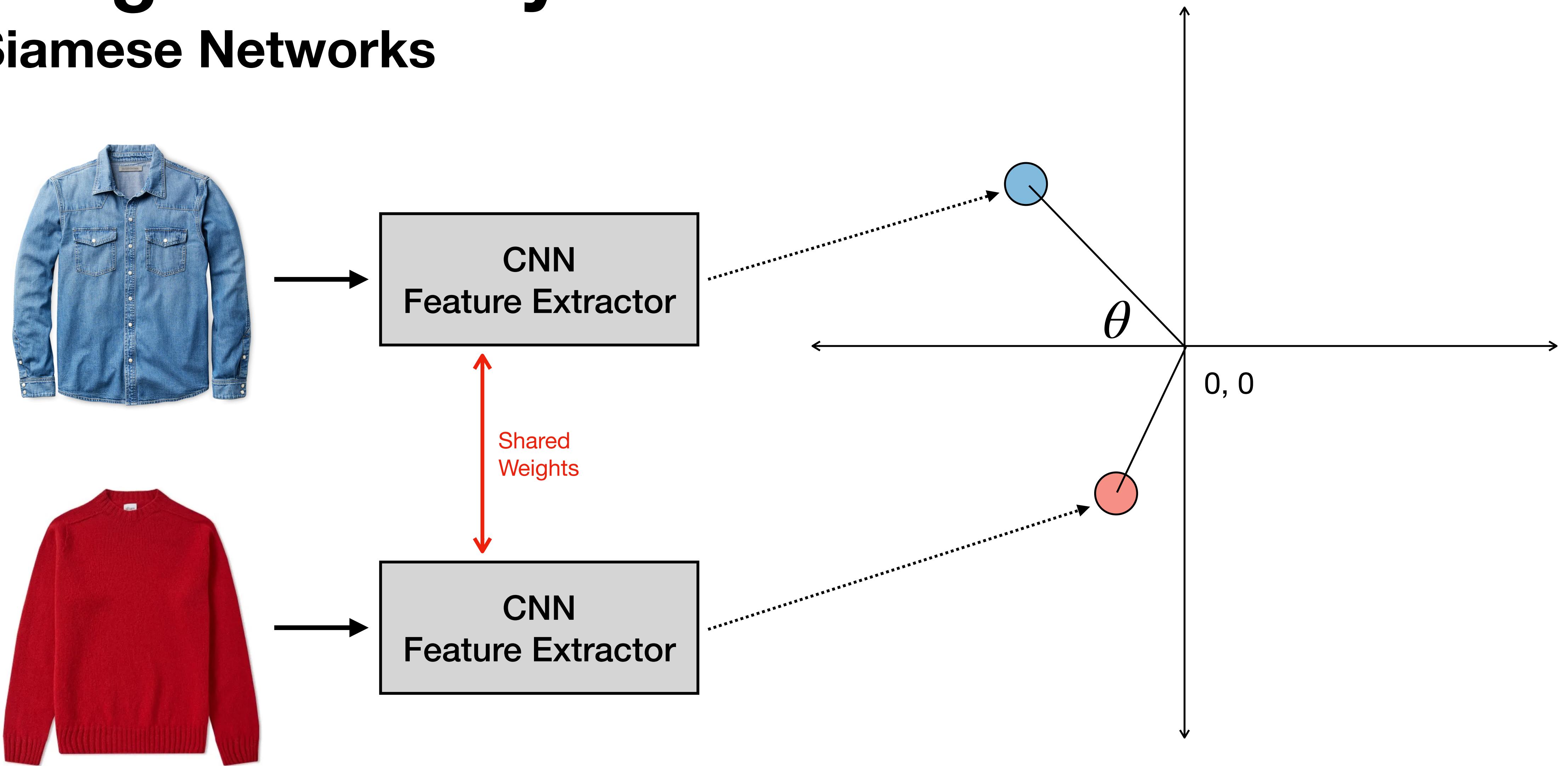
# Image Similarity

## Siamese Networks



# Image Similarity

## Siamese Networks



# Contrastive Loss

## Siamese Networks

Hadsell, Raia, Sumit Chopra, and Yann LeCun. "Dimensionality reduction by learning an invariant mapping." *2006 IEEE computer society conference on computer vision and pattern recognition (CVPR'06)*. Vol. 2. IEEE, 2006.

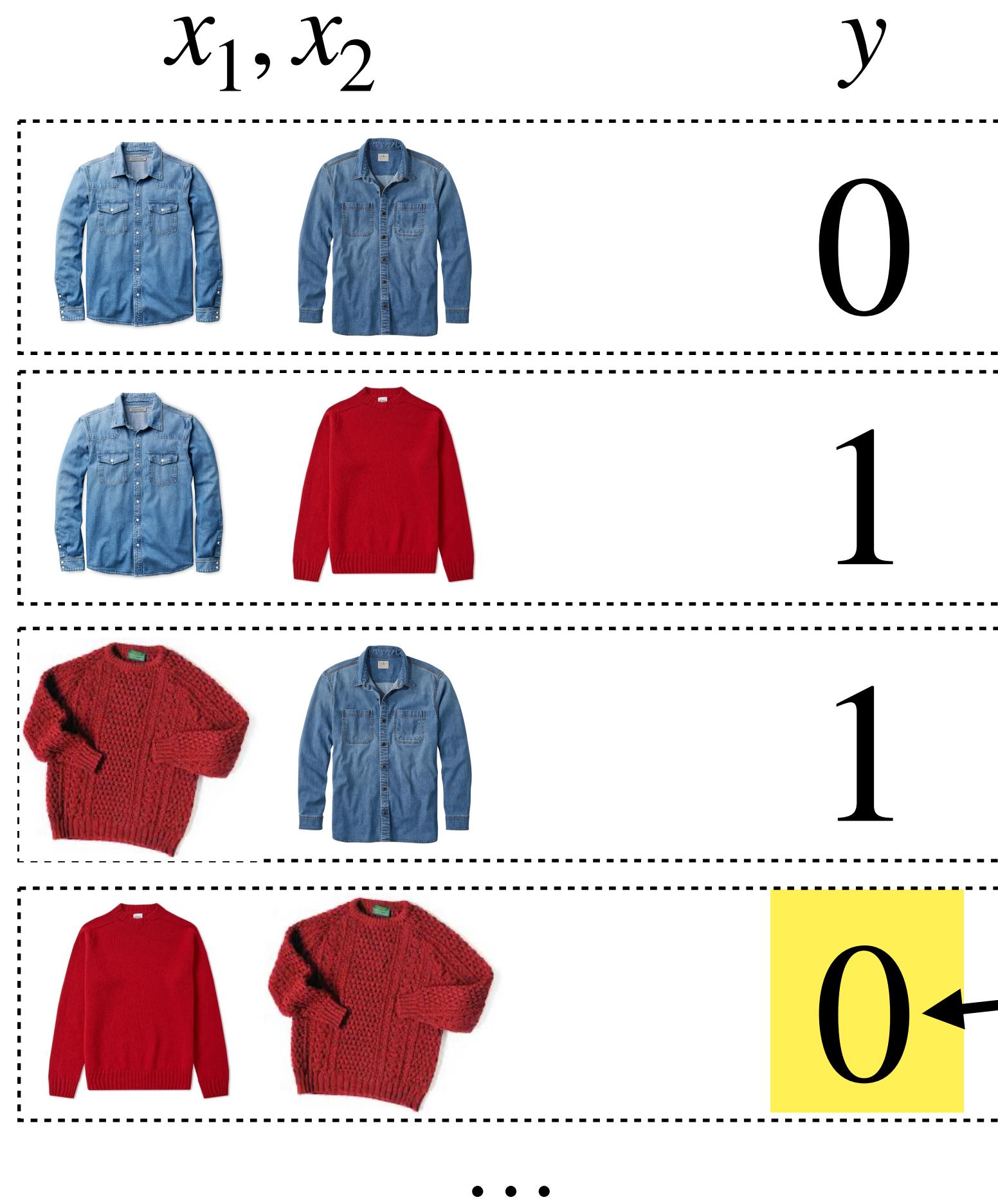
$$\mathcal{X} = \{ \text{ \}$$

Big bag of images

# Contrastive Loss

## Siamese Networks

Hadsell, Raia, Sumit Chopra, and Yann LeCun. "Dimensionality reduction by learning an invariant mapping." *2006 IEEE computer society conference on computer vision and pattern recognition (CVPR'06)*. Vol. 2. IEEE, 2006.



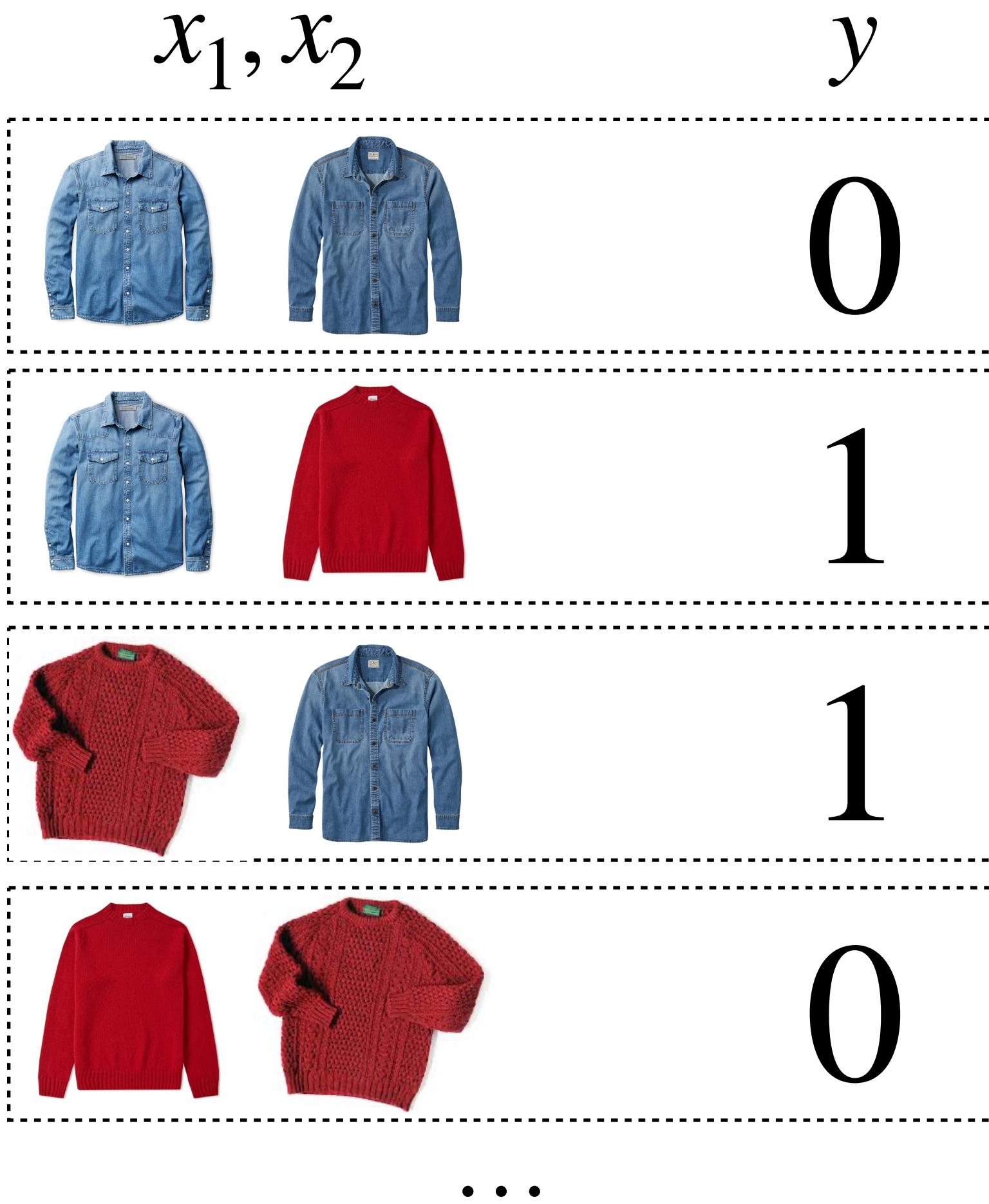
$$\mathcal{X} = \{ \quad \text{(blue denim shirt)} \quad \text{(blue denim shirt)} \quad \text{(red sweater)} \quad \text{(red sweater)} \quad \}$$

Combinations of images from  $\mathcal{X}$

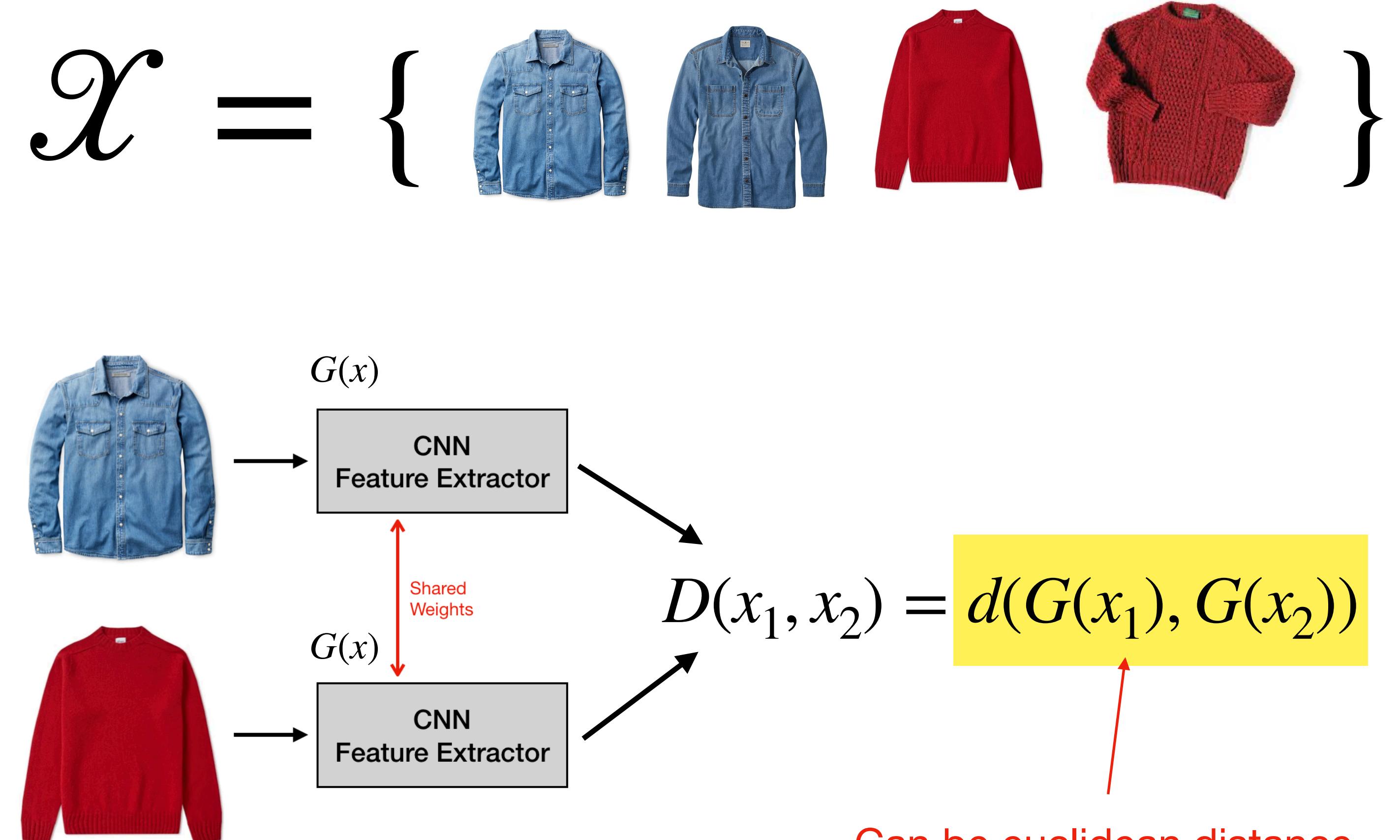
We must be able to tell if any two images from  $\mathcal{X}$  are similar or not

# Contrastive Loss

## Siamese Networks



Hadsell, Raia, Sumit Chopra, and Yann LeCun. "Dimensionality reduction by learning an invariant mapping." 2006 IEEE computer society conference on computer vision and pattern recognition (CVPR'06). Vol. 2. IEEE, 2006.

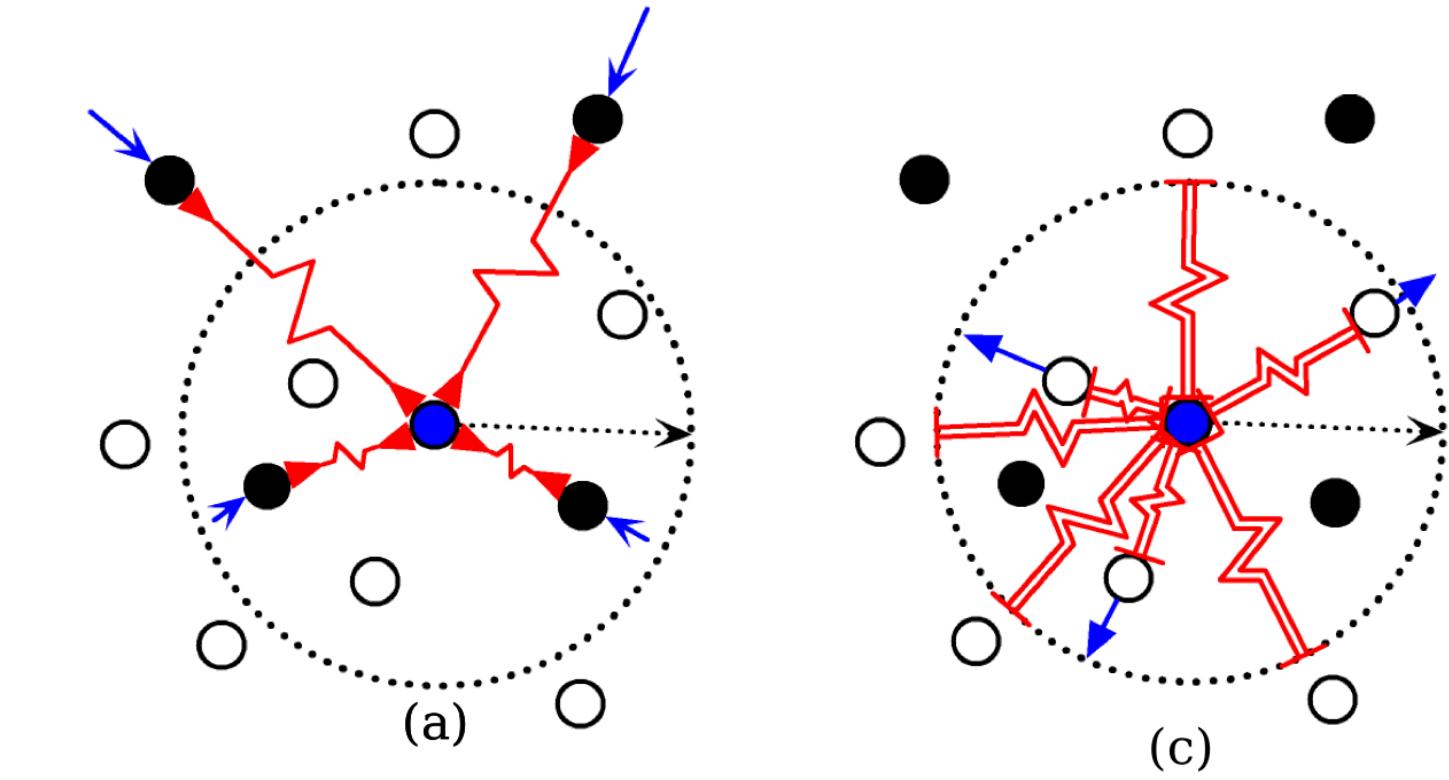
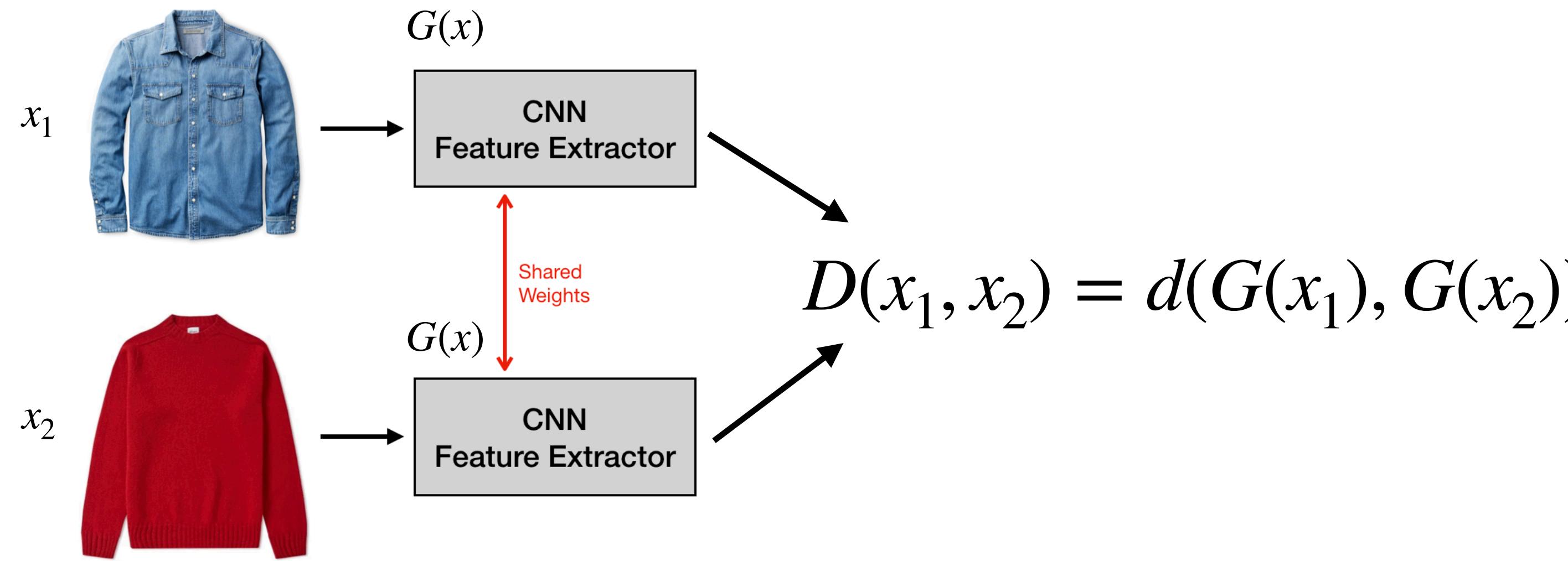


Can be euclidean distance  
or cosine distance

# Contrastive Loss

## Siamese Networks

Hadsell, Raia, Sumit Chopra, and Yann LeCun. "Dimensionality reduction by learning an invariant mapping." 2006 IEEE computer society conference on computer vision and pattern recognition (CVPR'06). Vol. 2. IEEE, 2006.



$$\mathcal{L}(x_1, x_2, y) = (1 - y)L_S(x_1, x_2) + (y)L_D(x_1, x_2)$$

Loss component for  
similar images

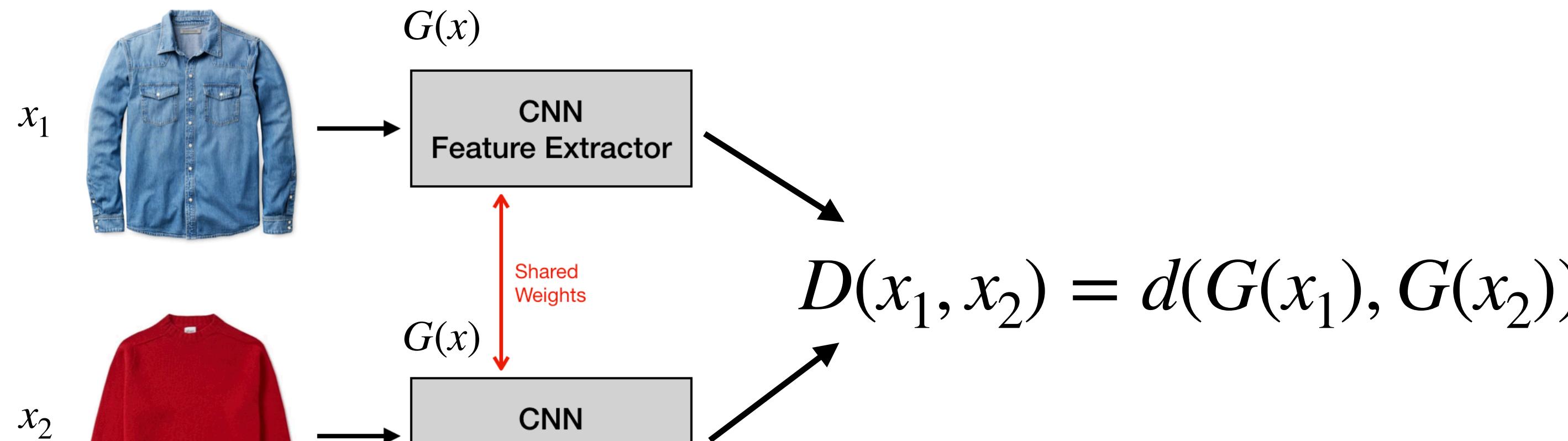
Loss component for  
dissimilar images

Hadsell, Raia, Sumit Chopra, and Yann LeCun. "Dimensionality reduction by learning an invariant mapping." 2006 IEEE computer society conference on computer vision and pattern recognition (CVPR'06). Vol. 2. IEEE, 2006.



# Contrastive Loss

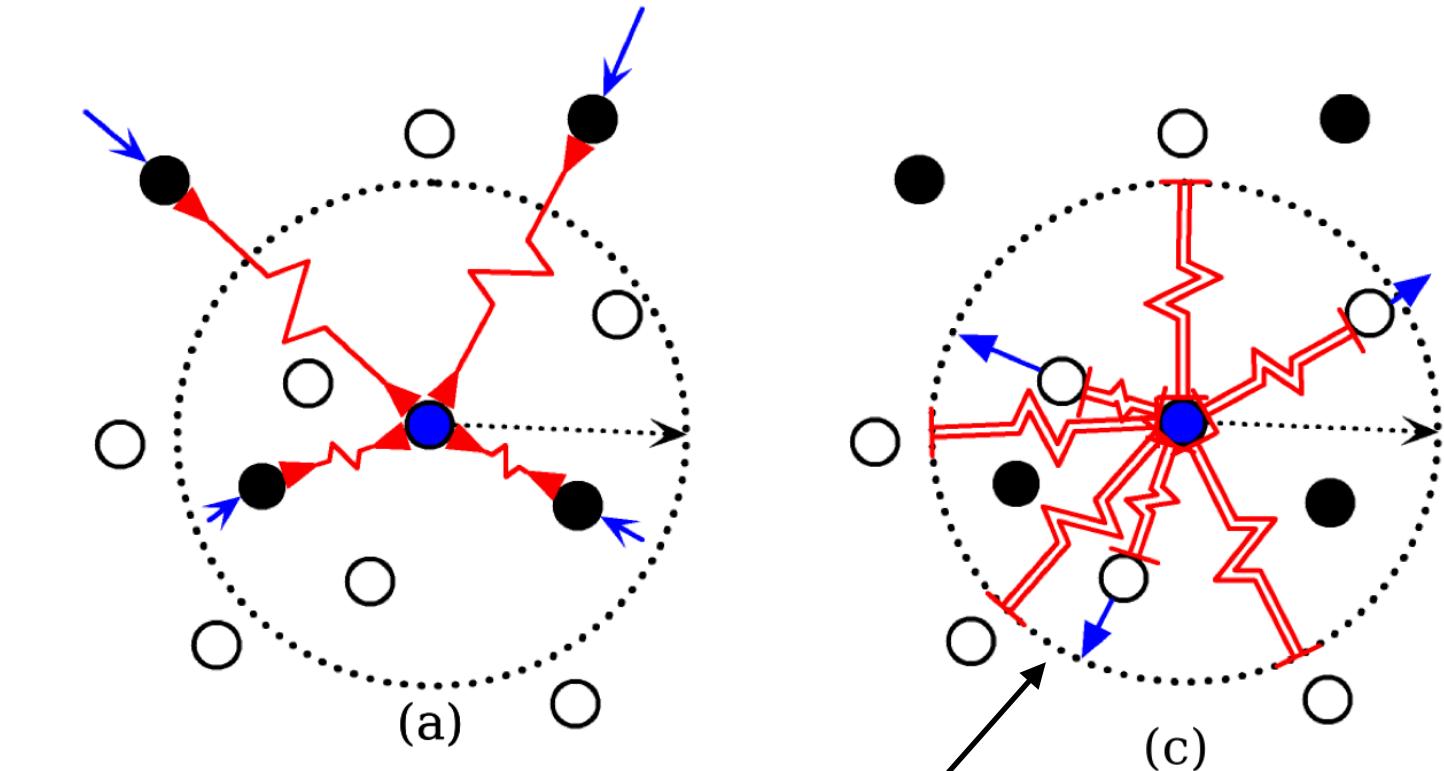
## Siamese Networks



$$D(x_1, x_2) = d(G(x_1), G(x_2))$$

$$\mathcal{L}(x_1, x_2, y) = (1 - y)L_S(x_1, x_2) + (y)L_D(x_1, x_2)$$

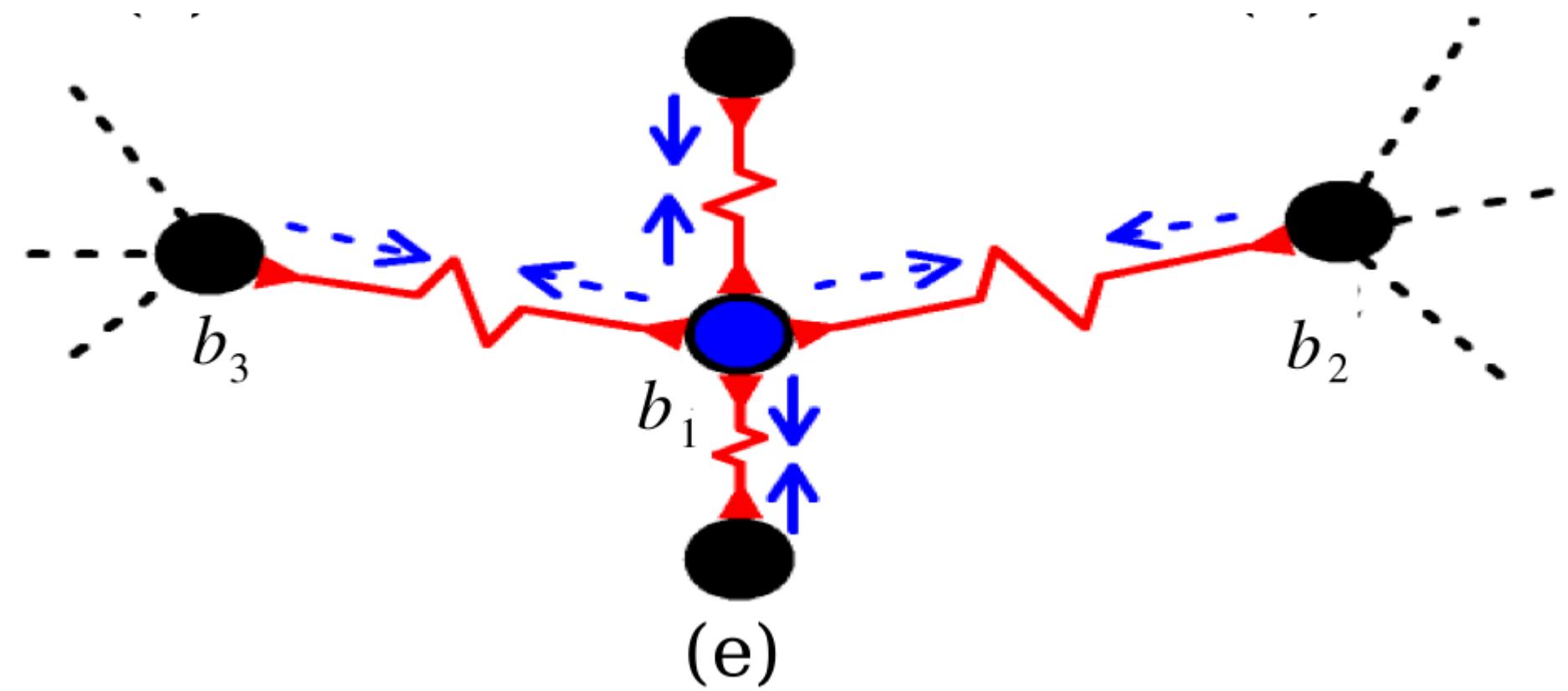
$$\mathcal{L}(x_1, x_2, y) = (1 - y)\frac{1}{2}D(x_1, x_2)^2 + (y)\frac{1}{2}\{\max(0, m - D(x_1, x_2))\}^2$$



max radius of the  
repulsive force

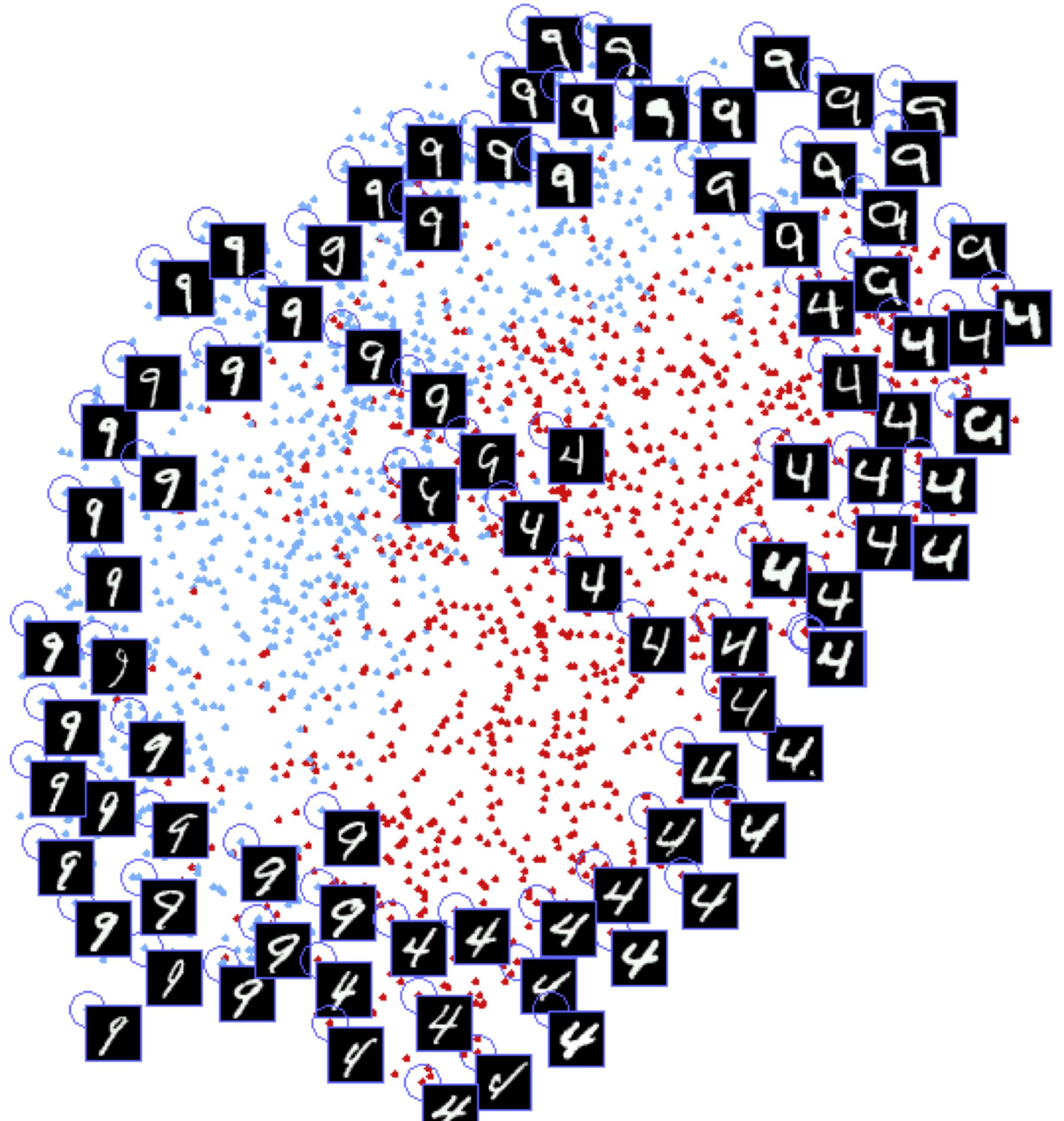
# Contrastive Loss

## Siamese Networks



Equilibrium was reached

Hadsell, Raia, Sumit Chopra, and Yann LeCun. "Dimensionality reduction by learning an invariant mapping." 2006 IEEE computer society conference on computer vision and pattern recognition (CVPR'06). Vol. 2. IEEE, 2006.



# Face Recognition

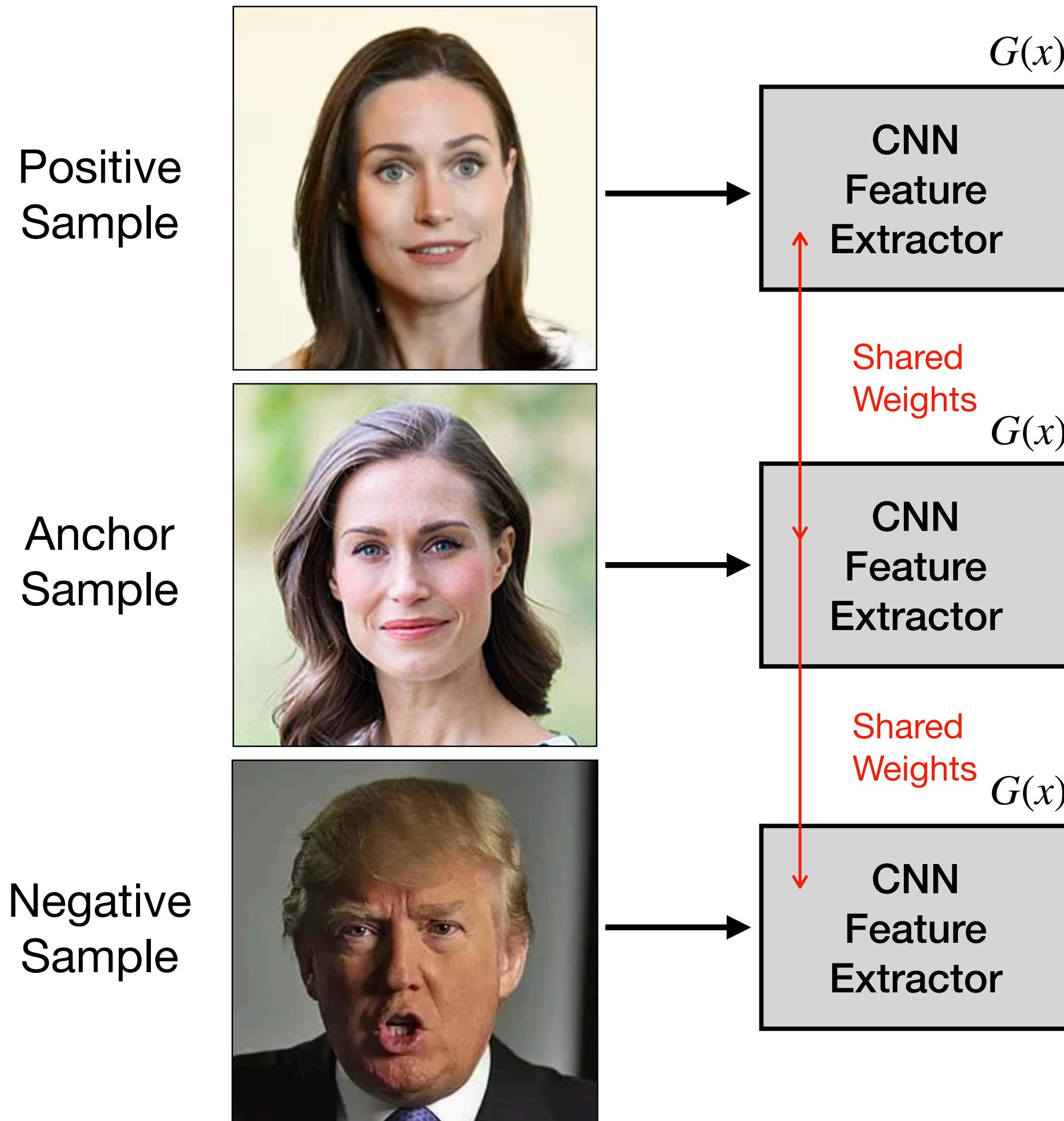
- A case of image similarity
- Large number of classes (Open-set)
  - few samples / class
- **Samples belonging to the same class should be isolated from other classes**



Schroff, Florian, Dmitry Kalenichenko, and James Philbin. "Facenet: A unified embedding for face recognition and clustering." *Proceedings of the IEEE conference on computer vision and pattern recognition*. 2015.

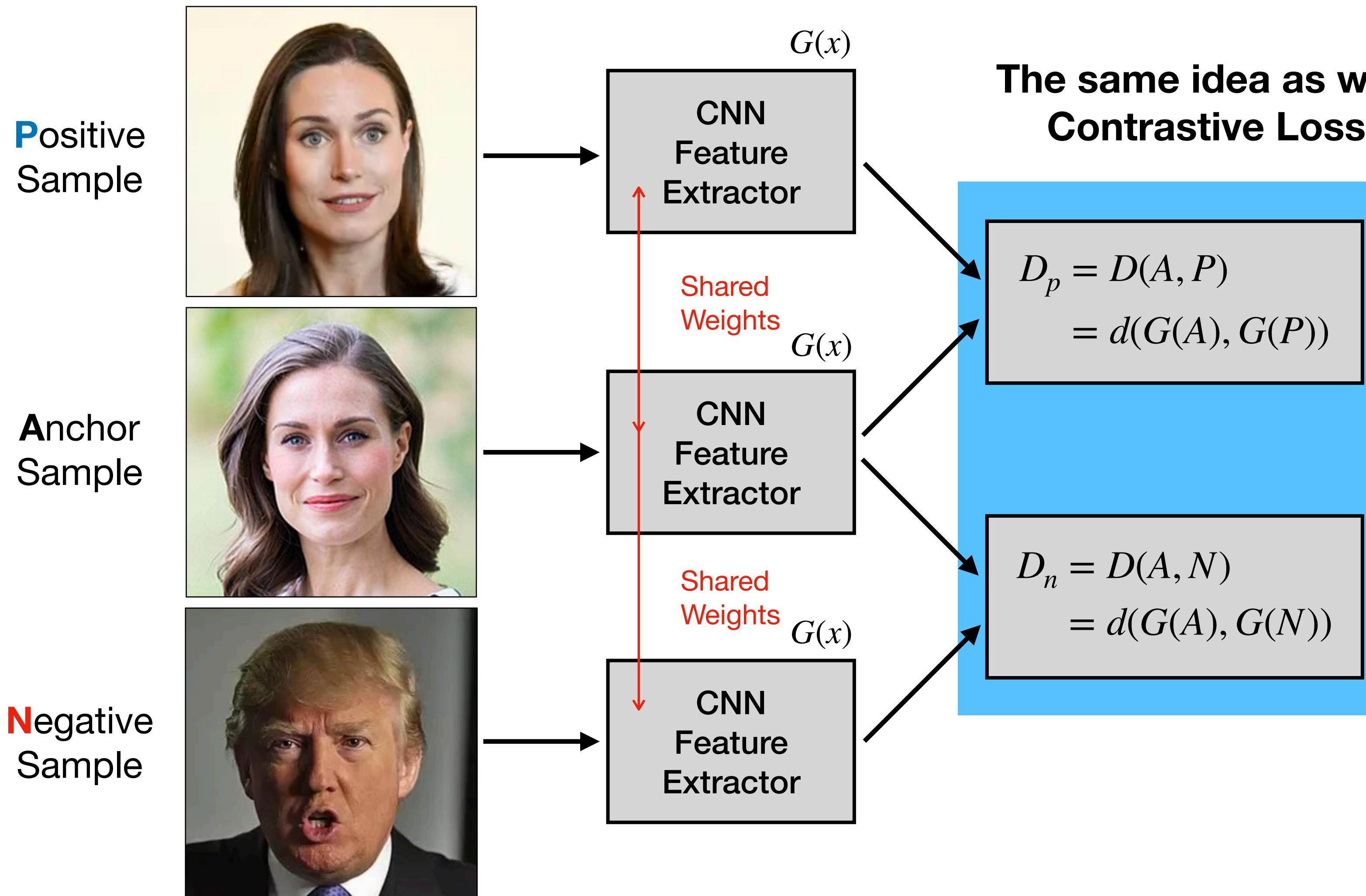
# Face Recognition

## Triplet Loss



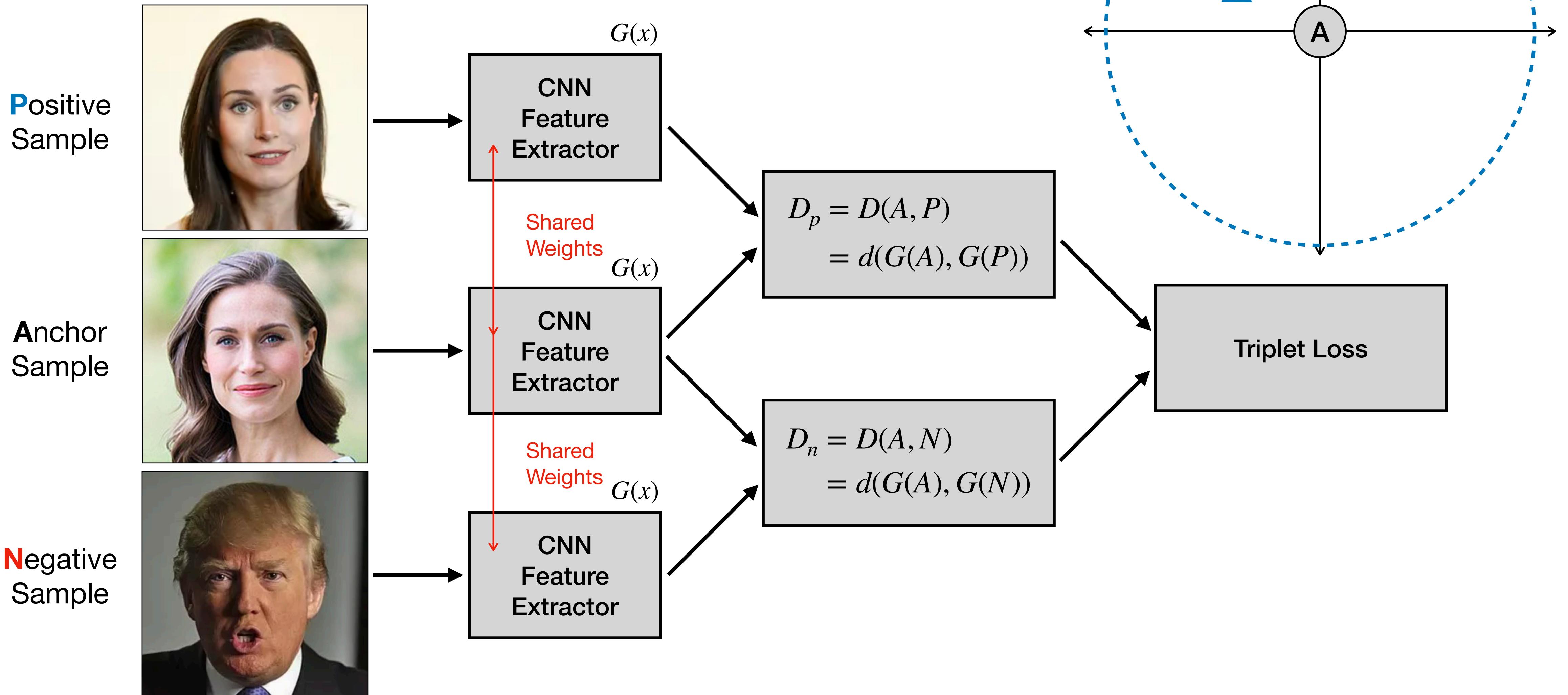
# Face Recognition

## Triplet Loss



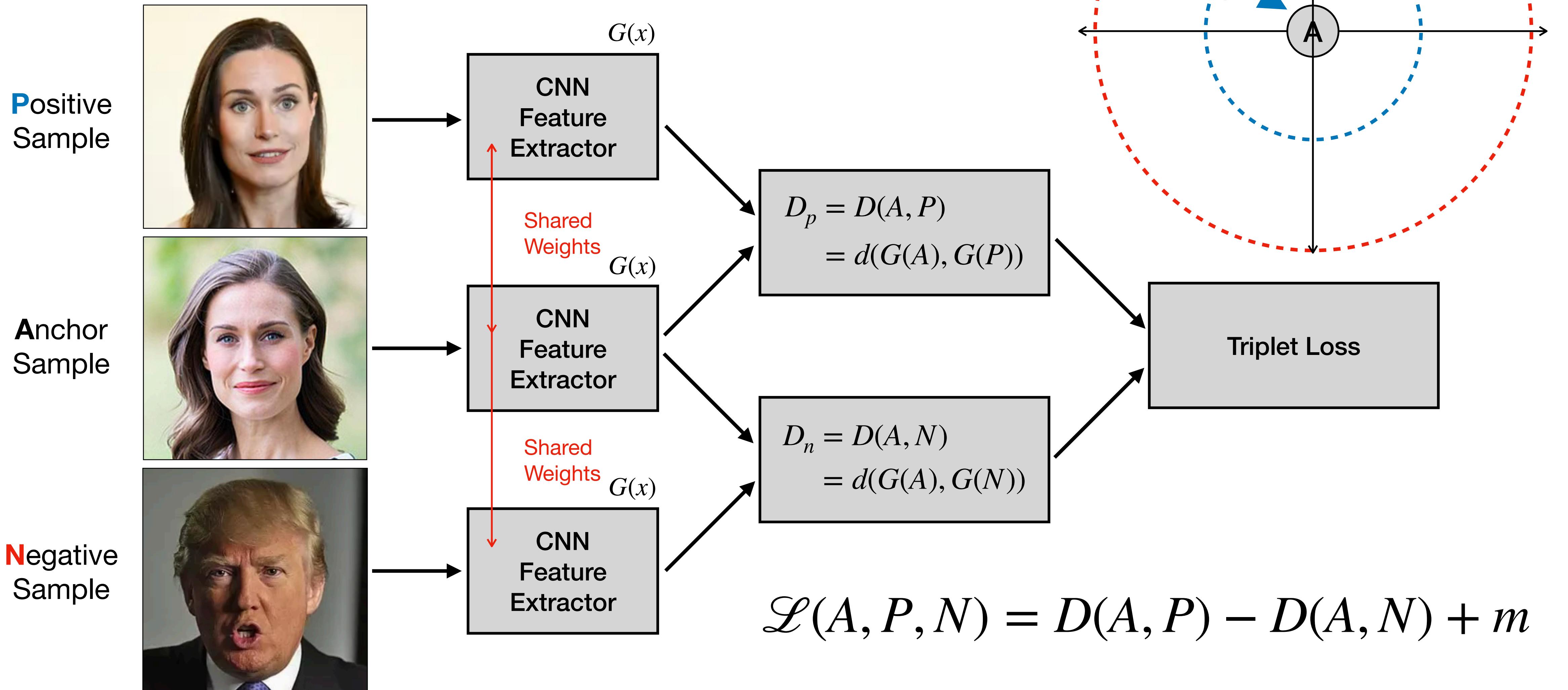
# Face Recognition

## Triplet Loss



# Face Recognition

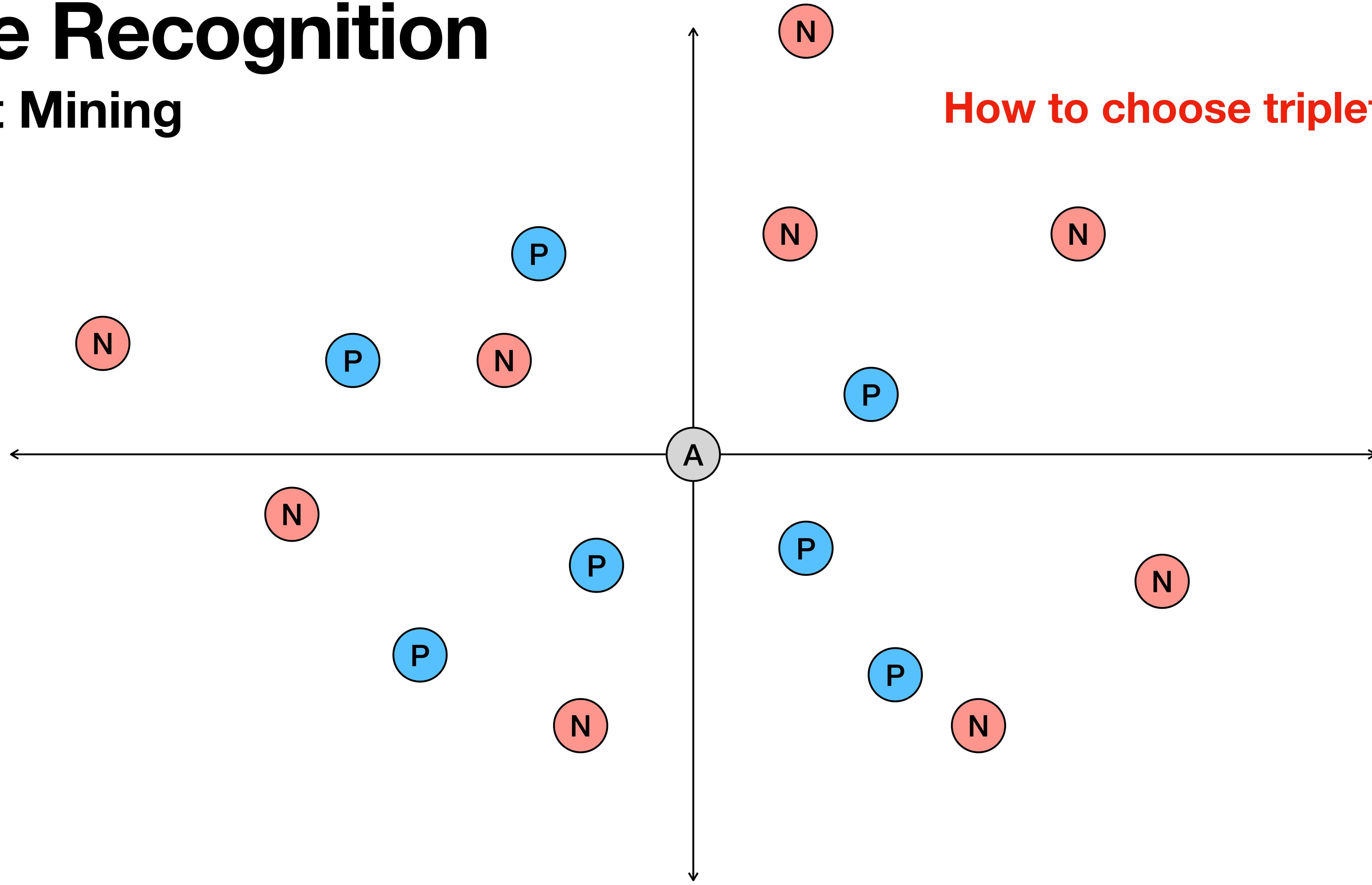
## Triplet Loss



# Face Recognition

## Triplet Mining

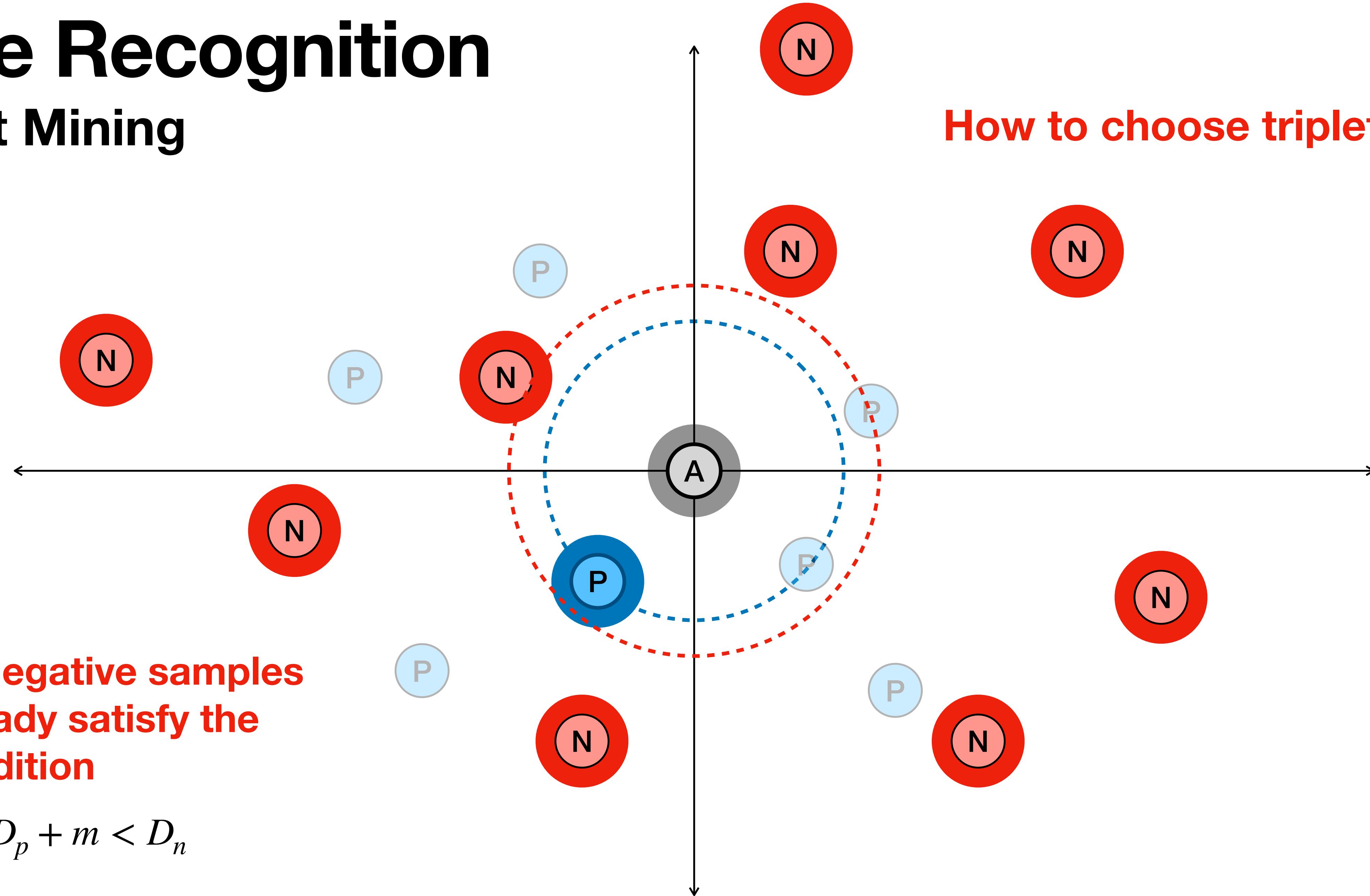
How to choose triplets ?



# Face Recognition

## Triplet Mining

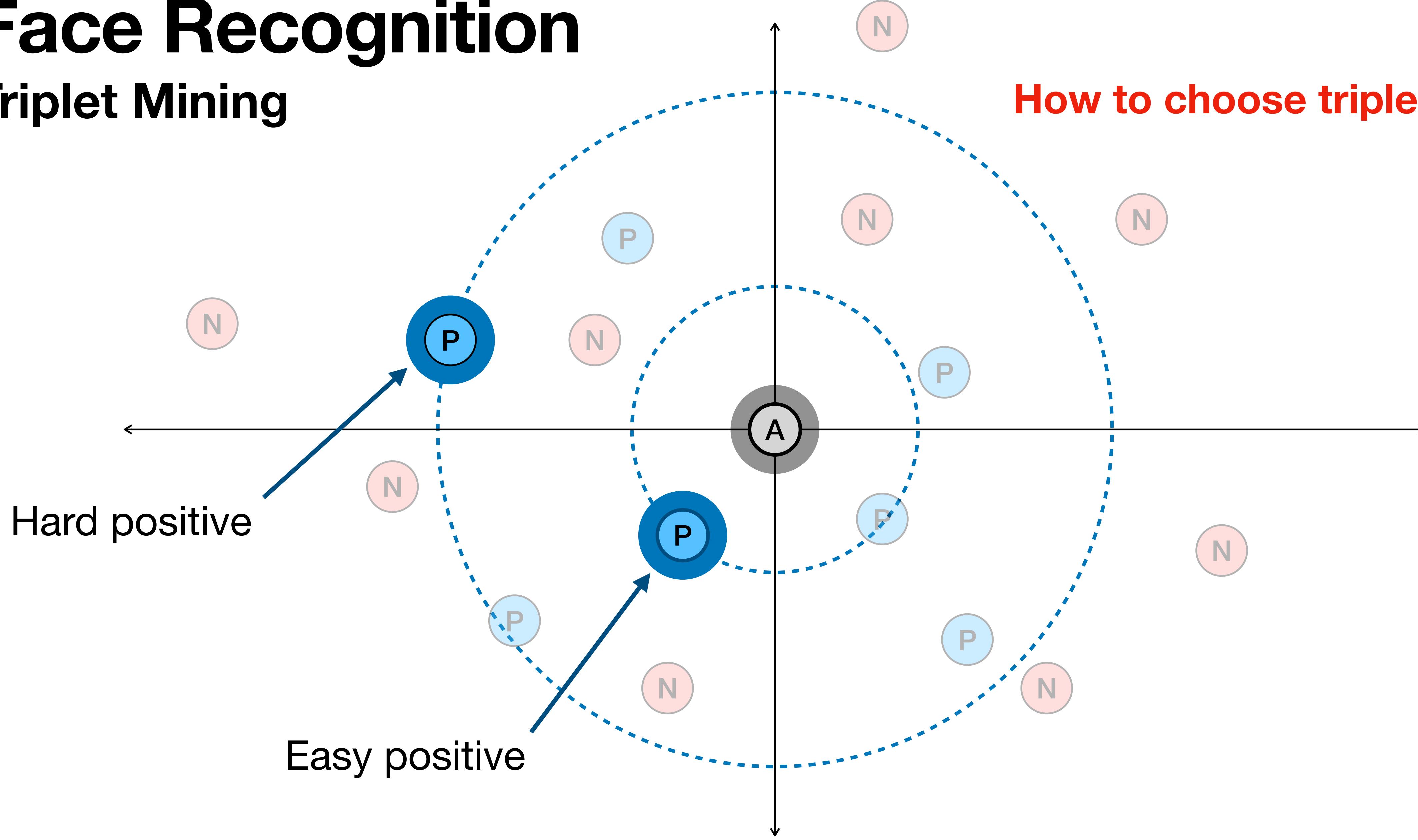
How to choose triplets ?



# Face Recognition

## Triplet Mining

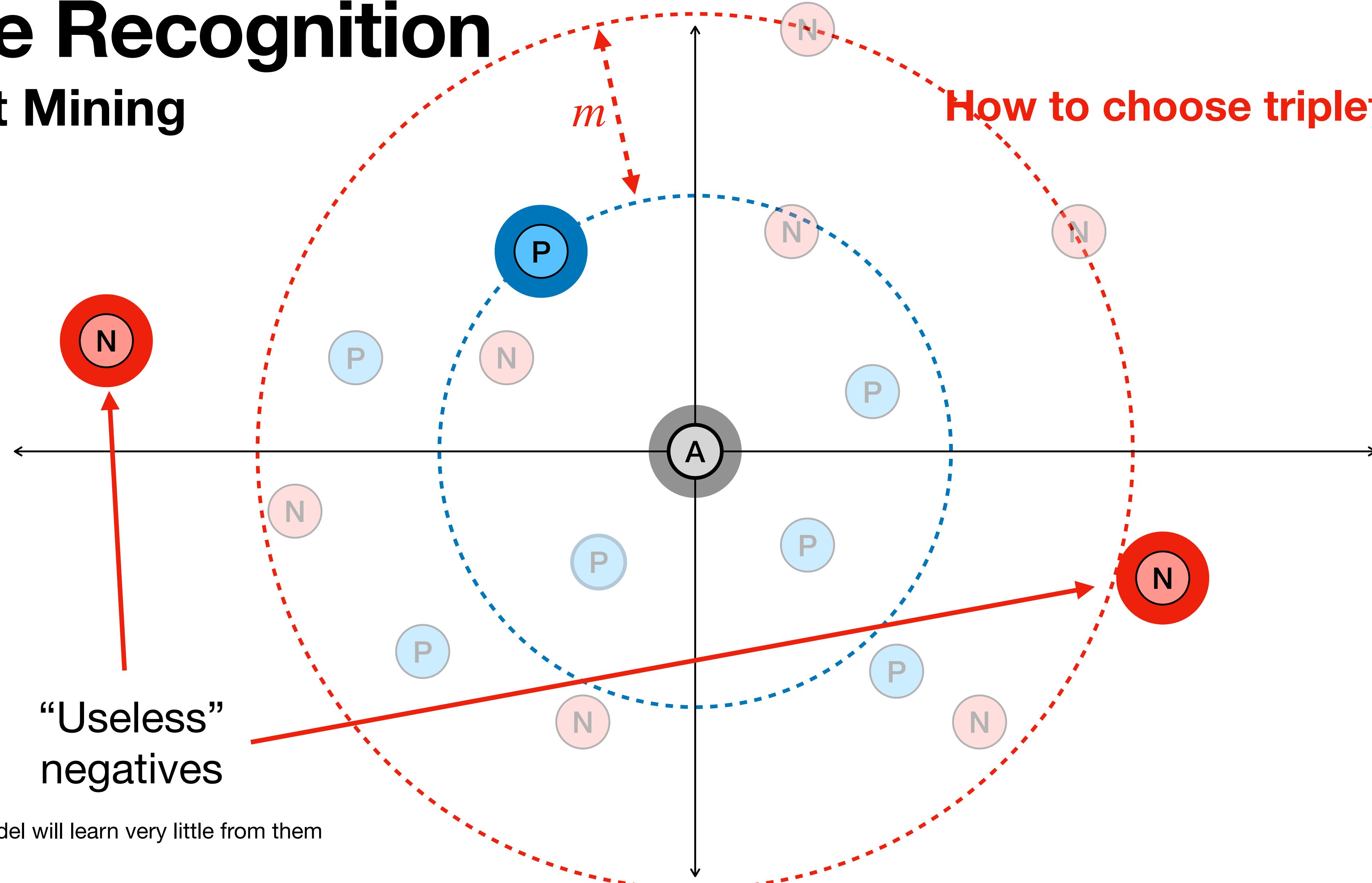
How to choose triplets ?



# Face Recognition

## Triplet Mining

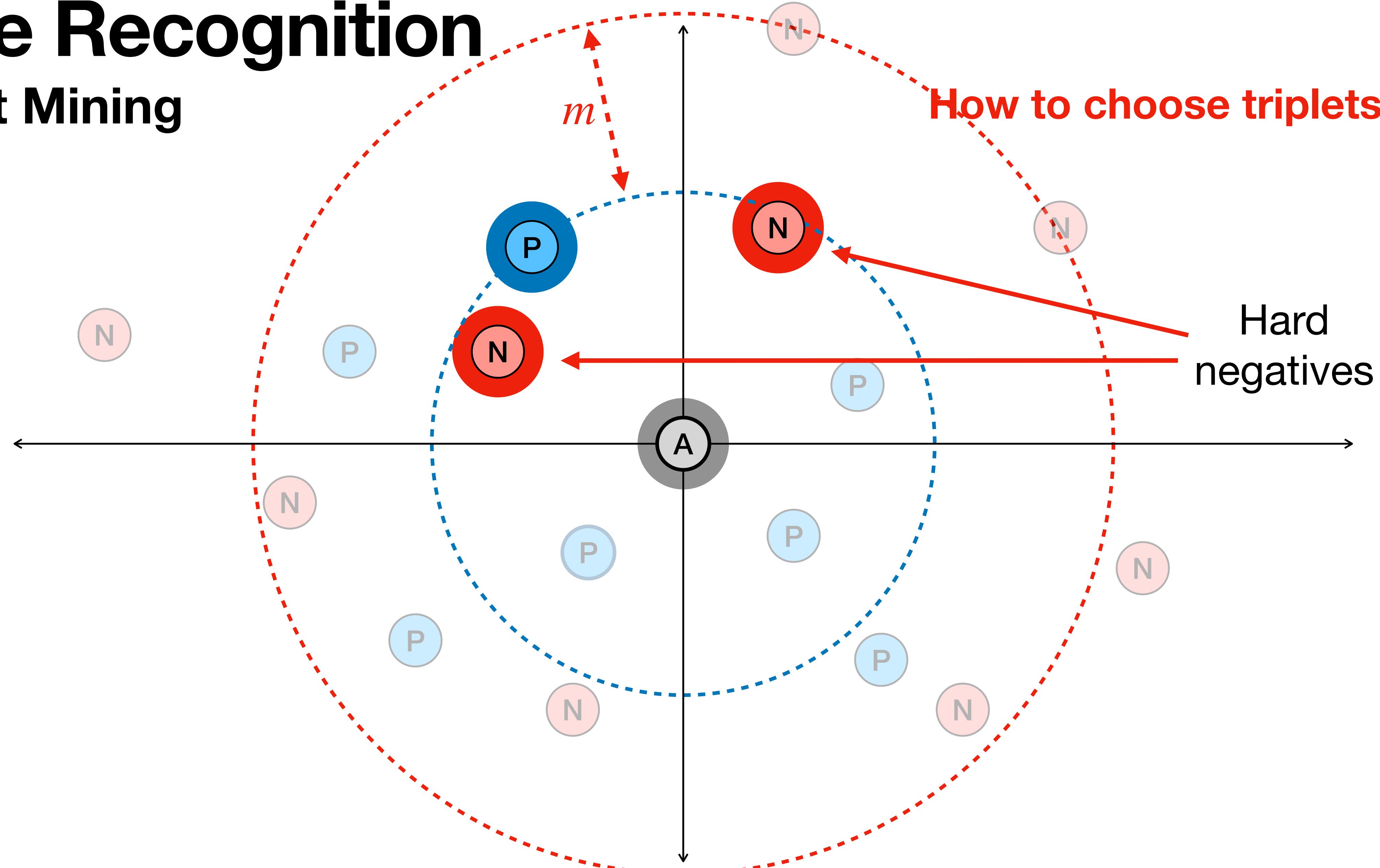
How to choose triplets ?



# Face Recognition

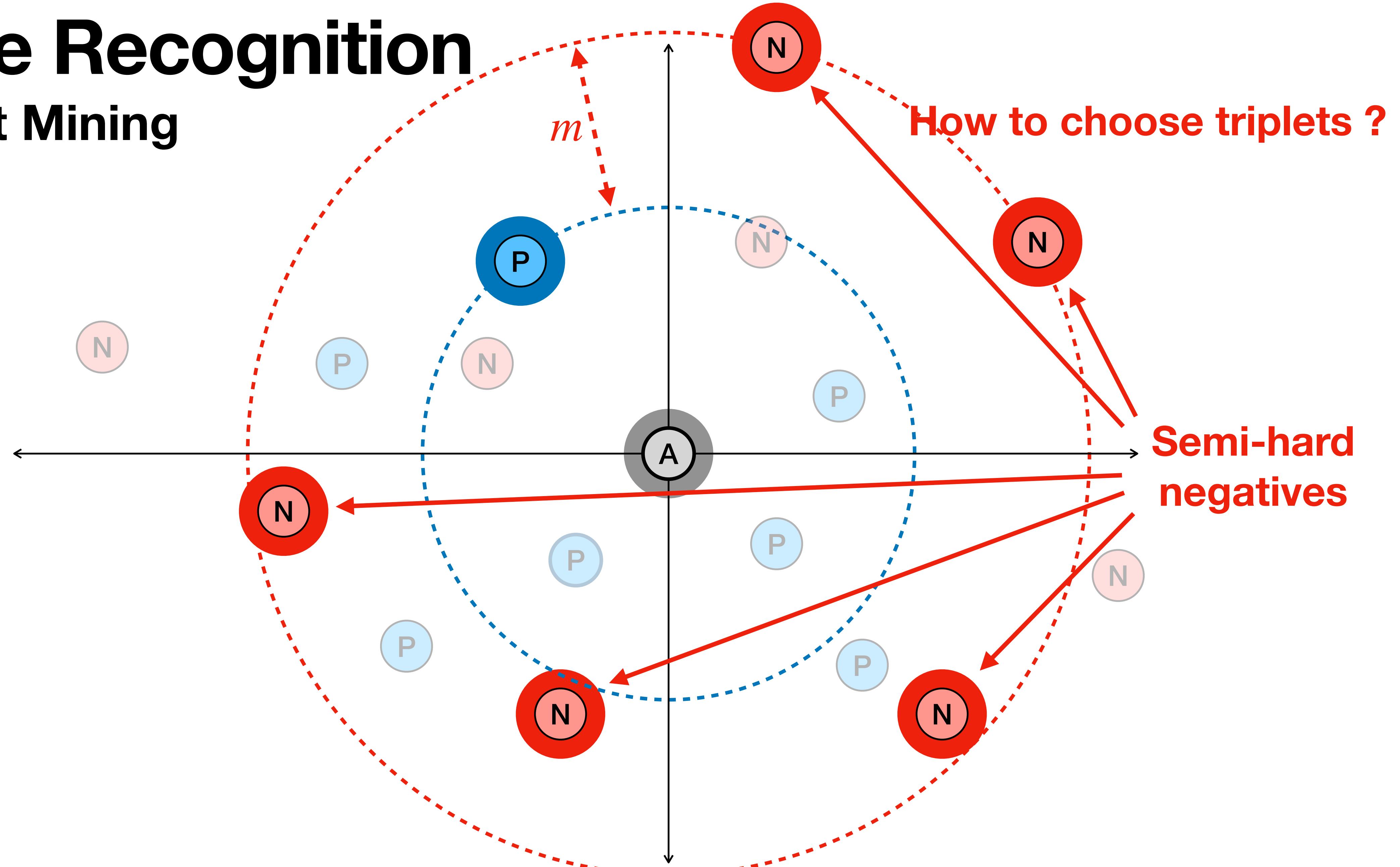
## Triplet Mining

How to choose triplets ?



# Face Recognition

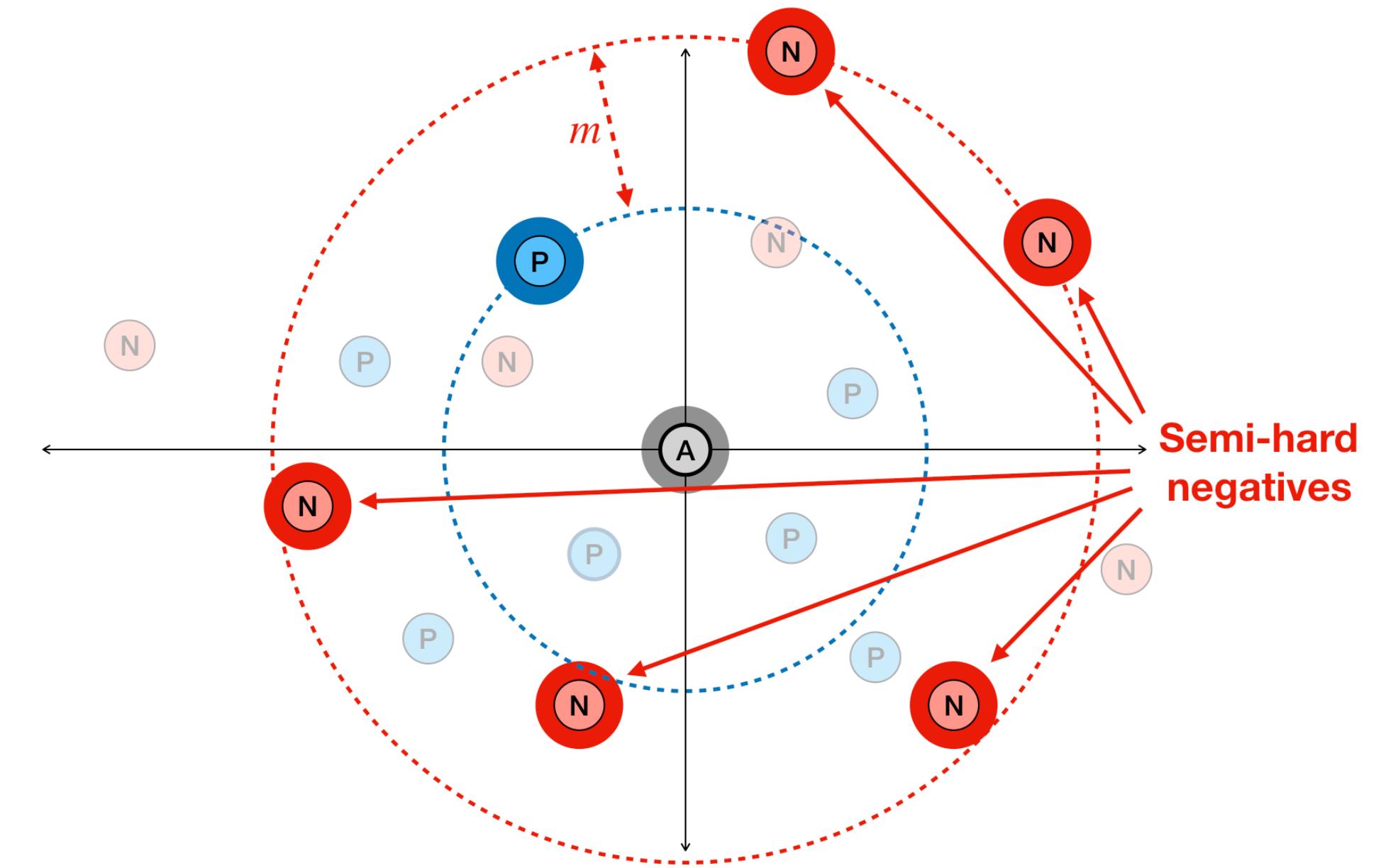
## Triplet Mining



# Face Recognition

## Triplet Mining

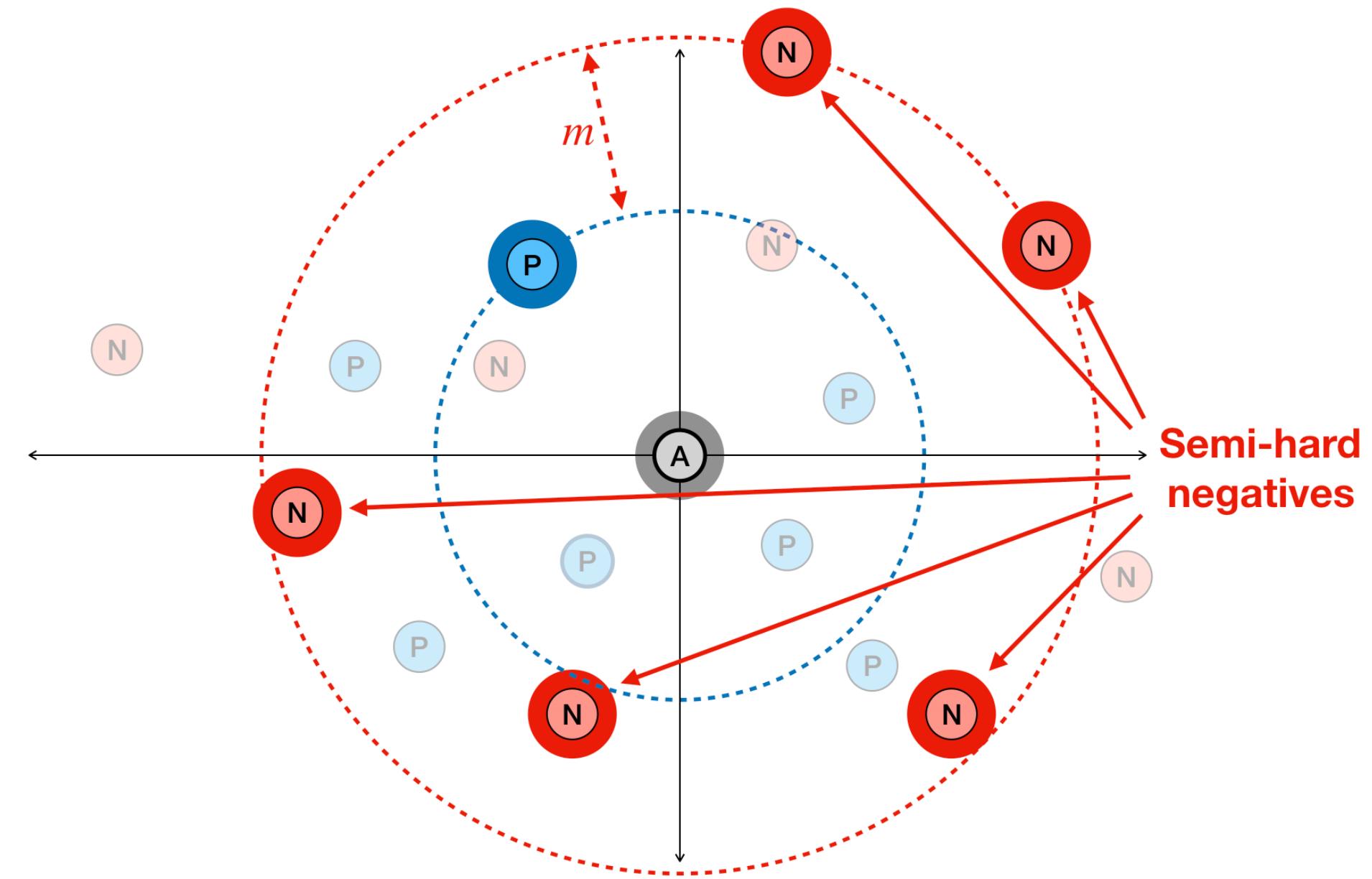
- **Offline**
  - Difficult with large datasets
  - Representations change as training progresses



# Face Recognition

## Triplet Mining

- Offline
  - Difficult with large datasets
  - Representations change as training progresses
- **Online**
  - Randomly sample a larger “mini-batch”
  - Compute distance matrix
  - Select useful triplets



# Face Recognition

## Image Batch

1. Randomly select identity

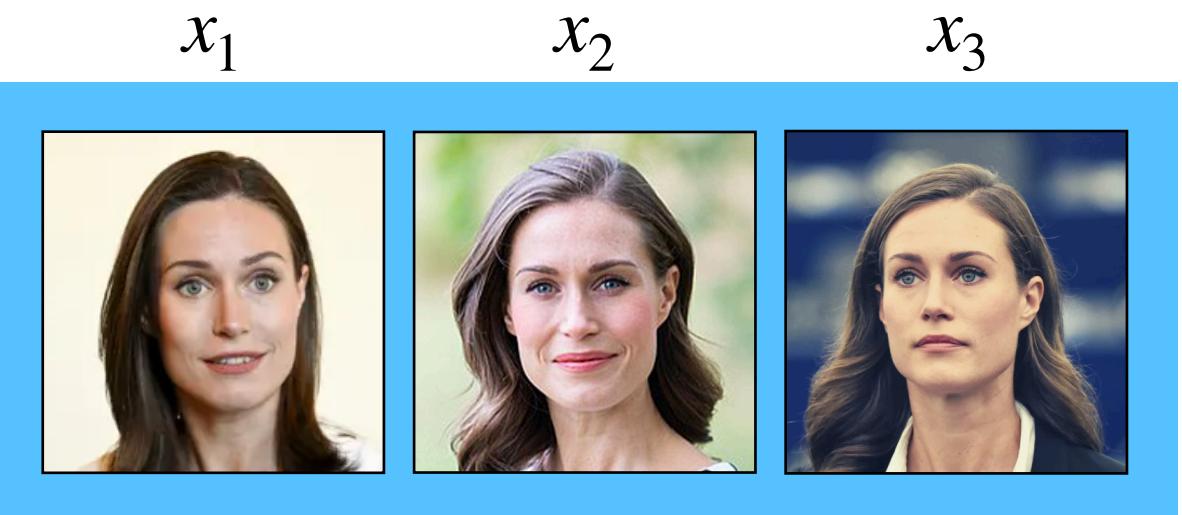
$x_1$



# Face Recognition

## Image Batch

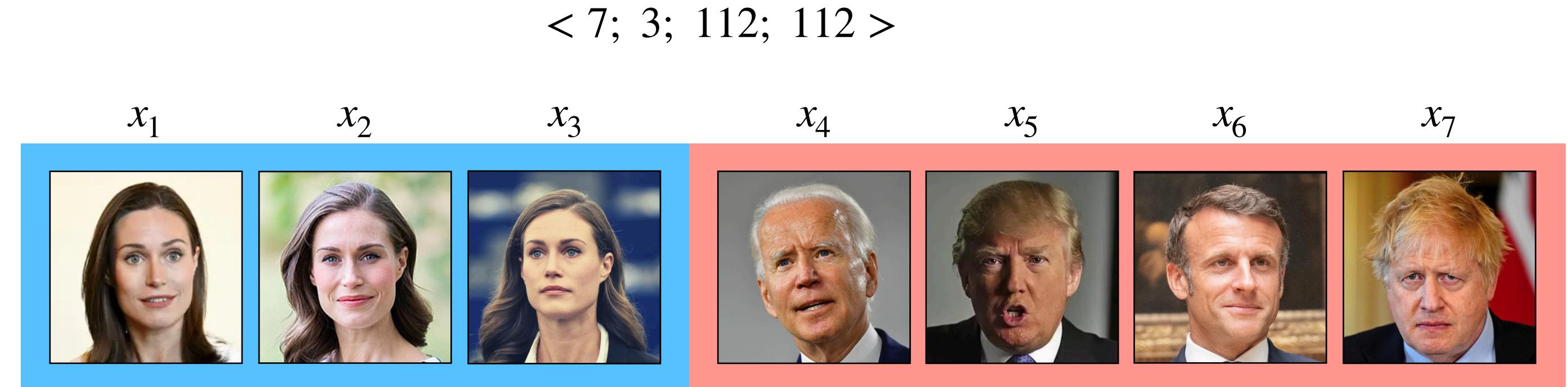
1. Randomly select identity
2. Select more samples of the same identity (positives)



# Face Recognition

## Image Batch

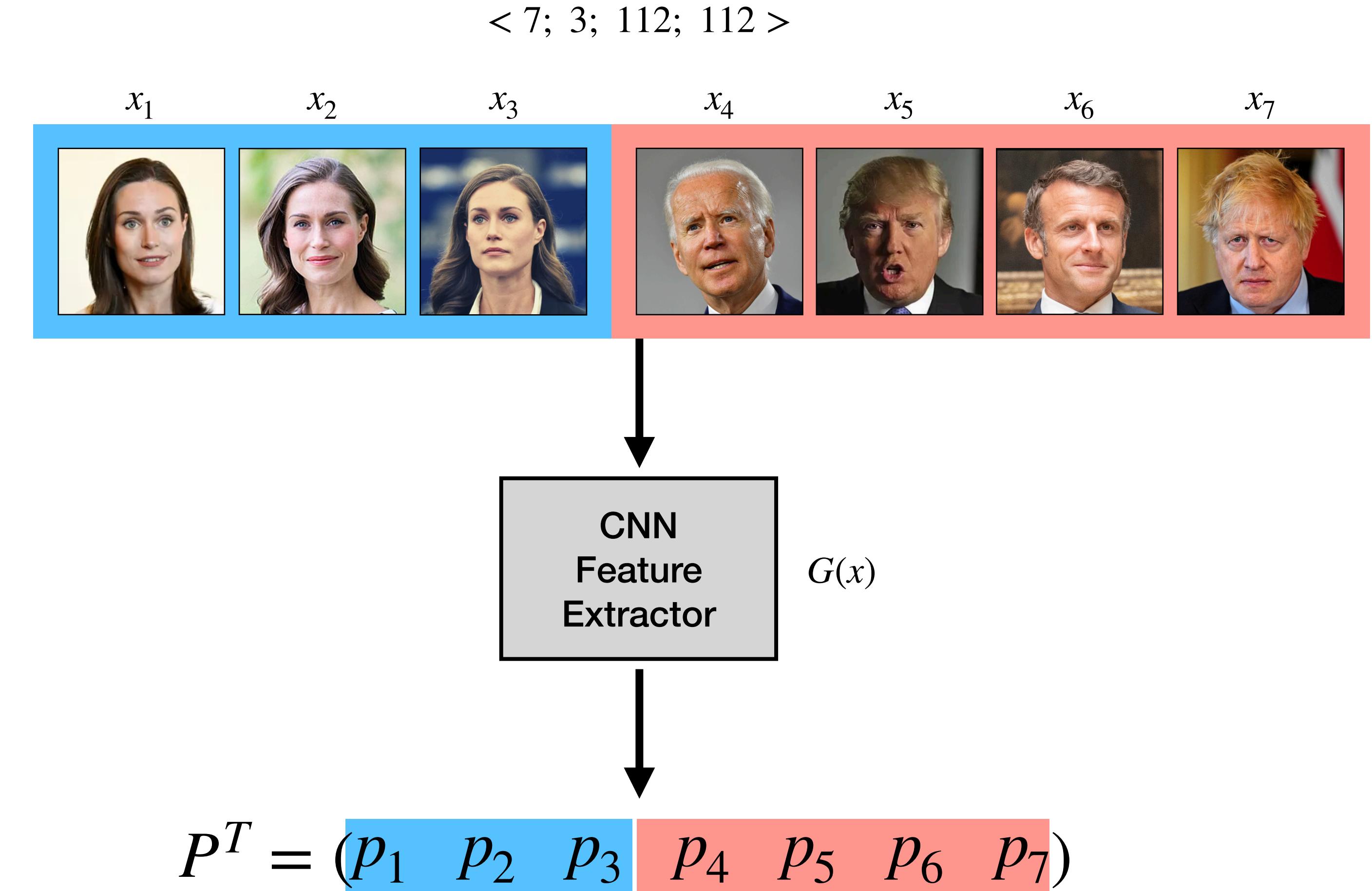
1. Randomly select identity
2. Select more samples of the same identity (positives)
3. Fill the rest of the batch with random negative samples



# Face Recognition

## Image Batch

1. Randomly select identity
2. Select more samples of the same identity (positives)
3. Fill the rest of the batch with random negative samples
4. Project into lower-dimension representation



Projected samples live on the  
surface of a hyper-sphere

$$\|p_i\| = 1$$

$< 7; 384 >$

# Face Recognition

## Distance Matrix

$$\|p_i\| = 1$$

< 7; 384 >

$$P^T = (p_1 \ p_2 \ p_3 \ | \ p_4 \ p_5 \ p_6 \ p_7)$$

1. Compute 2D matrix with mutual distances of all elements

$$D = \begin{bmatrix} d(p_1, p_1) & d(p_1, p_2) & d(p_1, p_3) & \dots & d(p_1, p_7) \\ d(p_2, p_1) & d(p_2, p_2) & d(p_2, p_3) & \dots & d(p_2, p_7) \\ d(p_3, p_1) & d(p_3, p_2) & d(p_3, p_3) & \dots & d(p_3, p_7) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ d(p_7, p_1) & d(p_7, p_2) & d(p_7, p_3) & \dots & d(p_7, p_7) \end{bmatrix}$$

**Distances between positive samples** →

$$\|p_i\| = 1$$

# Face Recognition Distance Matrix

< 7; 384 >

$$P^T = (p_1 \ p_2 \ p_3 \ | \ p_4 \ p_5 \ p_6 \ p_7)$$

1. Compute 2D matrix with mutual distances of all elements

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If we are using cosine distance

$$d(a, b) = 1 - a \cdot b$$

$$\|p_i\| = 1$$

# Face Recognition Distance Matrix

< 7; 384 >

$$P^T = (p_1 \ p_2 \ p_3 \ | \ p_4 \ p_5 \ p_6 \ p_7)$$

1. Compute 2D matrix with mutual distances of all elements

$$D = 1 - \begin{bmatrix} p_1p_1 & p_1p_2 & p_1p_3 & \cdots & p_1p_7 \\ p_2p_1 & p_2p_2 & p_2p_3 & \cdots & p_2p_7 \\ p_3p_1 & p_3p_2 & p_3p_3 & \cdots & p_3p_7 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ p_7p_1 & p_7p_2 & p_7p_3 & \cdots & p_7p_7 \end{bmatrix}$$



# Face Recognition

## Distance Matrix

$$\|p_i\| = 1$$

$< 7; 384 >$

$$P^T = (p_1 \ p_2 \ p_3 \ | \ p_4 \ p_5 \ p_6 \ p_7)$$

1. Compute 2D matrix with mutual distances of all elements

$$D = 1 - \begin{bmatrix} p_1p_1 & p_1p_2 & p_1p_3 & \cdots & p_1p_7 \\ p_2p_1 & p_2p_2 & p_2p_3 & \cdots & p_2p_7 \\ p_3p_1 & p_3p_2 & p_3p_3 & \cdots & p_3p_7 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ p_7p_1 & p_7p_2 & p_7p_3 & \cdots & p_7p_7 \end{bmatrix}$$

**Cosine distance matrix**

$< 7; 7 >$

**Cosine similarity matrix**

$$= 1 - PP^T$$

$< 7; 384 > \quad < 384; 7 >$

Outer product



# Face Recognition

## Distance Matrix

$$\|p_i\| = 1$$

$< 7; 384 >$

$$P^T = (p_1 \ p_2 \ p_3 \ | \ p_4 \ p_5 \ p_6 \ p_7)$$

1. Compute 2D matrix with mutual distances of all elements

Expresses how good each of the  $p_i$  vectors is aligned with all of the others

**Cosine similarity matrix**

$$D = 1 - \begin{bmatrix} p_1p_1 & p_1p_2 & p_1p_3 & \cdots & p_1p_7 \\ p_2p_1 & p_2p_2 & p_2p_3 & \cdots & p_2p_7 \\ p_3p_1 & p_3p_2 & p_3p_3 & \cdots & p_3p_7 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ p_7p_1 & p_7p_2 & p_7p_3 & \cdots & p_7p_7 \end{bmatrix}$$

**Cosine distance matrix**

$< 7; 7 >$

$$= 1 - PP^T$$

$< 7; 384 > \quad < 384; 7 >$

Outer product

# Face Recognition

## Triplet Selection

$$m = 0.3$$

### 1. Select Anchor

D	$p_1$	$p_2$	$p_3$	$p_4$	$p_5$	$p_6$	$p_7$
$p_1$		0.3	0.9	0.4	0.6	0.7	1.4
$p_2$	0.3		0.8	0.2	0.1	0.7	0.9
$p_3$	0.9	0.8		0.1	0.4	0.6	0.8
$p_4$	0.4	0.2	0.1		0.8	0.2	0.3
$p_5$	0.6	0.1	0.4	0.8		0.3	0.4
$p_6$	0.7	0.7	0.6	0.2	0.3		0.6
$p_7$	1.4	0.9	0.8	0.3	0.4	0.6	

# Face Recognition

## Triplet Selection

$$m = 0.3$$

### 1. Select Anchor

D	$p_1$	$p_2$	$p_3$	$p_4$	$p_5$	$p_6$	$p_7$
$p_1$		0.3	0.9	0.4	0.6	0.7	1.4
$p_2$	0.3		0.8	0.2	0.1	0.7	0.9
$p_3$	0.9	0.8		0.1	0.4	0.6	0.8
$p_4$	0.4	0.2	0.1		0.8	0.2	0.3
$p_5$	0.6	0.1	0.4	0.8		0.3	0.4
$p_6$	0.7	0.7	0.6	0.2	0.3		0.6
$p_7$	1.4	0.9	0.8	0.3	0.4	0.6	

# Face Recognition

## Triplet Selection

$$m = 0.3$$

1. Select Anchor

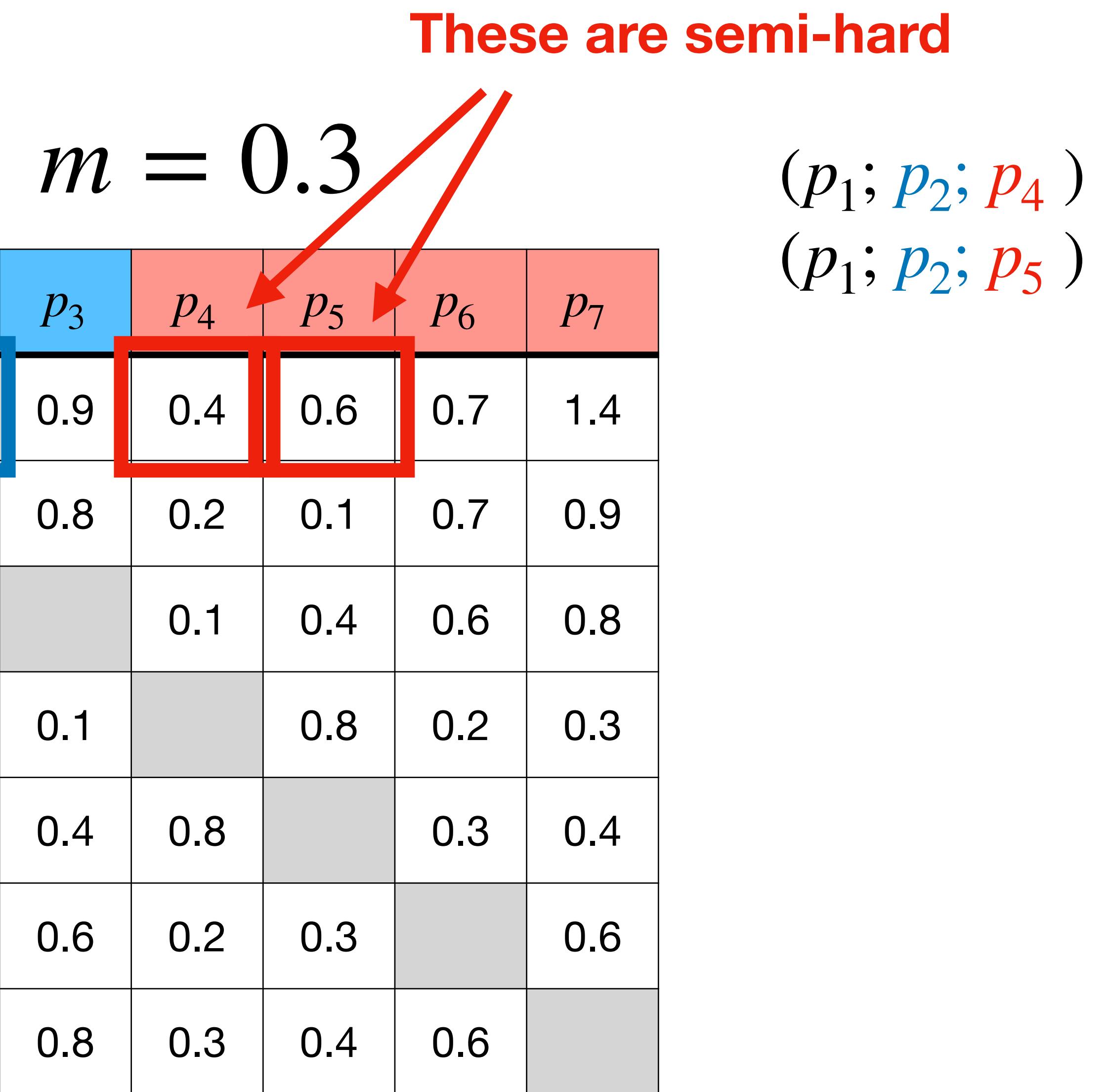
2. Combinations of Positives

D	$p_1$	$p_2$	$p_3$	$p_4$	$p_5$	$p_6$	$p_7$	
$p_1$		0.3		0.9	0.4	0.6	0.7	1.4
$p_2$	0.3		0.8	0.2	0.1	0.7	0.9	
$p_3$	0.9	0.8		0.1	0.4	0.6	0.8	
$p_4$	0.4	0.2	0.1		0.8	0.2	0.3	
$p_5$	0.6	0.1	0.4	0.8		0.3	0.4	
$p_6$	0.7	0.7	0.6	0.2	0.3		0.6	
$p_7$	1.4	0.9	0.8	0.3	0.4	0.6		

# Face Recognition

## Triplet Selection

1. Select Anchor
2. Combinations of Positives
3. Decide if proper Negatives are available



# Face Recognition

## Triplet Selection

$$m = 0.3$$

1. Select Anchor
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3. Decide if proper Negatives are available

D	$p_1$	$p_2$	$p_3$	$p_4$	$p_5$	$p_6$	$p_7$
$p_1$		0.3	0.9	0.4	0.6	0.7	1.4
$p_2$	0.3		0.8	0.2	0.1	0.7	0.9
$p_3$	0.9	0.8		0.1	0.4	0.6	0.8
$p_4$	0.4	0.2	0.1		0.8	0.2	0.3
$p_5$	0.6	0.1	0.4	0.8		0.3	0.4
$p_6$	0.7	0.7	0.6	0.2	0.3		0.6
$p_7$	1.4	0.9	0.8	0.3	0.4	0.6	

$(p_1; p_2; p_4)$   
 $(p_1; p_2; p_5)$   
 $(p_1; p_3; p_4)$   
 $(p_1; p_3; p_5)$   
 $(p_1; p_3; p_6)$

# Face Recognition

## Triplet Selection

$$m = 0.3$$

1. Select Anchor
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$p_5$	0.6	0.1	0.4	0.8		0.3	0.4
$p_6$	0.7	0.7	0.6	0.2	0.3		0.6
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$(p_1; p_2; p_4)$   
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 $(p_1; p_3; p_6)$   
 $(p_2; p_1; p_4)$   
 $(p_2; p_1; p_5)$

# Face Recognition

## Triplet Selection

$$m = 0.3$$

1. Select Anchor
2. Combinations of Positives
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$p_4$	0.4	0.2	0.1		0.8	0.2	0.3
$p_5$	0.6	0.1	0.4	0.8		0.3	0.4
$p_6$	0.7	0.7	0.6	0.2	0.3		0.6
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# Face Recognition

## Triplet Selection

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3. Decide if proper Negatives are available

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$p_5$	0.6	0.1	0.4	0.8		0.3	0.4
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$(p_1; p_2; p_4)$   
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...

# Face Recognition

## Triplet Selection

$$m = 0.3$$

1. Select Anchor
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 $(p_2; p_3; p_5)$   
 $(p_2; p_3; p_6)$   
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...

# **Are there any questions ?**