POLITECNICO
MILANO 1863

Exploration in Policy Search by Multiple Importance Sampling

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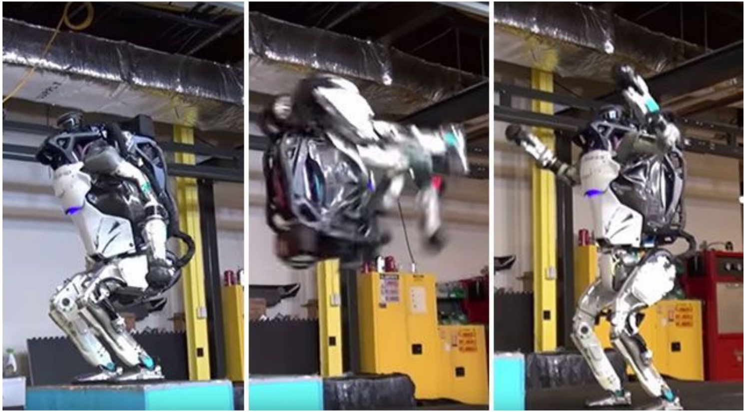
How can robots learn to backflip?



Algorithmic Trading

Self-driving Cars

Plan

1. Basics of Reinforcement Learning
2. Exploration in Policy Search
3. Problem Formalization
4. OPTIMIST

The Reinforcement Learning Framework

Environment

$\mathcal{P}, \mathcal{R}$

The Reinforcement Learning Framework

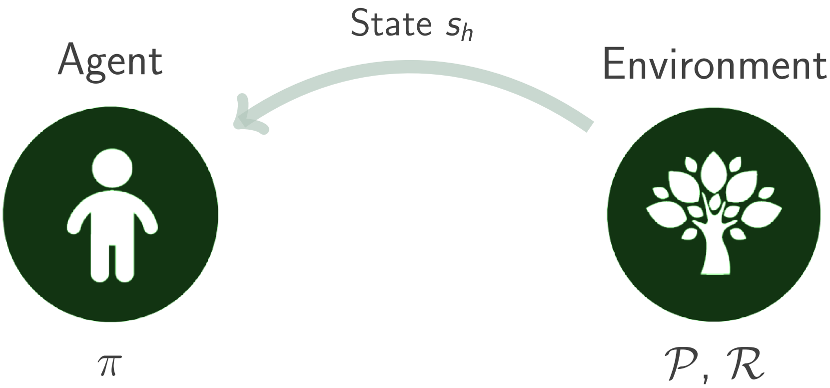
Agent

π

Environment

$\mathcal{P}, \mathcal{R}$

The Reinforcement Learning Framework



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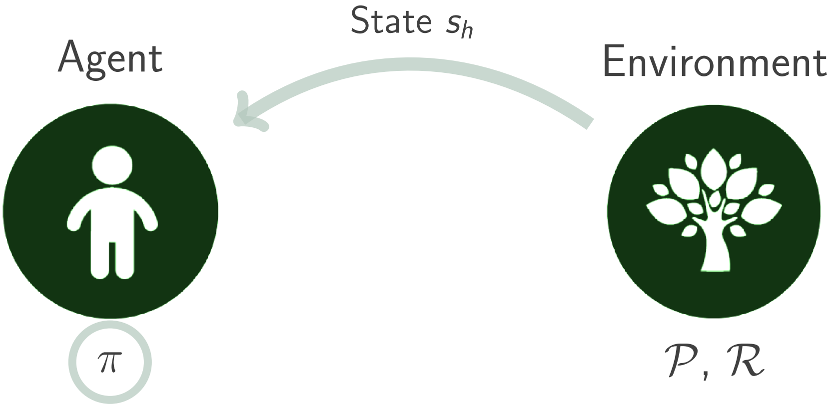

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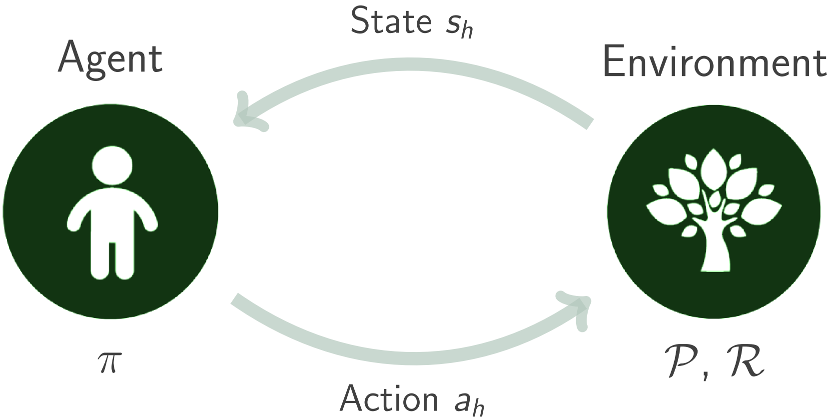

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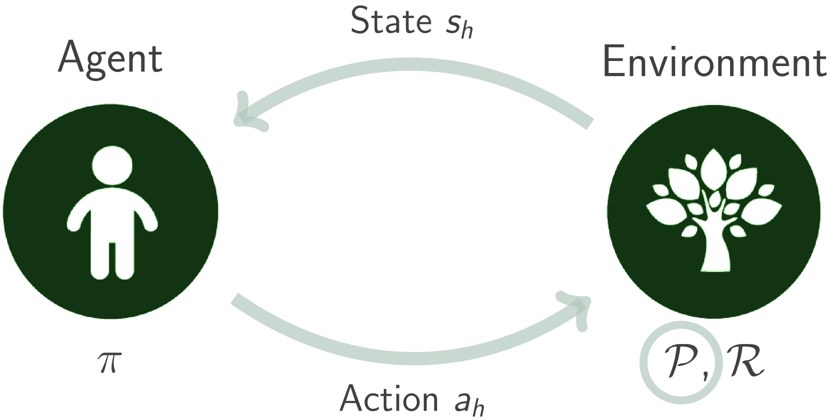

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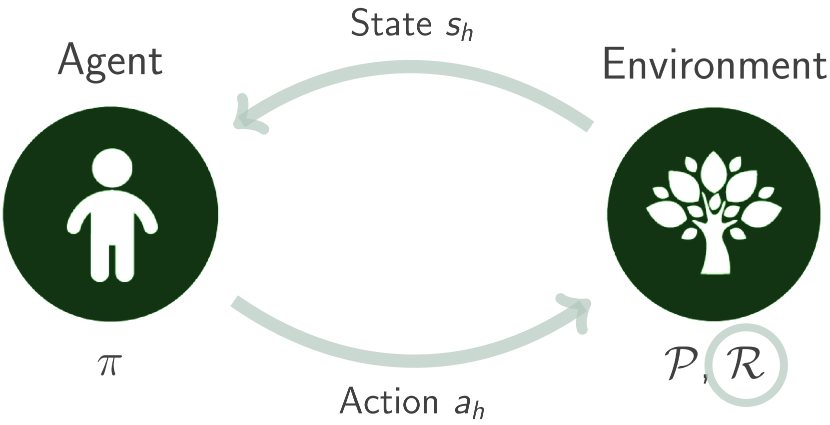

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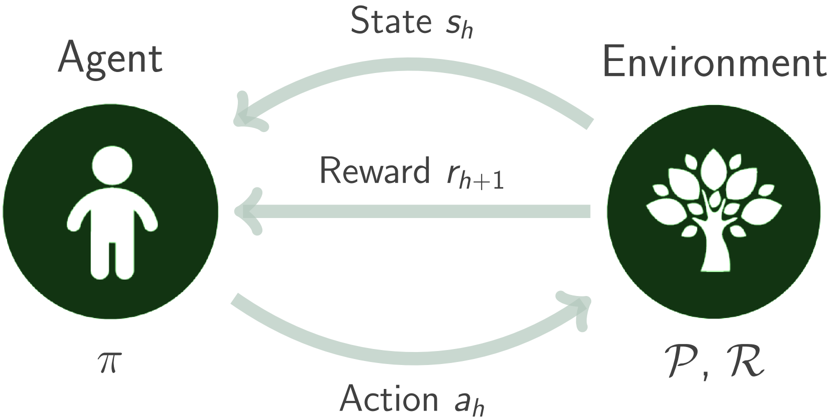

Policy Search Formulation

Cumulative return of a trajectory τ:

$$\mathcal{R}(\tau) = \sum_{h=0}^{H-1} \gamma^h r_{h+1}, \text{ with } \tau = [s_0, a_0, s_1, a_1, \ldots, s_{H-1}, a_{H-1}]$$

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Parametric policy:

$$\pi_{\boldsymbol{\theta}} : \mathcal{S} \to \Delta(\mathcal{A}), \text{ i.e., } \pi_{\boldsymbol{\theta}}(a|s) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2}\left(\frac{a - \boldsymbol{\theta}^T \phi(s)}{\sigma}\right)^2\right)$$

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Performance:

$\mu(\boldsymbol{\theta}) = \mathop{\mathbb{E}}_{\tau \sim p_{\boldsymbol{\theta}}} [\mathcal{R}(\tau)]$, where $p_{\boldsymbol{\theta}}$ is the **distribution over trajectories** $\tau \in \mathcal{T}$ induced by $\pi_{\boldsymbol{\theta}}$

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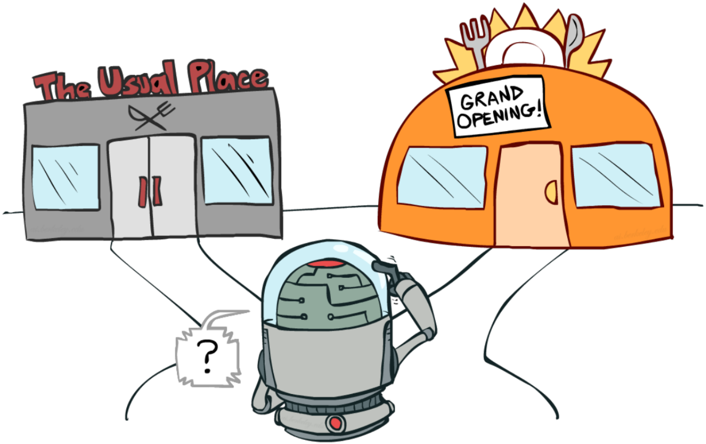
Objective:

$$\boldsymbol{\theta}^* = \arg\max_{\boldsymbol{\theta} \in \Theta} \mu(\boldsymbol{\theta}).$$

Exploration VS Exploitation



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Undirected exploration

Generate actions based on randomness, without any knowledge of the learning process.

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- Count-based techniques.

Plan

Problem Formalization

Decision Set or Arms Set

The parameter space $\Theta \subseteq \mathbb{R}^d$.

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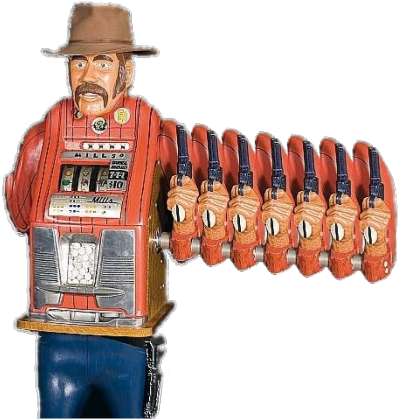
Goal

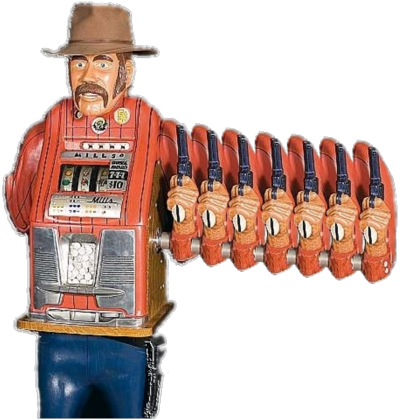
Minimize $\mathit{Regret}(T) = \sum_{t=0}^{T} \mu(\boldsymbol{\theta}^*) - \mu(\boldsymbol{\theta}_t)$, where $\boldsymbol{\theta}^* = \arg\max_{\boldsymbol{\theta} \in \Theta} \mu(\boldsymbol{\theta})$

Problem Formulation



Multi Armed Bandits

Problem Formulation



Multi Armed Bandits

- Simpler framework;

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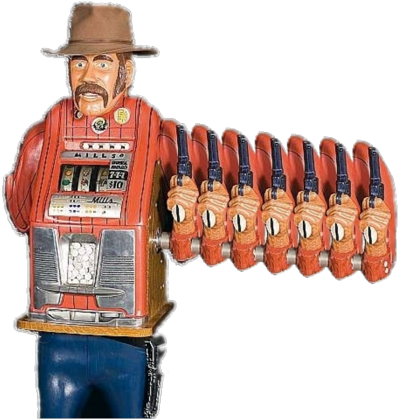

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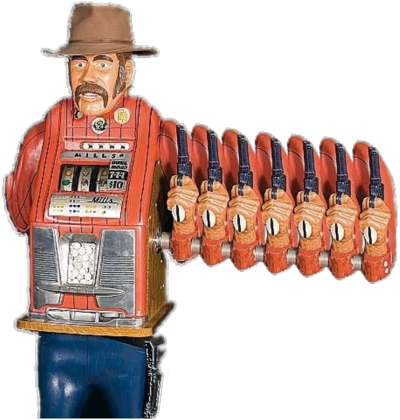

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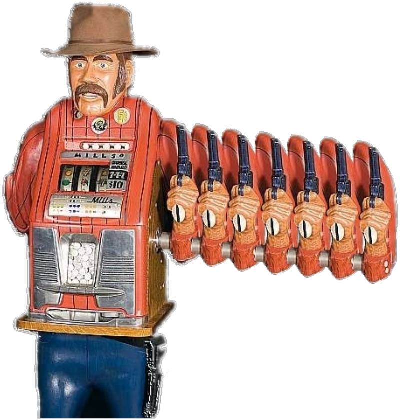

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Desideratum

sub-linear $Regret(T) \Leftrightarrow \lim_{T\to\infty} Regret(T)/T = 0$

E.g. $Regret(T) = \mathcal{O}(\log T)$

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Optimistic Policy opTImization via MIS with Truncation

Algorithm 1 OPTIMIST

1: **Input:** initial parameters $\boldsymbol{\theta}_0$, confidence schedule $(\delta_t)_{t=1}^{T}$, order $\epsilon \in (0, 1]$

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Algorithm 2 OPTIMIST

1: **Input:** initial parameters $\boldsymbol{\theta}_0$, confidence schedule $(\delta_t)_{t=1}^{T}$, order $\epsilon \in (0, 1]$

2: Draw trajectory $\tau_0 \sim p_{\boldsymbol{\theta}_0}$ and observe return $\mathcal{R}(\tau_0)$

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Algorithm 3 OPTIMIST

Input: initial parameters $\boldsymbol{\theta}_0$, confidence schedule $(\delta_t)_{t=1}^{T}$, order $\epsilon \in (0, 1]$
Draw trajectory $\tau_0 \sim p_{\boldsymbol{\theta}_0}$ and observe return $\mathcal{R}(\tau_0)$
for $t = 1, \dots, T$ **do**

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Algorithm 4 OPTIMIST

1: **Input:** initial parameters $\boldsymbol{\theta}_0$, confidence schedule $(\delta_t)_{t=1}^{T}$, order $\epsilon \in (0, 1]$
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3: **for** $t = 1, \dots, T$ **do**
4: Select arm $\boldsymbol{\theta}_t = \arg\max_{\boldsymbol{\theta} \in \Theta} B_t^{\epsilon}(\boldsymbol{\theta}, \delta_t)$

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5: Draw trajectory $\tau_t \sim p_{\boldsymbol{\theta}_t}$ and observe return $\mathcal{R}(\tau_t)$
6: **end for**

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Algorithm 6 OPTIMIST

1: **Input:** initial parameters $\boldsymbol{\theta}_0$, confidence schedule $(\delta_t)_{t=1}^{T}$, order $\epsilon \in (0, 1]$
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6: **end for**

$$\mu(\boldsymbol{\theta}) \leqslant B_t^{\epsilon}(\boldsymbol{\theta}, \delta_t) \;=\; \check{\mu}_t^{MIS}(\boldsymbol{\theta}) \;+\; \|f\|_{\infty} \left(\sqrt{2} + \frac{4}{3}\right) \left(\frac{d_{1+\epsilon}(p_{\boldsymbol{\theta}_t} \| \Phi_t) \;\log \frac{1}{\delta_t}}{t} \right)^{\frac{\epsilon}{1+\epsilon}}$$

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Upper Confidence Bound

$$\boxed{\mu(\boldsymbol{\theta}) \leqslant B_t^{\epsilon}(\boldsymbol{\theta}, \delta_t)} = \check{\mu}_t^{MIS}(\boldsymbol{\theta}) + \|f\|_{\infty} \left(\sqrt{2} + \frac{4}{3}\right) \left(\frac{d_{1+\epsilon}(p_{\boldsymbol{\theta}_t} \| \Phi_t) \ \log \frac{1}{\delta_t}}{t} \right)^{\frac{\epsilon}{1+\epsilon}}$$

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Input: initial parameters $\boldsymbol{\theta}_0$, confidence schedule $(\delta_t)_{t=1}^{T}$, order $\epsilon \in (0,1]$
Draw trajectory $\tau_0 \sim p_{\boldsymbol{\theta}_0}$ and observe return $\mathcal{R}(\tau_0)$
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end for

Truncated Multiple Importance Sampling Estimator

$$\mu(\boldsymbol{\theta}) \leqslant B_t^{\epsilon}(\boldsymbol{\theta}, \delta_t) = \boxed{\check{\mu}_t^{MIS}(\boldsymbol{\theta})} + \|f\|_\infty \left(\sqrt{2} + \frac{4}{3}\right) \left(\frac{d_{1+\epsilon}(p_{\boldsymbol{\theta}_t} \| \Phi_t) \log \frac{1}{\delta_t}}{t}\right)^{\frac{\epsilon}{1+\epsilon}}$$

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Exploration Bonus

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Thank you for your attention!

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