POLITECNICO
MILANO 1863

Exploration in Policy Search by Multiple Importance Sampling

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Co-supervisors: Matteo Papini, Alberto Maria Metelli

April 16th, 2019

Reinforcement Learning

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Exploitation VS Exploration

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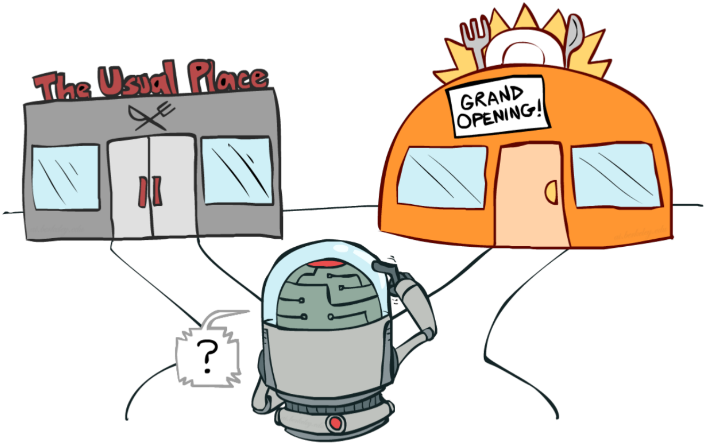

Plan

The Reinforcement Learning Framework

Environment

$\mathcal{P}$

The Reinforcement Learning Framework

Agent

π

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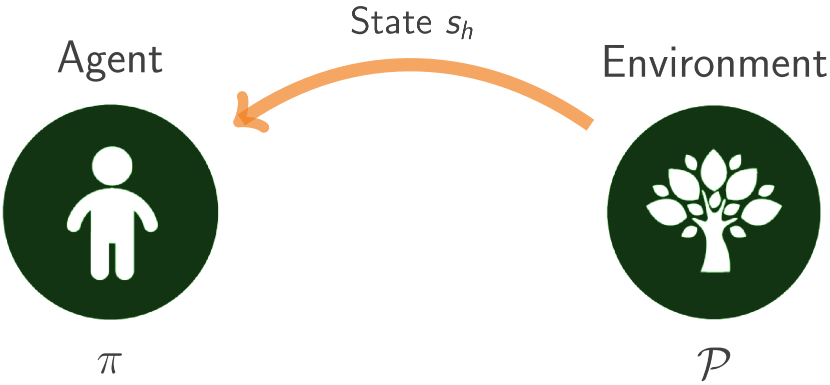
The Reinforcement Learning Framework



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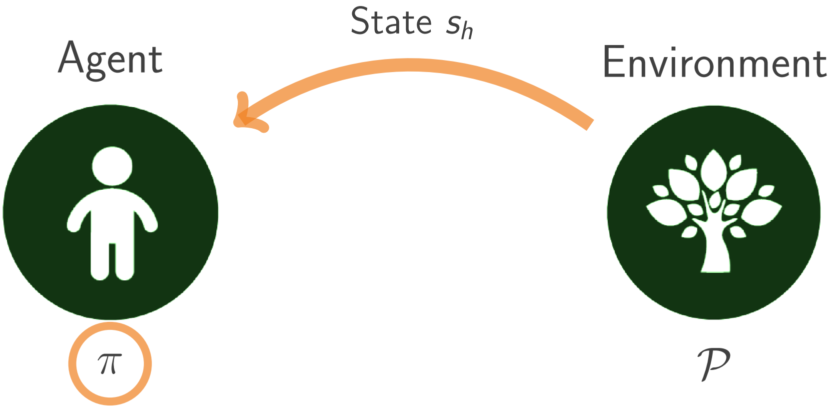

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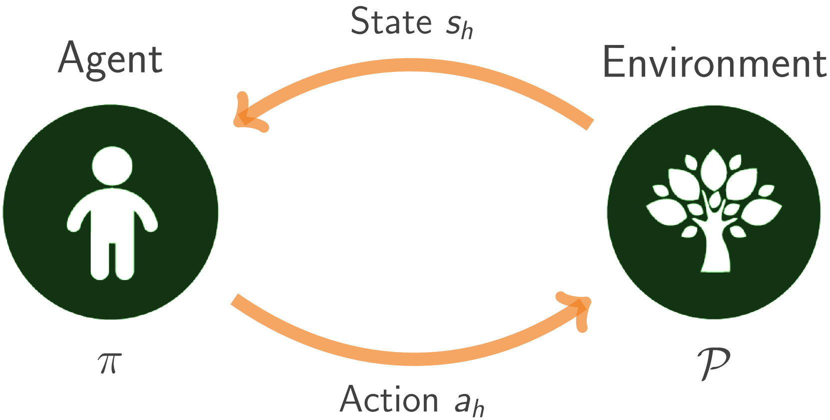

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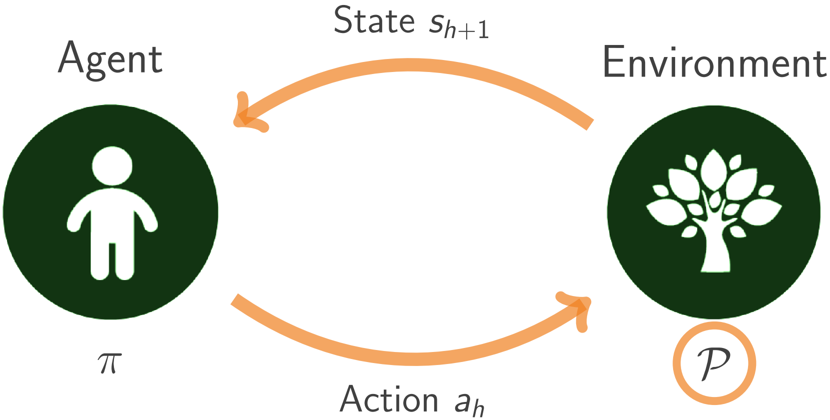

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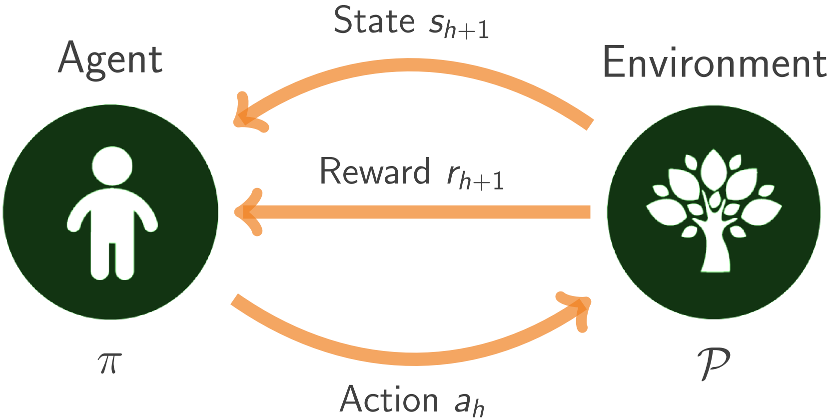

Policy Search Formulation

Parametric policy:

$$\pi_{\boldsymbol{\theta}} : \mathcal{S} \to \Delta(\mathcal{A}) \qquad \text{E.g.: } \pi_{\boldsymbol{\theta}}(a|s) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2}\left(\frac{a - \boldsymbol{\theta}^T s}{\sigma}\right)^2\right)$$

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$$\mu(\boldsymbol{\theta}) = \mathop{\mathbb{E}}_{\tau \sim \pi_{\boldsymbol{\theta}}} [\mathcal{R}(\tau)]$$

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$$\boldsymbol{\theta}^* = \arg\max_{\boldsymbol{\theta} \in \Theta} \mu(\boldsymbol{\theta}).$$

Plan

1. Basics of Reinforcement Learning
2. Exploration in Policy Search
3. Problem Formalization
4. OPTIMIST
5. Experiments
6. Conclusions

Exploration in Policy Search

Undirected exploration

Explore actions based on randomness, without any knowledge of the learning process.

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- E.g.: Count-based techniques [Bellemare et al., 2016].

Plan

Problem Formalization

Decision Set

The parameter space $\Theta \subseteq \mathbb{R}^d$.

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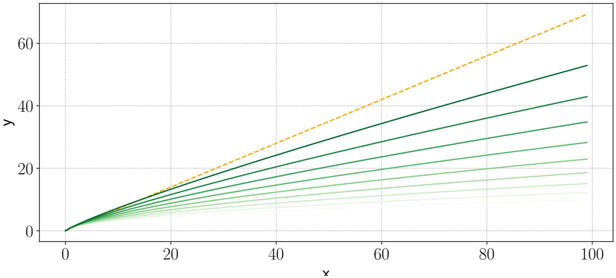
Goal

Minimize $\mathit{Regret}(T) = \sum_{t=0}^{T} \mu(\boldsymbol{\theta}^*) - \mu(\boldsymbol{\theta}_t), \quad \text{where } \boldsymbol{\theta}^* = \arg\max_{\boldsymbol{\theta} \in \Theta} \mu(\boldsymbol{\theta})$

Desideratum

Sub-linear Regret

1. for any linear function $f(T)$, for sufficiently large input T, $Regret(T)$ grows slower than $f(T)$;



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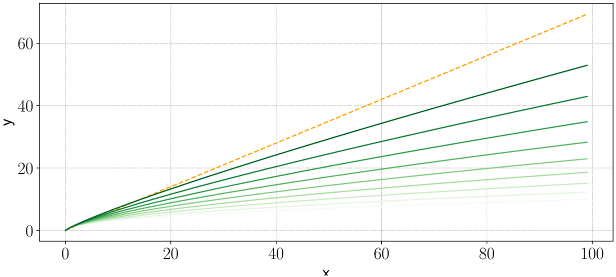

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1. for any linear function $f(T)$, for sufficiently large input T, $\mathit{Regret}(T)$ grows slower than $f(T)$;
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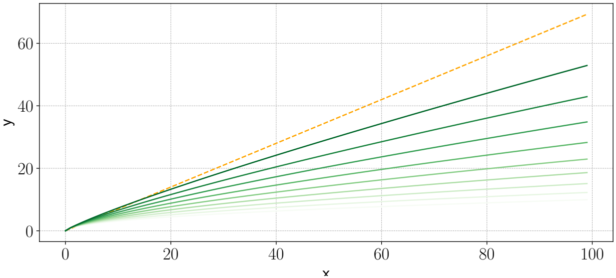
3. **Meaning**: after a certain number of iterations, the policy keeps improving.



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Optimistic Policy opTImization via Multiple Importance Sampling with Truncation

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Truncated Multiple Importance Sampling Estimator

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OPTIMIST

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Regret Analysis - Discrete Arms Set

Theorem 1 - Discrete Parameter Space:

$$Regret(T) \leqslant \Delta_0 + C_1\sqrt{T\left[v_1\left(2\log T + \log\frac{2\pi^2}{3\delta}\right)\right]}$$

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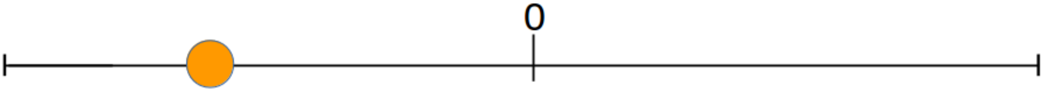
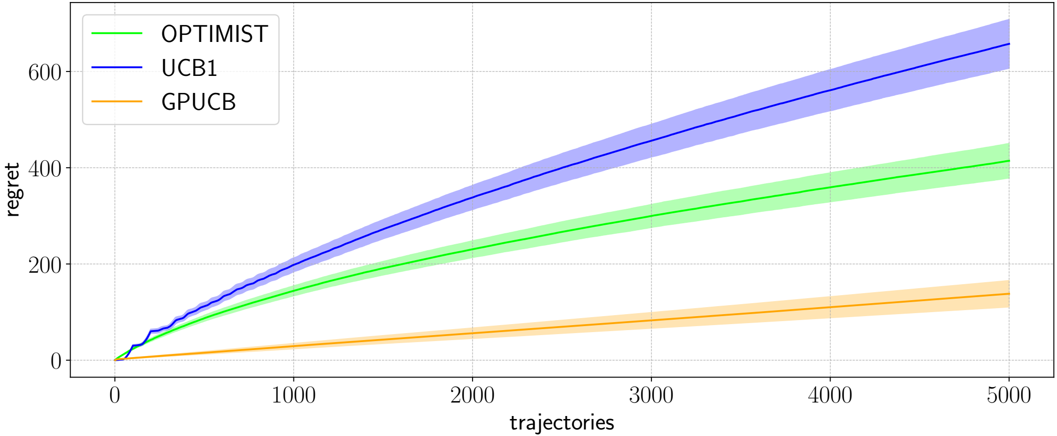
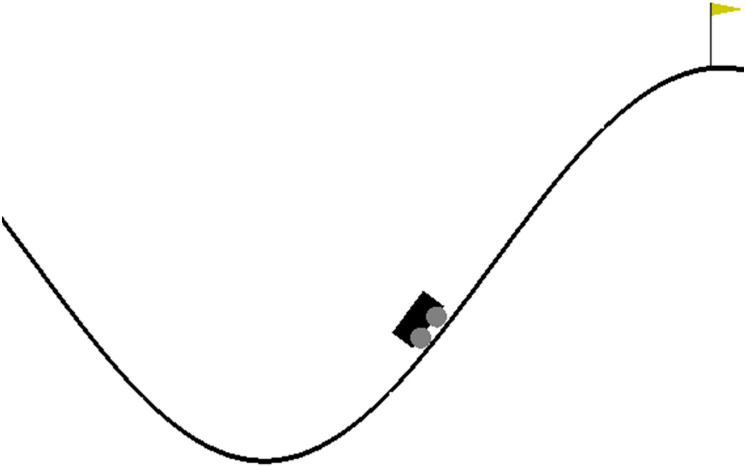
Linear Quadratic Gaussian Regulator



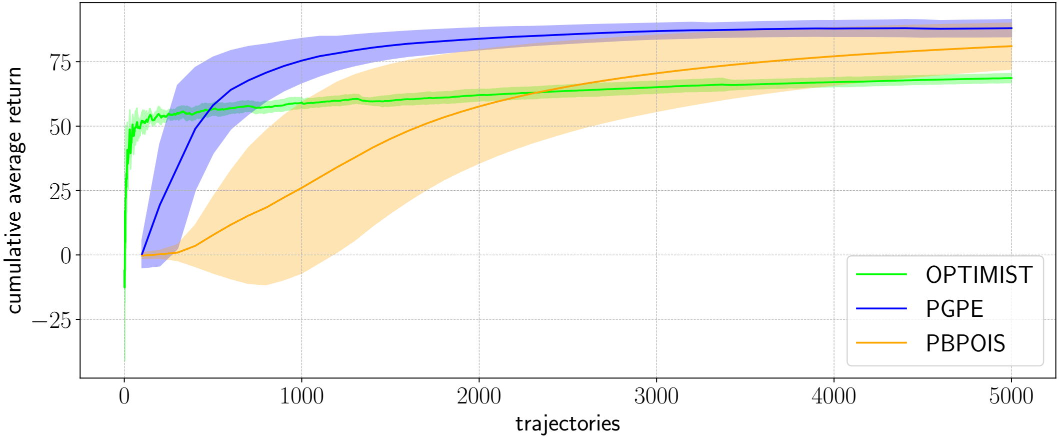
Linear Quadratic Gaussian Regulator - Regret



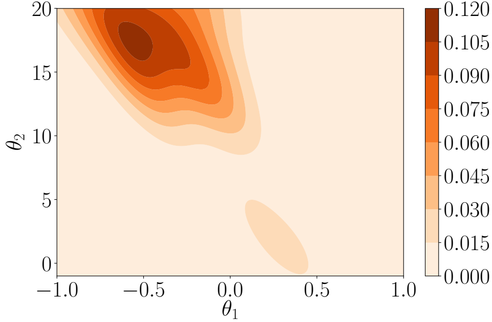
Mountain Car



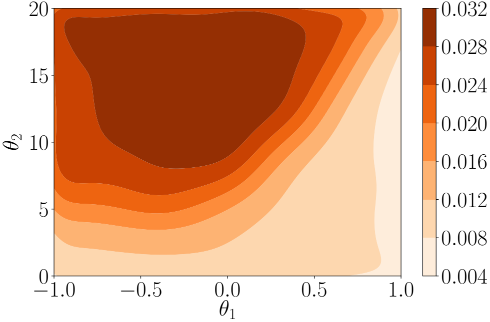
Mountain Car - Performance



Mountain Car - Parameter Space Exploration



PGPE



OPTIMIST

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Paper submission at **ICML2019** (International Conference on Machine Learning)

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Future Works

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Future Works

1. Optimization problem;
2. Posterior sampling.

Thank you for your attention!

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