Pick and Place Project Report

Wolfgang Steiner

wolfgang.steiner@gmail.com

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I. DH Parameters and Forward Kinematics

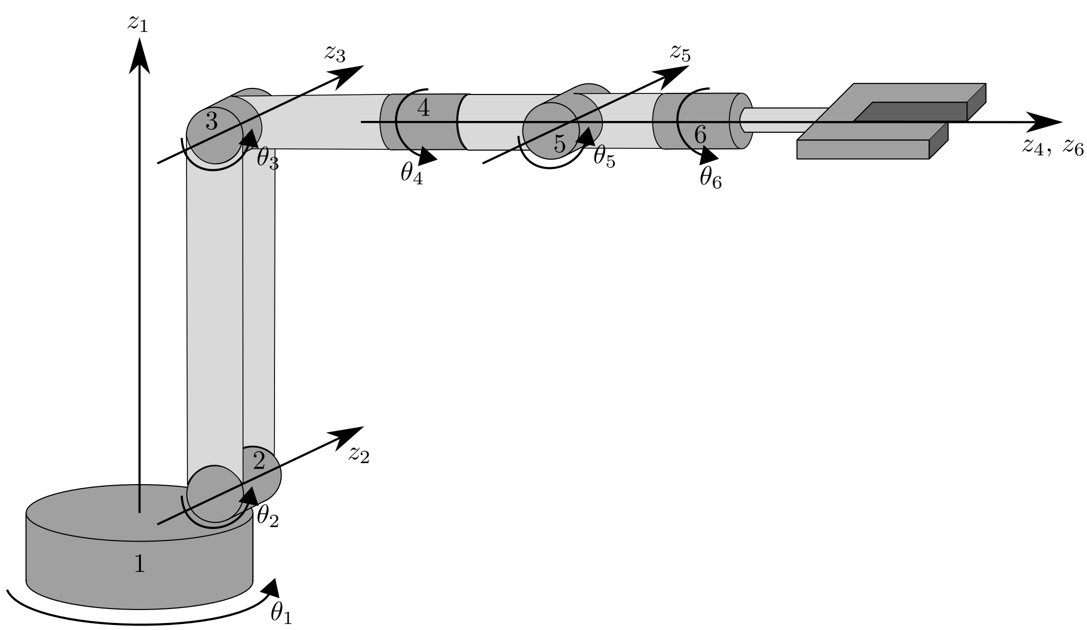

Figure 1: *Schematics of the Kuka KR210 arm.*

Table 1: *Modified DH parameters of the Kuka KR210 arm.*

	α_{i-1}	a_{i-1}	d_i	θ_i
${}^0_1T$	0	0	0.750	θ_1
${}^1_2T$	$-\frac{\pi}{2}$	0.350	0	$\theta_2 - \frac{\pi}{2}$
${}^2_3T$	0	1.250	0	θ_3
${}^3_4T$	$-\frac{\pi}{2}$	-0.054	1.500	θ_4
${}^4_5T$	$\frac{\pi}{2}$	0	0	θ_5
${}^5_6T$	$-\frac{\pi}{2}$	0	0	θ_6
${}^6_GT$	0	0	0.303	0

$$
{}^{i-1}_{i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-i} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-i} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}
$$

$$ {}^0_1T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2} $$

$$ {}^1_2T = \begin{bmatrix} c(\theta_2 - \frac{\pi}{2}) & -s(\theta_2 - \frac{\pi}{2}) & 0 & a_1 \\ 0 & 0 & 1 & 0 \\ -s(\theta_2 - \frac{\pi}{2}) & c(\theta_2 - \frac{\pi}{2}) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} s\theta_2 & c\theta_2 & 0 & a_1 \\ 0 & 0 & 1 & 0 \\ c\theta_2 & s\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3} $$

$$ {}^2_3T = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & a_2 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4} $$

$$ {}^3_4T = \begin{bmatrix} c\theta_4 & -s\theta_4 & 0 & a_3 \\ 0 & 0 & 1 & d_4 \\ -s\theta_4 & -c\theta_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5} $$

$$ {}^4_5T = \begin{bmatrix} c\theta_5 & -s\theta_5 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s\theta_5 & c\theta_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6} $$

$$ {}^5_6T = \begin{bmatrix} c\theta_6 & -s\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s\theta_6 & -c\theta_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{7} $$

$$ {}^6_GT = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_G \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{8} $$

II. Calculation of the Wrist Center Position

The orientation of the gripper is supplied to the inverse kinematic server as a quaternion from this, the rotation matrix relative to the base frame can be computed the function `tf.transformations.quaternion_matrix`, which is equivalent to the following operation:

$$ R_{gripper} = \begin{bmatrix} 1 - 2q_y^2 - 2q_z^2 & 2q_xq_y - 2q_zq_w & 2q_xq_z + 2q_yq_w \\ 2q_xq_y + 2q_zq_w & 1 - 2q_x^2 - 2q_z^2 & 2q_yq_z - 2q_xq_w \\ 2q_xq_z - 2q_yq_w & 2q_yq_z + 2q_xq_w & 1 - 2q_x^2 - 2q_y^2 \end{bmatrix} \cdot R_{corr} \tag{9} $$

Alternatively, the roll (ϕ), pitch (θ), and yaw (ψ) angles can be computed with the function `transformations.euler_from_quaternion`, by specifiying the `'rzyx'` option. This computes the Euler angles relative to the local frame in the order of a rotation about the z, y and x axes in this order. With these Euler angles, the total rotation matrix can be computed as:

$$ R_{gripper} = R_z(\psi)R_y(\theta)R_x(\phi)R_{corr} = \begin{bmatrix} c\theta c\psi & s\phi s\theta c\psi - s\psi c\phi & s\phi s\psi + s\theta c\phi c\psi \\ s\psi c\theta & s\phi s\theta s\psi + c\phi c\psi & -s\phi c\psi + s\theta s\psi c\phi \\ -s\theta & s\phi c\theta & c\phi c\theta \end{bmatrix} \cdot R_{corr} \tag{10} $$

With This rotation matrix, the total transformation matrix from the base frame to the gripper frame can be expressed as:

$$ T_{gripper} = \begin{bmatrix} s\phi s\psi + s\theta c\phi c\psi & -s\phi s\theta c\psi + s\psi c\phi & c\theta c\psi & p_x \\ -s\phi c\psi + s\theta s\psi c\phi & -s\phi s\theta s\psi - c\phi c\psi & s\psi c\theta & p_y \\ c\phi c\theta & -s\phi c\theta & -s\theta & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{11} $$

The third column of this transformation matrix represents the z-axis of the griper frame $N_{z,gripper}$. XXXXXXX

$$ \begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix} = \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} - \begin{bmatrix} c\theta c\psi \\ s\psi c\theta \\ -s\theta \end{bmatrix} \cdot d_G \tag{12} $$

III. Inverse Kinematics of the Wrist Center

The next step in the inverse kinematic problem lies in determining the angles θ_1, θ_2 and θ_3 in such a way that the wrist center of the Kuka arm lies at the coordinates w_x, w_y, and w_z. To achieve this, the angle θ_1 can easily be computed in the xy-plane of the base frame from:

$$\theta_1 = \mathrm{atan2}\left(w_y, w_x\right) \tag{13}$$

After computing θ_1, the problem of solving for θ_2 and θ_3 can be transformed into a second 2D problem in the xz-plane of the Kuka arm as shown in Fig. 2. For this, the wrist center is transformed into the reference frame of link 1:

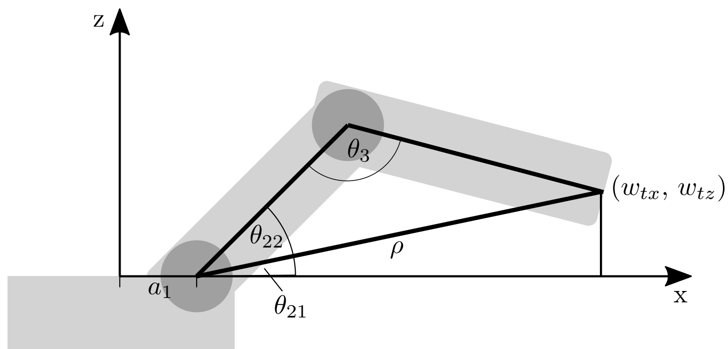

Figure 2: *Sideview of the Kuka arm.*

$$\begin{aligned} w_{tx} &= w_x \mathrm{c}\theta_1 + w_y \mathrm{s}\theta_1 \\ w_{ty} &= -w_x \mathrm{s}\theta_1 + w_y \mathrm{c}\theta_1 = 0 \\ w_{tz} &= w_z - d_1 \end{aligned} \tag{14}$$

$${}^4_1T = \begin{bmatrix} \mathrm{s}(\theta_2+\theta_3)\mathrm{c}\theta_4 & -\mathrm{s}\theta_4\mathrm{s}(\theta_2+\theta_3) & \mathrm{c}(\theta_2+\theta_3) & a_1 + a_2\mathrm{s}(\theta_2) + a_3\mathrm{s}(\theta_2+\theta_3) + d_4\mathrm{c}(\theta_2+\theta_3) \\ -\mathrm{s}\theta_4 & -\mathrm{c}\theta_4 & 0 & 0 \\ \mathrm{c}\theta_4\mathrm{c}(\theta_2+\theta_3) & -\mathrm{s}\theta_4\mathrm{c}(\theta_2+\theta_3) & -\mathrm{s}(\theta_2+\theta_3) & a_2\mathrm{c}(\theta_2) + a_3\mathrm{c}(\theta_2+\theta_3) - d_4\mathrm{s}(\theta_2+\theta_3) \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{15}$$

The entries of the rightmost column of the matrix ${}^4_1T$ represent the translation of the wrist center relative to the origin of link 1, so we can extract the following two equations from this:

$$w_{tx} - a_1 = a_2\mathrm{s}(\theta_2) + a_3\mathrm{s}(\theta_2+\theta_3) + d_4\mathrm{c}(\theta_2+\theta_3) \tag{16}$$

$$w_{tz} = a_2\mathrm{c}(\theta_2) + a_3\mathrm{c}(\theta_2+\theta_3) - d_4\mathrm{s}(\theta_2+\theta_3) \tag{17}$$

The angle θ_3 effectively defines the reach or radius of the robot arm, which has to be equal to the distance between link one and the transformed wrist center position in the plane of the arm. Thus by computing the squared distance we get:

$$(w_{tx} - a_1)^2 + w_{tz}^2 = a_2^2 + a_3^2 + d_4^2 + 2a_2a_3\mathrm{c}\theta_3 - 2a_2d_4\mathrm{s}\theta_3 \tag{18}$$

$$\frac{\rho^2 - \sigma^2}{2a_2} = a_3\mathrm{c}\theta_3 - d_4\mathrm{s}\theta_3 = \sqrt{a_3^2 + d_4^2} \cdot \sin(\theta_3 + \mathrm{atan2}(a_3, -d_4)) \tag{19}$$

$$\rho^2 = (w_{tx} - a_1)^2 + w_{tz}^2 \tag{20}$$

$$\sigma^2 = a_2^2 + a_3^2 + d_4^2 \tag{21}$$

Solving (19) for θ_3 finally yields:

$$\theta_3 = \text{asin}\left(\frac{\rho^2 - \sigma^2}{2a_2\sqrt{a_3^2 + d_4^2}}\right) - \text{atan2}(a_3, -d_4) \tag{22}$$

With θ_3 determined, it now becomes possible to solve for θ_2. This sub-problem can be further divided into the two angles θ_{21} and θ_{22} (cf. Fig. 2). The angle θ_{21} can be thought of as the angle required to place the tip of a straight arm with length ρ at the wrist center. It be can easily computed as:

$$\theta_{21} = \text{atan2}\left(w_{tz}, w_{tx} - a_1\right) \tag{23}$$

The angle θ_{22} can now be solved by utilizing the matrix element ${}^1_4T_{33}$ in (15). Here, the displacement w_{tz} has already been accounted for by θ_{21} so that this matrix element must be equal to zero for θ_{22}:

$$a_2\text{c}(\theta_{22}) + a_3\text{c}(\theta_{22} + \theta_3) - d_4\text{s}(\theta_{22} + \theta_3) = \alpha_{22}\sin(\theta_{22} + \phi_{22}) = 0 \tag{24}$$

Solving this equation for θ_{22} yields:

$$\theta_{22} = -\text{atan2}\left(-a_3\text{s}\theta_3 - d_4\text{c}\theta_3, a_2 + a_3\text{c}\theta_3 - d_4\text{s}\theta_3\right) + n\pi \tag{25}$$

Finally, the two angles θ_{21} and θ_{22} can be combined:

$$\theta_2 = \frac{\pi}{2} - \theta_{21} - \theta_{22} \tag{26}$$

IV. Inverse Kinematics of the Gripper Orientation

Once the first three joint angles have been determined, the inverse kinematics problem can be completed by computing the orientation of the tool relative to the wrist center. The desired orientation of the gripper is supplied to the IK solver in form of a quaternion **??**.

As the angles θ_1, θ_2 and θ_3 are already known from the previous steps, it is now possible to compute the rotation matrix ${}^3_6R$ by premultiplying the matrix $R_{gripper}$ with the inverse of the composite rotation ${}^0_3R$:

$${}^3_6R = ({}^0_1R \cdot {}^1_2R \cdot {}^2_3R)^{-1}R_{gripper} = \begin{bmatrix} -\text{s}\theta_4\text{s}\theta_6 + \text{c}\theta_4 * \text{c}\theta_5\text{c}\theta_6 & -\text{s}\theta_4\text{c}\theta_6 - \text{s}\theta_6\text{c}\theta_4\text{c}\theta_5 & -\text{s}\theta_5\text{c}\theta_4 \\ \text{s}\theta_5\text{c}\theta_6 & -\text{s}\theta_5\text{s}\theta_6 & \text{c}\theta_5 \\ -\text{s}\theta_4\text{c}\theta_5\text{c}\theta_6 - \text{s}\theta_6\text{c}\theta_4 & \text{s}\theta_4\text{s}\theta_6\text{c}\theta_5 - \text{c}\theta_4\text{c}\theta_6 & \text{s}\theta_4\text{s}\theta_5 \end{bmatrix} \tag{27}$$

From this matrix, θ_5 can be directly computed as:

$$\theta_5 = \pm\text{acos}({}^3_6R_{23}) \tag{28}$$

To guarantee that the gripper will point into the correct direction and not its opposite, it is important to compute θ_4 by satisfying *two* entries of the matrix ${}^3_6R$.

$$\frac{{}^3_6R_{33}}{-{}^3_6R_{13}} = \frac{\text{s}\theta_4\text{s}\theta_5}{\text{c}\theta_4\text{s}\theta_5} = \tan(\theta_4) \tag{29}$$

From this, θ_4 can be computed as follows. Here, I set $theta_4$ to zero for very small θ_5. In this case, the correct orientation of the gripper is achieved by θ_6.

$$\theta_4 \begin{cases} \text{atan2}\left({}^3_6R_{33}, -{}^3_6R_{13}\right) & \text{if } |\theta_5| > \epsilon \\ 0 & \text{if } |\theta_5| \leq \epsilon \end{cases} \tag{30}$$

Finally, θ_6 can be computed from one of two cases: If θ_5, and consequently θ_4 are close to zero, the first entry ${}^3_6R_{11}$ simplifies to cθ_6. Otherwise, θ_6 can easily be computed from ${}^3_6R_{21}$. Thus, we get the following expressions for computing θ_6:

$$\theta_6 = \begin{cases} \text{acos}\left(\frac{{}^3_6R_{21}}{\sin(\theta_5)}\right) & \text{if } |\theta_5| > \epsilon \\ \text{acos}\left({}^3_6R_{11}\right) & \text{if } |\theta_5| < \epsilon \end{cases} \tag{31}$$

V. Results

With the presented inverse kinematics procedure, the robot arm is able to reliably pick up the object and place it in the receptable. In order to measure the accuracy of the procedure, I calculate the root-mean-square error (*RMSE*) of the gripper position in the following way: at each time step I perform a forward kinematics pass with the calculated joint angles in order to compute the *predicted* gripper position $p_{pred,i}$, which is then compared to the *ground truth* position $p_{gt,i}$:

$$\mathrm{RMSE}(p) = \sqrt{\frac{1}{n}\sum_i (p_{pred,i} - p_{gt,i})^2} \tag{32}$$

The resulting RMSE values for three typical pick and place cycles are summarized in table Tab. 2. As can be seen, the RMSE values are in the magnitude of floating point rounding errors, thus supporting the correctness of the presented inverse kinematics procedure.

Table 2: *RMSE values of the predicted gripper position for three typical pick and place cycles.*

n	RMSE(p_x)/m	RMSE(p_y)/m	RMSE(p_z)/m
1	$1.38372517 \cdot 10^{-16}$	$2.50313645 \cdot 10^{-16}$	$1.01349032 \cdot 10^{-16}$
2	$8.69068332 \cdot 10^{-15}$	$1.21391017 \cdot 10^{-10}$	$4.44089210 \cdot 10^{-16}$
3	$1.64697269 \cdot 10^{-15}$	$2.23592115 \cdot 10^{-11}$	$3.59867566 \cdot 10^{-16}$