Position From the Nth Derivative With Respect to Time

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Summary

In this study, position problems were explored using advanced math techniques to get a greater understanding of the kinematics equations used in class. Before settling on the final procedure, several methods of solving the equation were attempted, partial derivation of recursive formulae, visualization of said formulae using matplotlib and python, graphing of 4-dimensional equations, etc. Constants along with Newton's method introduced a large amount of error; however, due to the nature of Euler-Cauchy equations, this could not be avoided.

Intro

Sitting in physics class watching my teacher complete problems on the board I began to wonder, why are we doing this? Who decided this would be the method to solve position problems. Everyone around me was dutifully memorizing how to complete each problem hindered with algebra based math, while my mind was fixated on how this applied to advanced math techniques. In this study position problems were explored using advanced math techniques to get a greater understanding of the kinematics equations used in class.

Purpose

Many students use a “plug-and-chug” method of applying the formulae and equations in AP Physics I. The purpose of this project is to better understand the underlying mathematics behind Newtonian kinematics and extend them further.

Procedure

Several methods of solving the equation were attempted, before the process described in the mathematics section.

- Partial derivation of recursive formulae
- Visualization of aforementioned formulae using matplotlib and python
- Graphing of 4-dimensional equations

Mathematics

$$x_f = x_i + vt + \frac{1}{2}at^2 + \frac{1}{6}jt^3 + \frac{1}{24}st^4 + \cdots$$

$$\begin{array}{lll} \text{coefficient} & 1, 1, \frac{1}{2}, \frac{1}{6}, \frac{1}{24}, \cdots & = \quad \frac{1}{0!}, \frac{1}{1!}, \frac{1}{2!}, \frac{1}{3!}, \frac{1}{4!}, \cdots \\ \text{exp. of t} & 0, 1, 2, 3, 4, \cdots & \\ \text{variable} & x_i, v, a, j, s, \cdots & = \quad \frac{d^0x(0)}{dt^0}, \frac{d^1x}{dt^1}, \frac{d^2x}{dt^2}, \frac{d^3x}{dt^3}, \frac{d^4x}{dt^4}, \cdots \end{array}$$

$$x_f = x_i + \sum_{n=0}^{P} n! \cdot x^{(n+1)}(t) \cdot t^n$$

where P is the order of the highest derivative

set the formula up as an Euler-Cauchy equation

$$-P! \cdot x(0) = -P! \cdot x(t) + \sum_{n=1}^{P} \frac{P!}{n!} \cdot t^n \cdot x^{(n)}(t)$$

let $x(t) = t^m$ where m is an unknown constant

$$\begin{aligned} \therefore x^{(1)}(t) &= mt^{m-1} \\ \therefore x^{(2)}(t) &= (m^2 - m)t^{m-2} \\ \therefore x^{(3)}(t) &= (m^3 - 3m^2 + 2m)t^{m-3} \\ \vdots \quad &\vdots \quad \vdots \\ \therefore x^{(n)}(t) &= t^{m-n} \cdot \textstyle\prod_{k=0}^{n-1} m - k \end{aligned}$$

substitute this in for the previous equation

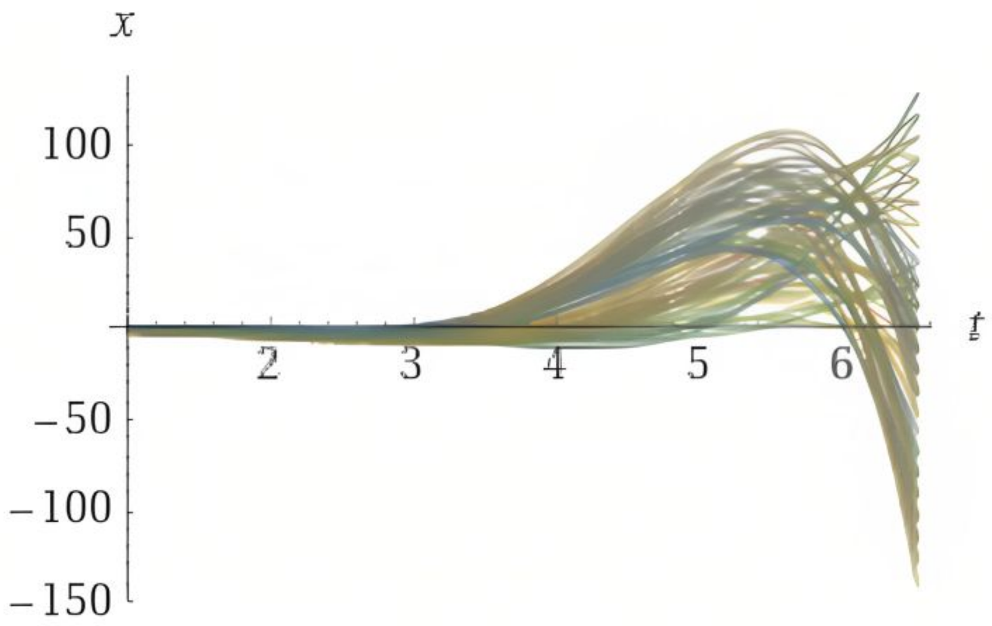
$$-P! \cdot x(0) = -P! \cdot t^m + \sum_{n=1}^{P} \left(\frac{P!}{n!} \cdot t^m \cdot \prod_{k=0}^{n-1} (m-k) \right)$$

solve this nth degree polynomial using Newton's method

$$x(t) = x(0) + \sum_{n=1}^{P-1} c_n t^{r_n}$$

where c_n is the nth constant and r_n is the nth root of the polynomial

Data



(sampling $x(1)$, $x'(1)$, $x''(1)$, $x^{(3)}(1)$, $x^{(4)}(1)$ and $x^{(5)}(1)$)

Via Wolfram Alpha

Conclusion

Due to the nature of differential equations, several constants were introduced. The number of these constants increased as the order of the equation increased. This leads to a direct increase in the possible error of the function as the order of the equation increases. Additionally, the utilization of Newton’s method leaves great room for human error.

Future Work

In the future, visualization of the formula with interactive sampling would provide an even deeper understanding of Newtonian kinematic equations.

References

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