Position From the Nth Derivative With Respect to Time

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RESEARCH QUESTION:

Is there a generalized form of Newtonian kinematics for nth-degree derivatives of displacement.

The overarching goal of this project is to better understand the underlying mathematics of Newtonian kinematics.

METHODOLOGY

$$x_f = x_i + vt + \frac{1}{2}at^2 + \frac{1}{6}jt^3 + \frac{1}{24}st^4 + \cdots$$

$$\begin{array}{lll}
\text{coefficient} & 1, 1, \frac{1}{2}, \frac{1}{6}, \frac{1}{24}, \cdots & = \frac{1}{0!}, \frac{1}{1!}, \frac{1}{2!}, \frac{1}{3!}, \frac{1}{4!}, \cdots \\
\text{exp. of t} & 0, 1, 2, 3, 4, \cdots & \\
\text{variable} & x_i, v, a, j, s, \cdots & = \frac{d^0x(0)}{dt^0}, \frac{d^1x}{dt^1}, \frac{d^2x}{dt^2}, \frac{d^3x}{dt^3}, \frac{d^4x}{dt^4}, \cdots
\end{array}$$

$$x_f = x_i + \sum_{n=0}^{P} n! \cdot x^{(n+1)}(t) \cdot t^n$$

where P is the order of the highest derivative

set the formula up as an Euler-Cauchy equation

$$-P! \cdot x(0) = -P! \cdot x(t) + \sum_{n=1}^{P} \frac{P!}{n!} \cdot t^n \cdot x^{(n)}(t)$$

let $x(t) = t^m$ where m is an unknown constant

$$\begin{aligned}
\therefore x^{(1)}(t) &= mt^{m-1} \\
\therefore x^{(2)}(t) &= (m^2 - m)t^{m-2} \\
\therefore x^{(3)}(t) &= (m^3 - 3m^2 + 2m)t^{m-3} \\
\vdots \quad & \vdots \quad \vdots \\
\therefore x^{(n)}(t) &= t^{m-n} \cdot \prod_{k=0}^{n-1} m - k
\end{aligned}$$

substitute this in for the previous equation

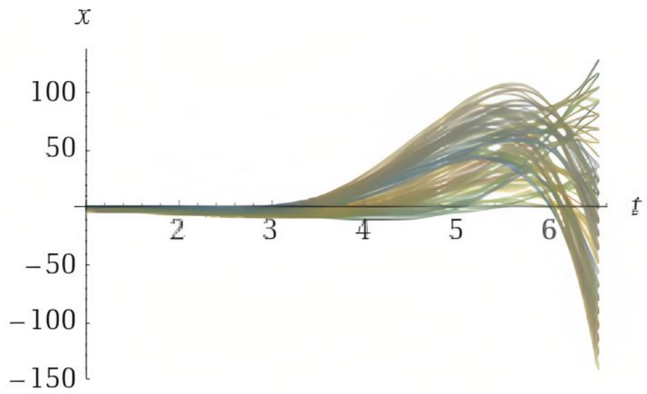
$$-P! \cdot x(0) = -P! \cdot t^m + \sum_{n=1}^{P} \left(\frac{P!}{n!} \cdot t^m \cdot \prod_{k=0}^{n-1} (m - k) \right)$$

solve this nth degree polynomial using Newton's method

$$x(t) = x(0) + \sum_{n=1}^{P-1} c_n t^{r_n}$$

where c_n is the nth constant and r_n is the nth root of the polynomial

DATA:



Via Wolfram Alpha

(sampling $x(1)$, $x'(1)$, $x''(1)$, $x^{(3)}(1)$, $x^{(4)}(1)$ and $x^{(5)}(1)$)

INTERPRETATION & CONCLUSIONS:

Due to the nature of differential equations, several constants were introduced. The number of these constants increased as the order of the equation increased. This leads to a direct increase in the possible error of the function as the order of the equation increases. Additionally, the utilization of Newton's method leaves great room for human error.

In the future, visualization of the formula with interactive sampling would provide an even deep understanding of the Newtonian kinematic equations.