Methods 2 – Portfolio Assignment 2

- *Type:* Individual assignment
- *Due:* 10 April 2022, 23:59

```
pacman::p_load(tidyverse)
```

1. Square root function

Along the lines of section 6.4.2 (p. 247ff) in Gill's book, write an *R* function that calculates the square root of a given positive number. Your solution should contain:

```
# function called Nsqrt
Nsqrt <- function(n) {
  # Make variable x (start)
  x <- 4
  delta <- 0.00001
  # n = the number we're taking the square root of
  # x = the start x value when using the Newton-Raphson method
  while (abs(x^2-(n)) > delta) {

    x_new <- x - (x^2-n)/(2*x)

    x <- x_new
  }

  # Returning x which is now defined as x = sqrt(n)
  return(x)
}
```

You enter a value n that you want to calculate. Inside the function x is defined as $x = 4$. The value of x isn't important on my PC since it goes so fast anyway, but it might be a factor on slower computers. In that case I would've looked for a way to make x value depend on the n value so to have a start x value that is closer to the final value when using Newton's method which would result in faster computation.

Another function vector is δ where $\delta = 0.00001$ The lower the value of δ the more precise the final output but can also result in longer computation times.

Afterwards the function enters a while loop. Inside the loop you run the formula for newton's method with the function $f(x) = x^2 - n$ which comes out as $x_{new} = x - \frac{x^2-n}{2x}$. Then x gets defined as $x = x_{new}$ inside the while loop and the loop repeats. This works since $f(x) = x^2 - n$ equals 0 when $x = \sqrt{n}\ and\ n > 0$

Since $x = \sqrt{n} \Rightarrow x^2 = n \Rightarrow x^2 - n = 0$ If $|x^2 - n| < \delta$ the while loop ends and x gets outputted. This makes sense since if $|x^2 - n| < \delta$, then x must be really close to it's true value.

I'll now make a demonstration with the following examples where we'll try to find the values of $\sqrt{n} = \sqrt{25}$ and $\sqrt{n} = \sqrt{-25}$. I'll start with the first mentioned example using Newton's method where the starting x-value equals 4.

$x_{new} = 4 - \frac{4^2-25}{2*4} = 5.125$

$x = x_{new}$

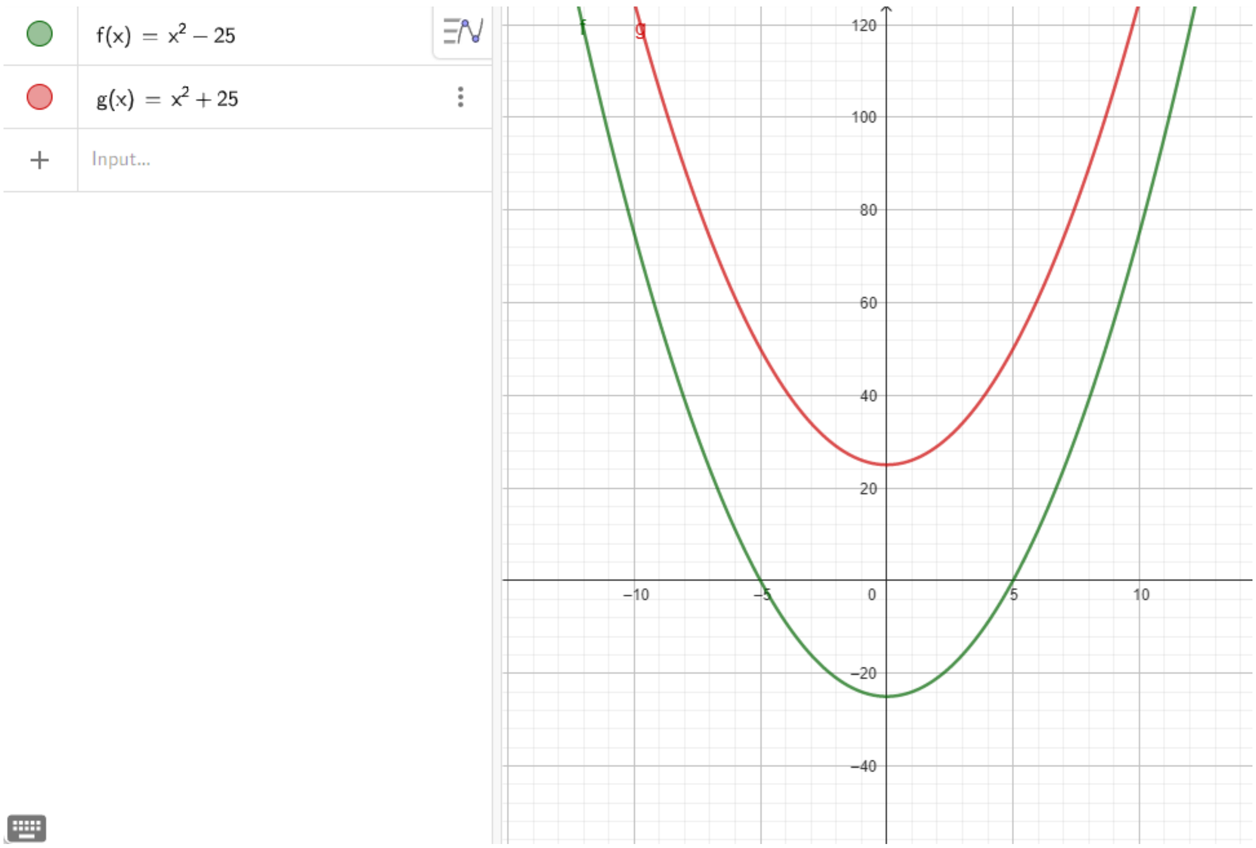
Loop repeats

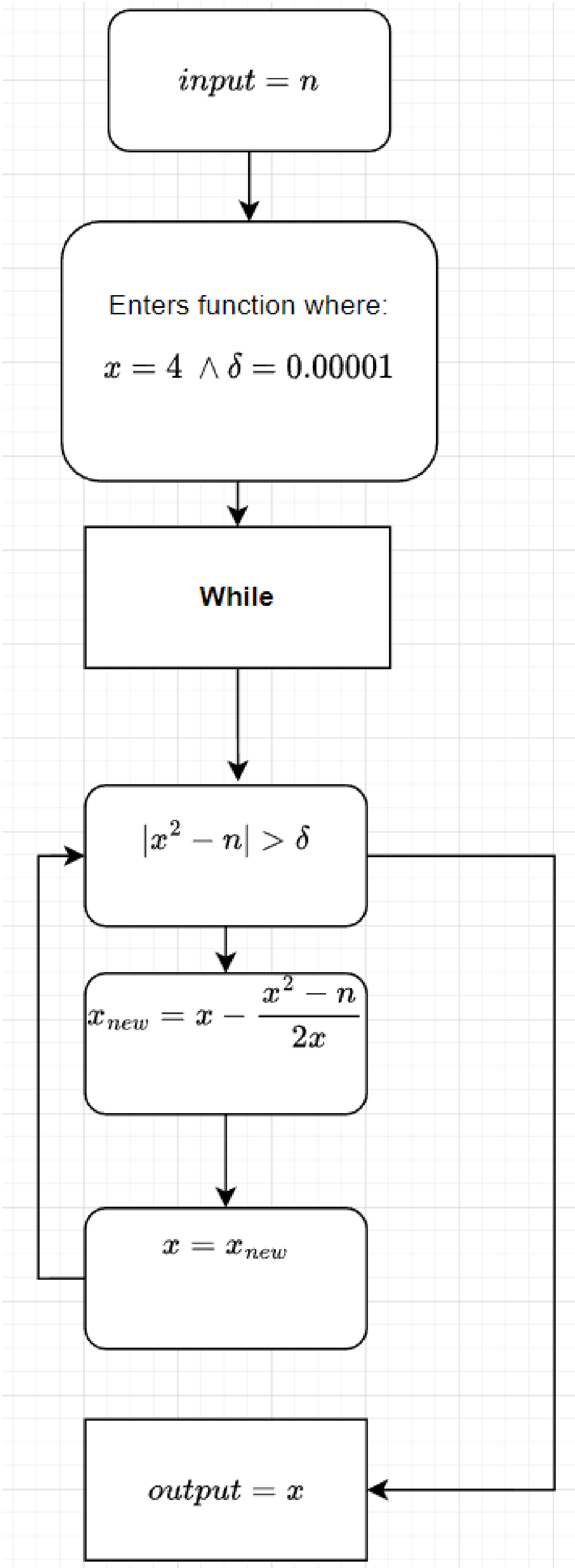
$x_{new} = 5.125 - \frac{5.125^2-25}{2*5.125}$

$x = x_{new}$

loop repeats until $|x^2 - 25| < \delta$ and x is outputted

In the second example $n < 0 \Rightarrow \ f(x) = x^2 + n > 0 \ \forall \ \mathbb{R}$. This means that you can't use the newton's method(or any method) to find the x value where $f(x) = 0$ and finding where $|x^2 - n| < \delta$ will therefore never be found. The result is an infinite loop where there's never given an output.




$input = n$
Enters function where:
$x = 4 \wedge \delta = 0.00001$
While
$|x^2 - n| > \delta$
$x_{new} = x - \frac{x^2 - n}{2x}$
$x = x_{new}$
$output = x$

The function works by you selecting some value you want to calculate the square root. For example $\sqrt{25}$

a) A quick introduction into what the function does and why it works

b) A discussion of the choices you made (e.g., starting point of the algorithm) Done this

c) A range of examples Two examples so far

d) A discussion of what happens when the program is applied to negative numbers Had an example with negative numbers where I explained.

2. Power series derivatives

The power series definitions (:= means "is defined as") of the exponential, sine, and cosine functions are

$$\exp(x) := \sum_{n=0}^{\infty} \frac{x^n}{n!},$$

$$\sin(x) := \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!},$$

$$\cos(x) := \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}.$$

Using these definitions, show that

$$\frac{\mathrm{d}}{\mathrm{d}x} \exp(x) = \exp(x),$$

$$\frac{\mathrm{d}}{\mathrm{d}x} \sin(x) = \cos(x),$$

$$\frac{\mathrm{d}}{\mathrm{d}x} \cos(x) = -\sin(x).$$

You can either use LaTeX or include photos of your (nicely) handwritten equations in the notebook. In any case, write down all intermediate steps.