3.1 Performing the vector multiplications

$$(1\ 1\ 1)\begin{pmatrix} a \\ b \\ c \end{pmatrix} = 1\cdot a + 1\cdot b + 1\cdot c = \underline{\underline{a+b+c}}$$

$$(1\ 1\ 1) \times \begin{pmatrix} a \\ b \\ c \end{pmatrix} \Leftrightarrow \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \times \begin{pmatrix} a \\ b \\ c \end{pmatrix} = (1\cdot c - 1\cdot b,\ 1\cdot c - 1\cdot a,$$

$$1\cdot b - 1\cdot a) = \underline{\underline{(c-b,\ c-a,\ b-a)}}$$

$$(-1,\ 1,\ -1)\begin{pmatrix} 4 \\ 3 \\ 12 \end{pmatrix} = -1\cdot 4 + 3\cdot 1 + (-1)\cdot 12 = -4+3-12 = \underline{\underline{-13}}$$

$$(-1\ 1\ -1) \times \begin{pmatrix} 4 \\ 3 \\ 12 \end{pmatrix} \Leftrightarrow \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix} \times \begin{pmatrix} 4 \\ 3 \\ 12 \end{pmatrix} = (1\cdot 12 - (-1)\cdot 3,$$

$$-1\cdot 12 - (-1)\cdot 4,\ -1\cdot 3 - 1\cdot 4) = (12+3,\ -12+4,\ -3-4)$$

$$= \underline{\underline{(15,\ -8,\ -7)}}$$

$$(0\ 9\ 0\ 11)\begin{pmatrix} 123.98211 \\ 6 \\ -6392.38743 \\ -5 \end{pmatrix} = 0 + 9\cdot 6 + 0 + 11\cdot(-5)$$

$$= 54 - 55 = \underline{\underline{-1}}$$

$$(123.98211\ \ 6\ \ -6392.38743\ \ -5)\begin{pmatrix} 0 \\ 9 \\ 6 \\ 11 \end{pmatrix} = 0 + 9\cdot 6 + 0 + 11\cdot(-5)$$

$$= 54 - 55 = \underline{\underline{-1}}$$

3.7 Showing that premultiplication and post multiplication with the identity matrix are the same

↳ that is $\mathbf{I}\mathbf{X} = \mathbf{X}\mathbf{I}$

$$\mathbf{I} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & & \vdots \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & 0 & 1 \end{bmatrix} \qquad \mathbf{X} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & & \vdots \\ \vdots & & \ddots & \\ x_{n1} & \cdots & \cdots & x_{nn} \end{bmatrix}$$

$$\mathbf{I}\mathbf{X} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & & \vdots \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & & 1 \end{bmatrix} \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & & \vdots \\ \vdots & & \ddots & \\ x_{n1} & \cdots & \cdots & x_{nn} \end{bmatrix} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & & \vdots \\ \vdots & & \ddots & \\ x_{n1} & \cdots & \cdots & x_{nn} \end{bmatrix}$$

$$\mathbf{X}\mathbf{I} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & & \vdots \\ \vdots & & \ddots & \\ x_{n1} & \cdots & \cdots & x_{nn} \end{bmatrix} \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & & \vdots \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & & 1 \end{bmatrix} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & & \vdots \\ \vdots & & \ddots & \vdots \\ x_{n1} & \cdots & \cdots & x_{nn} \end{bmatrix}$$

3.10 Perform the vector/matrix multiplications

$$\begin{bmatrix} 1 & \frac{1}{2} & 2 \\ 1 & \frac{1}{3} & 5 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 0.1 \\ 0.2 \\ 0.3 \end{bmatrix} = \begin{bmatrix} 1 \cdot 0.1 + \frac{1}{2} \cdot 0.2 + 2 \cdot 0.3 \\ 1 \cdot 0.1 + \frac{1}{3} \cdot 0.3 + 5 \cdot 0.3 \\ 1 \cdot 0.1 + 1 \cdot 0.2 + 2 \cdot 0.3 \end{bmatrix} = \begin{bmatrix} 0.1 + 0.1 + 0.6 \\ 0.1 + 0.1 + 1.5 \\ 0.1 + 0.2 + 0.6 \end{bmatrix}$$

$$= \underline{\underline{\begin{bmatrix} 0.8 \\ 1.7 \\ 0.9 \end{bmatrix}}}$$

$$\begin{bmatrix} 9 & 7 & 5 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = [9 \cdot 0 + 7 \cdot 1 + 5 \cdot 0, 9 \cdot 1 + 0 \cdot 7 + 5 \cdot 0,$$

$$9 \cdot 0 + 0 \cdot 7 + 5 \cdot 1 = \underline{\underline{\begin{bmatrix} 7 & 9 & 5 \end{bmatrix}}}$$

3.10

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 9 \\ 7 \\ 5 \end{bmatrix} = \begin{bmatrix} 0\cdot 9 + 7\cdot 1 + 0\cdot 5 \\ 1\cdot 9 + 0\cdot 7 + 0\cdot 5 \\ 0\cdot 9 + 7\cdot 0 + 1\cdot 5 \end{bmatrix} = \begin{bmatrix} 7 \\ 9 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 3 & 1 \\ 3 & 1 & 3 \\ 1 & 3 & 3 \end{bmatrix} \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix} = \begin{bmatrix} 3\cdot\frac{1}{3} + 3\cdot\frac{1}{3} + 1\cdot\frac{1}{3} \\ 3\cdot\frac{1}{3} + 1\cdot\frac{1}{3} + 3\cdot\frac{1}{3} \\ 1\cdot\frac{1}{3} + 3\cdot\frac{1}{3} + 3\cdot\frac{1}{3} \end{bmatrix} = \begin{bmatrix} \frac{3}{3} + \frac{3}{3} + \frac{1}{3} \\ \frac{3}{3} + \frac{1}{3} + \frac{3}{3} \\ \frac{1}{3} + \frac{3}{3} + \frac{3}{3} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{7}{3} \\ \frac{7}{3} \\ \frac{7}{3} \end{bmatrix}$$

3.11 Matrix multiplications

$$\begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 3\cdot 2 + (-3)\cdot 0 & 3\cdot 1 + (-3)\cdot 0 \\ -3\cdot 2 + 3\cdot 0 & -3\cdot 1 + 3\cdot 0 \end{bmatrix} = \begin{bmatrix} 6 & 3 \\ -6 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 4 & 7 \\ 3 & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 0\cdot 4 + 1\cdot 3 + 1\cdot 1 & 0\cdot 7 + 0\cdot 1 + 1\cdot 2 \\ 1\cdot 4 + 3\cdot 0 + 1\cdot 1 & 1\cdot 7 + 0\cdot 0 + 1\cdot 2 \\ 1\cdot 4 + 3\cdot 1 + 0\cdot 1 & 1\cdot 7 + 0\cdot 1 + 0\cdot 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 \\ 5 & 9 \\ 7 & 7 \end{bmatrix}$$

3.11 Matrix multiplications

$$\begin{bmatrix} 3 & 1 & -2 \\ 6 & 3 & 4 \end{bmatrix} \begin{bmatrix} 4 & 7 \\ 3 & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 3\cdot 4 + 1\cdot 3 + (-2)\cdot 1 & 3\cdot 7 + 1\cdot 0 + (-2)\cdot 2 \\ 6\cdot 4 + 3\cdot 3 + 4\cdot 1 & 6\cdot 7 + 3\cdot 0 + 4\cdot 2 \end{bmatrix}$$

$$= \begin{bmatrix} 13 & 17 \\ 37 & 50 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 1\cdot 1 + 0\cdot 3 & 1\cdot 0 + 0\cdot 1 \\ -3\cdot 1 + 1\cdot 3 & -3\cdot 0 + 1\cdot 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -9 \\ -1 & -4 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} -4 & -1 & -3 \\ -4 & 0 & -8 \end{bmatrix} =$$

$$\begin{bmatrix} -1\cdot(-4) + (-9)\cdot(-4) & -1\cdot(-1) + (-9)\cdot 0 & -1\cdot(-3) + (-9)(\dots \\ -1\cdot(-4) + (-4)\cdot(-4) & -1(-1) + (-4)\cdot 0 & -1\cdot(-3) + (-4)(\dots \\ 1\cdot(-4) + 2\cdot(-4) & 1\cdot(-1) + 2\cdot 0 & 1\cdot(-3) + 2\cdot(-8) \end{bmatrix}$$

$$= \begin{bmatrix} 40 & 1 & 75 \\ 20 & 1 & 35 \\ -12 & -1 & -19 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & \infty \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ -\infty & -\infty \end{bmatrix}$$

3.13

Utilize that $OR \cdot PO \cdot PR = OPR$

To ensure that the dimensionalities add up the following transformation is neccessary

$$(OR' PO')' \cdot PR = OPR$$

$$OR' = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad PO' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix} = OR' \cdot PO'$$

$$(OR' \cdot PO') \cdot PR =$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 2 \end{bmatrix} = \underline{\begin{bmatrix} 2 & 2 & 1 \\ 1 & 2 & 2 \\ 1 & 0 & 3 \end{bmatrix}}$$

3.13

Multiplying any of three matrices by a constant has the same effect on the outcome because of the properties of the iner product

3.22 Vectorization & vector norm

$$\overset{n}{\mathbf{X}} = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 3 \\ 3 & 1 & 2 \\ 4 & 3 & 6 \\ 5 & 5 & 5 \\ 6 & 7 & 6 \\ 7 & 9 & 9 \\ 8 & 8 & 8 \\ 9 & 8 & 3 \end{bmatrix} \qquad vec(\overset{n}{\mathbf{X}}) = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 2 \\ 4 \\ 1 \\ 3 \\ 5 \\ 7 \\ 9 \\ 8 \\ 8 \\ 1 \\ 3 \\ 2 \\ 6 \\ 5 \\ 6 \\ 9 \\ 8 \\ 3 \end{bmatrix}$$

Vector norm p. 93

$$\|\mathbf{V}\| = (v_1^2 + v_2^2 + \dots + v_n^2)^{\frac{1}{2}}$$

$$\|\mathbf{V}\|^2 = (1^2 + 2^2 + 3^2 \dots 9^2 + 8^2 + 3^2)^{\frac{1}{2}} = 874$$

3.23 Start with the angle between two vectors

$$\cos(\theta) = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} \iff \cos^2(\theta) = \frac{\mathbf{u}^2 \cdot \mathbf{v}^2}{\|\mathbf{u}\|^2\|\mathbf{v}\|^2} \iff$$

$$1 - \sin^2(\theta) = \frac{\mathbf{u}^2 \cdot \mathbf{v}^2}{\|\mathbf{u}\|^2\|\mathbf{v}\|^2} \iff \|\mathbf{u}\|^2\|\mathbf{v}\|^2 - \|\mathbf{u}\|^2\|\mathbf{v}\|^2 \sin^2(\theta) = \mathbf{u}^2 \cdot \mathbf{v}^2$$

$$\|\mathbf{u}\|^2\|\mathbf{v}\|^2 - \mathbf{u}^2 \cdot \mathbf{v}^2 = \|\mathbf{u}\|^2\|\mathbf{v}\|^2 \sin^2(\theta) \iff$$

$$\|\mathbf{u} \times \mathbf{v}\|^2 = \|\mathbf{u}\|^2\|\mathbf{v}\|^2 \sin^2(\theta)$$

$$\|\mathbf{u} \times \mathbf{v}\|^2 = \|\mathbf{u}\|\|\mathbf{v}\| \sin(\theta)$$