5.7

In order to find the derivative of

$$y = \frac{(2x^3-3)^{\frac{5}{2}}}{(x^2-1)^{\frac{2}{3}}(9x^2-1)^{\frac{1}{2}}}$$, we use logarithmic differentiation

so that we instead take the derivative of

$$\log(y) = \frac{5}{2}\log(2x^3-3) - \frac{2}{3}\log(x^2-1) - \frac{1}{2}\log(9x^2-1)$$

we know that $(\log(y))' = \frac{1}{y}\frac{d}{dx}y$, which together with the chain and power rule enables us to write that

$$\frac{1}{y}\frac{d}{dx}y = \frac{5}{2}\left(\frac{1}{2x^3-3}\right)(6x^2) - \frac{2}{3}\left(\frac{1}{x^2-1}\right)(2x) - \frac{1}{2}\left(\frac{1}{9x^2-1}\right)(18x)$$

$$= \frac{15x^2}{2x^3-3} - \frac{\frac{4}{3}x}{x^2-1} - \frac{9x}{9x^2-1}$$

Combine to one fraction by having the same denominator

$$= \frac{15x^2(x^2-1)(9x^2-1) - \frac{4}{3}x\cdot(2x^3-3)(9x^2-1) - 9x(2x^3-3)(x^2-1)}{(2x^3-3)(x^2-1)(9x^2-1)}.$$

= This can be simplified to

$$\frac{15x^2 - 31x + 63x^3 - \frac{388}{3}x^4 + 93x^6}{(2x^3-3)(x^2-1)(9x^2-1)}$$

By finally multiplying both sides by y we get

$$\frac{d}{dx}y = \frac{15x^2 - 31x + 63x^3 - \frac{388}{3}x^4 + 93x^6}{(2x^3-3)^{\frac{2}{3}}(x^2-1)^{\frac{1}{3}}(9x^2-1)^{\frac{1}{2}}}$$