AARHUS UNIVERSITY

Methods 2 - 4

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Math basics – Gill, chapter 1

- Arithmetic
- Notation
- Functions
- Polynomials
- Exponents
- Logarithms

The Summation Operator

- $\rightarrowtail$ If $X_1, X_2, \ldots, X_n$ are n numerical values,
- $\rightarrowtail$ then their sum can be represented by $\sum_{i=1}^{n} X_i$,
- $\rightarrowtail$ where i is an indexing variable to indicate the starting and stopping points in the series $X_1, X_2, \ldots, X_n$.

The Product Operator

- $\rightarrowtail$ If $X_1, X_2, \ldots, X_n$ are n numerical values,
- $\rightarrowtail$ then their product can be represented by $\prod_{i=1}^{n} X_i$,
- $\rightarrowtail$ where i is an indexing variable to indicate the starting and stopping points in the series $X_1, X_2, \ldots, X_n$.

Numbers

Symbol	Explanation
$\Re$	the set of real numbers
$\Re^+$	the set of positive real numbers
$\Re^-$	the set of negative real numbers
$\mathcal{I}$	the set of integers
$\mathcal{I}^+$ or $\mathbb{Z}^+$	the set of positive integers
$\mathcal{I}^-$ or $\mathbb{Z}^+$	the set of negative integers
$\mathbf{Q}$	the set of rational numbers
$\mathbf{Q}^+$	the set of positive rational numbers
$\mathbf{Q}^-$	the set of negative rational numbers
$\mathcal{C}$	the set of complex numbers (those based on $\sqrt{-1}$).

Symbols

Symbol	Explanation
$\neg$	logical negation statement
$\in$	is an element of, as in $3 \in \mathcal{I}^+$
$\ni$	such that
$\therefore$	therefore
$\because$	because
$\implies$	logical "then" statement
$\iff$	if and only if, also abbreviated "iff"
$\exists$	there exists
$\forall$	for all
$\between$	between
$\parallel$	parallel
$\angle$	angle

Symbols

Symbol	Explanation
$\emptyset$	the empty set (sometimes used with the Greek phi: ϕ)
$\cup$	union of sets
$\cap$	intersection of sets
$\setminus$	subtract from set
$\subset$	subset
$\complement$	complement

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Symbols

Symbol	Explanation
$\propto$	is proportional to
$\doteq$	equal to in the limit (approaches)
$\perp$	perpendicular
∞	infinity
∞^+, $+\infty$	positive infinity
∞^-, $-\infty$	negative infinity
$\sum$	summation
$\prod$	product
$\lfloor\ \rfloor$	floor: round down to nearest integer
$\lceil\ \rceil$	ceiling: round up to nearest integer
$\vert$	given that: $X \vert Y = 3$

Symbols

Symbol	Explanation
$\vee$	maximum of two values
$\max()$	maximum value from list
$\wedge$	minimum of two values
$\min()$	minimum value from list
$\operatorname*{argmax}_x f(x)$	the value of x that maximizes the function $f(x)$
$\operatorname*{argmin}_x f(x)$	the value of x that minimizes the function $f(x)$

Symbols

Symbol	Meaning
$<$	less than
$\leq$	less than or equal to
$\ll$	much less than
$>$	greater than
$\geq$	greater than or equal to
$\gg$	much greater than
$\approx$	approximately the same
$\cong$	approximately equal to
$\lessapprox$	approximately less than (also $\lesssim$)
$\gtrapprox$	approximately greater than (also $\gtrsim$)
$\equiv$	equivalent by assumption

Functions

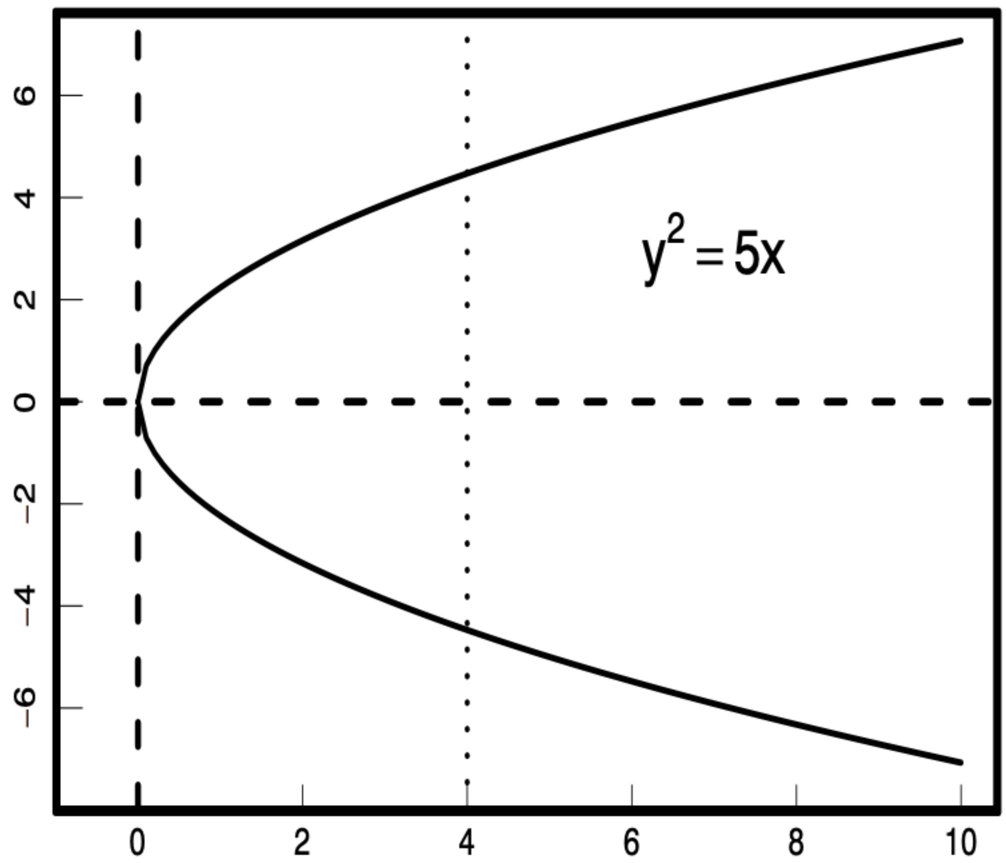
Properties of Functions, Given for $g(x) = y$

$\rightarrowtail$ A function is **continuous** if it has no "gaps" in its mapping from x to y.

$\rightarrowtail$ A function is **invertible** if its reverse operation exists: $g^{-1}(y) = x$, where $g^{-1}(g(x)) = x$.

Functions

Fig. 1.2. A Relation That Is Not a Function



Key Properties of Powers and Exponents

$\rightarrowtail$	Zero Property	$x^0 = 1$
$\rightarrowtail$	One Property	$x^1 = x$
$\rightarrowtail$	Power Notation	$\text{power}(x, a) = x^a$
$\rightarrowtail$	Fraction Property	$\left(\frac{x}{y}\right)^a = \left(\frac{x^a}{y^a}\right) = x^a y^{-a}$
$\rightarrowtail$	Nested Exponents	$(x^a)^b = x^{ab}$
$\rightarrowtail$	Distributive Property	$(xy)^a = x^a y^a$
$\rightarrowtail$	Product Property	$x^a \times x^b = x^{a+b}$
$\rightarrowtail$	Ratio Property	$x^{\frac{a}{b}} = (x^a)^{\frac{1}{b}} = \left(x^{\frac{1}{b}}\right)^a = \sqrt[b]{x^a}$

Basic Properties of Logarithms

$\rightarrowtail$	Zero/One	$\log_b(1) = 0$
$\rightarrowtail$	Multiplication	$\log(x \cdot y) = \log(x) + \log(y)$
$\rightarrowtail$	Division	$\log(x/y) = \log(x) - \log(y)$
$\rightarrowtail$	Exponentiation	$\log(x^y) = y\log(x)$
$\rightarrowtail$	Basis	$\log_b(b^x) = x$, and $b^{\log_b(x)} = x$