

Thursday January 16

# Assignment 1.1

Due Next Thursday January 23

*See Website for Details*

***The Coulomb Force***

and

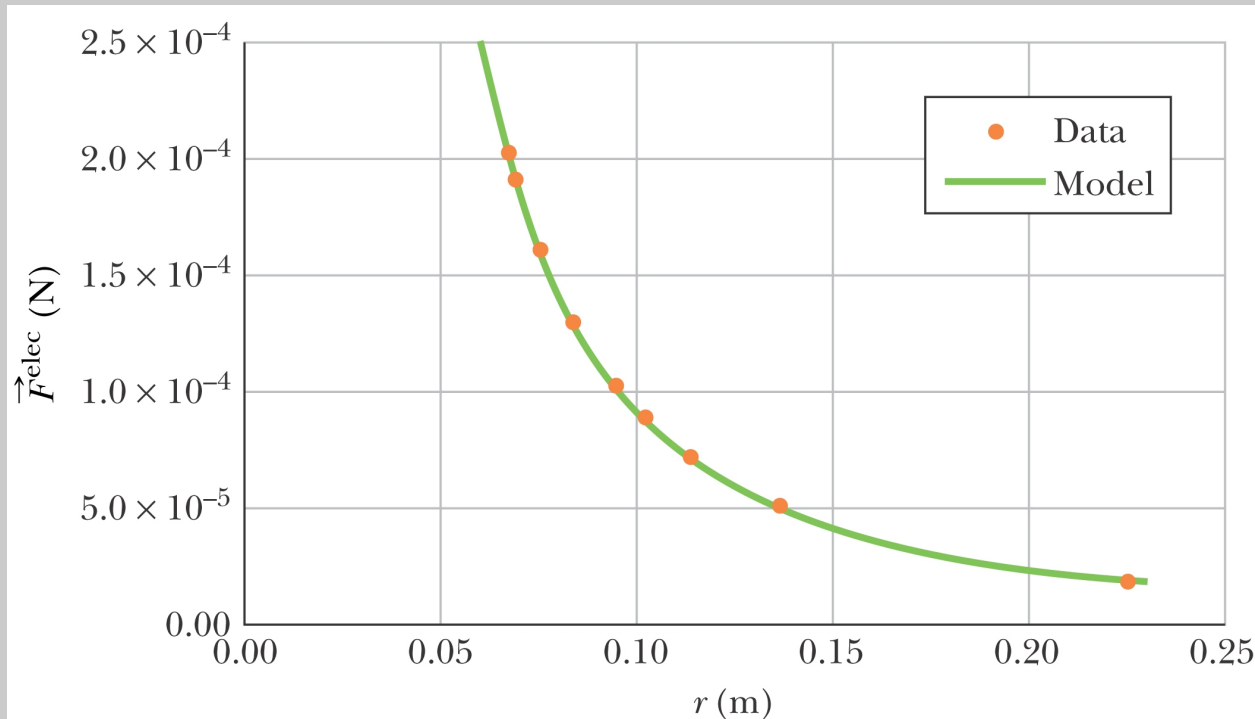
***Coulomb's Law***

# Coulomb's Law



**Direction of  $F$ : Opposites Attract, Likes Repel**

**Force as a function of Distance**



$$F = k \frac{|q_1 q_2|}{r^2}$$

$$k = 8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}$$

# *Computational Example*

## *Coulomb's Law in 1-Dimension*

Consider 2 electrons separated by a distance of 15 cm.

A) What will be the force experienced by one electron due to the presence of the other?

B) What will be its acceleration if it is unrestrained?

# Computational Example

## *Coulomb's Law in 1-Dimension*

### SETUP

*Draw Diagram*

*List Known Variables*

*List Target Variable*

*Select/write Applicable Equation*

### SOLUTION

*Solve Applicable Equation for Target Variable*

*Substitute Known Variables and Compute*

### REVIEW

*Check Values (does the answer seem reasonable?)*

*Sig Figs and Units*

# Computational Example

## Coulomb's Law in 1 dimension

Consider two electrons separated by a distance of 15 cm. (a) What will be the magnitude of the force experienced by one electron due to the presence of the other? (b) Will this force be repulsive or attractive? (c) What will be the acceleration of either electron if it is released?

### SETUP

#### Known Variables

$$r = 1.5 \times 10^{-1} \text{ m}$$

$$q_e = -1.602 \times 10^{-19} \text{ C}$$

$$m_e = 9.11 \times 10^{-31} \text{ kg}$$

#### Target Variable

a)  $F = ?$

c)  $a = ?$

#### Applicable Equation(s)

a)  $F = k \frac{|q_e q_e|}{r^2}$

c)  $F = m_e a$

### SOLUTION

- a) Solving the applicable equation for the target variable and substituting the known variables (as well as the value for the constant  $k$ ) we obtain

$$F = \left( 8.99 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \right) \frac{|(-1.602 \times 10^{-19} \text{ C})(-1.602 \times 10^{-19} \text{ C})|}{(1.5 \times 10^{-1} \text{ m})^2} = 1.0 \times 10^{-26} \text{ N}$$

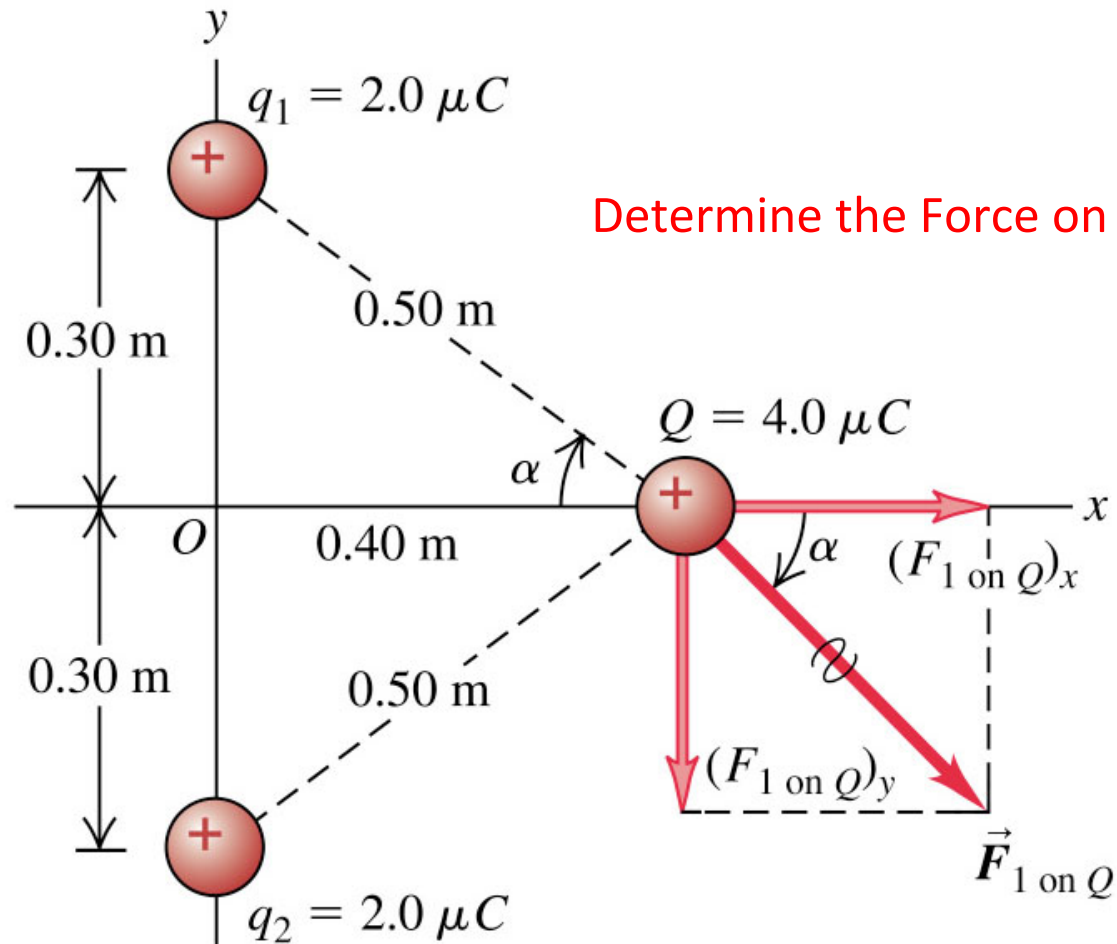
- b) The force is repulsive since it involves two electrons, which have the same charge.
- c) Solving the applicable equation (Newton's Second Law) for the target variable and substituting the known variables yields

$$a = \frac{F}{m_e} = \frac{1.0 \times 10^{-26} \text{ N}}{9.11 \times 10^{-31} \text{ kg}} = 1.1 \times 10^4 \frac{\text{kgm/s}^2}{\text{kg}} = 1.1 \times 10^4 \text{ m/s}^2$$

### REVIEW

The units are correct as well as the number of significant figures (2). The force experienced by one electron located 15 cm from another electron is quite small, but the small mass of an electron allows for a large acceleration.

# Coulomb's Law in 2-Dimensions



Determine the Force on  $Q$  due to  $q_1$  and  $q_2$



# Force due to multiple charges

## (method)

Compute Force Magnitude from Coulombs Law

$$F_{1 \text{ on } Q} = k \frac{|q_1 Q|}{r_1^2}$$

Get direction from geometry and 'attraction rule'

Compute Force Vector Components

$$F_{(1 \text{ on } Q)_x} = F_{1 \text{ on } Q} \cos(\theta) \quad F_{(1 \text{ on } Q)_y} = F_{1 \text{ on } Q} \sin(\theta)$$

Compute Force Vector

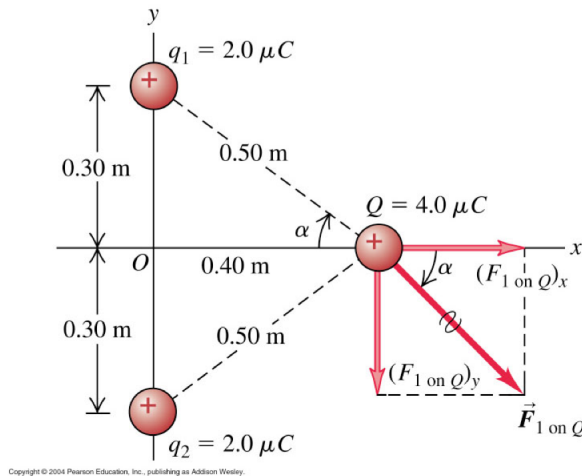
$$\vec{F}_{1 \text{ on } Q} = F_{(1 \text{ on } Q)_x} \hat{i} + F_{(1 \text{ on } Q)_y} \hat{j}$$

Repeat for each particle, then add by components to get Net Force

# Computational Example

## Coulomb's Law in 2 dimensions

Coulomb's Law in two dimensions. Determine the Force on  $Q$  due to  $q_1$  and  $q_2$



### SETUP

#### Known Variables

$$q_1 = q_2 = 2.0 \times 10^{-6} \text{ C}$$

$$Q = 4.0 \times 10^{-6} \text{ C}$$

$$r_1 = r_2 = 0.50 \text{ m}$$

#### Target Variable

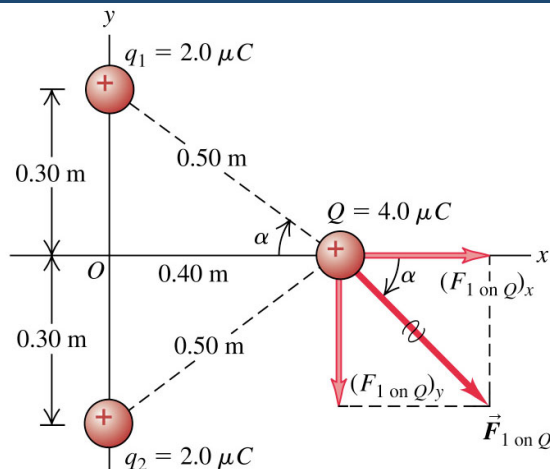
$$F_{\text{on } Q} = ?$$

#### Applicable Equation

$$F = k \frac{|qQ|}{r^2}$$

# Computational Example

## Coulomb's Law in 2 dimensions



### SOLUTION

We will first determine force on  $Q$  due to  $q_1$  and then the force on  $Q$  due to  $q_2$ . The net force on  $Q$  will be the vector sum of these.

Determine the magnitude of the Coulomb force on  $Q$  due to  $q_1$ :

$$F_1 = k \frac{|q_1 Q|}{r_1^2} = \left( 8.99 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \right) \frac{(2.0 \times 10^{-6} \text{ C})(4.0 \times 10^{-6} \text{ C})}{(0.50 \text{ m})^2} = 0.29 \text{ N}$$

Determine the x and y-components of  $F_1$ :

$$F_{1,x} = F_1 \cos \alpha = (0.29 \text{ N}) \frac{0.4 \text{ m}}{0.5 \text{ m}} = 0.23 \text{ N}$$

$$F_{1,y} = F_1 \sin \alpha = (0.29 \text{ N}) \frac{-0.3 \text{ m}}{0.5 \text{ m}} = -0.17 \text{ N}$$

Examination of the geometry reveals symmetry that we can take advantage of to reduce our workload. Note first that  $q_1$  and  $q_2$  have the same magnitude and they are both positive, located the same distance away from  $Q$ . This means that they will produce the same magnitude of force on  $Q$ . More specifically,  $q_1$  and  $q_2$  have the same x-coordinate location and opposite y-coordinate locations relative to  $Q$ . Therefore, the x-components of their forces will add while the y-components of their forces will cancel each other out (before proceeding, convince yourself that this is correct.) So, by symmetry, we have

$$F_{2,x} = F_{1,x} = 0.23 \text{ N} \text{ and } F_{1,y} = -F_{2,y}.$$

# Computational Example

## Coulomb's Law in 2 dimensions

The force on  $Q$  due to  $q_1$  and  $q_2$  is determined by adding the vector components due to each of them.

$$F_x = \sum_i F_{i,x} = F_{1,x} + F_{2,x} = 0.23 \text{ N} + 0.23 \text{ N} = 0.46 \text{ N}$$

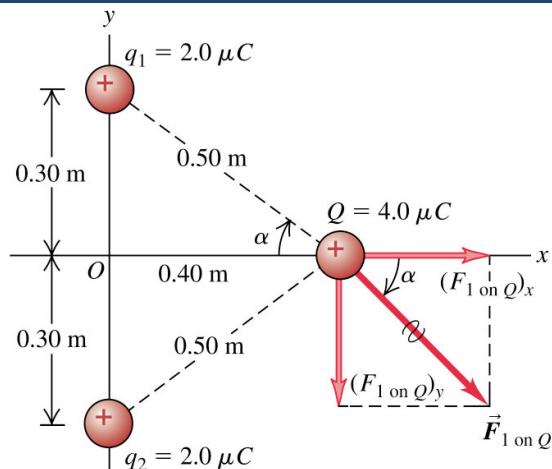
$$F_y = \sum_i F_{i,y} = F_{1,y} + F_{2,y} = 0 \text{ N}$$

We now have the vector components for the resultant vector,  $F$ . The final step is to express the final answer in unit vector notation.

$$\vec{F}_{\text{on } Q} = F_x \hat{i} + F_y \hat{j} = (0.46 \text{ N}) \hat{i}$$

### REVIEW

We have checked our setup and mathematics as well as the units and all appear to be correct and reasonable. We took advantage of the symmetry of the geometry to save ourselves a little bit of work along the way.



**Lab #1**

**Electrostatics**