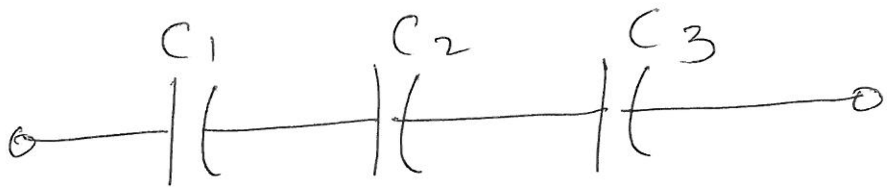
Series and Parallel Combinations

Examples :

a) Series Connections

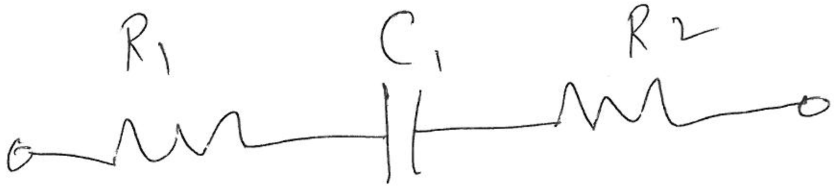
(i)

(ii)



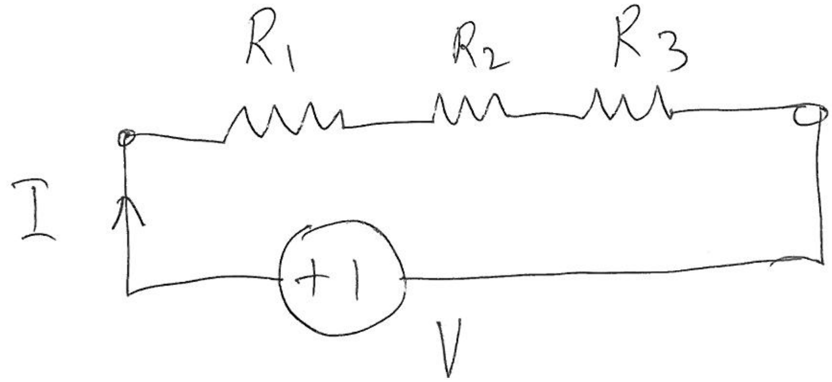
(iii)

(iv)



(v)

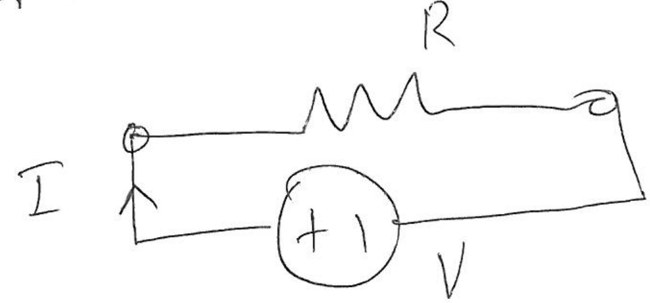
(i)



using KVL we can show

$$(R_1 + R_2 + R_3) I = V$$

Then we can derive an equivalent circuit as shown below



$$R I = V$$

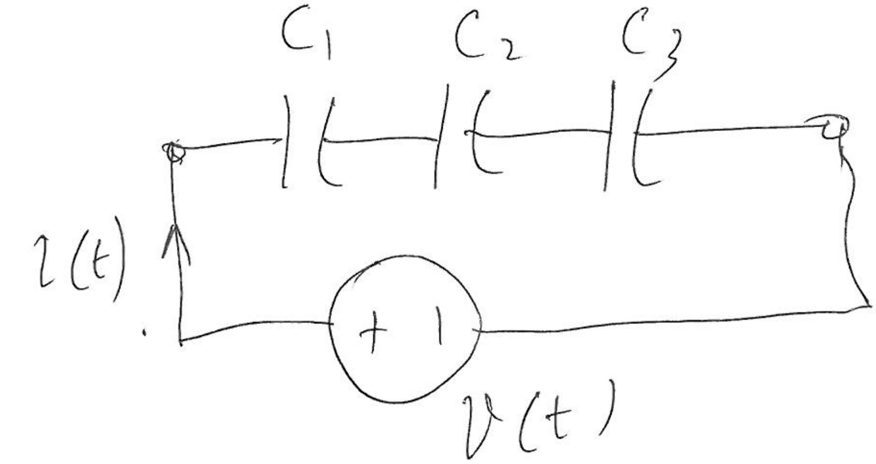
If we have the same I in both circuits then we can show that

$$R = R_1 + R_2 + R_3$$

ie <u>Resistors in series add</u>

(3)

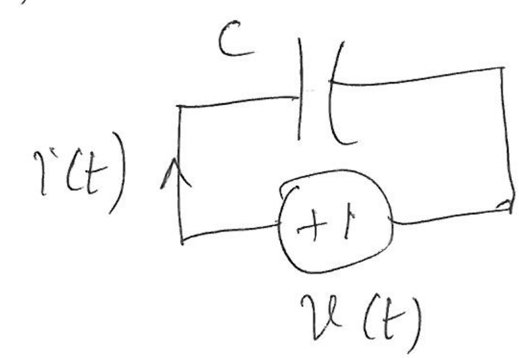
(ii)



Using KVL we can write

$$v(t) = \frac{1}{C_1}\int i(t)\,dt + \frac{1}{C_2}\int i(t)\,dt + \frac{1}{C_3}\int i(t)\,dt \qquad -(1)$$

We will construct a circuit with one capacitor that will have the same current as the circuit with three capacitors

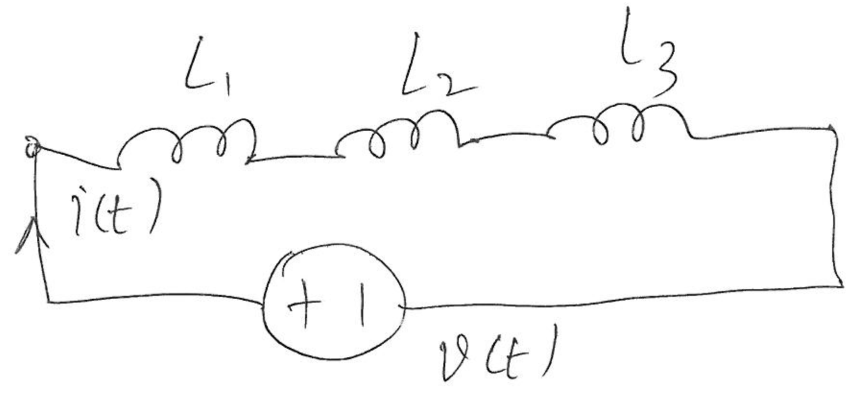


$$v(t) = \frac{1}{C}\int i(t)\,dt \qquad -(2)$$

This leads to $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

Capacitors in Series add reciprocally

(iii)

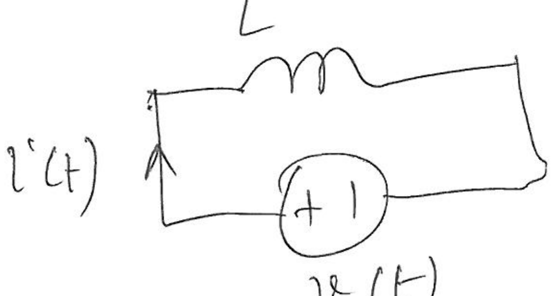


Using KVL we can write

$$v(t) = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + L_3 \frac{di}{dt}$$

$$= (L_1 + L_2 + L_3) \frac{di}{dt}$$

We can construct an equivalent circuit with one inductor L that will have the same voltage and current



$$v(t) = L \frac{di}{dt}$$

This leads to $L = L_1 + L_2 + L_3$

Inductors in series add

(iv)

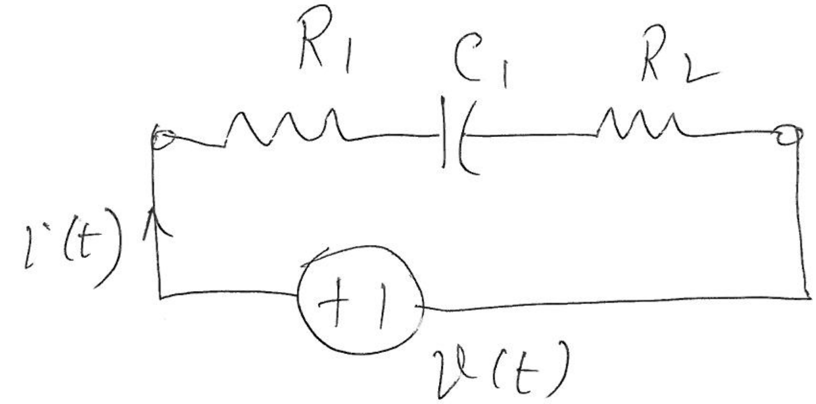


Fig 1

This is a special circuit and we will handle it in two ways

a) $v(t)$ is an arbitrary time function

Then $v(t) = R_1\, i(t) + \frac{1}{C_1}\int i(t)\,dt + R_2\, i(t)$

$$= \left(R_1 + R_2 + \frac{1}{C_1 D}\right) i(t)$$

Note: $\frac{1}{D}$ is the integral operator

b) $v(t) = A e^{j\omega_0 t}$ — AC circuit

In this case we can replace the capacitor with its impedance $\frac{1}{j\omega_0 C}$. We redraw the circuit as shown in Fig 2

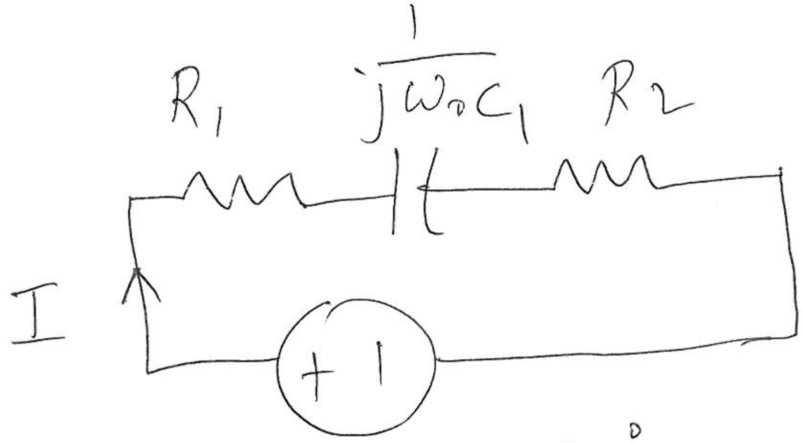


Fig 2

$V = A\angle 0^\circ$ -

We now explain this. $Ae^{j(\omega_0 t+\theta)}$ is a complex sinusoidal signal with amplitude A and initial angle θ°. In Fig 2, this angle is 0° ie $\theta = 0^\circ$.

~~We rep~~ V and I are <u>phasor</u> representations of voltage and current when we deal with an AC circuit we simply write the KVL equation as

$$V = R_1 I + \frac{1}{j\omega_0 C_1} I + R_2 I$$

$$= \left(R_1 + \frac{1}{j\omega_0 C_1} + R_2\right) I$$

We can write this as

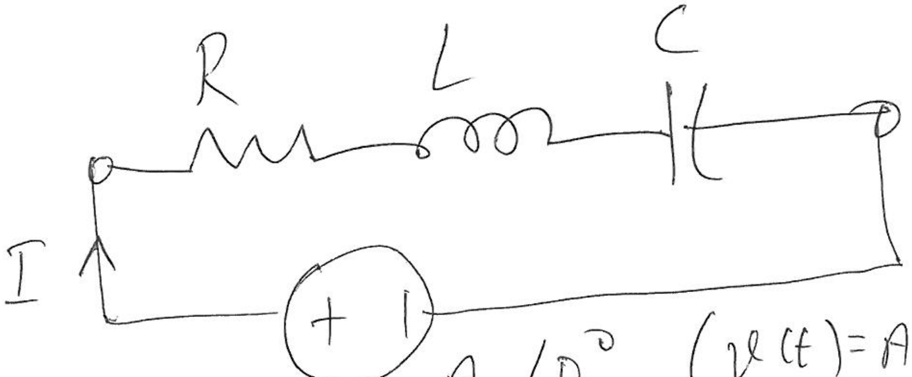
$$V = Z I \quad \text{where}$$

$$Z = R_1 + R_2 + \frac{1}{j\omega_0 C}$$

Z is called the impedance of the circuit. In this series combination we can say that

<u>Impedances in Series add</u>

(v)



<u>AC analysis</u>

$V = A\angle 0^\circ \quad (v(t) = A e^{j\omega_0 t})$

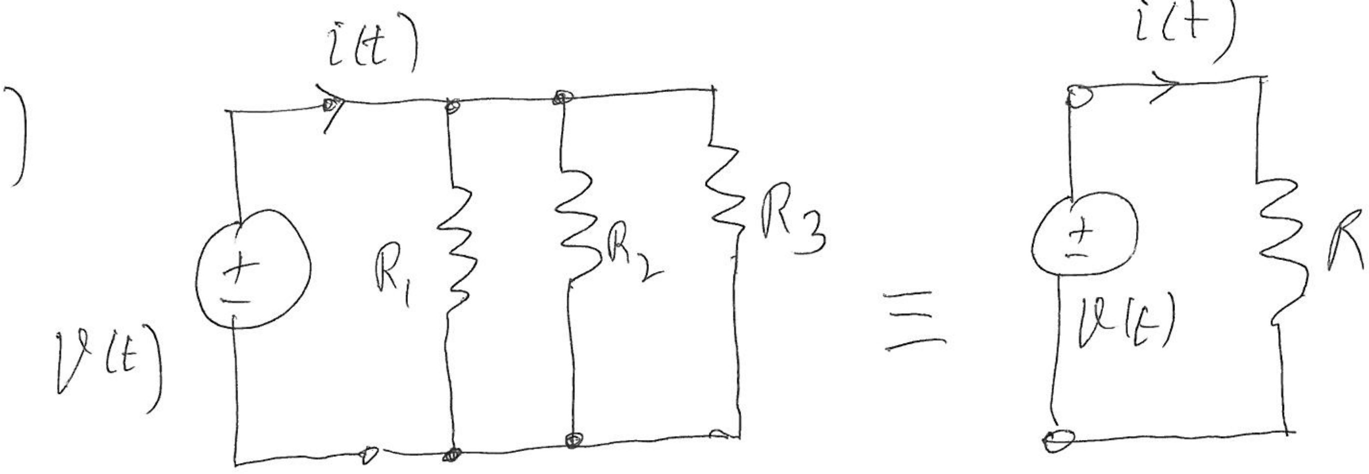
Using KVL, $V = \left(R + j\omega_0 L + \frac{1}{j\omega_0 C}\right) I$

$$V = Z I, \quad Z = R + j\omega_0 L + \frac{1}{j\omega_0 C}$$

<u>Impedances in Series add</u>

Parallel Combinations

(i)



using KCL we can write

$$i(t) = \frac{v(t)}{R_1} + \frac{v(t)}{R_2} + \frac{v(t)}{R_3} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) v(t)$$

For the circuit on the right we get

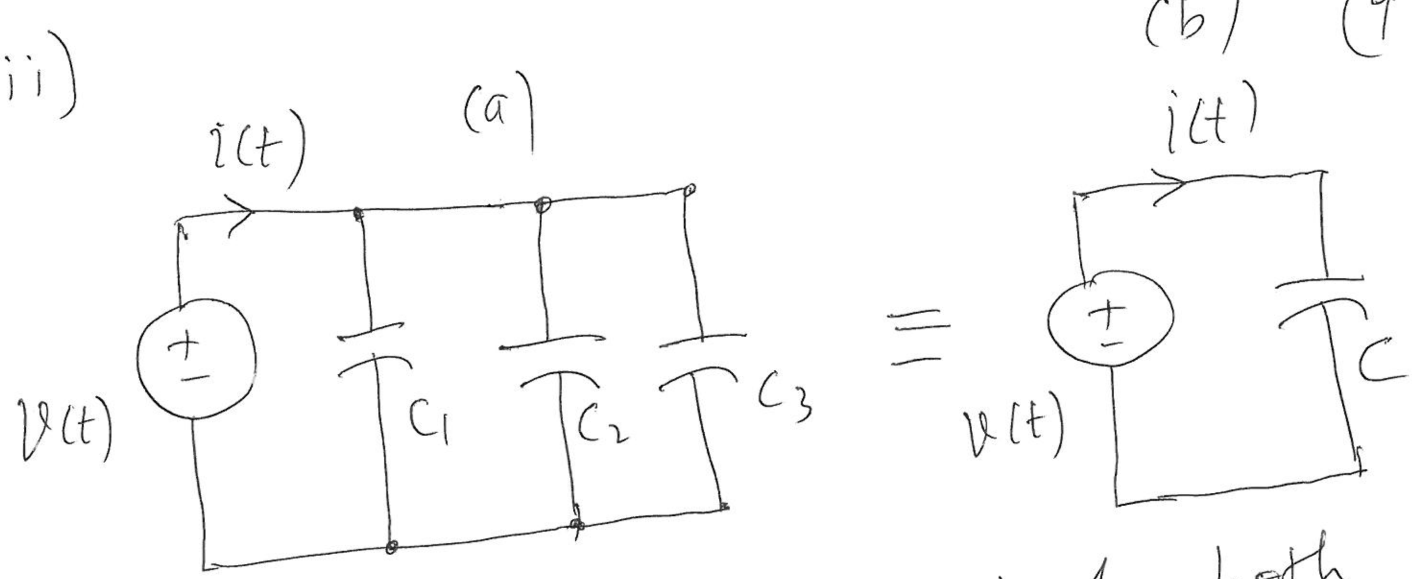
$$i(t) = \frac{v(t)}{R}$$

If $v(t)$ and $i(t)$ are the same in both circuits then we can show that

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Resistors in parallel add reciprocally

(ii)



Using KCL we can write for both circuits

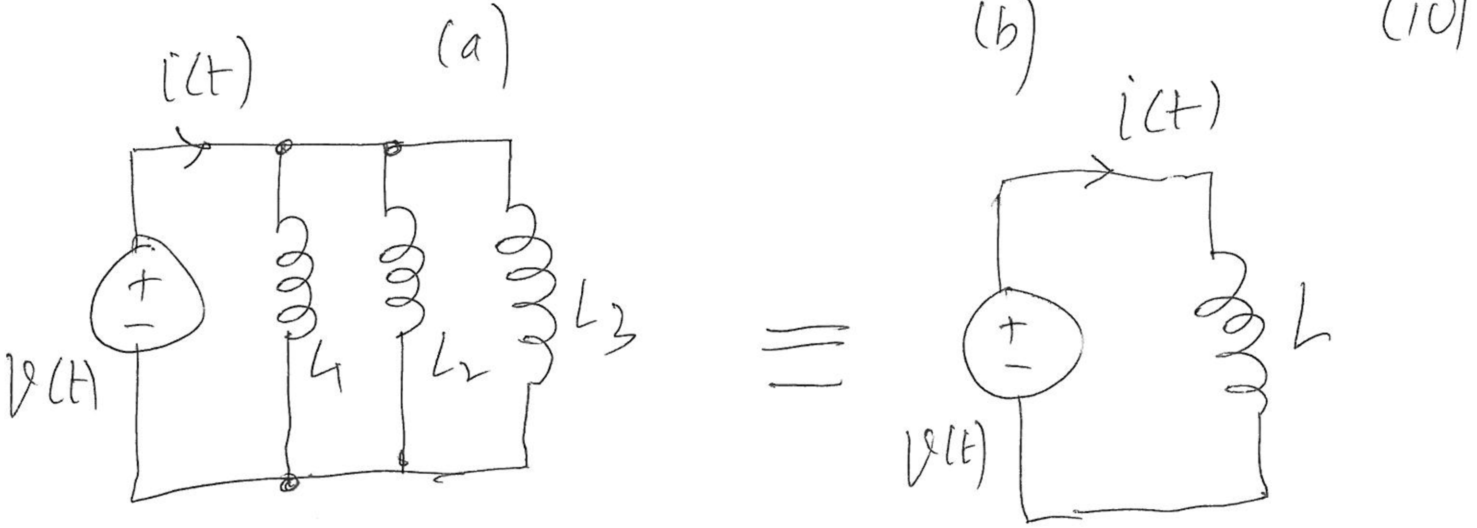
$$i(t) = C_1\frac{dv}{dt} + C_2\frac{dv}{dt} + C_3\frac{dv}{dt} \quad \text{for (a)}$$

$$i(t) = C\frac{dv}{dt} \quad \text{for (b)}$$

If i(t) and v(t) are the same in both circuits then we can establish the following relationship:

$$C = C_1 + C_2 + C_3$$

ie <u>Capacitors in parallel add</u>



using KCL we get

$$i(t) = \frac{1}{L_1}\int v(t)\,dt + \frac{1}{L_2}\int v(t)\,dt + \frac{1}{L_3}\int v(t)\,dt$$

$$= \left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}\right)\int v(t)\,dt \longrightarrow (a)$$

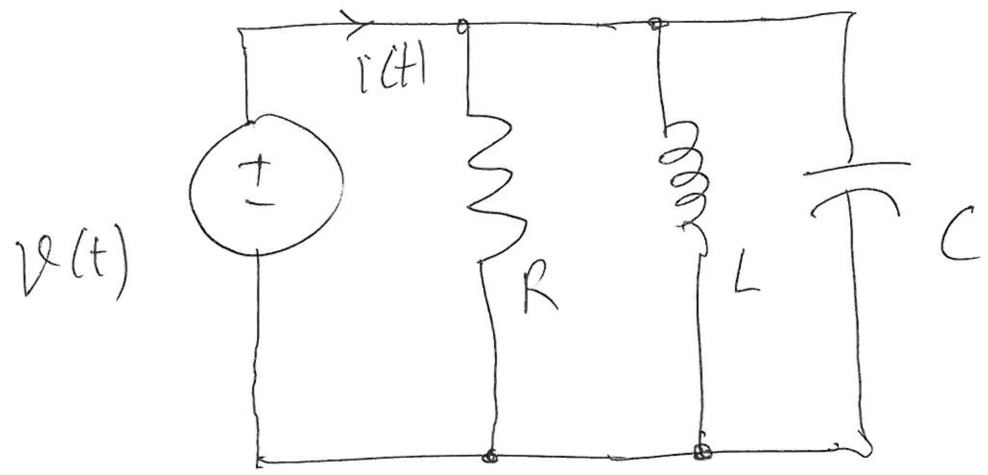
$$i(t) = \frac{1}{L}\int v(t)\,dt \longrightarrow (b)$$

If v(t) and i(t) are the same for both circuits then we can show that

$$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

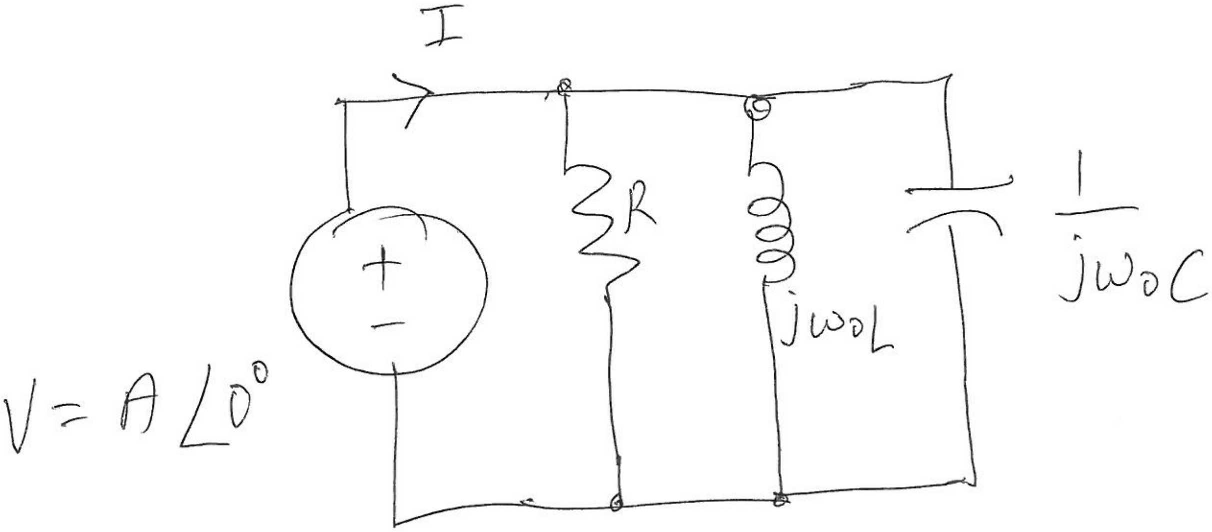
Inductors in parallel add reciprocally

Special Case – AC Circuits



For <u>AC Circuits</u> $v(t) = A e^{j(\omega_0 t)}$

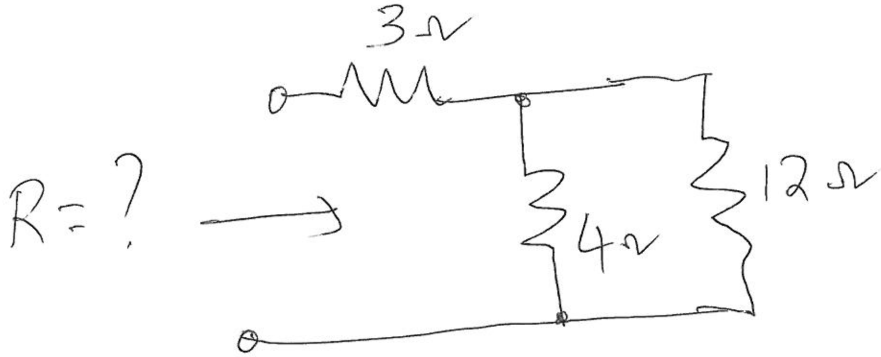
When $v(t)$ is an AC signal we redraw the circuit



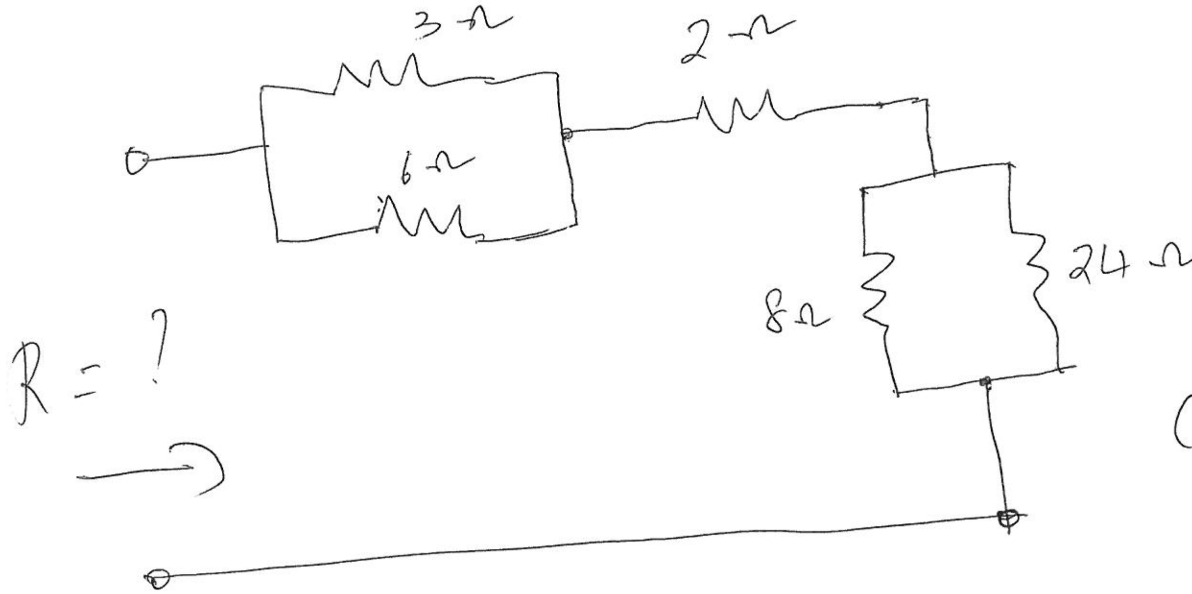
<u>KCL:</u> $I = \left(\frac{1}{R} + \frac{1}{j\omega_0 L} + j\omega_0 C\right) V$

ie Impedances in parallel add reciprocally

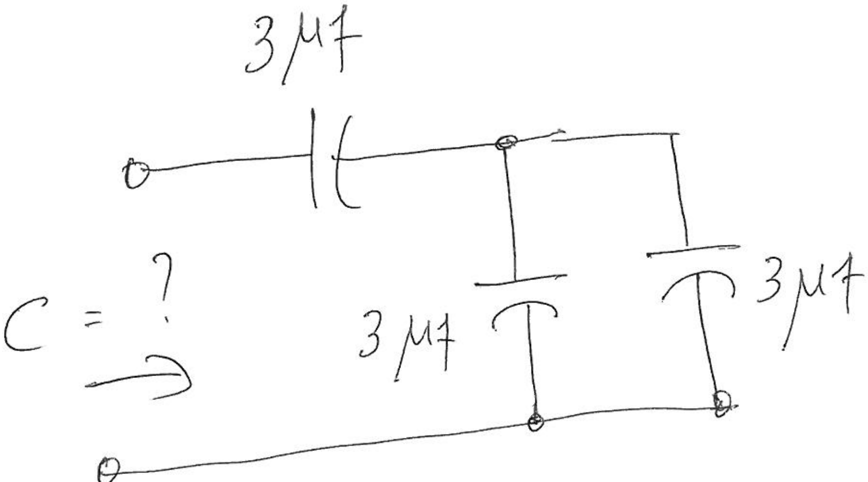
Practice problems



(a)



(b)



(c)