~~Fall 2013~~ ECE 210 Soln to Practice Problems - 1

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1 a) $Z = 1 + j2 \quad |Z| = \sqrt{5} \quad \angle Z = \tan^{-1}\frac{2}{1} \simeq 63^\circ$

b) $Z = -1 + j2 \quad |Z| = \sqrt{5} \quad \angle Z \ \tan^{-1}\frac{2}{-1} \simeq 117^\circ$

c) $Z = 1 - j2 \quad |Z| = \sqrt{5} \quad \angle Z = \tan^{-1}\frac{-2}{1} \simeq -63^\circ$

d) $Z = -1 - j2 \quad |Z| = \sqrt{5} \quad \angle Z = \tan^{-1}\frac{-2}{-1} \simeq -117^\circ$

e) $e^{j\pi} = 1\angle 180^\circ$, f) $e^{j\frac{\pi}{2}} = 1\angle 90^\circ$

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2) a) $5e^{j\frac{\pi}{6}} = 5\cos\left(\frac{\pi}{6}\right) + j5\sin\frac{\pi}{6}$

$Re = \frac{5\sqrt{3}}{2}$, $Im = \frac{5}{2}$

b) $2e^{-j\frac{\pi}{3}} \rightarrow Re = 2\cos\left(\frac{\pi}{3}\right) = 1$

$Im = -2\sin\left(\frac{\pi}{3}\right) = -\sqrt{3}$

c) $e^{j\frac{\pi}{4}} = \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}$

d) $\sqrt{3 + j4} = \sqrt{5e^{j\theta}} = \sqrt{5}\, e^{j\frac{\theta}{2}}$

$= \sqrt{5}\angle 27^\circ$

$\theta = \tan^{-1}\frac{4}{3} \simeq 53^\circ$

$Re = \sqrt{5}\cos(27^\circ)$, $Im = \sqrt{5}\sin(27^\circ)$

e) $(3 - j4)^{1/3} = \left[5e^{-j\theta}\right]^{1/3} = 5^{1/3} e^{-j\frac{\theta}{3}}$

$\theta = \tan^{-1} -\frac{4}{3} \simeq -53^\circ$

$Re = 5^{1/3} \cos(18^\circ)$, $Im = 5^{1/3} \sin(18^\circ)$

f) $$\frac{2+j3}{3+j4} = \frac{\sqrt{13}\angle \tan^{-1} 3/2}{5\angle \tan^{-1} 4/3} = \frac{\sqrt{13}}{5}\angle 56^\circ - 53^\circ = \frac{\sqrt{13}}{5}\angle 3^\circ$$

$Re = \frac{\sqrt{13}}{5}\cos(3^\circ)$, $Im = \frac{\sqrt{13}}{5}\sin(3^\circ)$

g) $$(2+j3)(3+j4) = 5\sqrt{13}\angle \tan^{-1}\frac{3}{2} + \tan^{-1}\frac{4}{3}$$

$$= 5\sqrt{13}\angle 109^\circ$$

$Re = 5\sqrt{13}\cos(109^\circ)$, $Im = 5\sqrt{13}\sin(109^\circ)$

h) $$(2+j3)e^{j\frac{\pi}{3}} = \sqrt{13}\angle 56^\circ + 60^\circ = \sqrt{13}\angle 116^\circ$$

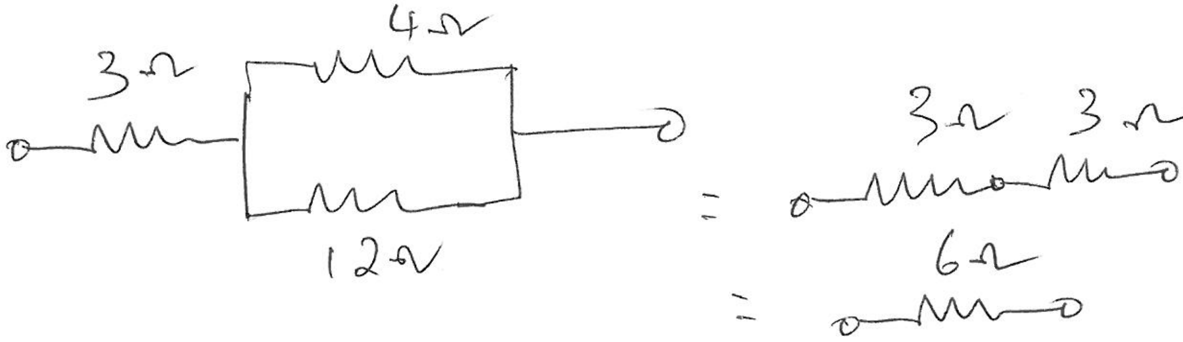
$Re = \sqrt{13}\cos(116^\circ)$, $Im = \sqrt{13}\sin(116^\circ)$

i) $$\frac{e^{j\frac{\pi}{4}}}{3+j4} = \frac{1\angle 45^\circ}{5\angle 53^\circ} = 0.2\angle -8^\circ$$

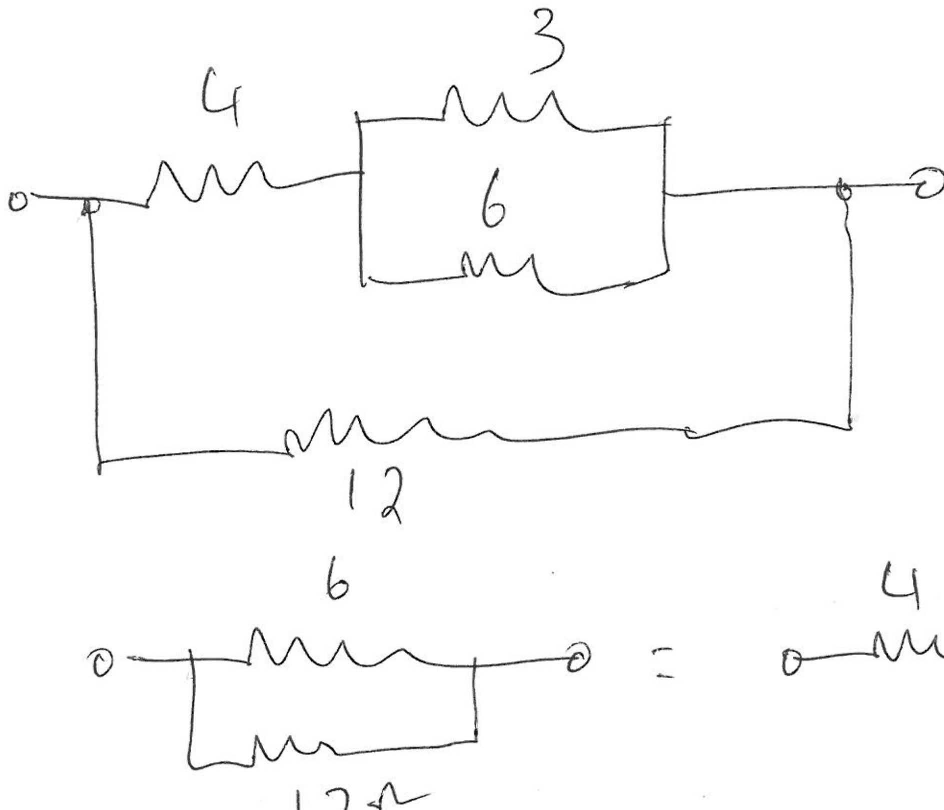
$Re = 0.2\cos(8^\circ)$

$Im = -0.2\sin(8^\circ)$

3) a)


3Ω
4Ω
12Ω
=
3Ω 3Ω
=
6Ω

b)


4
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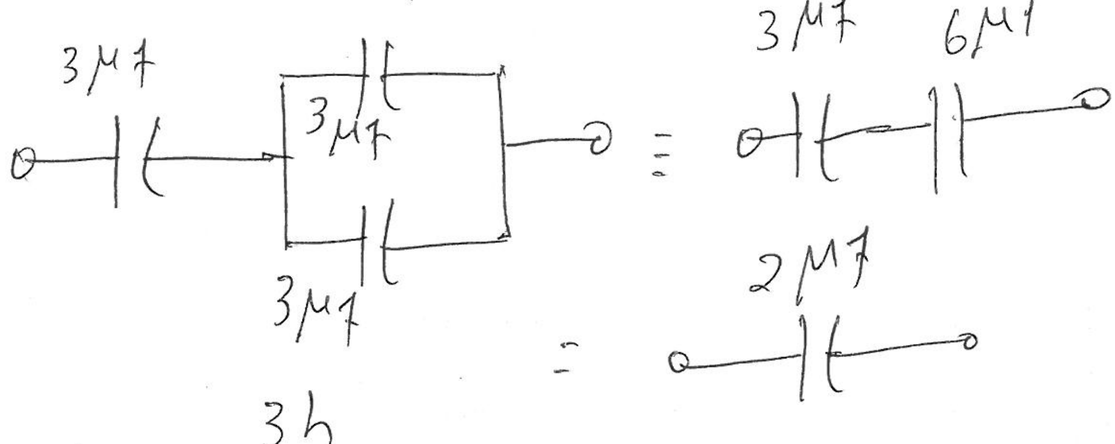
c)


3μf
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3μf 6μf
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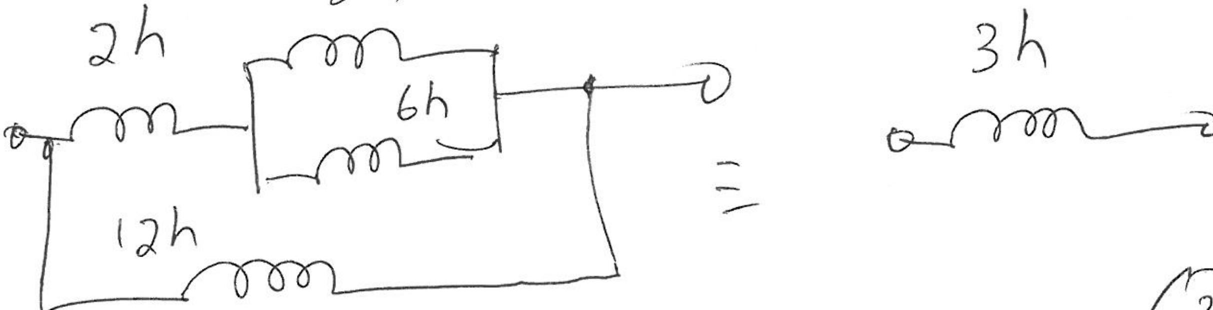
d)


2h
3h
6h
12h
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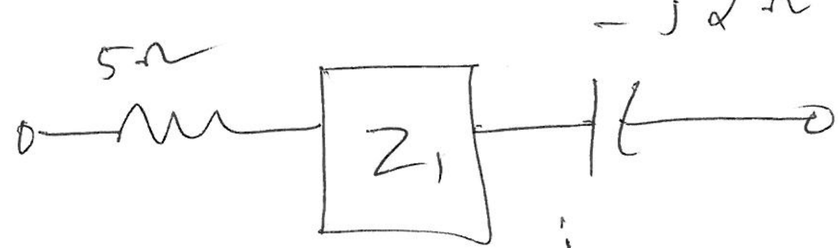
3h

3 e)



$Z_1 = \frac{j}{1+j}$

$Z = 5 + \frac{j}{1+j} - j2$

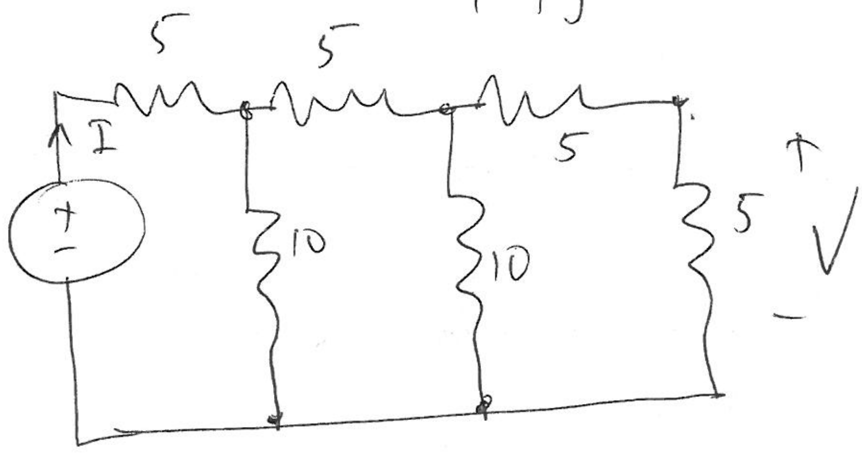
$\frac{5(1+j) + j - j2(1+j)}{1+j}$

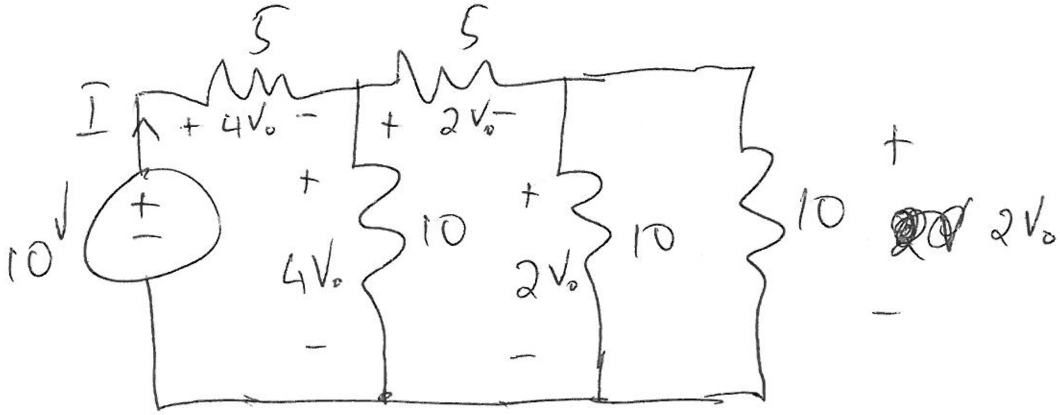
$= \frac{5 + j5 + j - j2 + 2}{1+j} = \frac{7+j4}{1+j}$

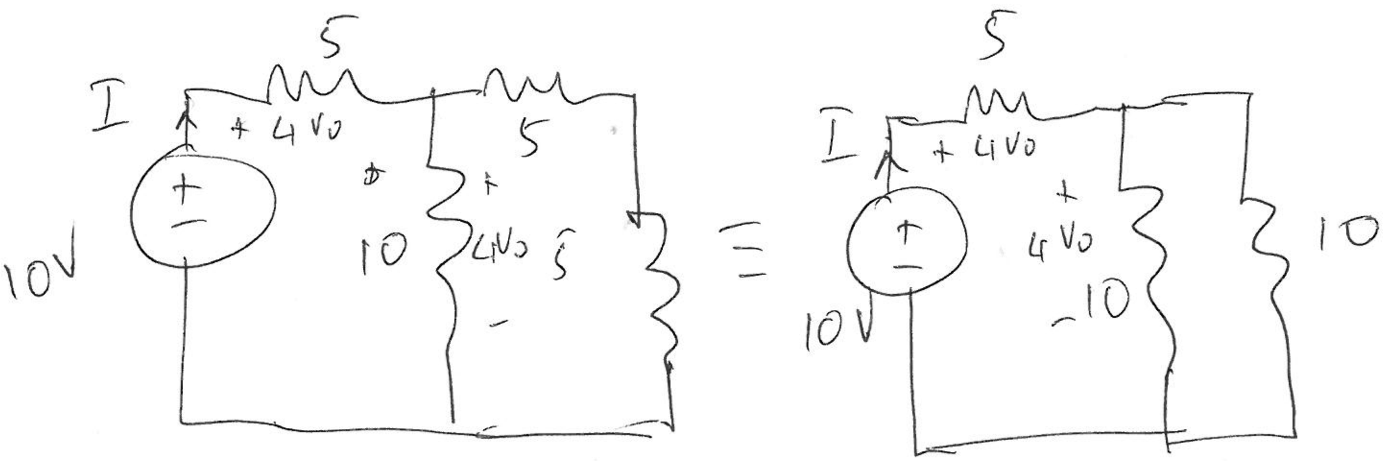
$\frac{(7+j4)(1-j)}{2}$

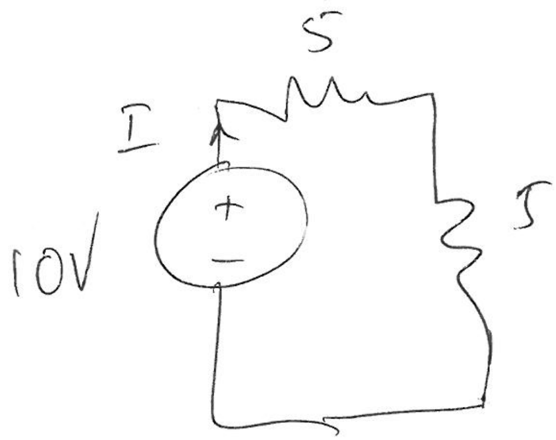
$\frac{11-j3}{2} = 5.5 - j1.5$

4)





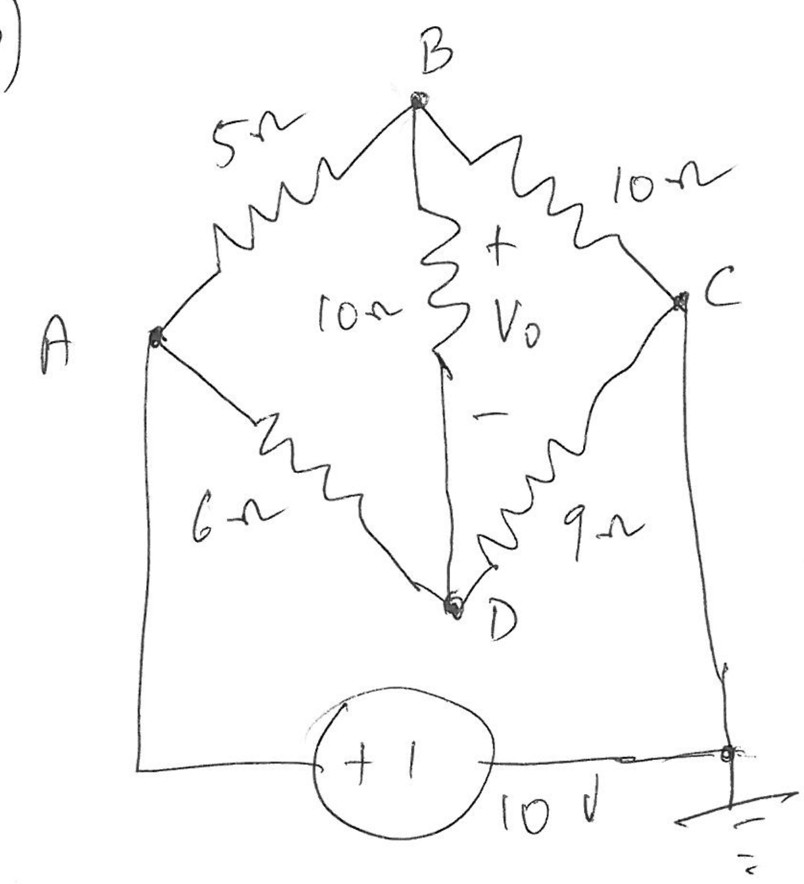




$$I = \frac{10V}{10\Omega} = 1A$$

$$8V_0 = 10V, \quad V_0 = \frac{10}{8} = \frac{5}{4}V$$

5b)



Find V_o

C is Ref node

<u>Node A</u> : $V_A = 10V$

<u>Node B</u> : $-\frac{V_A}{5} + \left(\frac{1}{5}+\frac{1}{10}+\frac{1}{10}\right)V_B - \frac{1}{10}V_D = 0$

$-2V_A + 4V_B - V_D = 0$

$4V_B - V_D = 2V_A = 20$ —— ①

<u>Node D</u> : $-\frac{V_A}{6} - \frac{V_B}{10} + \left(\frac{1}{6}+\frac{1}{9}+\frac{1}{10}\right)V_D = 0$

<u>multiply by 90</u> $-15V_A - 9V_B + 34V_D = 0$ ②

$-9V_B + 34V_D = 15V_A = 150$ —— ⑥

① $4V_B - V_D = 20$ Multiply by 34 and add

② $-9V_B + 34V_D = 150$

$$136V_B - 34V_D = 680$$
$$-9V_B + 34V_D = 150$$

Adding $127V_B = 830$

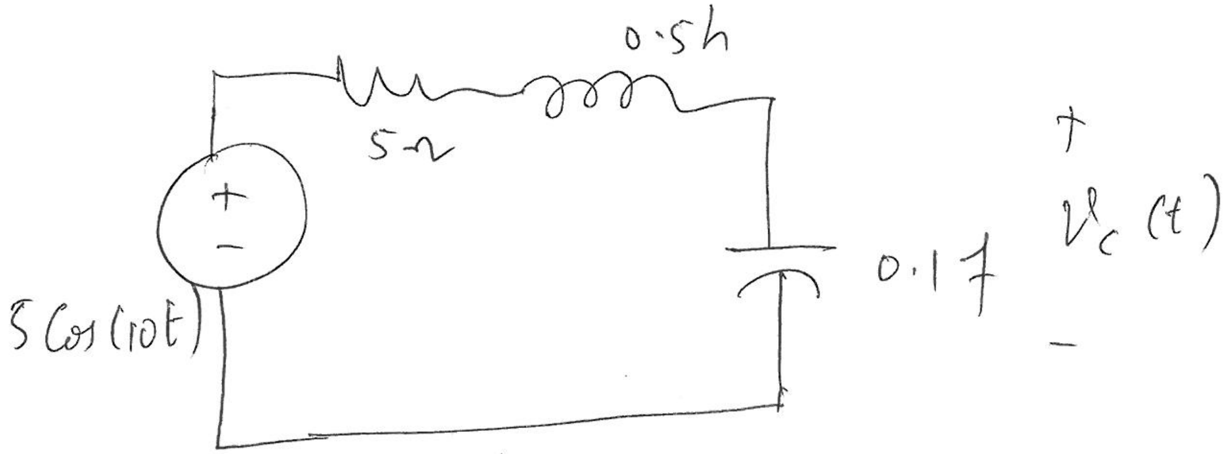
$$V_B = \frac{830}{127} = 6.53\,V$$

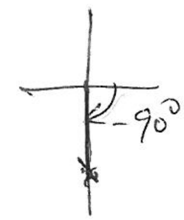
From ① $V_D = 4V_B - 20$

$$= \frac{(830)(4)}{127} - 20 = \frac{3320}{127} - 20$$
$$= 6.14\,V$$

$$V_D = 6.14\,V$$

$$\boxed{V_o = V_B - V_D = 6.53 - 6.14 = 0.39\,V}$$





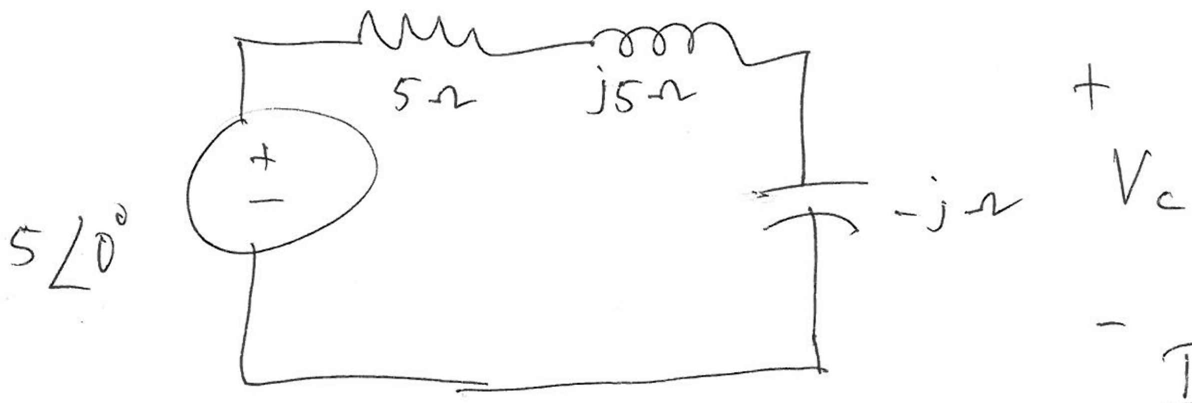
$$\Downarrow$$

$$\omega_0 = 10 \text{ r/s}$$

$$5\Omega \rightarrow 5\Omega$$

$$0.5h \rightarrow j\omega_0 L = j(10)(0.5) = j5$$

$$0.1F \rightarrow \frac{1}{j\omega_0 C} = \frac{-j}{(10)(0.1)} = -j$$



$$I = \frac{5}{5+j4}$$

$$V_C = \frac{-j}{5+j5-j} 5\angle 0^\circ$$

$$= \frac{-j5\angle 0^\circ}{5+j4} = \frac{5\angle -90^\circ}{\sqrt{41}\angle 39^\circ}$$

$$V_c = \frac{5}{\sqrt{41}} \angle -129^\circ = \frac{5}{6.42} \angle -129^\circ = 0.78\angle -129^\circ$$

8

$$V_C = 0.78\angle -129^\circ$$

$$\boxed{v_C(t) = 0.78\cos(10t - 129^\circ)}$$