Thursday January 23

Important Dates

Unit 1

Assignment 1.1	-	**TONIGHT**
Assignment 1.2	-	Next Thursday, January 30
Assignment 1.3	-	Tuesday February 4

Unit 1 Exam - Thursday February 6

TUESDAY

Quiz 2

Electric Field Calculation

TODAY

Quiz 1

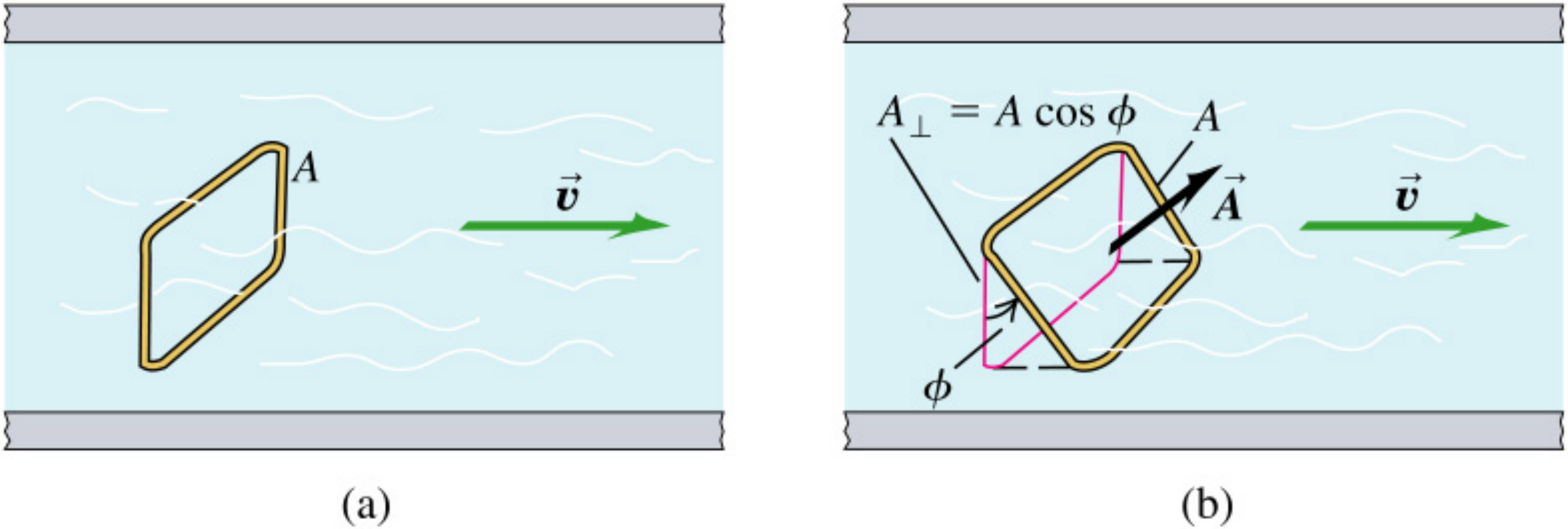
Coulomb's Law

Gauss's Law

Chapter 22

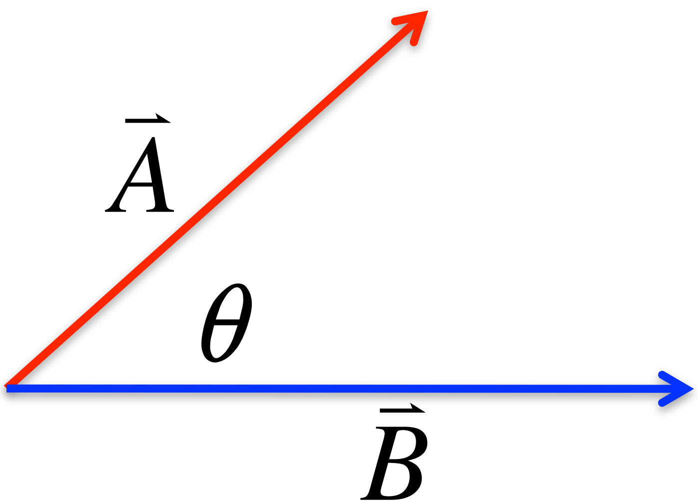
The concept of ***Electric Flux***

Flux



$$\Phi = \vec{v}\cdot\vec{A}$$

Dot Product
review



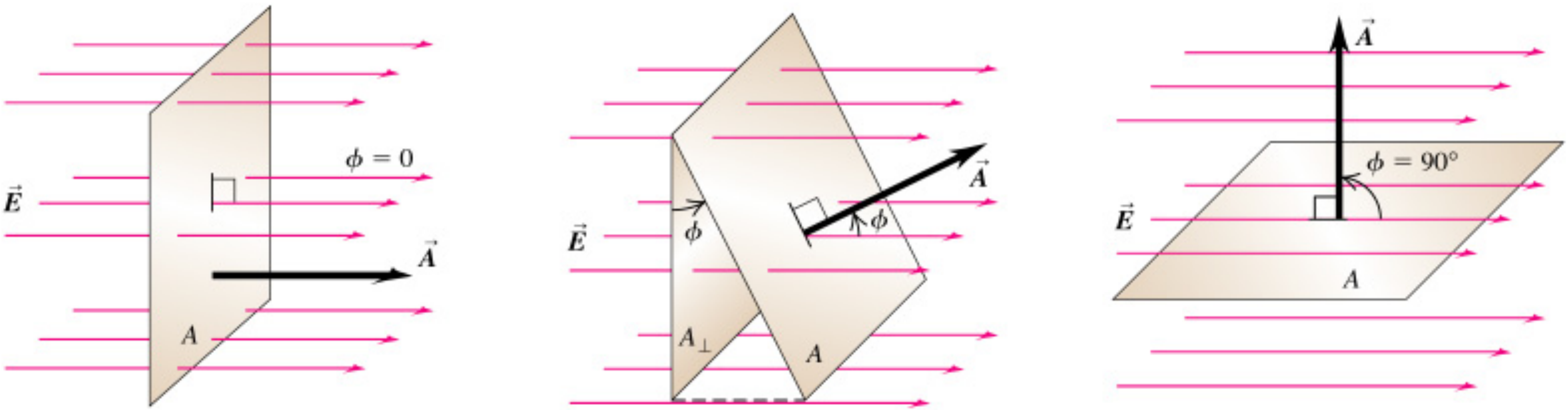
$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + \cdots$$

The Dot Product is a ***Scalar***, so
Electric Flux is a ***scalar***

Electric Flux

$$\boxed{\Phi_E = \vec{E} \cdot \vec{A}}$$

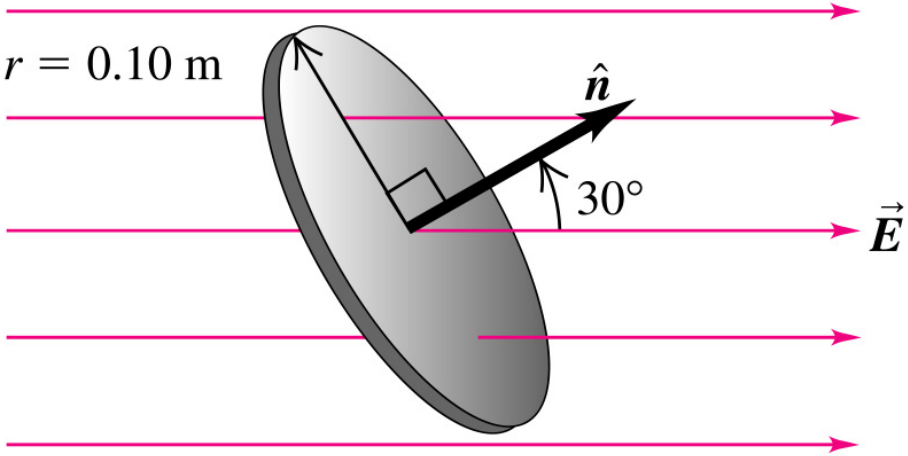


(a) Surface face-on to electric field
$\vec{E}$ and $\vec{A}$ parallel
angle between $\vec{E}$ and $\vec{A}$ is $\phi = 0$
flux $\Phi_E = \vec{E} \cdot \vec{A} = EA$

(b) Surface tilted from face-on orientation
by an angle ϕ
angle between $\vec{E}$ and $\vec{A}$ is ϕ
flux $\Phi_E = \vec{E} \cdot \vec{A} = EA \cos \phi$

(c) Surface edge-on to electric field
$\vec{E}$ and $\vec{A}$ perpendicular angle
between $\vec{E}$ and $\vec{A}$ is $\phi = 90°$
flux $\Phi_E = \vec{E} \cdot \vec{A} = EA \cos 90° = 0$

Calculate the Electric Flux through the surface if E has Magnitude 2.5 N/C



$$\Phi_E = \vec{E} \cdot \vec{A} = EA\cos\theta$$

$$\Phi_E = \underbrace{(2.5\ \mathrm{N/C})}_{E}\,\underbrace{\pi(0.1\ \mathrm{m})^2}_{A}\cos 30 = 6.8\times10^{-2}\ \mathrm{Nm^2/C}$$

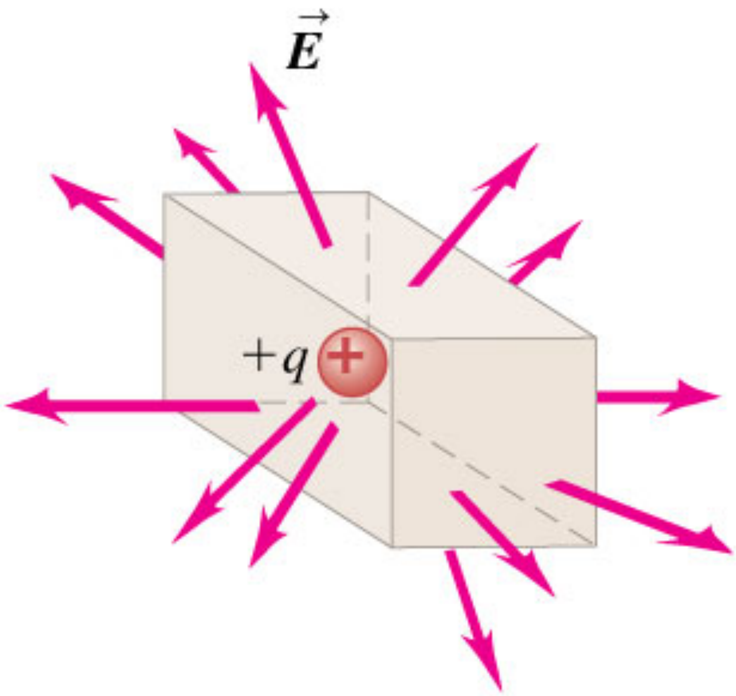
Probing the Electric Field on a ***Closed Surface***

and

Examining the ***Net Electric Flux*** through the closed surface

Electric Flux

Positive Charge in the box,
Outward Flux



Negative Charge in the box,
Inward Flux

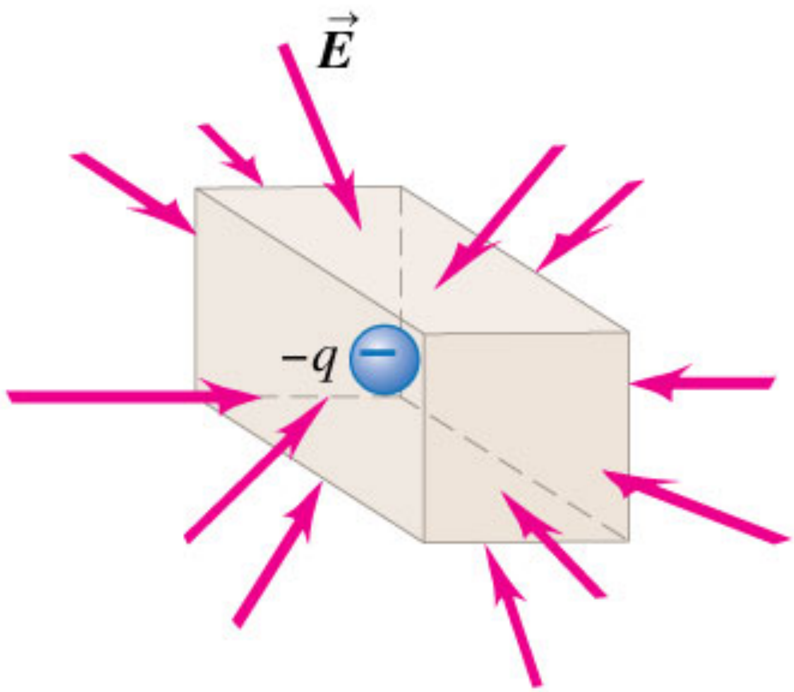
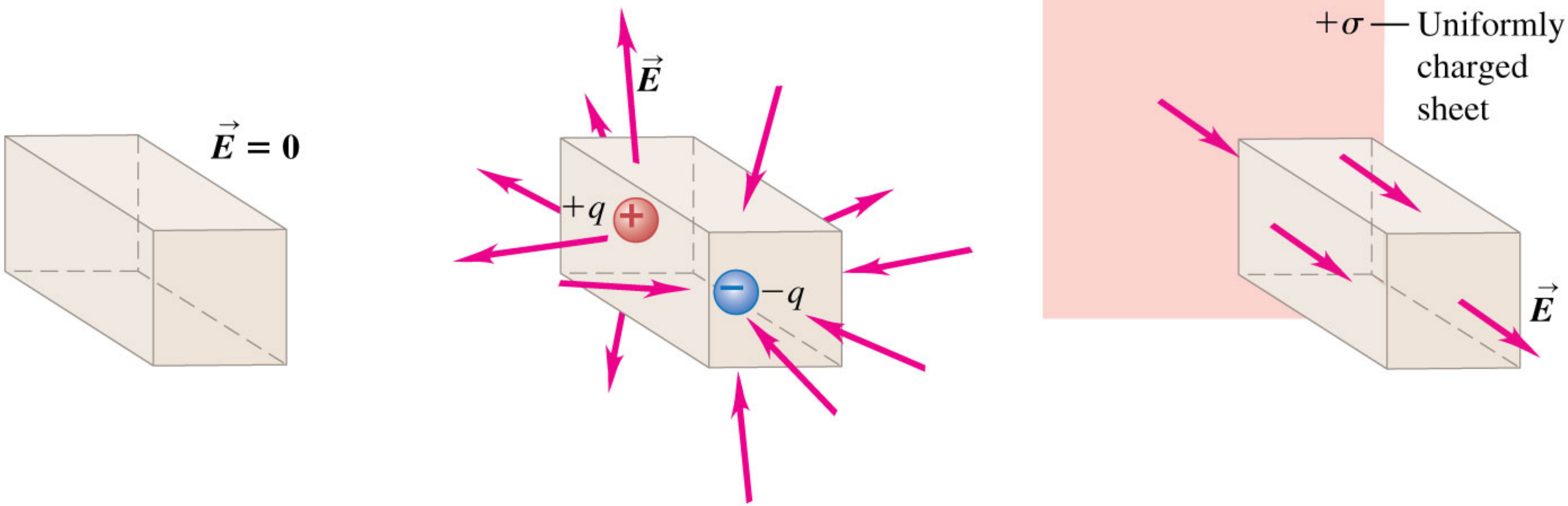


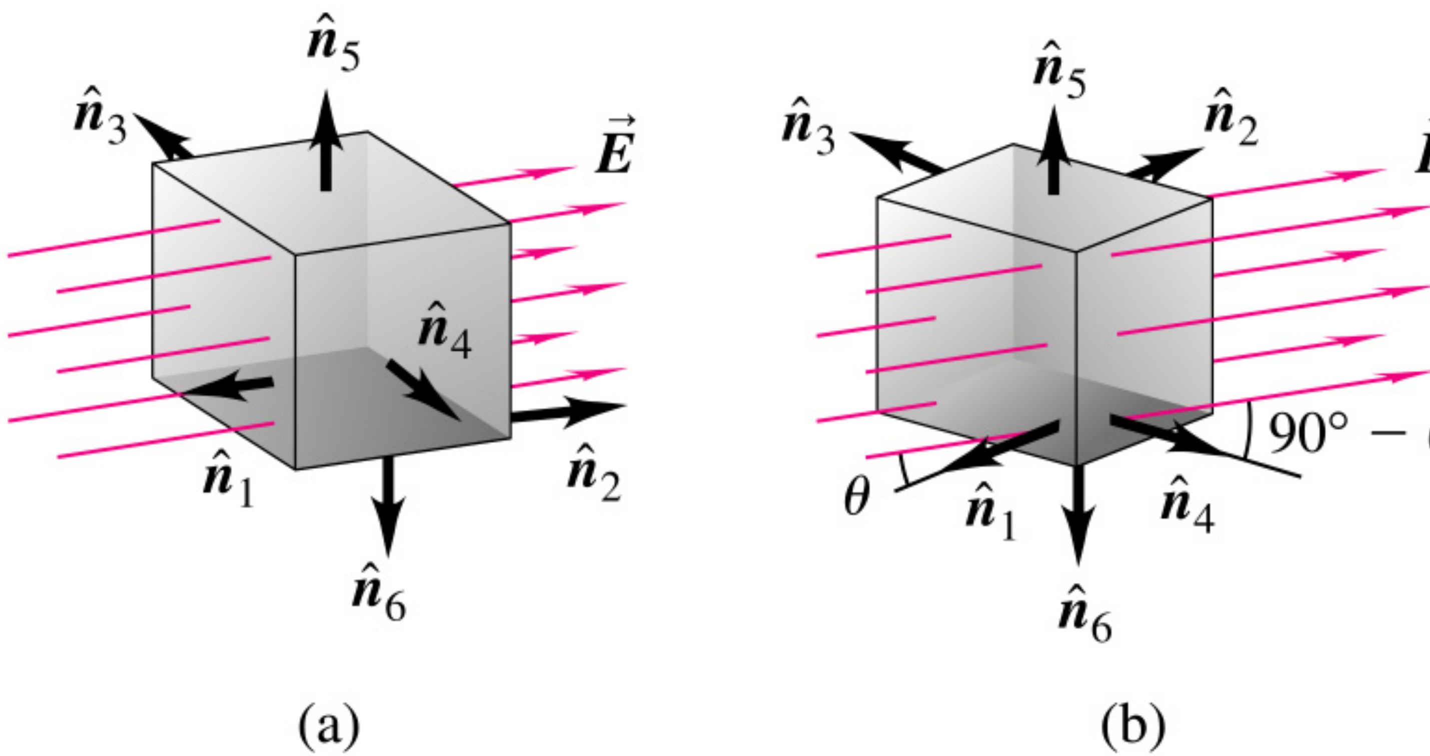
Figure 22.3

(a) No charge inside box, zero flux

(b) Zero *net* charge inside box, inward flux cancels outward flux.

(c) No charge inside box, inward flux cancels outward flux.





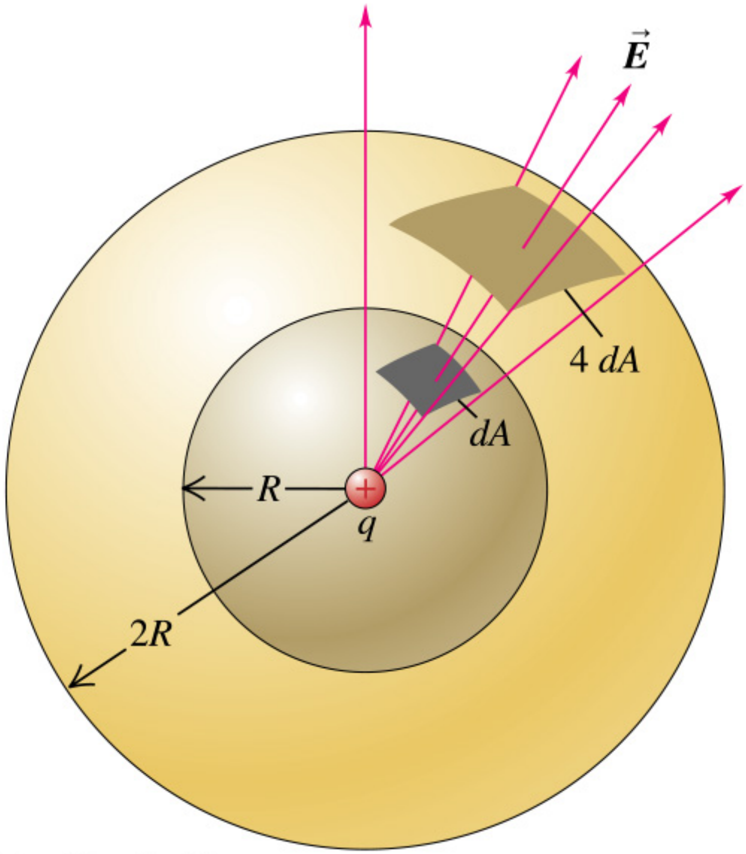
$$\Phi_E = \sum_i \vec{E} \cdot \vec{A}_i = \sum_i \vec{E} \cdot \left(A_i \hat{n}_i \right)$$

Electric Flux

general definition

$$\Phi_{\text{net}} = \sum_i \vec{E} \cdot \vec{A}_i$$

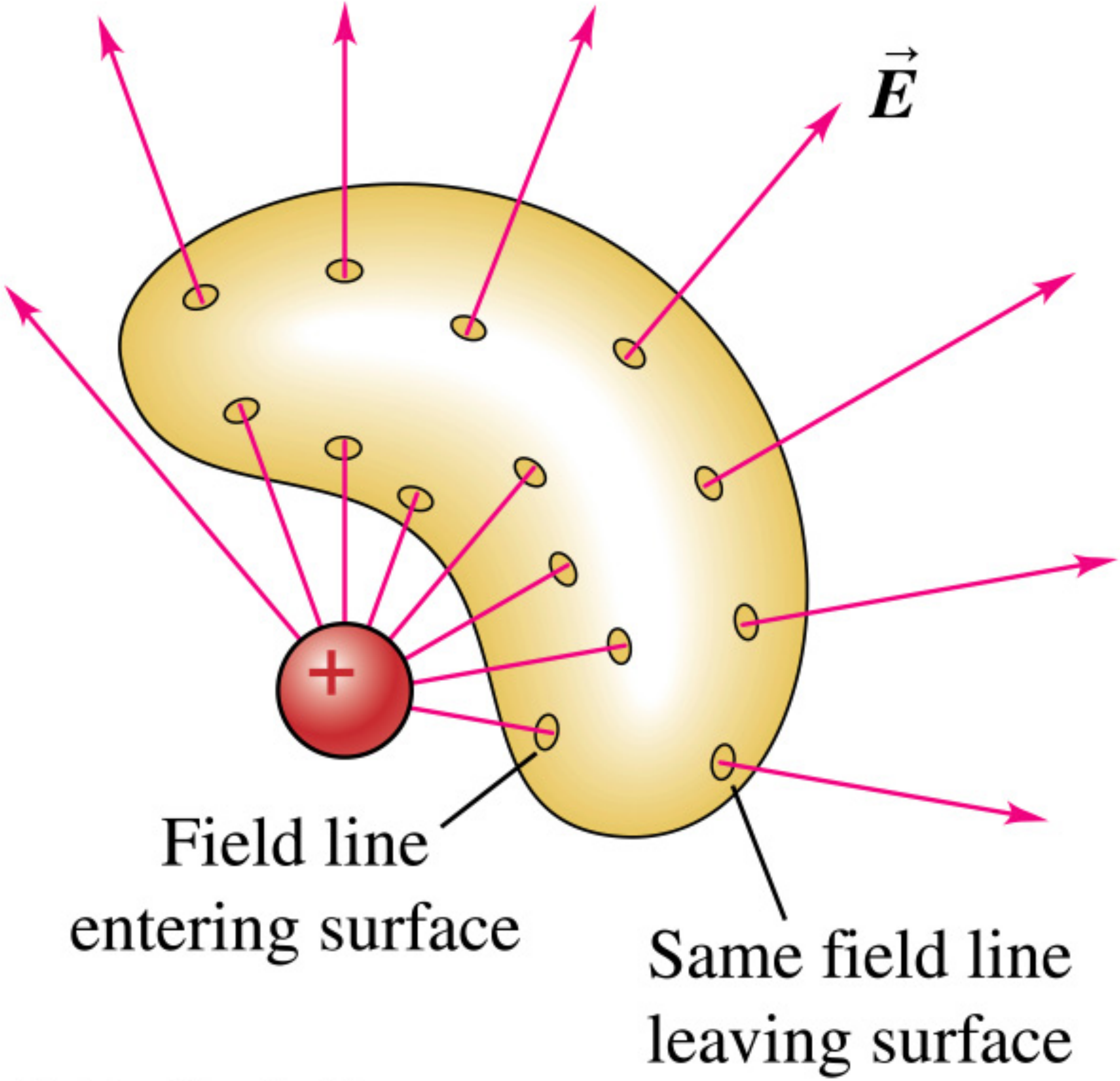
$$\Phi_{\text{net}} = \oint_{\text{closed surface}} \vec{E} \cdot d\vec{A}$$



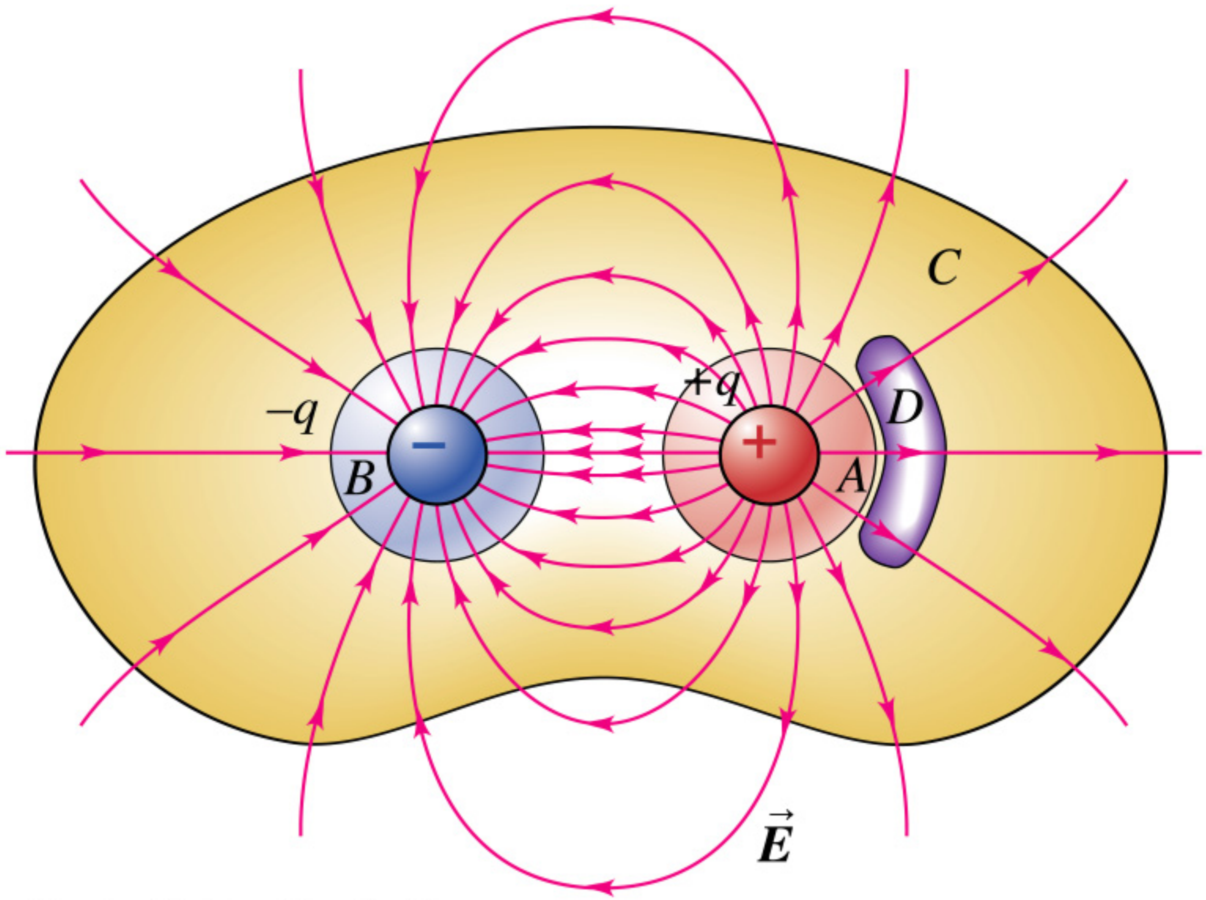
$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$

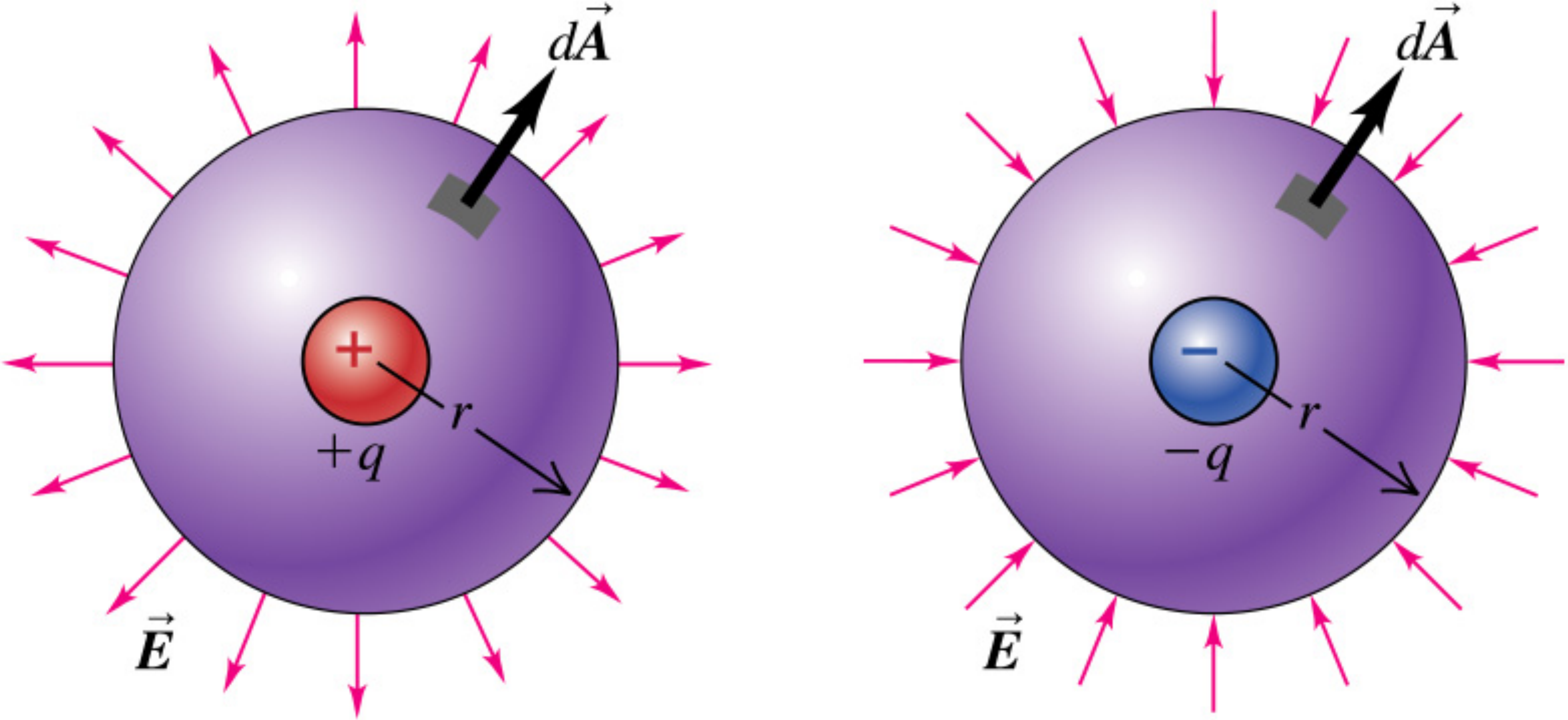
How does the flux through the smaller sphere compare to the flux through the larger sphere?

The electric field at the inner sphere is 4x greater than at the outer sphere
while
The surface area of the inner sphere is 1/4 that of the outer sphere


$\vec{E}$
Field line entering surface
Same field line leaving surface

Determine the Electric Flux through each surface





(a) Gaussian surface around positive charge: positive (outward) flux

(b) Gaussian surface around negative charge: negative (inward) flux

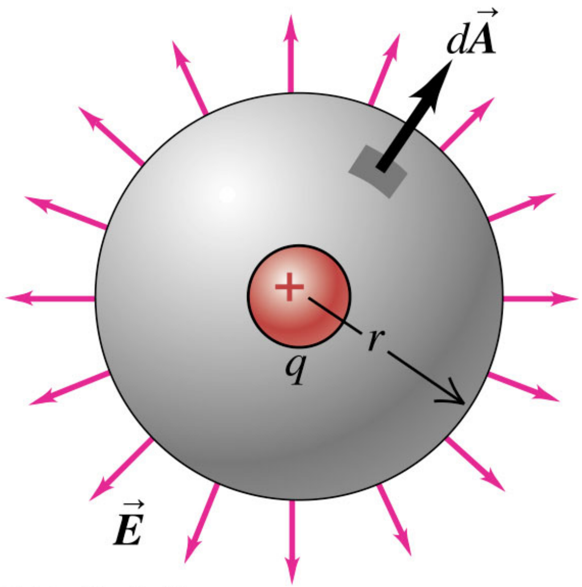
Electric Flux

through a Closed Surface

1. Whether there is a net *outward* or *inward* flux through the closed surface depends on the *sign* of the enclosed charge.

2. Charges outside the surface give a zero net flux through the surface.

3. The net electric flux is directly proportional to the net amount of charge enclosed within the surface but is otherwise independent of the size (and shape) of the closed surface.



Quantitative Example

Find the Electric Flux through the Gaussian Sphere

$$q = 3.0\ \mu C,\ \ r = 0.20\ \text{m}$$

$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$

$$\Phi_E = \int E\, dA$$

$$\Phi_E = E \int dA$$

$$E = k\frac{q}{r^2} = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{q}{r^2}$$

$$\text{where } \varepsilon_0 = 8.85 \times 10^{-12}\ \ \text{C}^2/\text{Nm}^2$$

$$A = \int dA = 4\pi r^2$$

$$\boxed{\Phi_E = \frac{q}{4\pi\varepsilon_0 r^2} 4\pi r^2 = \frac{q}{\varepsilon_0}}$$

Gauss's Law

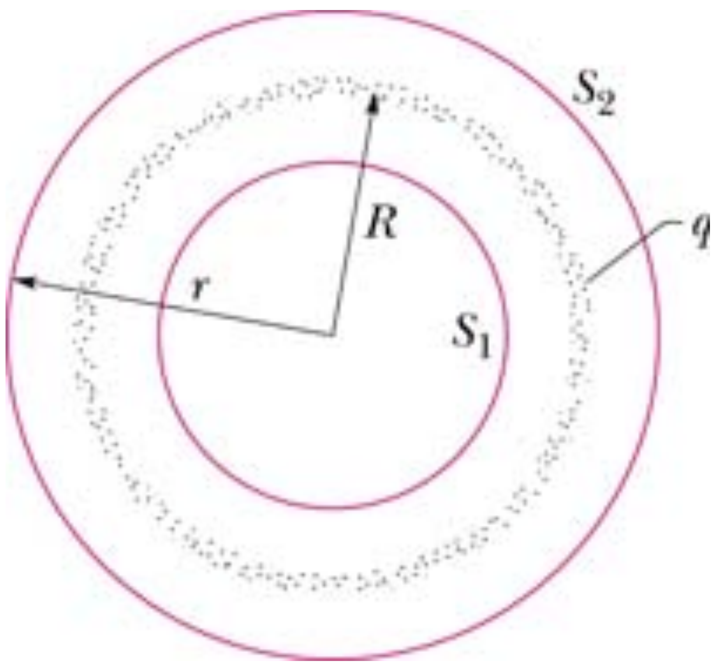
$$\Phi_{\text{net}} = \frac{q_{\text{enc}}}{\varepsilon_0}$$

$$\boxed{\int \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0}}$$

Using Gauss's Law to determine the Electric Field near a point charge

Spherical Symmetry

charged spherical shell



What is E at S_1?

$$\int \vec{E} \cdot d\vec{A} = EA_{S_1} = E4\pi\left(r_{S_1}\right)^2 = \frac{q_{\text{enc}}}{\varepsilon_0}$$

$$E = \frac{q_{\text{enc}}}{4\pi\varepsilon_0\left(r_{S_1}\right)^2} = 0$$

What is E at S_2?

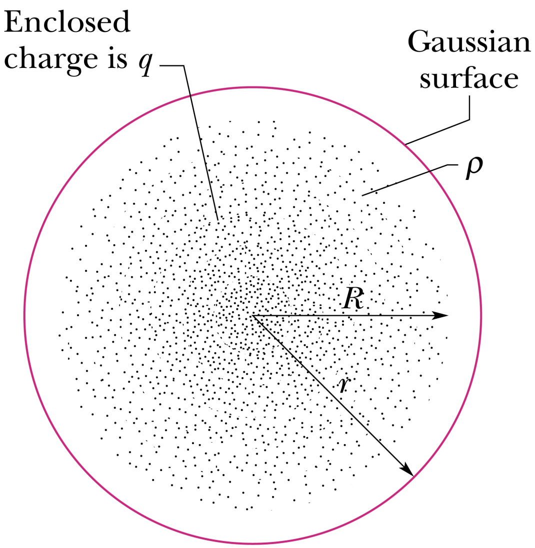
$$\int \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0}$$

$$E = \frac{q_{\text{enc}}}{4\pi\left(r_{S_2}\right)^2\varepsilon_0} = \frac{q}{4\pi\varepsilon_0\left(r_{S_2}\right)^2} = k\frac{q}{\left(r_{S_2}\right)^2}$$

This is the same expression as for a point charge!

Spherical Symmetry

Uniform Volume Charge Distribution

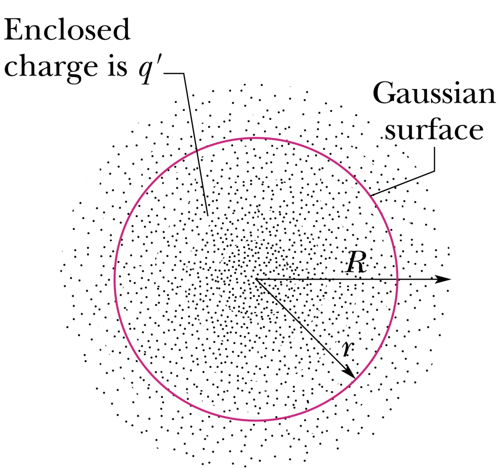


(a)

What is E at r>R?

$$\int \vec{E}\cdot d\vec{A} = EA_r = E4\pi r^2 = \frac{q_{\text{enc}}}{\varepsilon_0}$$

$$\boxed{E = \frac{q_{\text{enc}}}{4\pi r^2 \varepsilon_0}}$$



(b)

What is E at r<R?

$$\int \vec{E}\cdot d\vec{A} = EA_r = E4\pi r^2 = \frac{q_{\text{enc}}}{\varepsilon_0}$$

$$q_{\text{enc}} = \rho V_{\text{enc}} = \left(\frac{q}{V_{\text{total}}}\right)V_{\text{enc}} = \left(\frac{q}{\frac{4}{3}\pi R^3}\right)\frac{4}{3}\pi r^3 = q\frac{r^3}{R^3}$$

$$E4\pi r^2 = \frac{qr^3}{\varepsilon_0 R^3}$$

$$\boxed{E = \frac{qr^3}{4\pi r^2\varepsilon_0 R^3} = \left(\frac{q}{4\pi\varepsilon_0 R^3}\right)r}$$

Using Gauss's Law to determine the Electric Field…

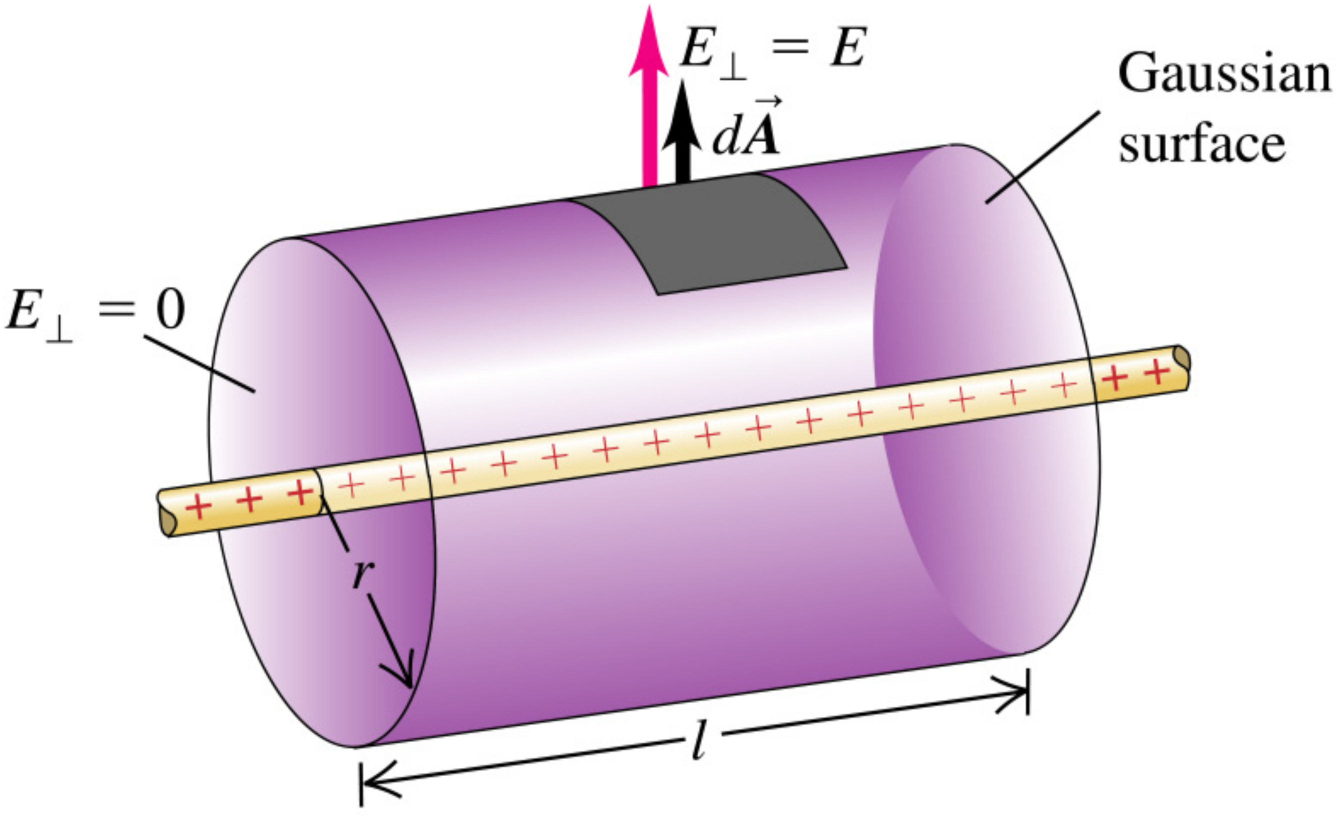
near an infinite, *thin* sheet of charge with surface charge density σ

Calculate the E-Field Near a Long Charged Wire

What would be an appropriate shape for the Gaussian Surface?

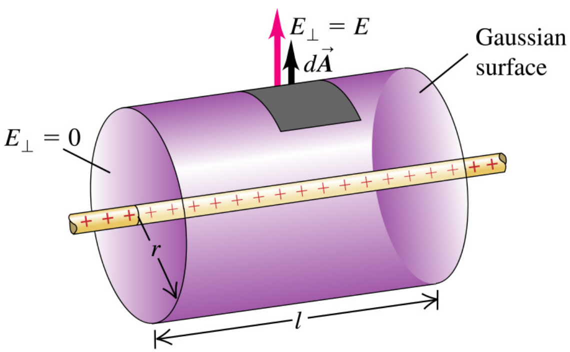
Using Gauss's Law to Calculate E-Field Near a Long Charged Wire

Cylindrical Symmetry



Using Gauss's Law to Calculate E-Field Near a Long Charged Wire

Cylindrical Symmetry



$$\Phi_{\text{net}} = \sum_i \vec{E} \cdot \vec{A}_i = \frac{q_{\text{enc}}}{\varepsilon_0}$$

First, note that at the two ends of the Gaussian Cylinder, E varies, but is always perpendicular to A. So, …

$$\Phi_{net} = EA_{cylinder\ wall} = \frac{q^{enclosed}}{\varepsilon_0}$$

$$E = \frac{q^{enclosed}}{\varepsilon_0 A_{cylinder\ wall}}$$

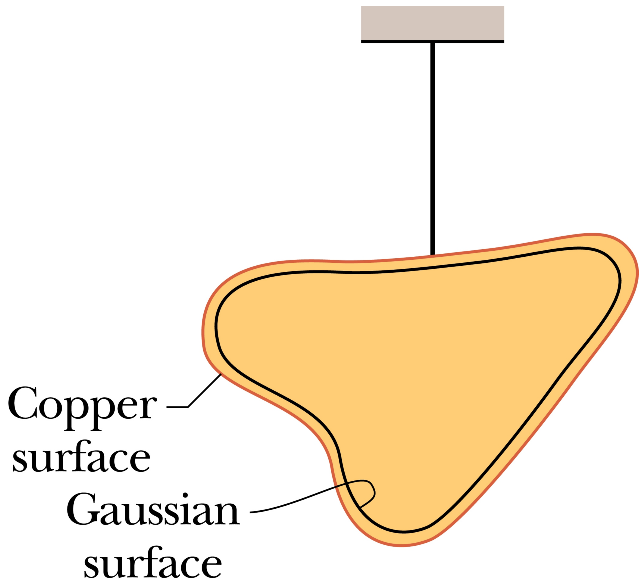
$$\lambda = \frac{q^{enclosed}}{l} \Rightarrow q^{enclosed} = \lambda l$$

$$A_{\text{cylinder wall}} = 2\pi r l$$

$$E = \frac{\lambda l}{\varepsilon_0 2\pi r l}$$

$$\boxed{E = \frac{\lambda}{\varepsilon_0 2\pi r}}$$

E-field near a long line charge



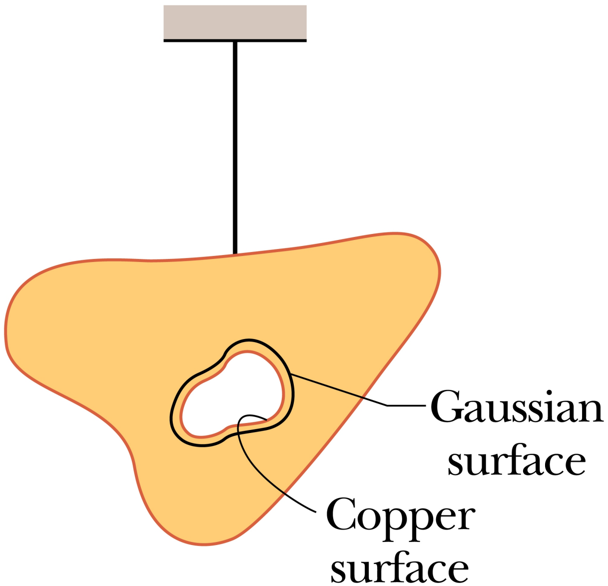
How is Charge Distributed on/in a Conductor?

What must be the value of the excess charge enclosed within the Gaussian Surface?

All excess charge MUST settle on the Surface of the Conductor.

Charge Distribution in/on a conductor with a cavity…

What must be the value of the excess charge enclosed within the Gaussian Surface?



(b)

All excess charge MUST settle on the (outer) Surface of the Conductor.

Finish Electric Field Line Lab