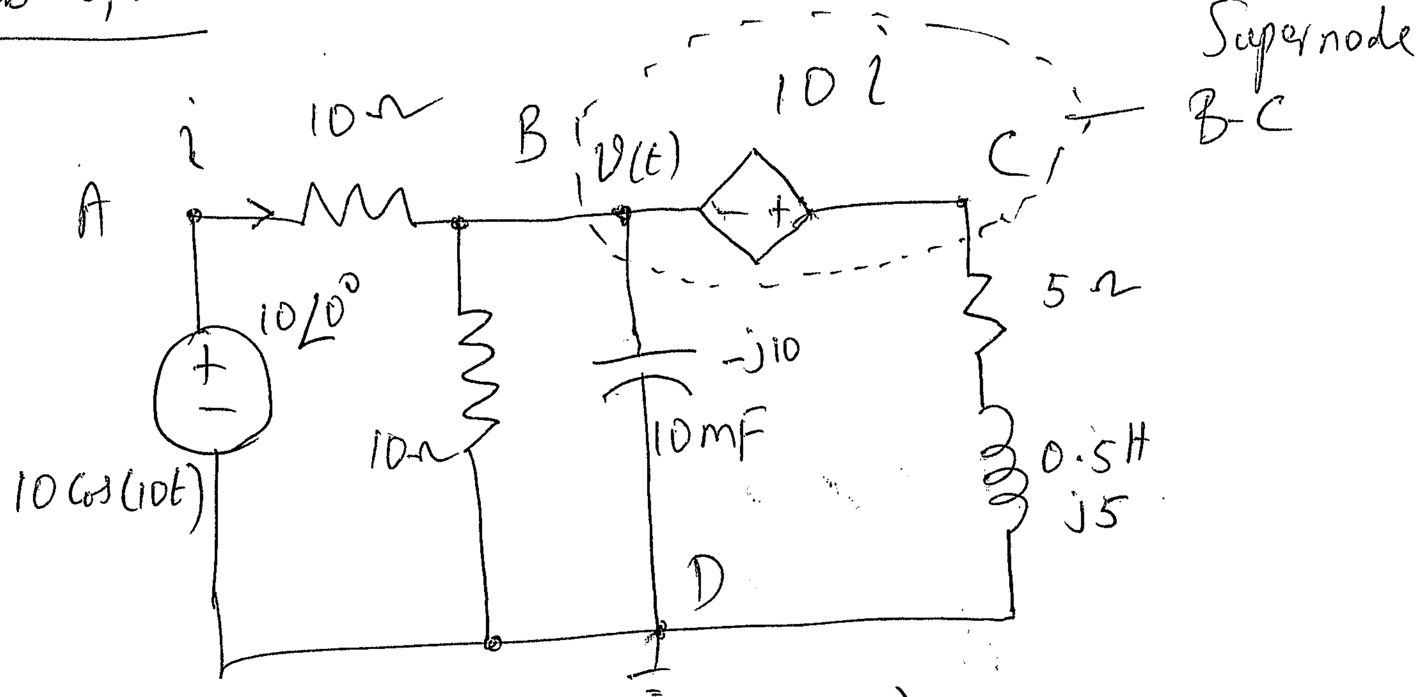
ECE 210    A C Circuit with Supernode    ①

Feb 25, 2013




<u>Prob</u>: Find $i$ and $v(t)$

1) Calculate Impedances of circuit components

$R : 10\,\Omega \longrightarrow 10\,\Omega$

$L : 0.5H \longrightarrow j\omega L = j(10)(0.5) = j5\,\Omega$

$C : 10\,mF \longrightarrow \dfrac{1}{j\omega C} = \dfrac{1}{j(10)(10 \times 10^{-3})} = -j10\,\Omega$

<u>Method 1</u> : Use nodal analysis

4 nodes — A, B, C and D

Facts : 1) $V_A = 10\angle 0^\circ$, 2) B − C is a Supernode

3) $i = \dfrac{10 - V_B}{10}$, 4) $V_C - V_B = 10i = 10 - V_B$

Super node equation (B - C)

$$\left(\frac{1}{10} + \frac{1}{10} + \frac{1}{-j10}\right) V_B + \frac{1}{5+j5} V_C = \frac{V_A}{10} = 1$$

$$(2+j) V_B + \frac{10}{5+j5} V_C = 10$$

$$(2+j) V_B + \frac{2}{1+j} V_C = 10$$

Since $V_C - V_B = 10 - V_B \Rightarrow V_C = 10$

$$(2+j) V_B = 10 - \frac{2}{1+j} V_C = 10 - \frac{2}{1+j} 10$$

$$= 10\left[1 - \frac{2}{1+j}\right]$$

$$(2+j) V_B = \cancel{100} \; 10 \frac{-1+j}{1+j}$$

$$V_B = 10 \frac{-1+j}{(2+j)(1+j)}$$

$$= \cancel{100} \; \frac{10\sqrt{2} \angle 135^\circ}{\sqrt{5} \angle 26^\circ \sqrt{2} \angle 45^\circ}$$

$$= \frac{10\sqrt{2}}{\sqrt{5}\sqrt{2}} \angle +135^\circ - 26^\circ - 45^\circ = \frac{10}{\sqrt{5}} \angle 64^\circ$$

$$\boxed{v(t) = \frac{10}{\sqrt{5}} \cos(10t + 64^\circ)}$$

$$i = \frac{10 - V_B}{10} = 1 - \frac{V_B}{10}$$

$$= 1 - \frac{-1+j}{(2+j)(1+j)}$$

$$= \frac{(2+j)(1+j) - (-1+j)}{(2+j)(1+j)}$$

$$= \frac{2 + j3 - \cancel{1} + \cancel{1} - j}{(2+j)(1+j)} = \frac{2+j2}{(2+j)(1+j)} = \frac{2}{2+j}$$

$$= \frac{2 \angle -26^\circ}{\sqrt{5}}$$

$$\boxed{i(t) = \frac{2}{\sqrt{5}} \cos(10t - 26^\circ)}$$

# Solve problem using Loop Analysis

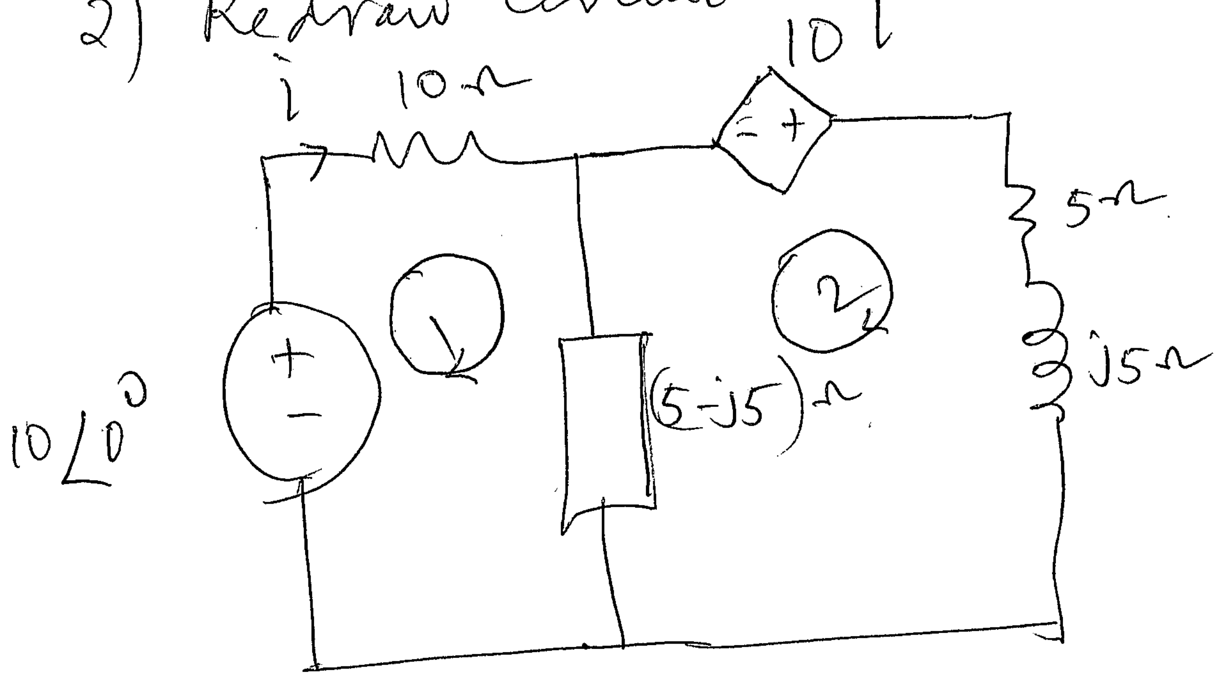
1) ~~Comp~~ Combine parallel combination of $10\,\Omega$ and $-j10$ into a single impedance

$$\cancel{(10)}\;(10)\,||\,(-j10) = \frac{(10)(-j10)}{10 - j10}$$

$$= \frac{-j10}{1-j} = \frac{-j10}{1-j}\cdot\frac{1+j}{1+j} = \frac{-j10+10}{2}$$

$$= 5 - j5$$

2) Redraw Circuit



**Loop 1 :** $(15-j5)I_1 - (5-j5)I_2 = 10\angle 0^\circ$ — (1)

**Loop 2 :** $-(5-j5)I_1 + 10I_2 - 10i = 0$

$i = I_1$

**Loop 2 :** $-(5-j5)I_1 + 10I_2 - 10I_1 = 0$

ie $-(15-j5)I_1 + 10I_2 = 0$ — (2)

Adding eqns (1) & (2) we get

$-(5-j5)I_2 + 10I_2 = 10\angle 0^\circ$

$(5+j5)I_2 = 10\angle 0^\circ$

$I_2 = \dfrac{10\angle 0^\circ}{5+j5} = \dfrac{2}{\sqrt{5}}\angle -45^\circ$

Also from eqn (2)

$(15-j5)I_1 = 10I_2$

$$I_1 = \frac{10}{15 - j5} I_2 = \frac{2}{3 - j} I_2$$

$$= \frac{2}{3 - j} \cdot \frac{10}{5 + j5} = \frac{2}{3 - j} \cdot \frac{2}{1 + j}$$

$$= \frac{4}{(3 - j)(1 + j)} = \frac{2}{\sqrt{5}} \angle -26^\circ$$

$$V_B = 10 - 10 I_1 = 10 \left[ 1 - \frac{4}{(3 - j)(1 + j)} \right]$$

$$= 10 \left[ \frac{(3 - j)(1 + j) - 4}{(3 - j)(1 + j)} \right]$$

$$= 10 \left[ \frac{4 + j2 - 4}{(3 - j)(1 + j)} \right] = \frac{j20}{(3 - j)(1 + j)}$$

$$= \frac{20 \angle 90^\circ}{\sqrt{10} \angle -18.4^\circ \sqrt{2} \angle 45^\circ} = \frac{10}{\sqrt{5}} \angle 63^\circ$$

$$\boxed{\begin{aligned} v(t) &= \frac{10}{\sqrt{5}} \cos(10t + 63^\circ) \\ i(t) &= \frac{2}{\sqrt{5}} \cos(10t - 26^\circ) \end{aligned}}$$

It is clear that in this problem nodal analysis is a more efficient method to find $v(t)$ and $i(t)$