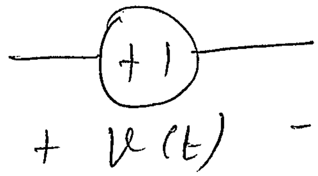
ECE 210 Circuits ~~W 2013~~

Handout - 1 1^{st} week of lectures

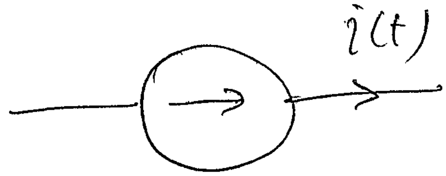
Electrical Components

1) Voltage Source



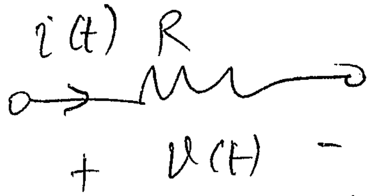
Examples : Batteries, Solar Cells, AC Supply etc.

2) Current Source



Current Sources are convenient as models for analysis.. Current Sources do not exist as a stand~~alone~~ alone unit, like a battery.

3) Resistor

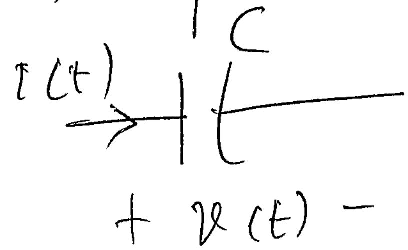


Resistor has a resistance determined according to Ohm's Law : $v(t) = R\, i(t)$

These are available in a variety of forms

4) Capacitor and energy

A Capacitor is a charge storage device and its capacity to hold a charge is governed by its Capacitance C



$$q(t) = C\,v(t)$$

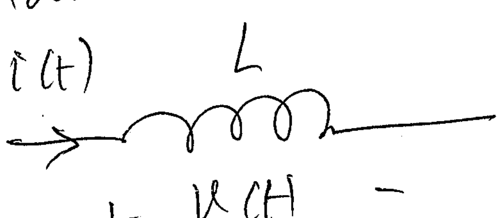
$$i(t) = \frac{dq}{dt} = C\,\frac{dv}{dt}$$

or $v(t) = \frac{1}{C}\int i(t)\,dt$,

$$\text{Energy} = \frac{1}{2} C\, v^2(t)$$

5) Inductor

An inductor is an energy storage device and its energy is dependendant on its inductance

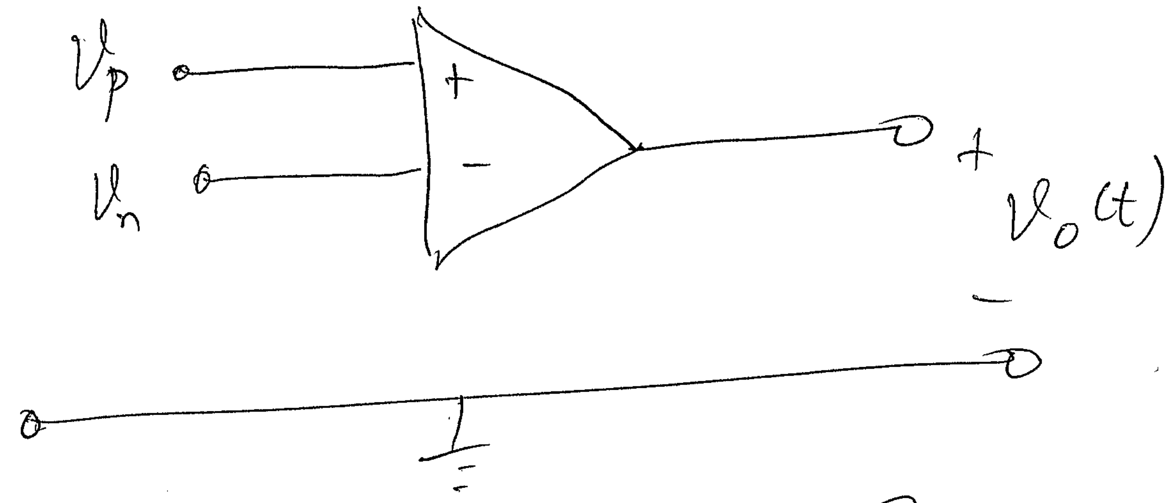


$$v(t) = L\,\frac{di}{dt}$$

$$\text{Energy} = \frac{1}{2} L\, i^2(t)$$

6) Amplifier

An amplifier is an active device used to amplify small signals



$$v_o(t) = A\left[v_p(t) - v_n(t)\right]$$

This device is also called an Operational Amplifier if <u>A is very large</u>

7) Other devices include transformer, transistor, diodes, thermistor, photocell These will not be covered in ECE 210 I'll touch upon transistor, diode and transformer.

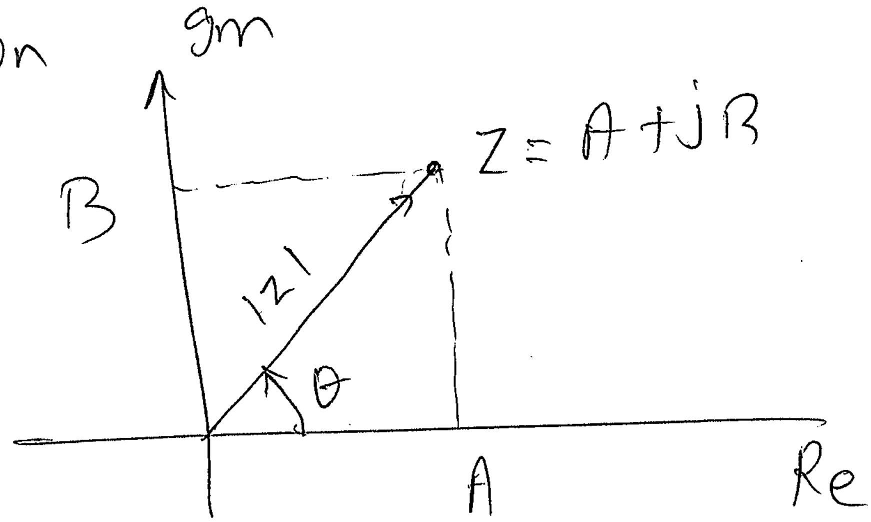
Complex numbers

A Complex number is represented as

$$Z = A + jB, \quad j = \sqrt{-1}$$

Since $\sqrt{-1}$ does not exist, we call it an impaginary number. A is the real part and B is the imaginary part

Graphically, it can be represented as shown



Z can also described according to its length (absolute value, magnitude etc.) and its orientation θ

The absolute value of Z is written as $|Z| = \sqrt{A^2+B^2}$. The orientation is written as $\theta = \tan^{-1}\frac{B}{A}$. Thus

$$Z = A+jB = \sqrt{A^2+B^2}\, e^{j\theta}$$

$Z = A+jB$ is the cartesian form

$Z = \sqrt{A^2+B^2}\, e^{j\theta}$ is the polar form

Examples: $Z = 1+j1 = \sqrt{2}\, e^{j\frac{\pi}{4}}$

$$Z = 1-j1 = \sqrt{2}\, e^{-j\frac{\pi}{4}}$$

$$Z = -1+j1 = \sqrt{2}\, e^{j\frac{3\pi}{4}}$$

$$Z = -1-j1 = \sqrt{2}\, e^{j\frac{5\pi}{4}}$$

Complex Arithmetic

$$Z_1 = 3 + j4, \quad Z_2 = 2 - j3$$

$$Z_1 + Z_2 = (3+2) + j(4-3) = 5 + j1$$

$$Z_1 - Z_2 = (3-2) + j(4+3) = 1 + j7$$

$$Z_1 Z_2 = (3+j4)(2-j3) = 6 + j8 - j9 + 12$$

$$= 18 + j1$$

$$\frac{Z_1}{Z_2} = \frac{3+j4}{2-j3} = \frac{3+j4}{2-j3} \cdot \frac{2+j3}{2+j3}$$

$$= \frac{6 + j8 + j9 - 12}{4+9} = \frac{-6 + j17}{13}$$

$$Z_1 = 3 + j4 = \sqrt{3^2+4^2}\, e^{j\theta_1}, \quad \theta_1 = \tan^{-1}\frac{4}{3}$$

$$Z_2 = 2 - j3 = \sqrt{2^2+3^2}\, e^{j\theta_2}, \quad \theta_2 = -\tan^{-1}\frac{3}{2}$$

$$Z_1 Z_2 = 5\sqrt{13}\, e^{j(\theta_1+\theta_2)}$$

$$\frac{Z_1}{Z_2} = \frac{5}{\sqrt{13}} e^{j(\theta_1 - \theta_2)}$$

Note: Sympolically, we can write

$$|Z| e^{j\theta} = |Z| \angle \theta$$

Practice problems

Find Real and imaginary parts of

a) $e^{j\frac{\pi}{4}}$, $e^{-j\frac{\pi}{2}}$, $e^{j\pi}$, $\sqrt{3+j4}$

b) Given $Z_1 = 3+j4$, $Z_2 = 1-j1$

find $Z_1 + Z_2$, $Z_1 - Z_2$, $Z_1 Z_2$, $\frac{Z_1}{Z_2}$

Complex Sinusoid $e^{j\omega_0 t}$

$$Z = e^{j\omega_0 t} = \cos(\omega_0 t) + j\sin(\omega_0 t)$$

(Euler's formula)

$$|Z| = \sqrt{\cos^2(\omega_0 t) + \sin^2(\omega_0 t)} = 1$$

i.e. The length or the absolute value is <u>always 1</u>, no matter what ω_0 and t are.

$$\angle Z = \omega_0 t$$

The angle increases linearly with 't'.

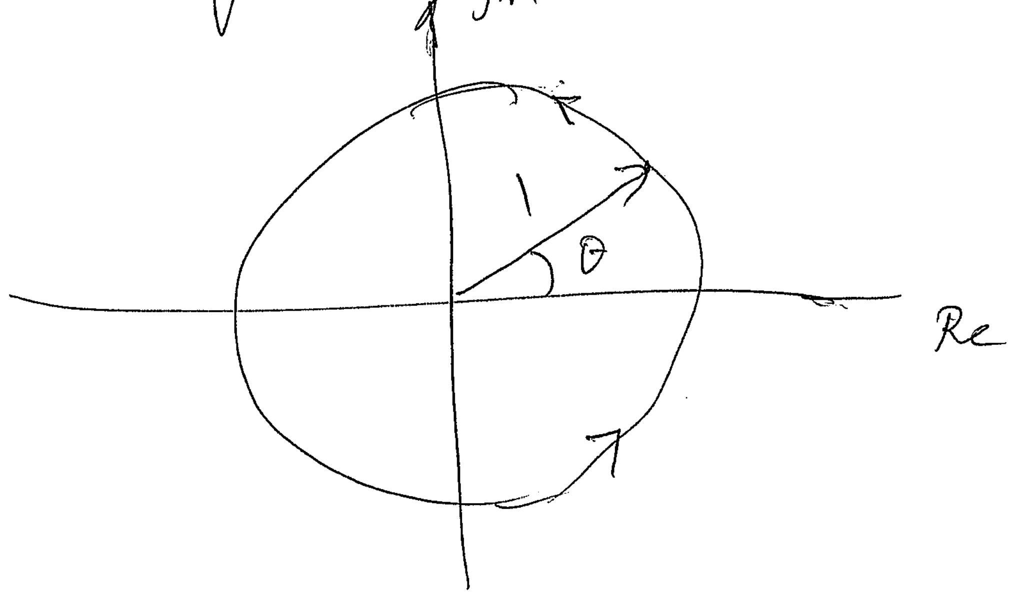
But you must realize that

<u>$e^{j\omega_0 t}$ is periodic</u>

$$e^{j\omega_0 t} = e^{j(\omega_0 t + 2K\pi)}$$

where K is any integer.

<u>Graphically</u> $e^{j\omega_0 t}$ travels around a circle of unit radius, as t increases



<u>Note</u> : You must fully understand these properties, as we will use them extensively for AC circuit analysis