

# Today's Session Summary

- Tower crane
- Forces and Equilibrium

# Crane Optimization



# Crane Optimization

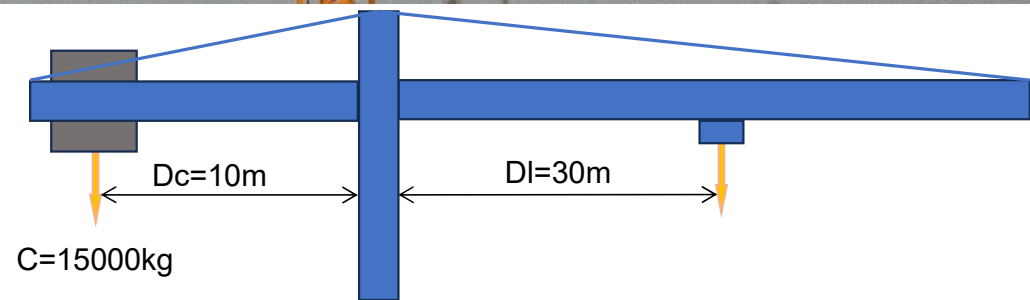
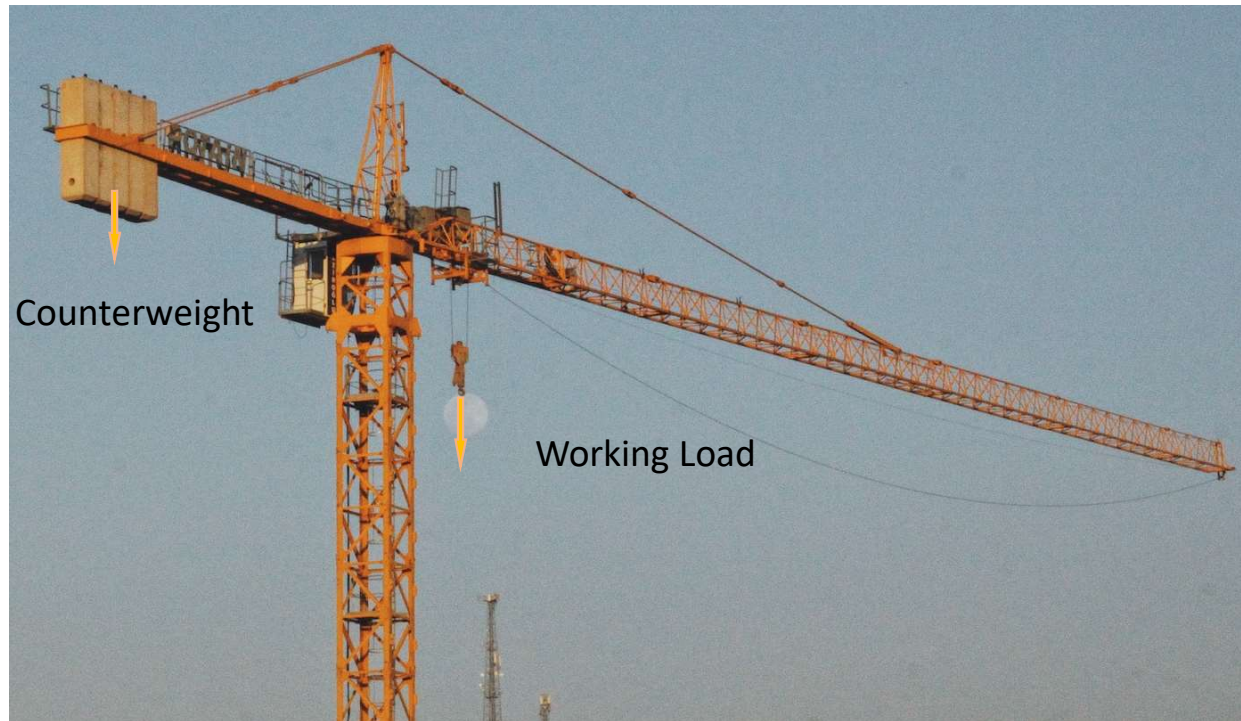
Equilibrium - Balance

Load Displacement

Structure

Elements

Optimization



From Equations to Innovation: Modeling and Optimization in Engineering

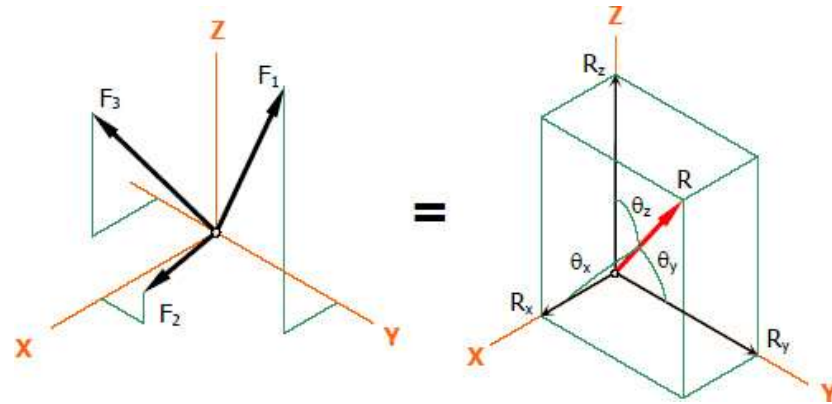


# Forces and Equilibrium

## Concurrent Forces

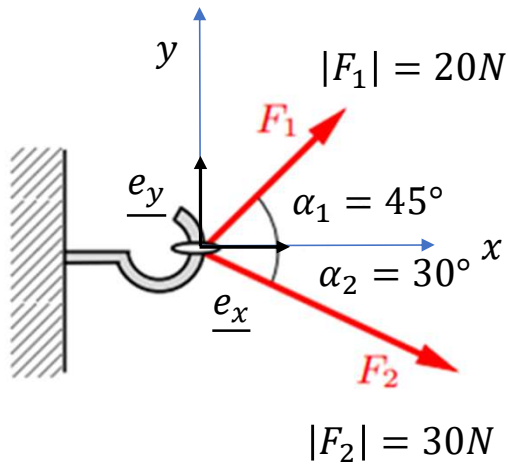
Resultant force

$$\begin{bmatrix} \sum_i F_{ix} \\ \sum_i F_{iy} \\ \sum_i F_{iz} \end{bmatrix} = \begin{bmatrix} R_x \\ R_y \\ R_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$



# Forces and Equilibrium

## Planar Concurrent Forces



$$\underline{R} = \underline{F}_1 + \underline{F}_2$$

$$R_x \underline{e}_x + R_y \underline{e}_y = F_{1x} \underline{e}_x + F_{1y} \underline{e}_y + F_{2x} \underline{e}_x + F_{2y} \underline{e}_y$$

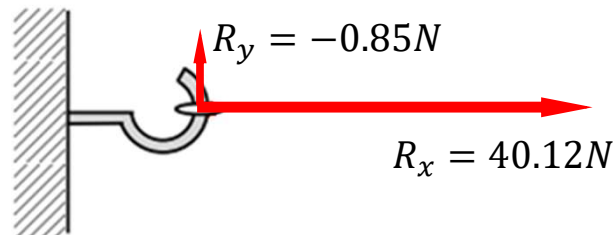
$$\underline{e}_x) \quad R_x = F_{1x} + F_{2x} = 20\text{ N} \cos(45^\circ) + 30\text{ N} \cos(30^\circ)$$

$$R_x = 40.12\text{ N}$$

$$\underline{e}_y) \quad R_y = F_{1y} + F_{2y} = 20\text{ N} \sin(45^\circ) - 30\text{ N} \sin(30^\circ)$$

$$R_y = -0.85\text{ N}$$

Resultant force



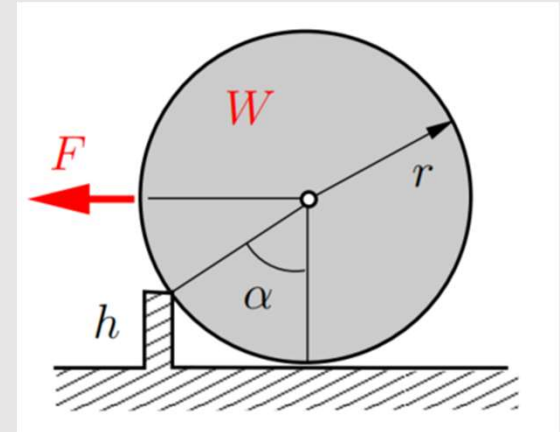
# Practice

- 1) Draw the free body diagram
- 2) Find  $F$  necessary to roll the cylinder over the obstacle

$$\underline{e_x}) \quad 0N = -F + R\sin(\alpha)$$

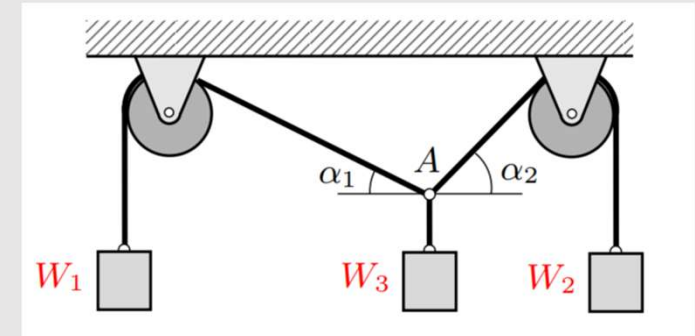
$$\underline{e_y}) \quad 0N = -W + R\cos(\alpha)$$

$$\frac{F}{W} = \tan(\alpha)$$



# Practice

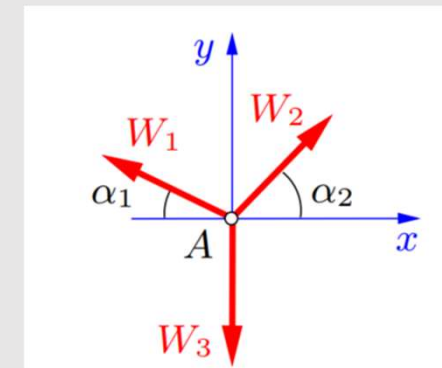
- 1) Draw the free body diagram for  $W_3$
- 2) Find  $\alpha_1$  and  $\alpha_2$  when  $W_1 = 2N$ ;  $W_2 = 2N$  and  $W_3 = 0N$
- 3) Find  $\alpha_1$  and  $\alpha_2$  when  $W_1 = 2N$ ;  $W_2 = 3N$  and  $W_3 = 3N$
- 4) Find  $\alpha_1$  and  $\alpha_2$  when  $W_1 = 2N$ ;  $W_2 = 2N$  and  $W_3 = 3N$



$$\underline{e_x}) \quad 0N = T_{1x} + T_{2x} = -W_1 \cos(\alpha_1) + W_2 \cos(\alpha_2)$$

$$\underline{e_y}) \quad 0N = T_{1y} + T_{2y} + T_{3y} = W_1 \sin(\alpha_1) + W_2 \sin(\alpha_2) - W_3$$

$$\cos(\alpha_1) = \frac{W_2}{W_1} \cos(\alpha_2) \qquad \sin(\alpha_1) = \frac{W_3 - W_2 \sin(\alpha_2)}{W_1}$$



# Practice

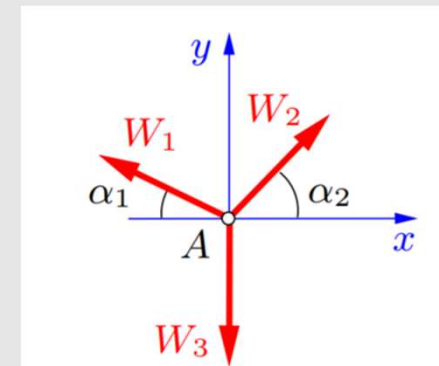
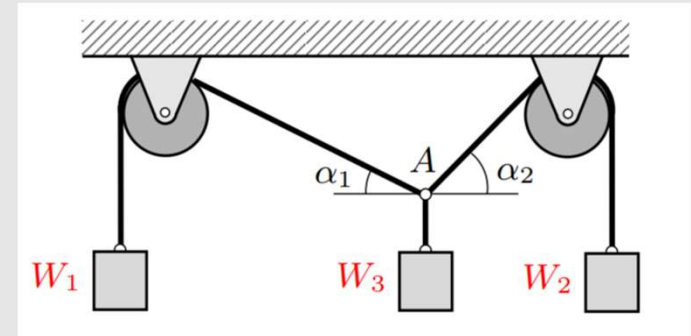
$$\cos(\alpha_1) = \frac{W_2}{W_1} \cos(\alpha_2) \quad \sin(\alpha_1) = \frac{W_3 - W_2 \sin(\alpha_2)}{W_1}$$

$$1 = \left( \frac{W_2}{W_1} \cos(\alpha_2) \right)^2 + \left( \frac{W_3 - W_2 \sin(\alpha_2)}{W_1} \right)^2$$

$$\left( \frac{W_1}{W_2} \right)^2 = (\cos(\alpha_2))^2 + \left( \frac{W_3}{W_2} - \sin(\alpha_2) \right)^2$$

$$\left( \frac{W_1}{W_2} \right)^2 = (\cos(\alpha_2))^2 + \left( \frac{W_3}{W_2} \right)^2 - 2 \frac{W_3}{W_2} \sin(\alpha_2) + (\sin(\alpha_2))^2$$

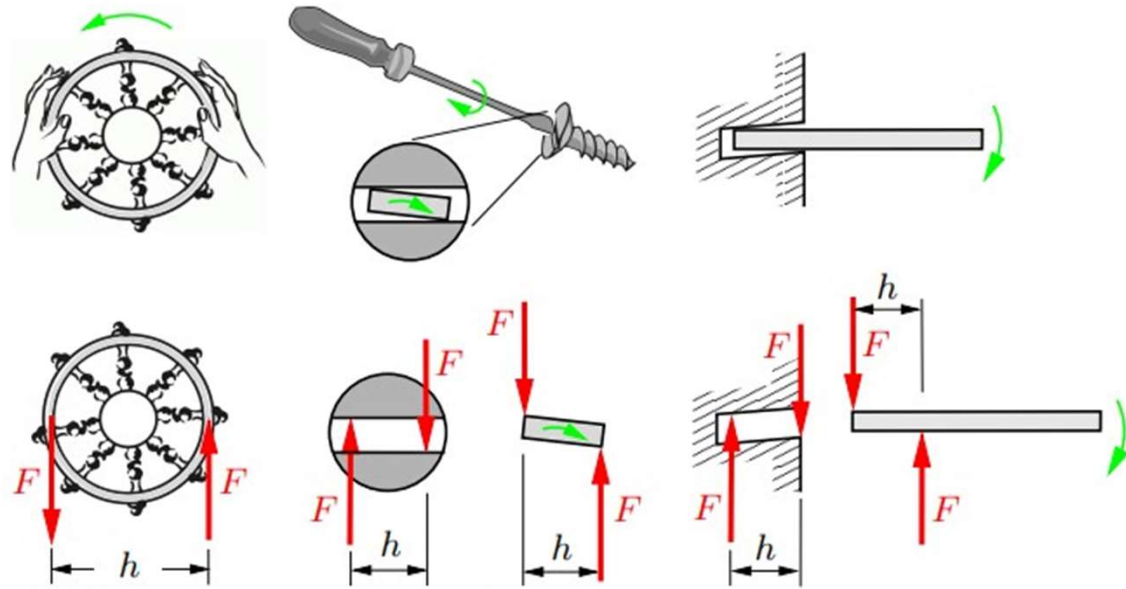
$$\alpha_2 = \arcsin \left( \frac{W_2}{2W_3} \left( 1 + \left( \frac{W_3}{W_2} \right)^2 - \left( \frac{W_1}{W_2} \right)^2 \right) \right)$$



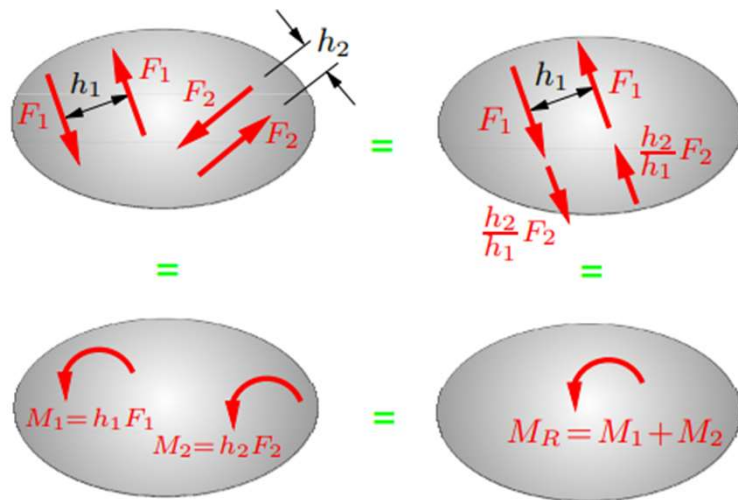


# Moments and Equilibrium

## Couples



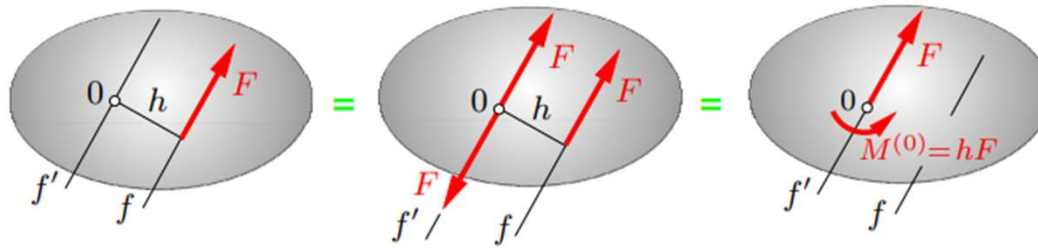
## Resulting moment



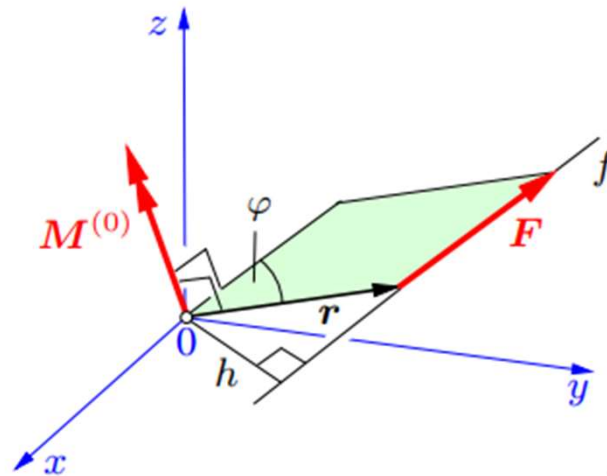
$$\sum_i M_i = M_R = 0$$

# Moments and Equilibrium

Moment of a force



$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$



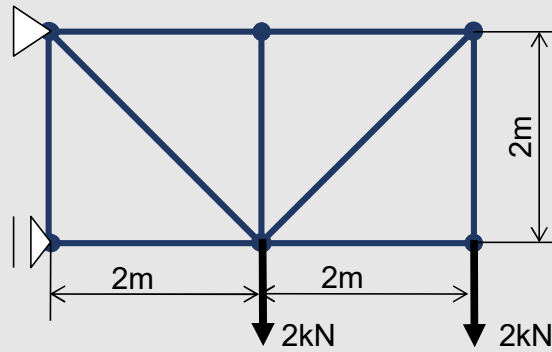
# Equilibrium

$$\sum F_i = 0, \quad \sum M_i^{(A)} = 0.$$

$$\begin{aligned} \sum F_{ix} &= 0, & \sum M_{ix}^{(A)} &= 0, \\ \sum F_{iy} &= 0, & \sum M_{iy}^{(A)} &= 0, \\ \sum F_{iz} &= 0, & \sum M_{iz}^{(A)} &= 0. \end{aligned}$$

# Practice

Equilibrium - Planar Example



# End Session 11