Today’s Session Summary

- Design Space
- Design requirements

Design Space

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Crane Design

Crane:

- *$Wc = 15000kg$*
- *Use proposed design space*

Material

- $E = 200GPa$
- $\rho = 7800\frac{kg}{m^3}$
- $\sigma_{adm} = 250MPa$

Objectives

- *Show extreme deformations*
- *Show stresses*
- *Show weight*
- *Buckling load factor*
- *Determine optimum structure considering admissible stress and buckling*

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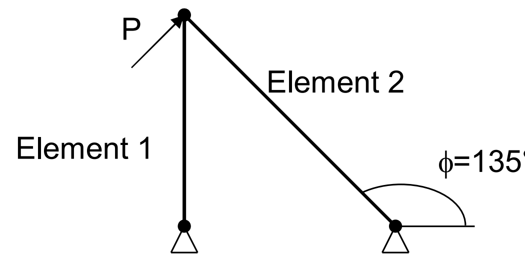
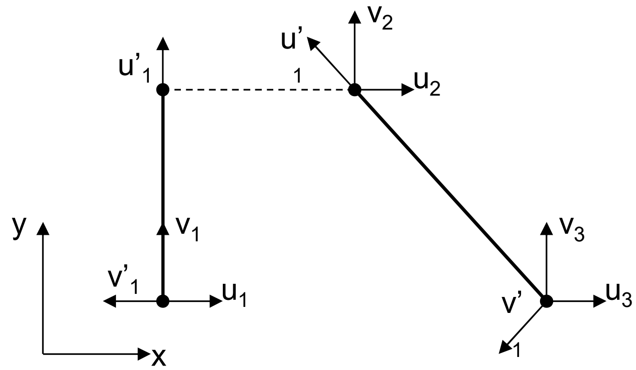
End Session 15

Today’s Session Summary

- Stresses in Rods
- Buckling
- Buckling load factor
- Design Conditions

Transformations – Planar Bar elements - Example





Local Stiffness matrices

$$[K_1'] = \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix} \quad \begin{Bmatrix} u'_1 \\ u'_2 \end{Bmatrix} \qquad k_1 = \frac{A_1 E_1}{L_1} \qquad A_1 = 1 \quad E_1 = 1 \quad L_1 = 1$$

$$[K_2'] = \begin{bmatrix} k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \quad \begin{Bmatrix} u'_1 \\ u'_2 \end{Bmatrix} \qquad k_2 = \frac{A_2 E_2}{L_2} \qquad A_2 = 1 \quad E_2 = 1 \quad L_2 = \sqrt{2}$$

Transformed matrices

$$[K_1] = [T_1]^T [K_1'][T_1] \qquad [K_1] = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad [K_1] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & k_1 & 0 & -k_1 \\ 0 & 0 & 0 & 0 \\ 0 & -k_1 & 0 & k_1 \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{bmatrix}$$

Where β=90°

$$[K_2] = [T_2]^T [K_2'][T_2] \qquad [K_2] = \begin{bmatrix} c\phi & 0 \\ s\phi & 0 \\ 0 & c\phi \\ 0 & s\phi \end{bmatrix} \begin{bmatrix} k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{bmatrix} c\phi & s\phi & 0 & 0 \\ 0 & 0 & c\phi & s\phi \end{bmatrix}$$

$$[K_2] = \begin{bmatrix} k_2 c^2\phi & k_2 \cdot c\phi \cdot s\phi & -k_2 c^2\phi & -k_2 \cdot c\phi \cdot s\phi \\ k_2 \cdot c\phi \cdot s\phi & k_2 s^2\phi & -k_2 \cdot c\phi \cdot s\phi & -k_2 s^2\phi \\ -k_2 c^2\phi & -k_2 \cdot c\phi \cdot s\phi & k_2 c^2\phi & k_2 \cdot c\phi \cdot s\phi \\ -k_2 \cdot c\phi \cdot s\phi & -k_2 s^2\phi & k_2 \cdot c\phi \cdot s\phi & k_2 s^2\phi \end{bmatrix} \begin{bmatrix} u_2 \\ v_2 \\ u_3 \\ v_3 \end{bmatrix}$$

Where $\beta = \phi$ =135°

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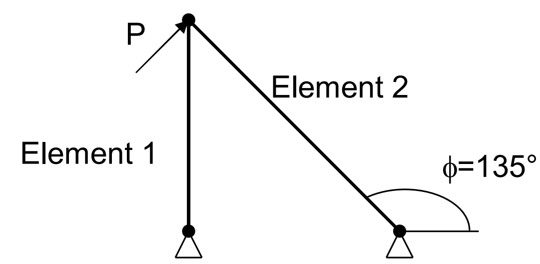
Transformations – Planar Bar elements - Example



Global Matrix

$$[K]=\begin{bmatrix}0 & 0 & 0 & 0 & 0 & 0\\ 0 & k_1 & 0 & -k_1 & 0 & 0\\ 0 & 0 & k_2c^2\phi & k_2\cdot c\phi\cdot s\phi & -k_2c^2\phi & -k_2\cdot c\phi\cdot s\phi\\ 0 & -k_1 & k_2\cdot c\phi\cdot s\phi & k_1+k_2s^2\phi & -k_2\cdot c\phi\cdot s\phi & -k_2s^2\phi\\ 0 & 0 & -k_2c^2\phi & -k_2\cdot c\phi\cdot s\phi & k_2c^2\phi & k_2\cdot c\phi\cdot s\phi\\ 0 & 0 & -k_2\cdot c\phi\cdot s\phi & -k_2s^2\phi & k_2\cdot c\phi\cdot s\phi & k_2s^2\phi\end{bmatrix}\begin{bmatrix}u_1\\ v_1\\ u_2\\ v_2\\ u_3\\ v_3\end{bmatrix}$$

External loads

$$\{R\}=\begin{Bmatrix}R_{x1}\\ R_{y1}\\ P_x\\ P_y\\ R_{x3}\\ R_{y3}\end{Bmatrix}$$

Reduced matrix

$$[K]=\begin{bmatrix}k_2c^2\phi & k_2\cdot c\phi\cdot s\phi\\ k_2\cdot c\phi\cdot s\phi & k_1+k_2s^2\phi\end{bmatrix}\quad\begin{bmatrix}u_2\\ v_2\end{bmatrix}$$

$$\{U\}=[K]^1\{R\}=\begin{bmatrix}\dfrac{\sqrt{2}}{4} & -\dfrac{\sqrt{2}}{4}\\ -\dfrac{\sqrt{2}}{4} & 1+\dfrac{\sqrt{2}}{4}\end{bmatrix}^{-1}\begin{Bmatrix}P\dfrac{\sqrt{2}}{2}\\ P\dfrac{\sqrt{2}}{2}\end{Bmatrix}=P\begin{Bmatrix}2+\sqrt{2}\\ \sqrt{2}\end{Bmatrix}$$

External reactions

$$\{R\}=[K]\{U\}=\begin{bmatrix}0 & 0 & 0 & 0 & 0 & 0\\ & 1 & 0 & -1 & 0 & 0\\ & & \sqrt{2}/4 & -\sqrt{2}/4 & -\sqrt{2}/4 & \sqrt{2}/4\\ & & & 1+\sqrt{2}/4 & \sqrt{2}/4 & -\sqrt{2}/4\\ & & & & \sqrt{2}/4 & -\sqrt{2}/4\\ & & & & & \sqrt{2}/4\end{bmatrix}P\begin{bmatrix}0\\ 0\\ 2+\sqrt{2}\\ \sqrt{2}\\ 0\\ 0\end{bmatrix}=P\begin{bmatrix}0\\ -\sqrt{2}\\ \sqrt{2}/2\\ \sqrt{2}/2\\ -\sqrt{2}/2\\ \sqrt{2}/2\end{bmatrix}$$

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Stresses

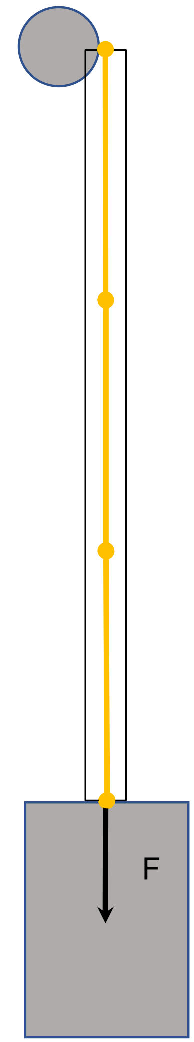
$$\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \frac{L_e}{A_e E_e} \begin{Bmatrix} 0 \\ 2.5qL_e + F \\ 4qL_e + F \\ 4.5qL_e + F \end{Bmatrix}$$

$$\sigma_e = E_e \varepsilon_e = E_e \frac{\Delta L}{L_e} = E_e \left(-\frac{u_1}{L_e} + \frac{u_2}{L_e} \right)$$

$$\sigma_I = E_I \frac{L_I}{A_I E_I} \left(-\frac{0}{L_I} + \frac{2.5qL_I + F}{L_I} \right)$$

$$\sigma_{II} = E_{II} \frac{L_{II}}{A_{II} E_{II}} \left(-\frac{2.5qL_{II} + F}{L_{II}} + \frac{4qL_{II} + F}{L_{II}} \right)$$

$$\sigma_{III} = E_{III} \frac{L_{III}}{A_{III} E_{III}} \left(-\frac{4qL_{II} + F}{L_{III}} + \frac{4.5qL_{III} + F}{L_{III}} \right)$$



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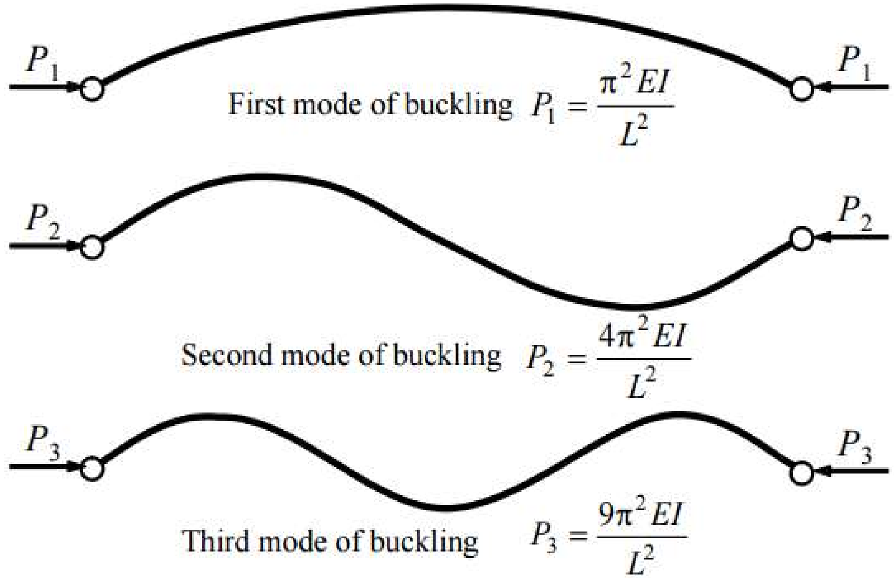
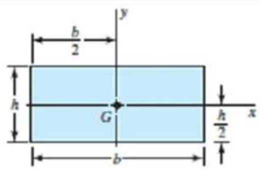
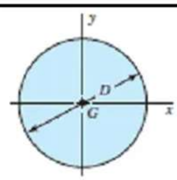
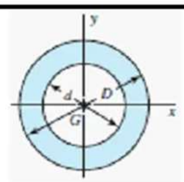
Buckling

Buckling Load

2^{nd} Moment of Area



Cross Section		Cross-Sectional Area (A)	Major Axis Moment of Area (Ix)	Minor Axis Moment of Area (Iy)
Rectangle		bh	$bh^3/12$	$b^3h/12$
Circle		$\pi D^2/4$	$\pi D^4/64$	$\pi D^4/64$
Circular Tube		$\pi/4(D^2 - d^2)$	$\pi/64(D^4 - d^4)$	$\pi/64(D^4 - d^4)$

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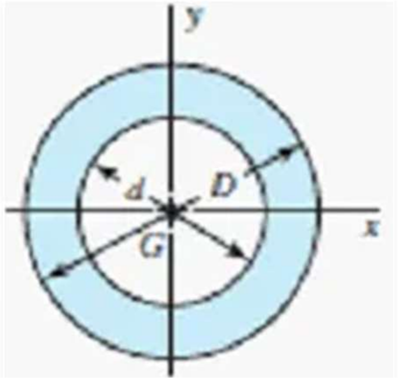
Design Conditions

$$-\sigma_{adm} \leq \sigma_e \leq \sigma_{adm}$$

$$-P_{critic} < P_e$$

$$\sigma_e = E_e\left(-\frac{u_1}{L_e} + \frac{u_2}{L_e}\right)$$



$$P_e = \sigma_e \mathrm{A_e}$$

$$P_{critic} = \frac{\pi^2 E_e I_e}{L_e^2}$$

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