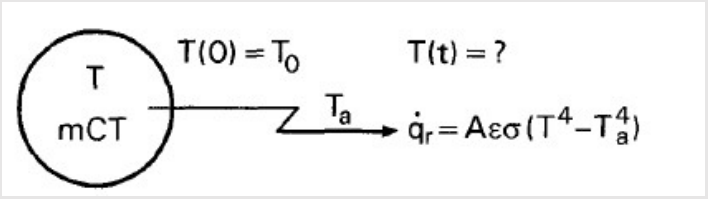
Today's Session Summary

- Review problems
- Finite Volume Method

Practice

$$y' - f(t, y) = 0$$

$$\frac{dT}{dt} + \frac{A\varepsilon\sigma}{mC}(T^4 - T_\infty^4) = 0$$



Forward

$$\frac{dT_n}{dt} = \frac{T_{n+1} - T_n}{\Delta t} \rightarrow \begin{cases} \frac{T_{n+1} - T_n}{\Delta t} = -\frac{A\varepsilon\sigma}{mC}\left({T_n}^4 - T_\infty^4\right) \\ T_{n+1} = T_n - \Delta t \frac{A\varepsilon\sigma}{mC}\left({T_n}^4 - T_\infty^4\right) \end{cases}$$

Explicit

Backward

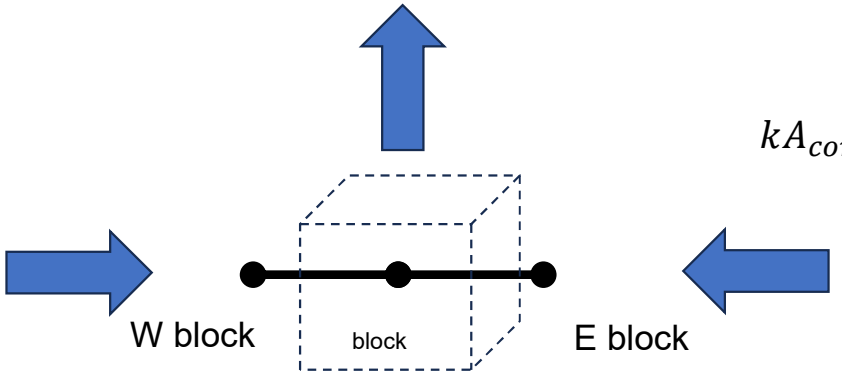
$$\frac{dT_{n+1}}{dt} = \frac{T_{n+1} - T_n}{\Delta t} \rightarrow \begin{cases} \frac{T_{n+1} - T_n}{\Delta t} = -\frac{A\varepsilon\sigma}{mC}\left({T_{n+1}}^4 - T_\infty^4\right) \\ T_{n+1} = T_n - \Delta t \frac{A\varepsilon\sigma}{mC}\left({T_{n+1}}^4 - T_\infty^4\right) \end{cases}$$

Implicit

Cell control volume

Heat Balance

$$hA_{convection}(T_{block} - T_{\infty}) = \dot{Q}_{convected}$$



$$kA_{conduction}\frac{T_{E\ block} - T_{block}}{Distance} = \dot{Q}_{E\ conducted}$$

$$kA_{conduction}\frac{T_{W\ block} - T_{block}}{Distance} = \dot{Q}_{W\ conducted}$$

$$kA_{conduction}\frac{T_{W\ block} - T_{block}}{Distance} + kA_{conduction}\frac{T_{E\ block} - T_{block}}{Distance} + hA_{convection}(T_{block} - T_{\infty}) = m_{block}C\dot{T}_{block}$$

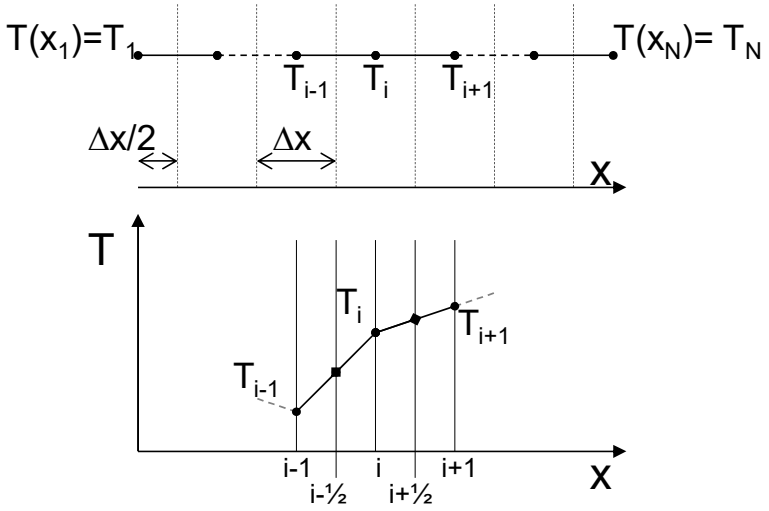
Finite Volume Method

Cell Equation

Eqn. System

$$a_i T_i - b_i T_{i+1} - c_i T_{i-1} = d_i$$

$$\underbrace{\begin{bmatrix} a_1 & -b_{1|} & & \\ -c_{2|} & a_2 & & \\ & & \ddots & -b_{N-1} \\ & & -c_N & a_N \end{bmatrix}}_{[A]} \underbrace{\begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ T_N \end{bmatrix}}_{\{T\}} = \underbrace{\begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_N \end{bmatrix}}_{\{D\}}$$

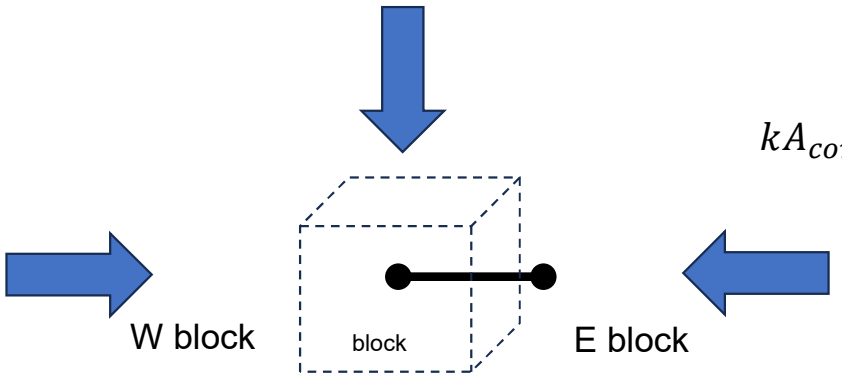


Frontal: $[A]\{T\} = \{D\}$

Boundary control volume

Heat Balance

$$hA_{convection}(T_\infty - T_{block}) = \dot{Q}_{convected}$$



$$kA_{conduction}\frac{T_{E\ block} - T_{block}}{Distance} = \dot{Q}_{E\ conducted}$$

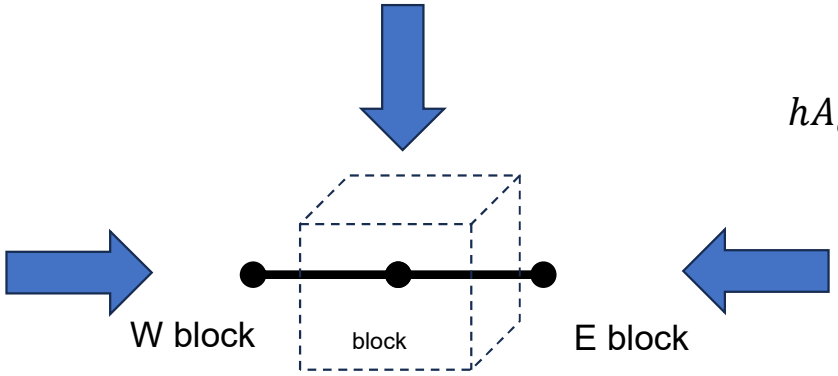
$$\dot{Q}_{W\ conducted\ fixed}$$

$$\dot{Q}_{W\ conducted\ fixed} + kA_{conduction}\frac{T_{E\ block} - T_{block}}{Distance} + hA_{convection}(T_\infty - T_{block}) = m_{block}C\dot{T}_{block}$$

Cell control volume

Heat Balance

$$hA_{convection}(T_\infty - T_{block}) = \dot{Q}_{convected}$$



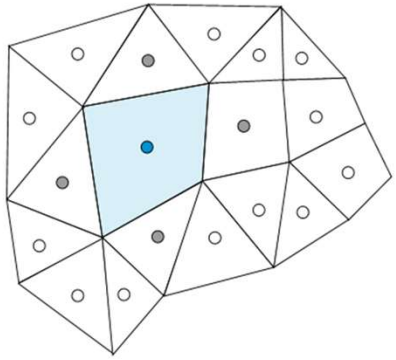
$$hA_{conduction}(T_\infty - T_{block}) = \dot{Q}_{E\ convected}$$

$$kA_{conduction}\frac{T_{W\ block} - T_{block}}{Distance} = \dot{Q}_{W\ conducted}$$

$$kA_{conduction}\frac{T_{W\ block} - T_{block}}{Distance} + h(A_{convection} + A_{conduction})(T_\infty - T_{block}) = m_{block}C\dot{T}_{block}$$

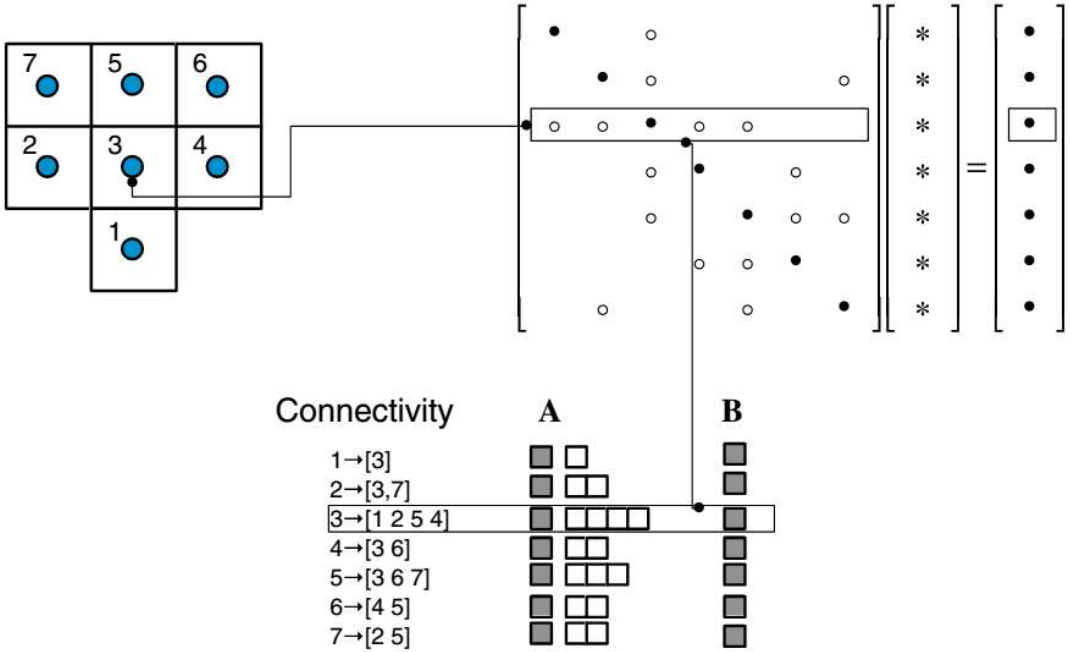
Variables Arrangement

Grid Information



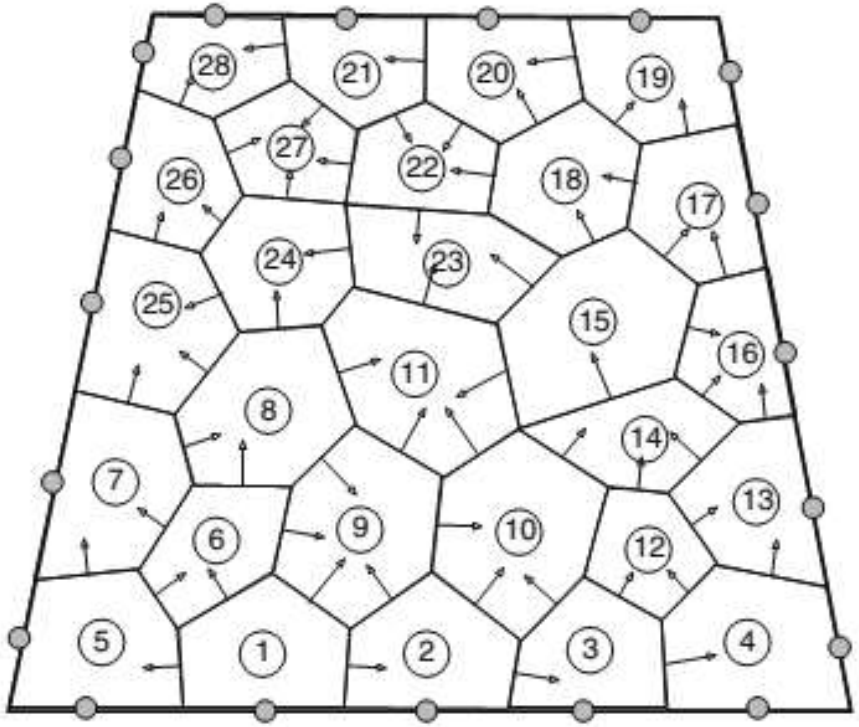
Cell-centered

Procedure



Moukalled, F., Mangani, L., Darwish, M., "The Finite Volume Method in Computational Fluid Dynamics", Springer International Publishing, 2015.

Topological Data - Unstructured Grids

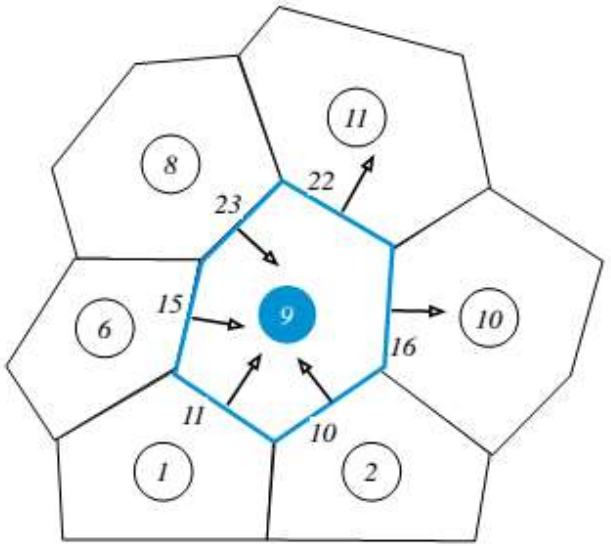


Element Connectivity

Neighbours
1 2 3 5 6
[10 11 8 6 1 2]

Faces
1 2 3 4 5 6
[16 22 23 15 11 10]

Nodes
1 2 3 4 5 6
[21 22 21 14 13 12]



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End Session 20