Chapter 6:
Math and Logic Puzzles

Cracking the code interview

Divisibility

- Positive integer can be decomposed into a product of primes:

$84 = 2^2 * 3^1 * 5^0 * 7^1 * 11^0 * 13^0 * 17^0 * \ldots$

Example

$x = 2^{j0} * 3^{j1} * 5^{j2} * 7^{j3} * 11^{j4} * \ldots$

$y = 2^{k0} * 3^{k1} * 5^{k2} * 7^{k3} * 11^{k4} * \ldots$

If x₩y (整除), then all $j_i <= k_i$

Example

Greatest Common Divisor 最大公因數

$$\gcd(x,y) = 2^{\min(j0,k0)} * 3^{\min(j1,k1)} * 5^{\min(j2,k2)} * \ldots$$

Least Common Multiple 最小公倍數

$$\mathrm{lcm}(x,y) = 2^{\max(j0,k0)} * 3^{\max(j1,k1)} * 5^{\max(j2,k2)} * \ldots$$

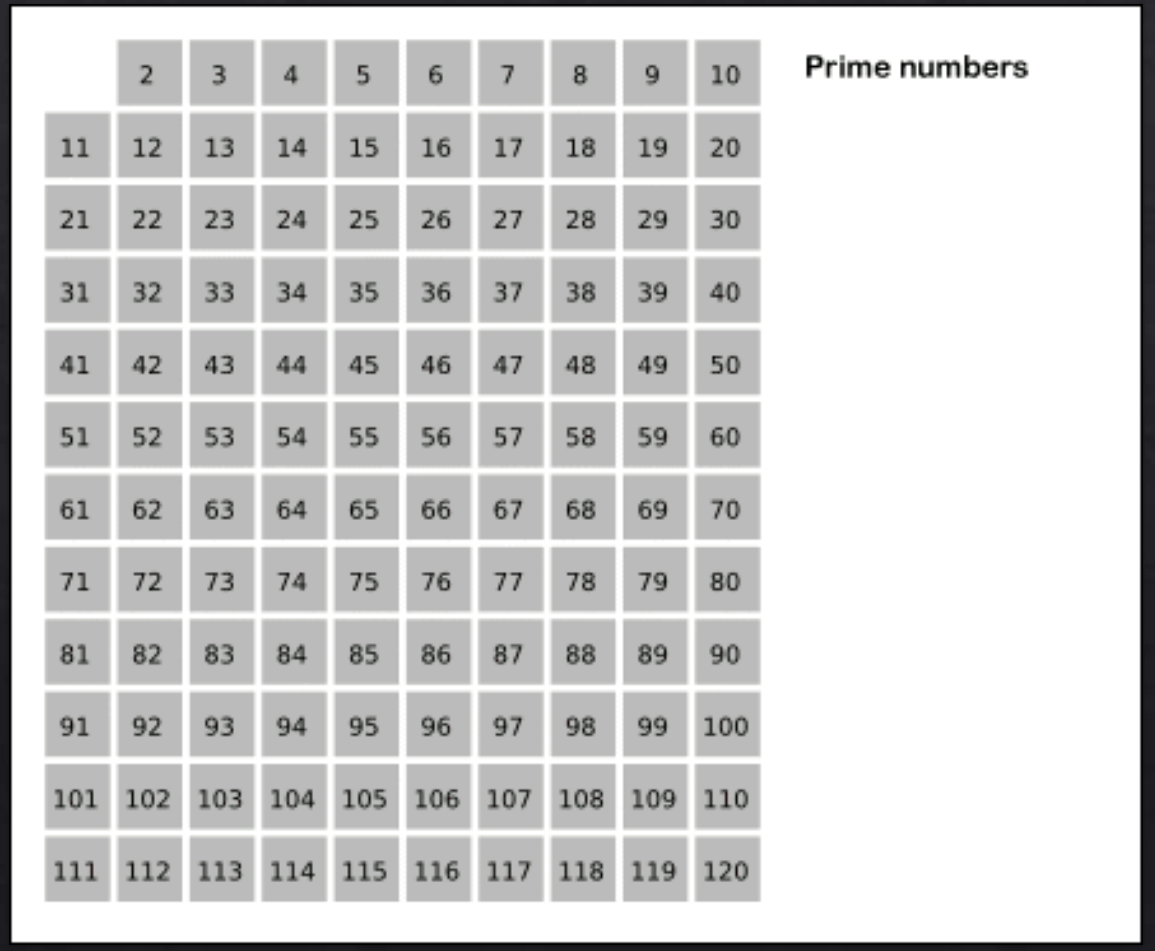
Question

gcd * lcm = ?

Prime

1. Greater than 1
2. Has no positive divisors other than 1 and itself

The Sieve of Eratosthenes



	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120

Probability

P(A and B) = P(B given A) P(A)

Example

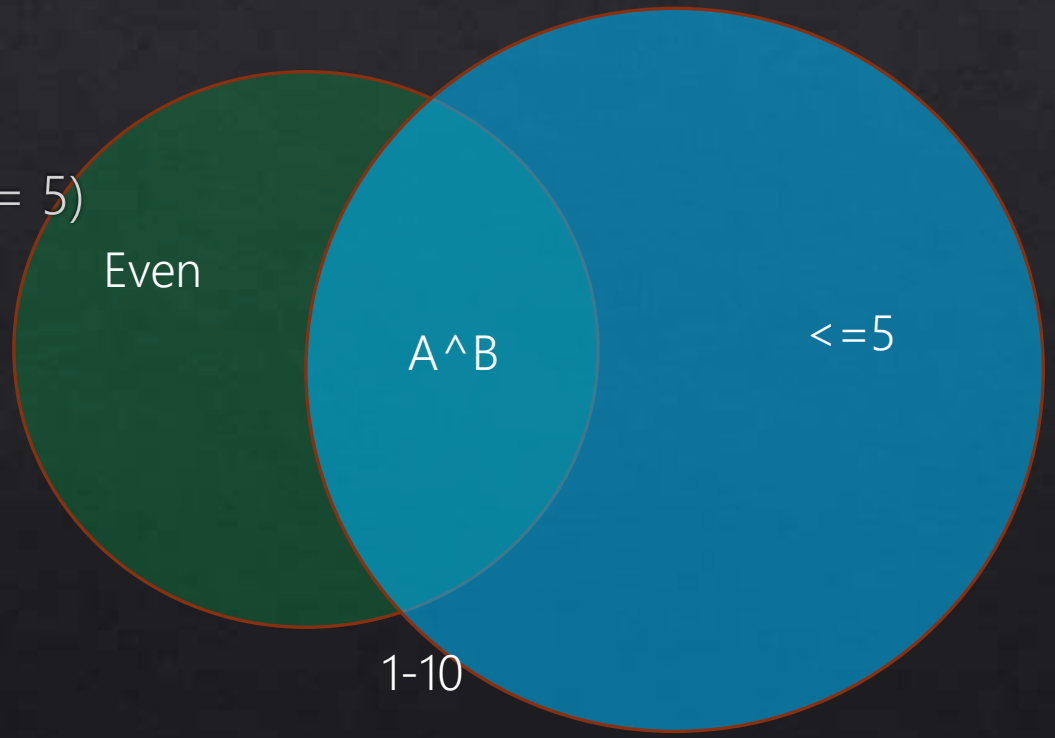
P(A and B) = P(B given A) P(A)

P(x is even and x <= 5)

= P(x is even given x <= 5) P(x <= 5)

= (2/5) * (1/2)

= 1/5



Probability

P(A or B) = P(A) + P(B) - P(A and B)

A or B

- Independence
- Mutual Exclusivity

Solving Problem

- Two ropes, and each takes exactly one hour to burn
- Q. How would you use them to time exactly 15 minutes?

“Nine balls" Question

- You have nine balls. Eight are of the same weight, and one is heavier.
- You are given a balance which tells you only whether the left side or the right side is heavier.
- Find the heavy ball in just two uses of the scale.