Xi'an Jiaotong-Liverpool University
西交利物浦大学

Module Code	Examiner	Department	Tel
INT201		Intelligent Science	

1st SEMESTER 2024/25 FINAL EXAMINATION

Undergraduate

Decision Computation and Language

TIME ALLOWED: *2 hours*

INSTRUCTIONS TO CANDIDATES

1. This is a close-book examination, which is to be written without books or notes.
2. Total marks available are 100.
3. Answer all questions. There is NO penalty for providing a wrong answer.
4. Only English solutions are accepted. Answer should be written in the answer booklet(s) provided.
5. All materials must be returned to the exam invigilator upon completion of the exam. Failure to do so will be deemed academic misconduct and will be dealt with accordingly.

Question 1

Indicate true or false of the following statements, and briefly justify your answers. (**30 Marks**)

(a) If a language A is regular, then it has an NFA. (**3 Marks**)

(b) If regular expression $R = 0(0 \cup 1)^*0$, then $L(R)$ is the language of all strings over $\Sigma = \{0, 1\}$ that begin and end with 0. (**3 Marks**)

(c) If A and B are regular languages, then $(\overline{A} \cup B)^*$ is regular. (**3 Marks**)

(d) The language $\{1^n0^n | n \geq 0\}$ has context-free grammar $G = (V, \Sigma, R, S)$, with $V = \{S\}$, $\Sigma = \{0, 1\}$, start variable S, and rules $S \to 1S0|0$. (**3 Marks**)

(e) If A is a context-free language that is also non-regular, then A has a CFG in Chomsky normal form. (**3 Marks**)

(f) Language A is regular only if there exists a pushdown automaton D such that A = L(D) (**3 Marks**)

(g) Language A is Turing-decidable only if there exists a Turing machine TM such that A = L(TM) (**3 Marks**)

(h) If Language A can be recognized by a nondeterministic Turing machine, A is TM-recognizable. (**3 Marks**)

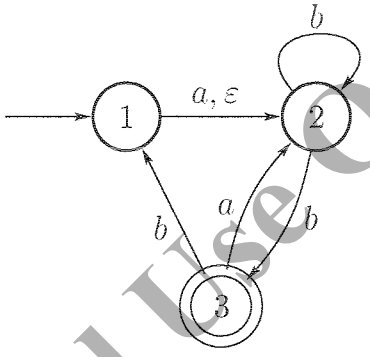
(i) With {0,1} as alphabets, the set of all languages is uncountable. (**3**

Marks)

(j) All context-free languages are closed under complementation. (**3 Marks**)

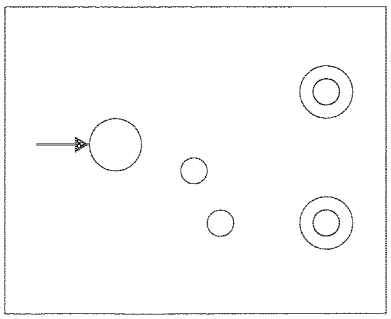
Question 2

Let N be the following NFA with $\Sigma = \{a, b\}$, and let $A = L(N)$. Convert the NFA N to its equivalent DFA that accepts language A. You only need to draw the graph.(**10 Marks**)



Question 3

Suppose that language A is recognized by NFA N (as shown in the below figure). Note that the internal transitions of N are not drawn. Draw the graph of an NFA for A^*. You do not need to give a 5-tuple description of your NFA for A^*. (**10 Marks**)



Question 4

The original CFG is shown as follows, and convert it to Chomsky normal form. **(15 Marks)**

$$S \to QSQ|aP$$
$$Q \to P|S$$
$$P \to b$$

Question 5

Consider the following languages $L_1 = \{a^i b^j c^k \mid i = j, i, j, k \geq 0\}$ and $L_2 = \{a^i b^j c^k \mid i = k, i, j, k \geq 0\}$ **(10 Marks)**

(a) Show L_2 is a context-free language by providing a context-free grammar. **(4 Marks)**

(b) What is the language of $L_1 \cap L_2$? **(2 Marks)**

(c) Is context-free language closed under union? Justify your answer. **(4 Marks)**

Question 6

Let $\Sigma = \{0, 1\}$, and consider the language $A = \{w1w \mid w \in \{0, 1\}^*\}$. The pumping lemma for context-free languages states:
If A is a context free language, then there is a number p (pumping length) where, if $s \in L$ with $\mid s \mid \geq p$, then there are strings u, v, x, y, z such that $s = uvxyz$, the following holds: (i) $uv^i xy^i z \in L$ for all $i \geq 0$,
(ii) $\mid vy \mid \geq 1$,
(iii) $|vxy| \leq p$ **(13 Marks)**

(a) Is the language A context-free? **(3 Marks)**

(b) If answered yes to (a), show a CFG that generates A, else use the pumping lemma to show it is not. **(10 Marks)**

Question 7

Let $\Sigma = \{0, 1\}$, and consider the language $A = \{\langle M \rangle \mid$M is a Turing machine and $L(M) = \{0, 1\}^*\}$. **(12 Marks)**

(a) Give the definition of a language being Turing recognizable but not decidable **(4 Marks)**

(b) Prove or disprove A is a decidable language. **(8 Marks)**

The end of the paper