Xi'an Jiaotong-Liverpool University
西交利物浦大学

Module Code	Examiner	Department	Tel
INT201		Intelligent Science	

1st SEMESTER 2024/25 RESIT EXAMINATION

Undergraduate

Decision Computation and Language

TIME ALLOWED: *2 hours*

INSTRUCTIONS TO CANDIDATES

1. This is a close-book examination, which is to be written without books or notes.
2. Total marks available are 100.
3. Answer all questions. There is NO penalty for providing a wrong answer.
4. Only English solutions are accepted. Answer should be written in the answer booklet(s) provided.
5. All materials must be returned to the exam invigilator upon completion of the exam. Failure to do so will be deemed academic misconduct and will be dealt with accordingly.

Question 1

Indicate true or false of the following statements, and briefly justify your answers. (**30 Marks**)

(a) A language is regular if and only if it is finite. (**3 Marks**)

(b) If a language A has an NFA, then A must be recognized by some PDA. (**3 Marks**)

(c) NFAs recognize nonregular languages. (**3 Marks**)

(d) The language $\{0^n1^n \mid n \geq 0\}$ has regular expression 0^*1^*. (**3 Marks**)

(e) If languages A and B are regular over the same alphabet, $A \cup B$ is regular. (**3 Marks**)

(f) All language recognized by a deterministic pushdown automaton are context free. (**3 Marks**)

(g) The set of all Turing-decidable languages is countable. (**3 Marks**)

(h) A language is either recognizable or co-recognizable. (**3 Marks**)

(i) Every subset of a decidable language is decidable. (**3 Marks**)

(j) All context-free languages are closed under complementation. (**3 Marks**)

Question 2

Give an NFA with exactly four states for the language $\{w \in \Sigma^* | w$ contains the substring $110\}$, where $\Sigma = \{0, 1\}$. You only need to draw the picture. **(12 Marks)**

Question 3

Suppose that A_1 is a language defined by a NFA $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ and A_2 is a language defined by a NFA $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$, where the alphabet Σ is the same for both languages. Let $A = A_1 \cup A_2$. Give a DFA M for A in terms of M_1 and M_2. Notably, you only need to present the description of each term rather than drawing the graph. **(14 Marks)**

Question 4

Suppose that we are in the process of converting a CFG G with $\Sigma = \{0, 1\}$ into Chomsky normal form. We have already applied some steps in the process, and we currently have the following CFG:

$$S_0 \to S$$
$$S \to A0AS|0AS1S|\epsilon$$
$$A \to 1A0S|\epsilon$$

In the next step, we want to remove the ϵ-rule $A \to \epsilon$. Give the CFG after carrying out just this one step. **(9 Marks)**

Question 5

Let $\Sigma = \{a, b\}$, a pushdown automaton is given by the diagram below. **(10 Marks)**

(a) What is the language that is being accepted by the PDA A? **(2 Marks)**

(b) Write a context-free grammar with only one non-terminal symbol S that

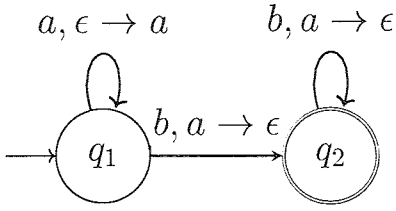

Figure 1: PDA A

accepts the same language of L(A) (**4 Marks**)

(c) Write a context-free grammar that accepts the complement language of L(A) (**4 Marks**)

Question 6

Let $\Sigma = \{a, b, c\}$, and consider the language $A = \{a^i b^j c^j \mid i \geq j \geq 0\}$. The pumping lemma for context-free languages states:
If A is a context free language, then there is a number p (pumping length) where, if $s \in L$ with $\mid s \mid \geq p$, then there are strings u, v, x, y, z such that $s = uvxyz$, the following holds: (i) $uv^i xy^i z \in L$ for all $i \geq 0$,
(ii) $\mid vy \mid \geq 1$,
(iii) $|vxy| \leq p$ (**13 Marks**)

(a) Is the language A context-free? (**3 Marks**)

(b) If answered yes to (a), show a CFG that generates A, else use the pumping lemma to show it is not. (**10 Marks**)

Question 7

Let $\Sigma = \{0, 1\}$, and consider the languages $A = \{\langle M \rangle \mid$M is a Turing machine and $L(M) = \{0, 1\}^*$ and $|M| \leq 1024\}$ and $B = \{\langle M \rangle \mid$M is a Turing machine and $L(M) = \{0, 1\}^*\}$. (**12 Marks**)

(a) Explain the language A in English (**4 Marks**)

(b) Prove or disprove A is a decidable language. (**6 Mark**)

(c) Is the language B Turing-recognizable (**2 Mark**)

The end of the paper