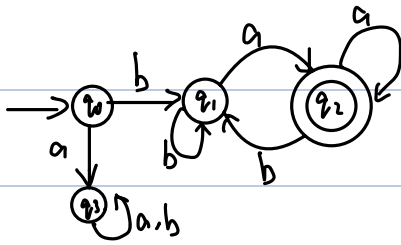


Q1.



Symbolic description: $M = \{Q, \Sigma, \delta, q, F\}$

The set of states is $Q = \{q_0, q_1, q_2, q_3\}$

The alphabet $\Sigma = \{a, b\}$

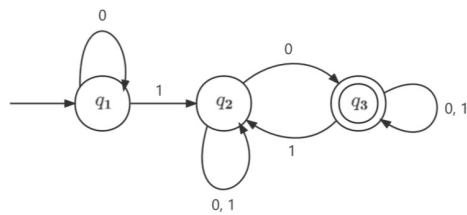
The start state q is q_0

The accept state is $F = \{q_2\}$

The transition function δ is shown by the transition table below:

	a	b
q_0	q_3	q_1
q_1	q_2	q_1
q_2	q_2	q_1
q_3	q_3	q_3

Q2.



$$\text{move}(q, a) = \{\delta(q, a)\}$$

$$E(\{q_1\}) = \{q_1\} = A$$

$$E(\text{move}(q_1, 0)) = E(\{q_1\}) = \{q_1\} = A$$

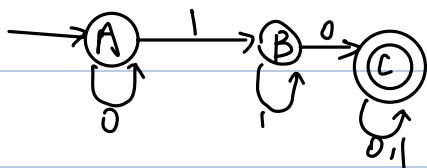
$$E(\text{move}(q_1, 1)) = E(\{q_2\}) = \{q_2\} = B$$

$$E(\text{move}(B, 0)) = E(\{q_2, q_3\}) = \{q_1, q_3\} = C$$

$$E(\text{move}(B, 1)) = E(\{q_2\}) = \{q_2\} = B$$

$$E(\text{move}(C, 0)) = E(\{q_1, q_3\}) = \{q_1, q_3\} = C$$

$$E(\text{move}(C, 1)) = E(\{q_2, q_1\}) = \{q_2, q_1\} = C$$

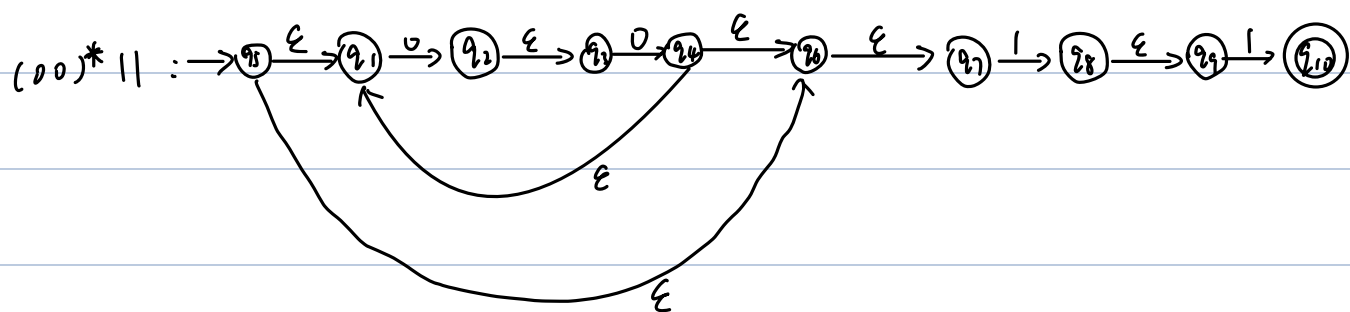
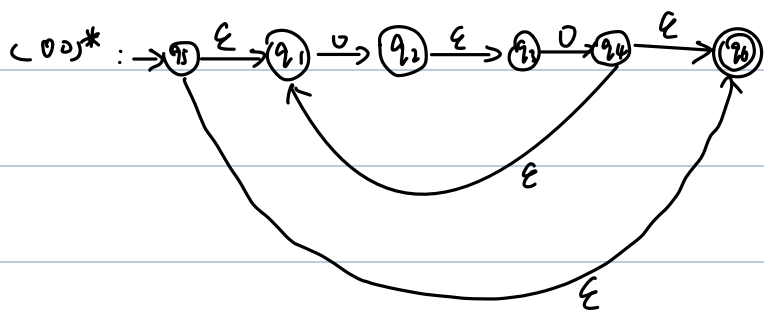
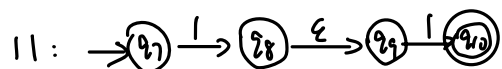
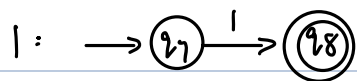
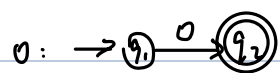


$$A: \{q_1\}$$

$$B: \{q_2\}$$

$$C: \{q_2, q_3\}$$

Q3.



Q4.

Suppose A_1 is regular, then it should satisfy pumping lemma.

Let p be the "pumping length" of the pumping lemma. Consider $w = a^p b$ then $s = www = a^p b a^p b a^p b$.

And $|s| = 3p + 3 > p$, so the pumping lemma will hold. Thus we can split the string s into 3 parts $s = xyz$ satisfying the conditions:

$$xy^i z \in A_1 \text{ for each } i \geq 0 \quad (1)$$

$$|y| > 0 \quad (2)$$

$$|xy| \leq p \quad (3)$$

Because $|xy| \leq p$, and first p symbols of s are a , so we can say that:

$$x = a^m \text{ for some } m \geq 0$$

$$y = a^n \text{ for some } n \geq 1$$

$$z = a^{p-m-n} b a^p b a^p b$$

$$\text{By condition (3), when } i=0, xy^0z = xz = a^m a^{p-m-n} b a^p b a^p b = a^{p-n} b a^p b a^p b$$

$$\text{Since } n \geq 1, p-n < p$$

so $a^{p-n} b \neq a^p b$, which means $xy^0z \notin A_1$, this contradicts to pumping lemma.

Hence our assumption that A_1 is regular is false. Therefore A_1 is nonregular.