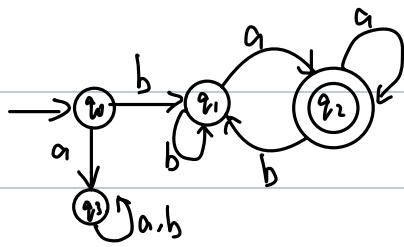


Q1.



Symbolic description:  $M = \{Q, \Sigma, \delta, q_0, F\}$

The set of states is  $Q = \{q_0, q_1, q_2, q_3\}$

The alphabet  $\Sigma = \{a, b\}$

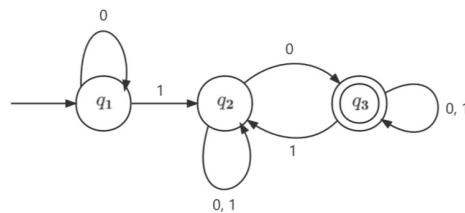
The start state  $q_0$  is  $q_0$

The accept state is  $F = \{q_2\}$

The transition function  $\delta$  is shown by the transition table below:

	a	b
$q_0$	$q_3$	$q_1$
$q_1$	$q_2$	$q_1$
$q_2$	$q_2$	$q_1$
$q_3$	$q_3$	$q_3$

Q2.



$$\text{move}(q, a) = \{\delta(q, a)\}$$

$$E(\{q_1\}) = \{q_1\} = A$$

$$E(\text{move}(q_1, 0)) = E(\{q_1\}) = \{q_1\} = A$$

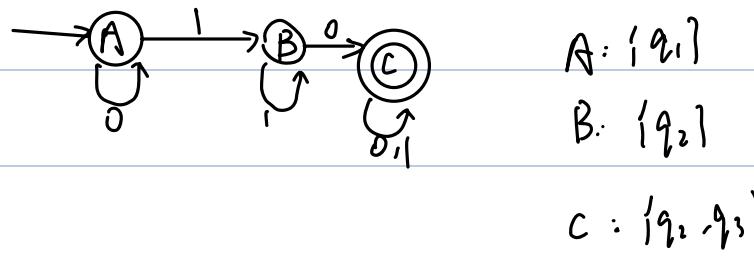
$$E(\text{move}(q_1, 1)) = E(\{q_2\}) = \{q_2\} = B$$

$$E(\text{move}(B, 0)) = E(\{q_2, q_3\}) = \{q_2, q_3\} = C$$

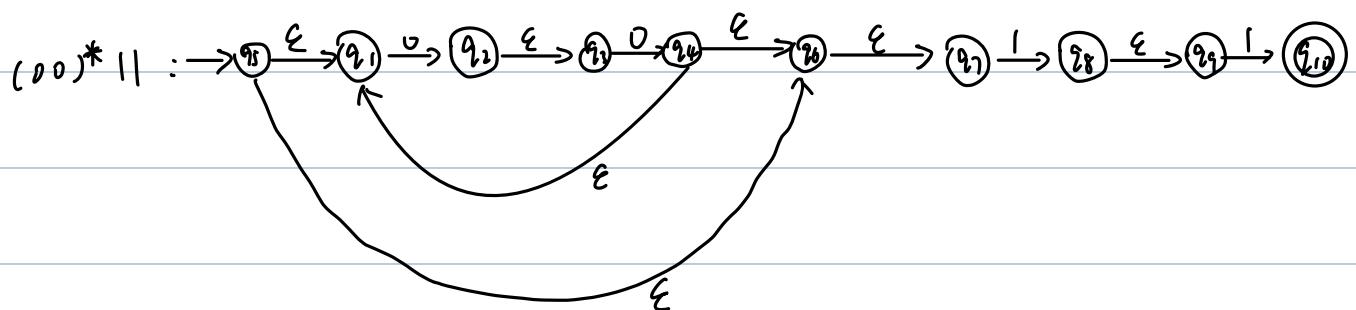
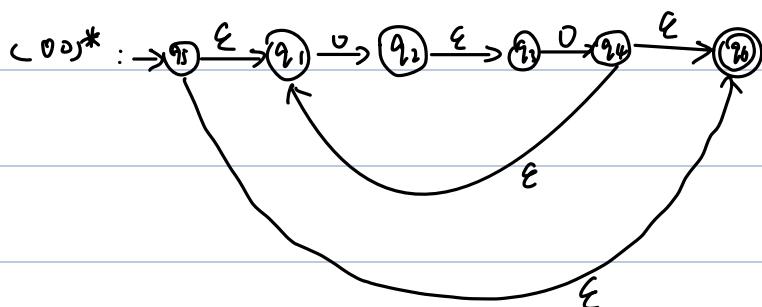
$$E(\text{move}(B, 1)) = E(\{q_3\}) = \{q_3\} = B$$

$$E(\text{move}(C, 0)) = E(\{q_1, q_3\}) = \{q_1, q_3\} = C$$

$$E(\text{move}(C, 1)) = E(\{q_2, q_3\}) = \{q_2, q_3\} = C$$



Q3.



Q4.

Suppose  $A_1$  is regular, then it should satisfy pumping lemma.

Let  $p$  be the "pumping length" of the Pumping Lemma. Consider  $w = a^p b$  then  $s = www = a^p b a^p b a^p b$ .

And  $|s| = 3p + 3 > p$ , so the Pumping lemma will hold. Thus we can split the string  $s$  into 3 parts  $s = xyz$  satisfying the conditions :

$xy^i z \in A_1$  for each  $i \geq 0$  (1)

$|y| > 0$  (2)

$|xy| \leq p$  (3)

Because  $|xy| \leq p$ , and first  $p$  symbols of  $s$  are  $a$ , so we can say that :

$x = a^m$  for some  $m \geq 0$

$y = a^n$  for some  $n \geq 1$

$z = a^{p-m-n} b a^p b a^p b$

By condition (3), when  $i=0$ ,  $xy^0 z = xz = a^m a^{p-m-n} b a^p b a^p b = a^{p-n} b a^p b a^p b$

Since  $n \geq 1$ ,  $p-n < p$

so  $a^{p-n} b \neq a^p b$ , which means  $xy^0 z \notin A_1$ , this contradicts to pumping lemma.

Hence our assumption that  $A_1$  is regular is false. Therefore  $A_1$  is nonregular