# New research in Hopfield Networks: A short intro

Joe El Khoury - GenAI Engineer · Follow
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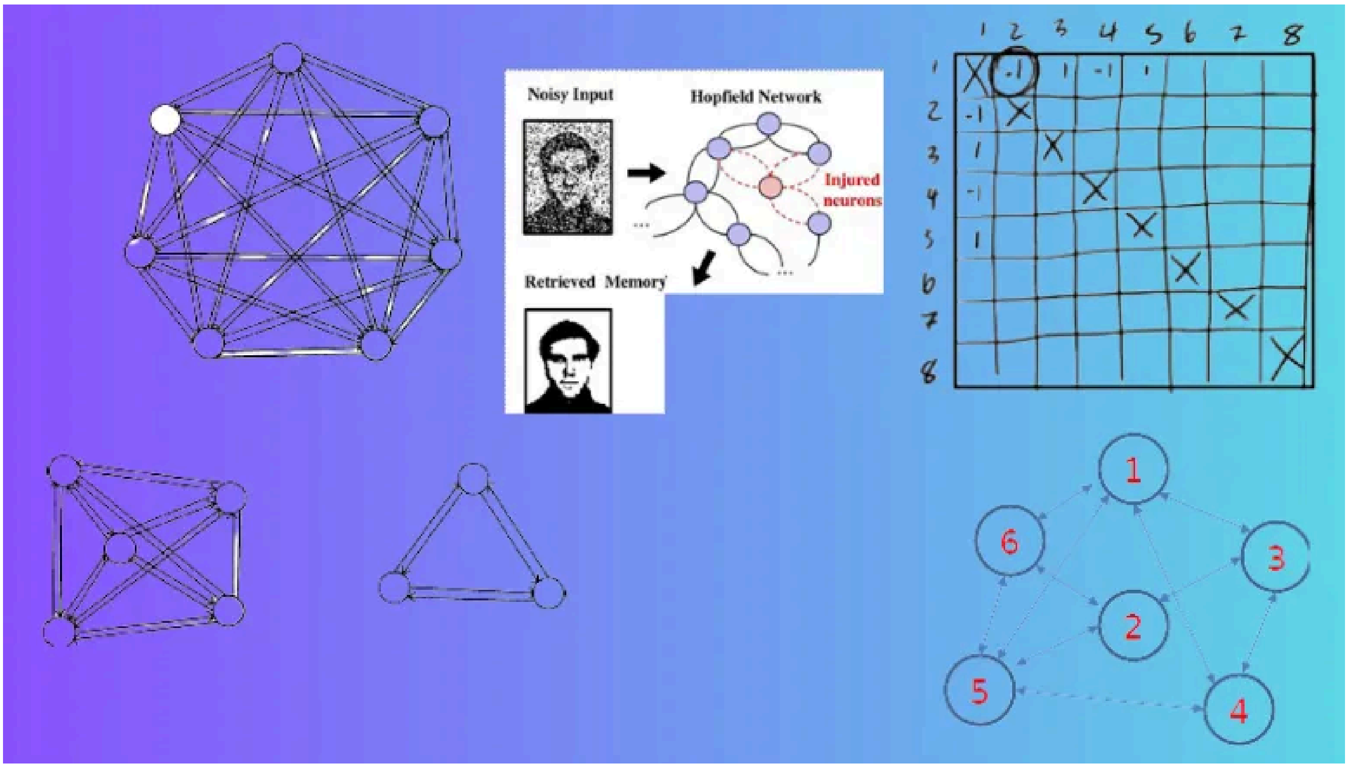
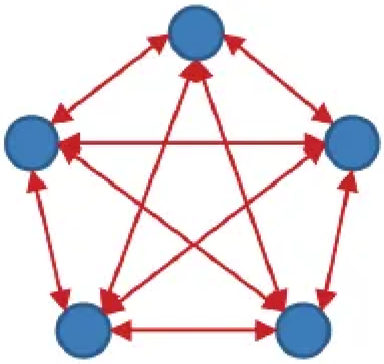



In the era of technological advancement, the ability of a system to recognize and recall patterns, especially when they're imperfect, is of immense value. The Hopfield network, a recurrent artificial neural network popularized by John Hopfield in 1982, lies at the heart of this capability. Hopfield Networks, originating from the convergence of statistical physics, neuroscience, and computer science, have established themselves as pivotal models for associative memory. With the advent and proliferation of deep learning, their relevance has been accentuated, particularly with the attention mechanism's introduction in the Transformer model. This network is unique due to its symmetric connections, ensuring the weight between any two nodes is mutual. Originally devised as a memory model, its unmatched potential lies in its capacity to reconstruct entire patterns from partial inputs without backpropagation and multi layers involved.

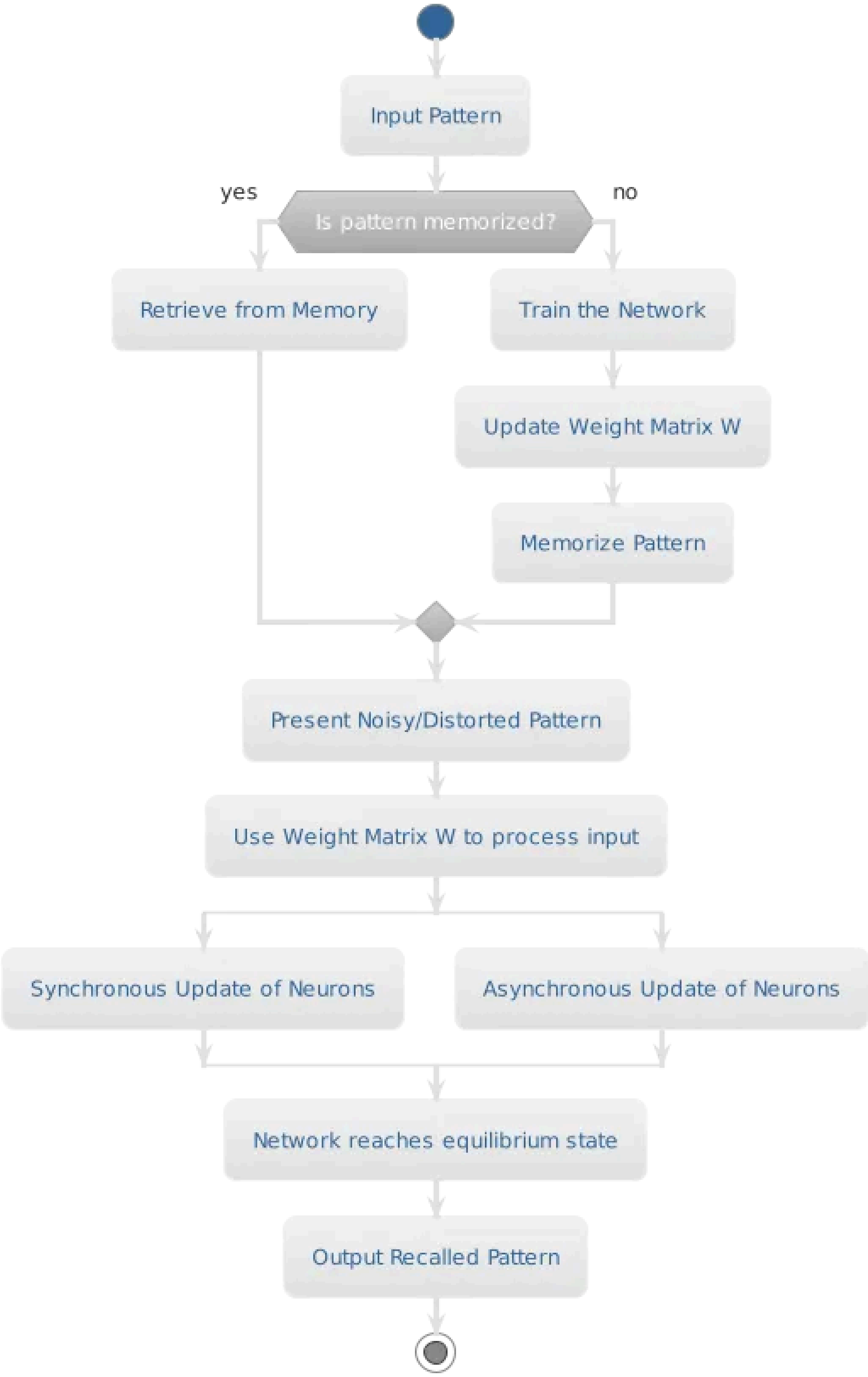
## Introduction

The Hopfield network is a fully connected recurrent neural network that can be used for associative memory tasks, such as pattern recognition and noise reduction in input data .



It was popularized by John Hopfield in 1982 and is characterized by symmetric connections between neurons, allowing for bidirectional information flow. The network can memorize specific patterns, and when a noisy or distorted pattern is input, it can recall the original pattern. The structure of a Hopfield network consists of N neurons, with each pair of neurons having connections, resulting in N*N connections, including self-connections . The connection weight from neuron i to neuron j is represented by a weight matrix W . Each neuron combines inputs from other connected neurons to compute a value and holds a binary value (1 or -1) . The output of the network is an N-dimensional vector containing 1 or -1 values . To store and recall patterns, Hopfield networks use a learning process that involves updating the weight matrix W based on the patterns to be memorized . Once the network is trained, it can recall the original pattern when a noisy or distorted input pattern is presented . The recall process involves updating the neuron values iteratively until the network reaches an equilibrium state, at which point the recalled output is obtained . There are two update methods for Hopfield networks: synchronous (updating all neurons simultaneously) and asynchronous (updating neurons one by one).

Thus, Hopfield networks are a type of recurrent neural network that can be used for associative memory tasks. They have a fully connected structure with symmetric connections between neurons, allowing for bidirectional information flow. The network can memorize specific patterns and recall the original pattern when a noisy or distorted input is presented. The learning and recall processes involve updating the neuron values and weight matrix based on the patterns to be memorized and the input patterns, respectively.




## Understanding the Hopfield Network

**The Recursive Neural Framework**

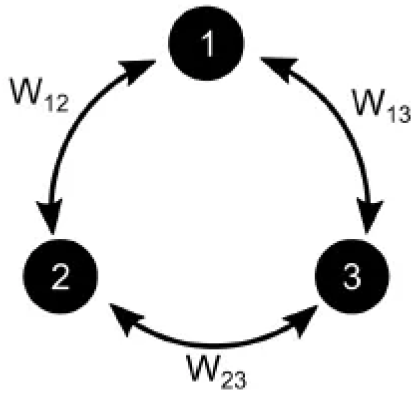
Hopfield Network is characterized as a fully connected recursive neural network. This implies that every neuron is interconnected, ensuring bidirectional information flow. The network operates predominantly based on an energy function. When updating individual neuron states, the energy function is steered in a decremental direction. The network's memory representation is in the form of "stable states," with specific network states gravitating towards these stable configurations.

**The Classical Hopfield Networks**

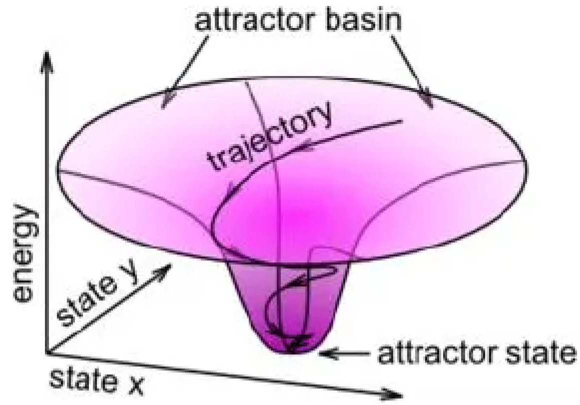
In this classical approach, each state within the network is signified by *N* neurons, each holding a binary value of ±1. The energy function encompasses the state of each neuron, the inter-neuronal weights, and the neuron count designated for storage.

A central aspect of this model is the update rule, which ensures that the energy function consistently gets revised in a decremental direction. This network is adept at pattern recognition and error correction. However, there are constraints, such as the limited number of storable patterns. Exceeding this limit can cause the energy function to collapse.

Thus to explain this more without going into formulas, the physicist John Hopfield realized that the basic idea of the Ising model could be used to describe the collective behaviour of neurons, with certain states of activity representing low energy states while assuming that all neurons do not connect to themselves while the output of each neuron is a "spike"(1) or "no spike" (-1) depending on the neuron's input then an energy function can be defined over states. It can be proven that if we randomly select a neuron at each time-step and update its activity, then the energy term will always decrease or stay the same.



Thus, to store a memory, we only need to decrease the energy of that activity pattern to make it an attractor state. which done using gradient descent, we take the partial of the energy with respect to the weights and this is called the Hebbian learning rule.



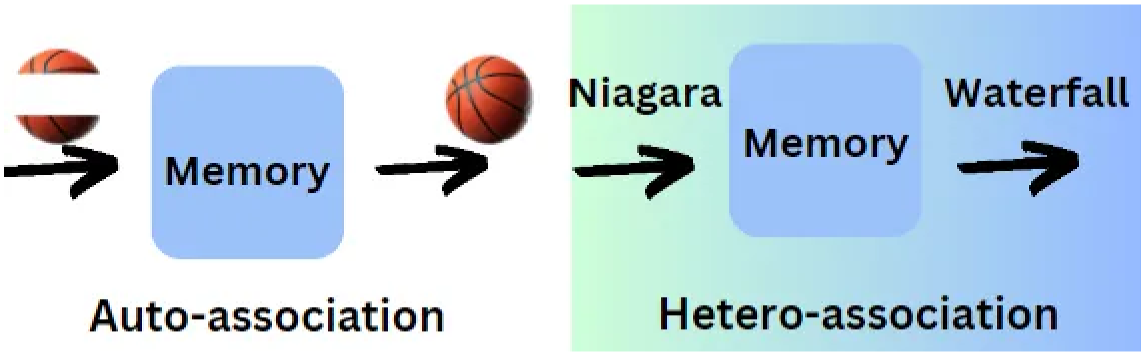

Using this weight update Hopfield networks can be trained to store multiple different memories, with a limit of roughly 0.138n memories, where n is the number of neurons.

Thus, the number of memories that can be stored increases with greater orthogonalization and sparsity in the patterns (making the representations a bit more local). Interestingly, this is what the dentate gyrus of the hippocampus seems to do.

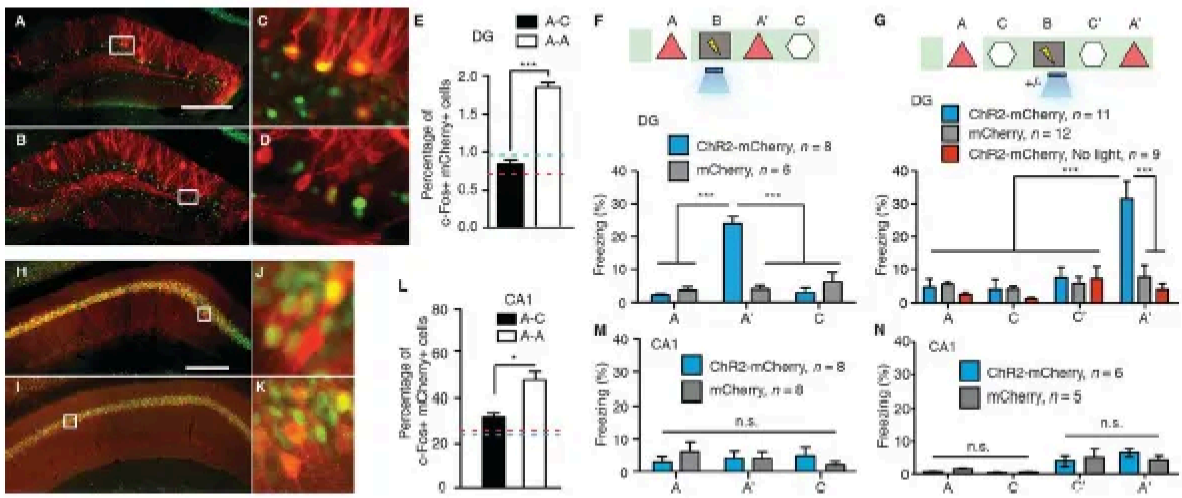
#### Modeling Associative Memory

We as humans we associate the faces with names, letters with sounds, or we can recognize the people even if they have sunglasses or if they are somehow elder now. or when we listen to the first seconds of a melody we assign it to an old song we listened to. The Hopfield Network's prowess as an associative memory model is undeniable.

> *Associative memory is a data collectively stored in the form of a memory or weight matrix, which is used to generate output that corresponds to a given input, can be either* ***auto-associative*** *or* ***hetero- associative memory****. Classification of associative memory is such that while the memory in which the associated input and output patterns differ are called Hetero Associative Memory, it is called Auto Associative Memory if they are the same.*




Hopfield networks uses auto-association while hopfield networks have seen significant advancements and improvements over the years, particularly in terms of memory capacity and correlations with attention mechanisms. The deep learning revolution prompted a re-evaluation of the Hopfield network concept, leading to significant advancements in memory capacity, as documented in research works by Krotov+16 and Demircigil+17 . Furthermore, correlations between Hopfield networks and attention mechanisms have been established in studies by Ramsauer+20 and ICLR2021 . The idea of leveraging single-body complexes for enhancing neuronal interactions was proposed in Burns+23 and ICLR2023 . These advancements have contributed to the development of modern Hopfield networks, which offer improved performance and more efficient storage and retrieval of information.



(Figure from Ramirez et al., 2013, Science, 341: 387-391.)

Real brains seems to work like hopfield

**Modern Hopfield Networks**

The modern incarnation of the Hopfield Network is a testimony to evolution. It touts:

- Amplified storage capacities.
- Revamped energy functions and update rules.
- Accelerated convergence.

The transition from binary to continuous state variables is particularly pronounced, especially when assimilating the Hopfield Network into deep learning architectures. This evolution is reminiscent of the Transformer formula, reinforcing the significance of the attention mechanism.

### Modern vs Classical Hopfield Networks

Classical Hopfield Networks are recurrent neural networks with dynamical trajectories converging to fixed point attractor states and described by an energy function. They have binary states (0 or 1) and are typically used for auto-association and optimization tasks (links 1, 2). However, classical Hopfield Networks have a linear scaling relationship between the number of input features and the number of stored memories, limiting their storage capacity (link). Modern Hopfield Networks, also known as Dense Associative Memories, break the linear scaling relationship by introducing stronger non-linearities in the energy function or neurons' activation functions. This leads to super-linear or even exponential memory storage capacity as a function of the number of feature neurons . Due to their continuous states, modern Hopfield Networks are differentiable and can be integrated into deep learning architectures.

### Transformer Formula and Attention Mechanism

The Transformer model is a deep learning architecture that relies on the parallel multi-head attention mechanism . The attention mechanism computes the output by mapping a query and a set of key-value pairs . In the Transformer model, the representation of a sequence is computed by relating different words in the same sequence using self-attention .

### Continuous State Variables in Hopfield Networks

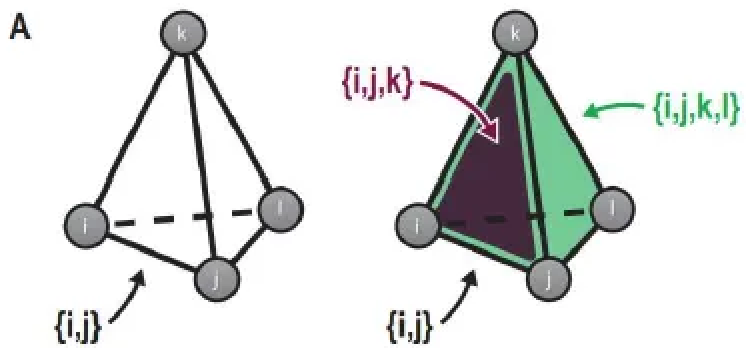
Continuous state variables in Hopfield Networks allow for differentiability and integration into deep learning architectures . Discrete-time, continuous-state Hopfield Networks have been studied and shown to have convergent activation dynamics . The transition from binary to continuous state variables in modern Hopfield Networks enables an abundance of new deep learning architectures and improved memory storage capacity.

## Hopfield Networks in Deep Learning Architectures

Modern Hopfield Networks can be integrated into deep learning architectures due to their differentiability and continuous states . This enables new deep learning architectures that leverage the associative memory capabilities of Hopfield Networks. For example, modern Hopfield Networks have been used for graph embedding, where they form basins of attraction around "prototypical node classes" that resemble many individual nodes from the training set .In summary, modern Hopfield Networks have amplified storage capacities, revamped energy functions and update rules, and accelerated convergence compared to classical Hopfield Networks. The transition from binary to continuous state variables allows for integration into deep learning architectures, reminiscent of the Transformer formula and attention mechanism.

while noting that all the improvement on hopfield network has led to increase its memory storage from 0.138*N (N for neurons) to 2^(N/2) while passing to exponential function (e^x).

#### The Simplical Hopfield Networks




Simplicial Hopfield networks extend traditional and modernHopfield networks by incorporating higher-order relationships in the form of simplices, which allows the network to capture more complex interactions between elements and store more memory patterns. This leads to improved performance in associative memory tasks compared to traditional pairwise Hopfield networks. In traditional Hopfield networks, connections between neurons represent pairwise relationships, limiting the network's ability to capture more complex interactions and store memory patterns. Simplicial Hopfield networks, on the other hand, incorporate higher-order relationships involving connections between groups of three or more neurons . These connections form structures like triangles (2-simplices) or tetrahedra (3-simplices), allowing the network to represent more complex interactions and store more memory patterns.Simplicial Hopfield networks outperform traditional pairwise Hopfield networks in terms of memory storage capacity . They can represent more complex interactions and store more memory patterns, leading to improved performance in associative memory tasks. Even when connections are limited to a small random subset, simplicial Hopfield networks still perform better than their pairwise counterparts . In summary, simplicial Hopfield networks offer advantages over traditional Hopfield networks by incorporating higher-order relationships, allowing for more complex interactions and increased memory storage capacity.

To better understand this concept in the context of Hopfield networks, let's consider a simple example. Suppose we have a Hopfield network that stores memory patterns of images. In a traditional pairwise Hopfield network, each connection between neurons represents the relationship between two pixels in the image. This means that the network can only capture pairwise relationships between pixels, limiting its ability to store complex patterns.In a simplicial Hopfield network, higher-order relationships can be used to represent more complex interactions between groups of pixels. For example, a triangle (2-simplex) could represent the relationship between three pixels that form a small triangular pattern in the image. By incorporating these higher-order relationships, the simplicial Hopfield network can store more complex memory patterns and achieve better performance in associative memory tasks compared to traditional pairwise Hopfield networks.In summary, simplicial Hopfield networks extend the traditional Hopfield network model by incorporating higher-order relationships in the form of simplices. This allows the network to capture more complex interactions between elements and store more memory patterns, leading to improved performance in associative memory tasks.

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## Some insights

Simplicial Hopfield networks could potentially enhance Transformer models by improving the attention mechanism. In the context of Transformers, the attention mechanism is crucial for capturing long-range dependencies and processing input data effectively, especially in natural language processing (NLP) and computer vision tasks . Simplicial Hopfield networks incorporate higher-order relationships in the form of simplices, allowing them to represent more complex interactions and store more memory patterns (as per the paper). By integrating the concepts of simplicial Hopfield networks into the attention mechanism of Transformer models, it may be possible to increase memory storage capacity and improve the model's ability to capture complex relationships in the input data. Research on simplicial Hopfield networks suggests that they could be a promising avenue for enhancing the attention mechanism in Transformer models. However, further exploration and experimentation are needed to determine the extent of the improvements and how they can be effectively integrated into real-world Transformer settings.

### Conclusion and Further Thoughts

The trajectory of Hopfield Networks, spanning from their rudimentary form to their contemporary avatars, underscores their profound connections with diverse scientific disciplines. The inclusion of continuous variables and the innovative concept of unitary complexes amplifies their potential.

Yet, it's imperative to recognize that the current model of the Hopfield Network might not always attain state-of-the-art benchmarks. However, the Transformer's relevance, especially in terms of energy minimization and associative memory, could be the harbinger of future advancements.




*I'm Joe, and my ambition is to lead the way to industry 5.0 performance. I'm always interested in new opportunities, so don't hesitate to contact me on my LinkedIn.*

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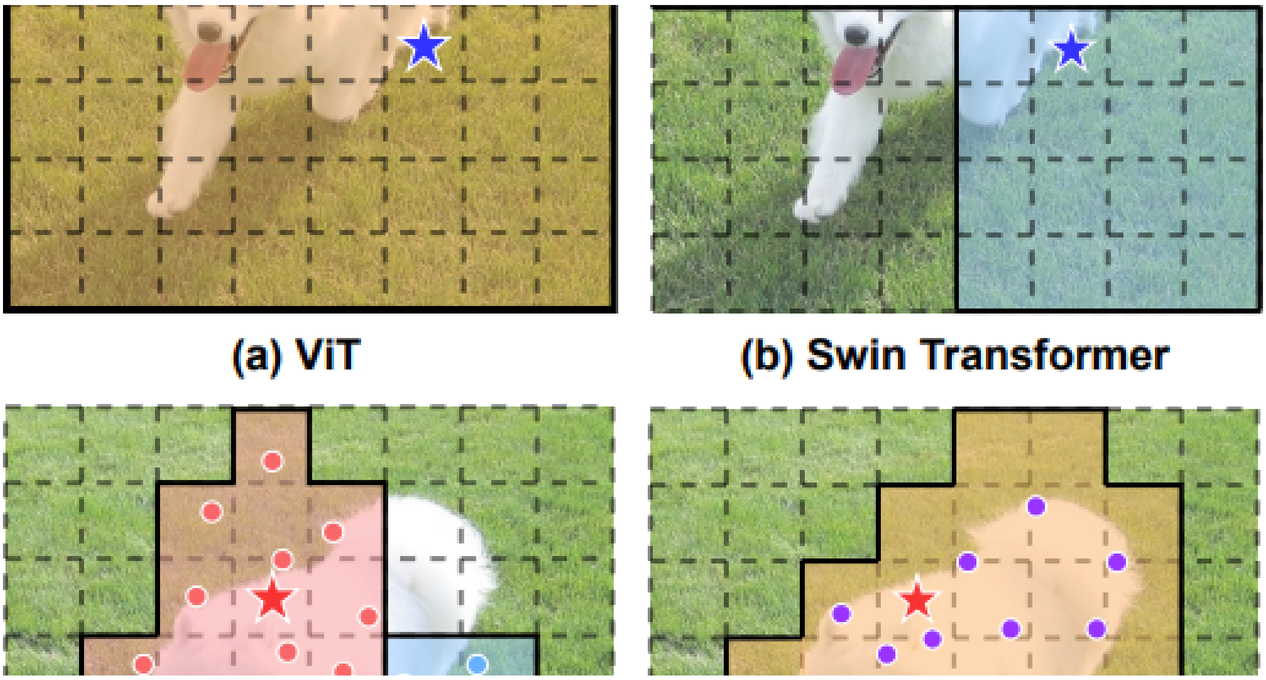


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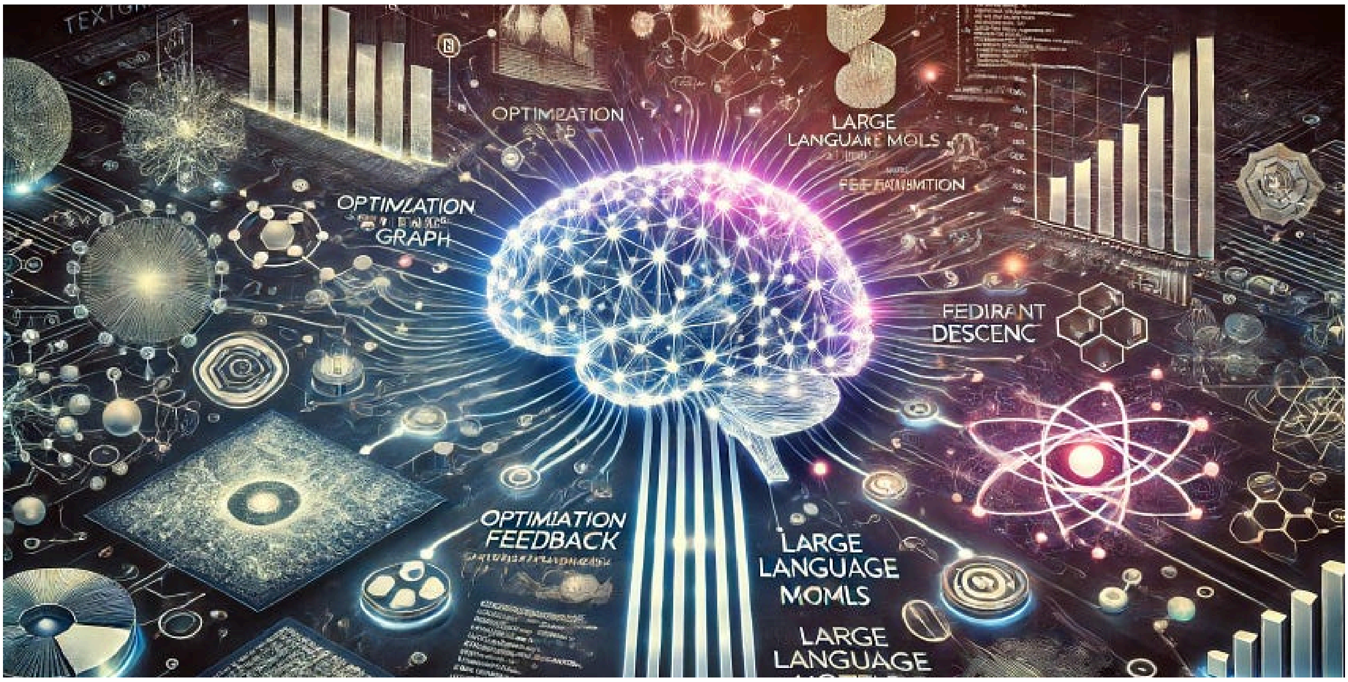


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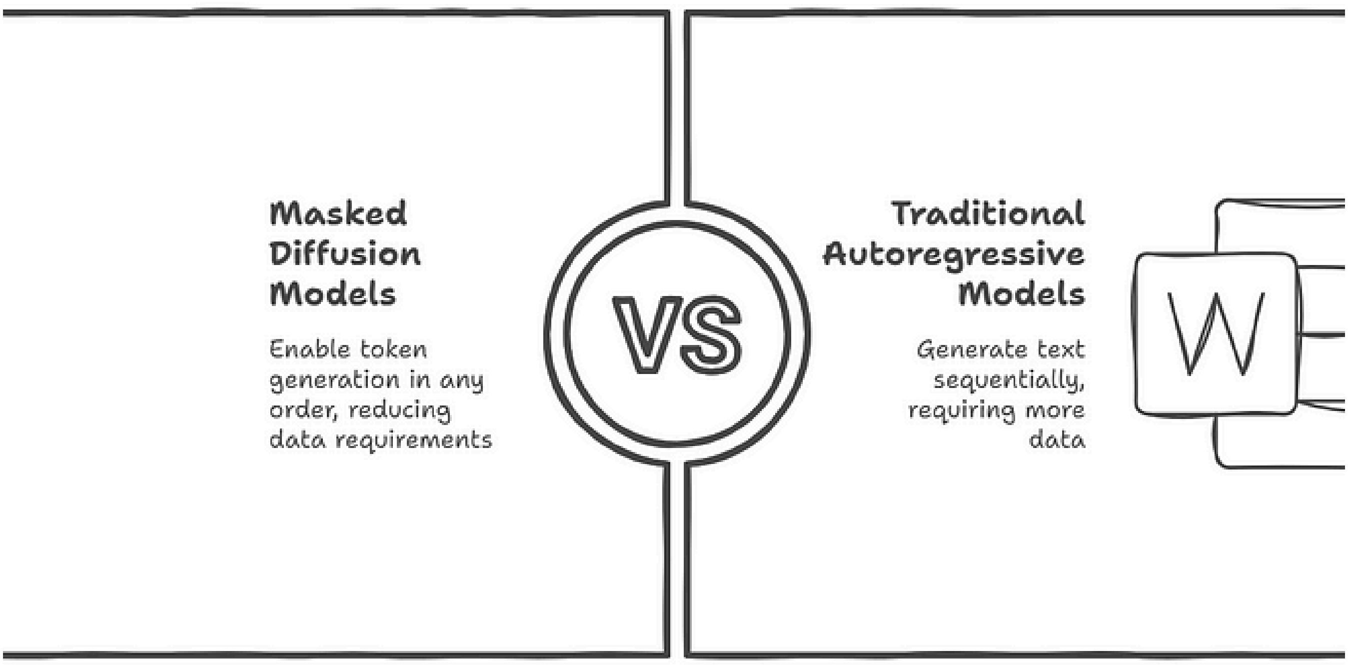

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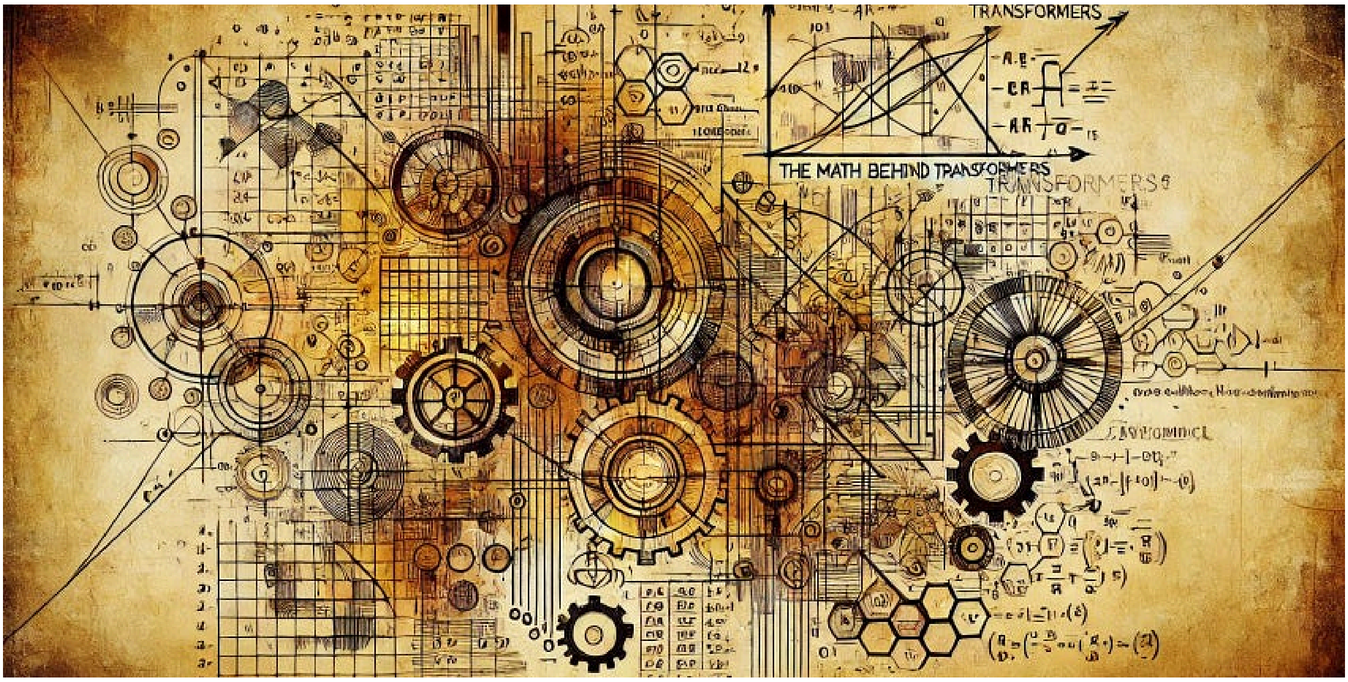


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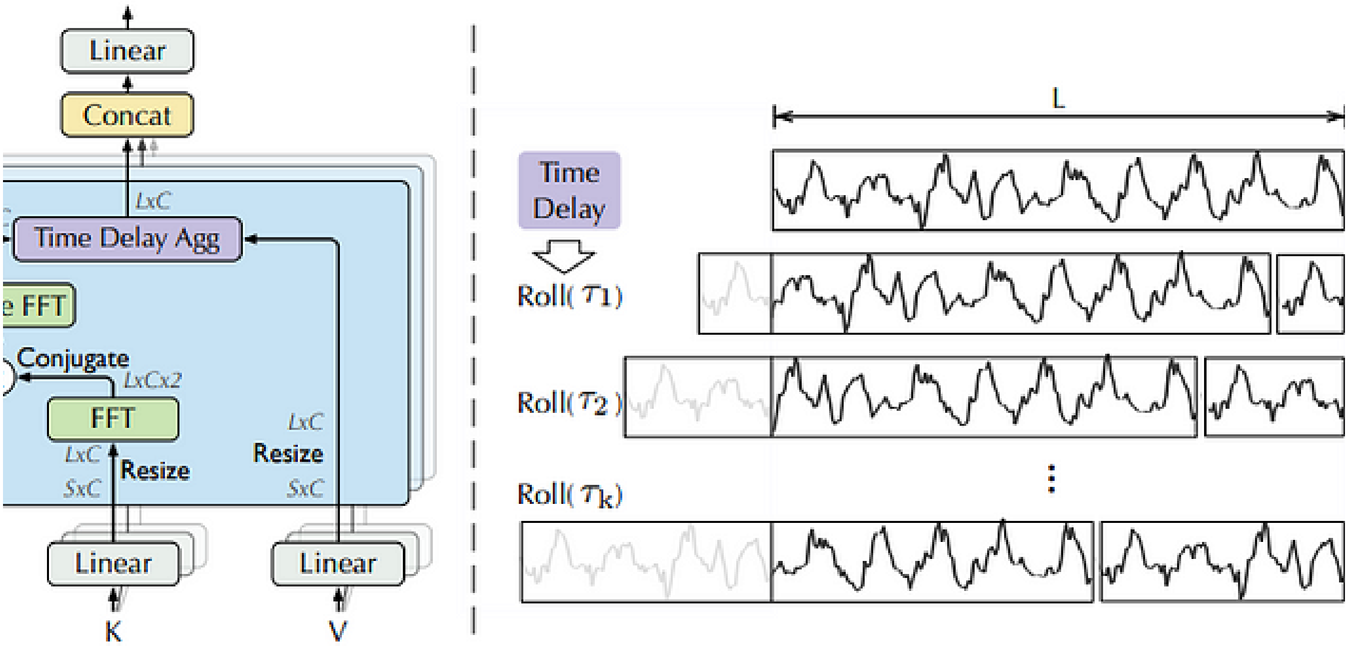


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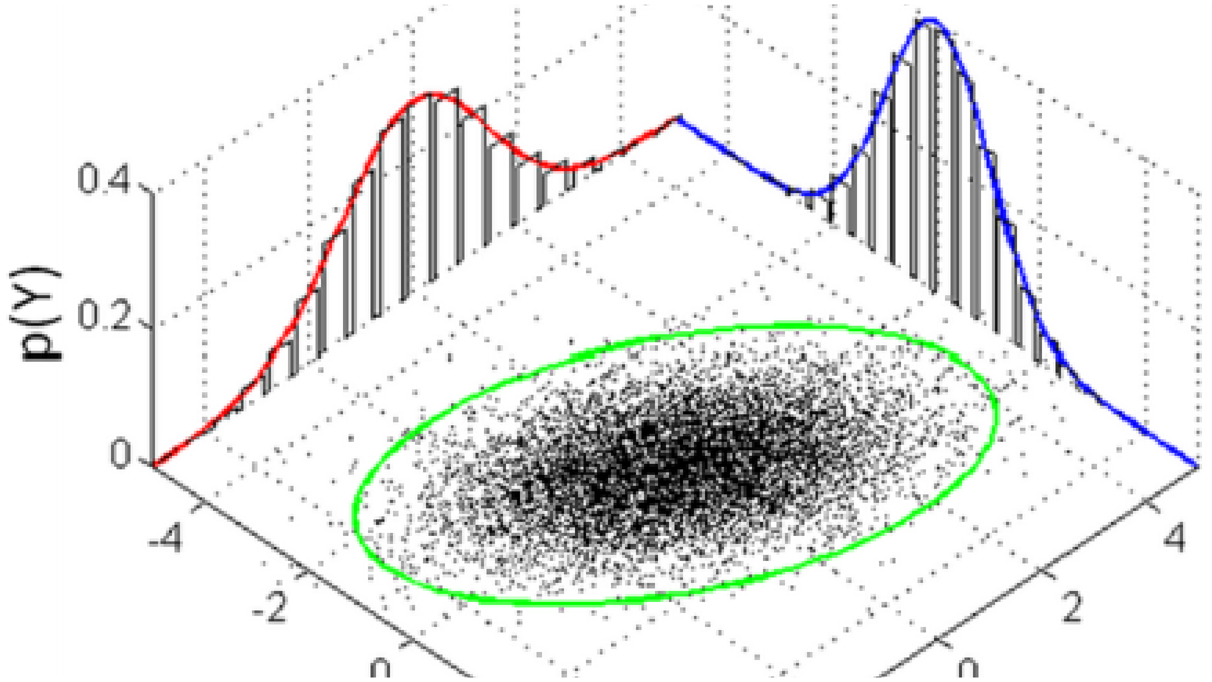


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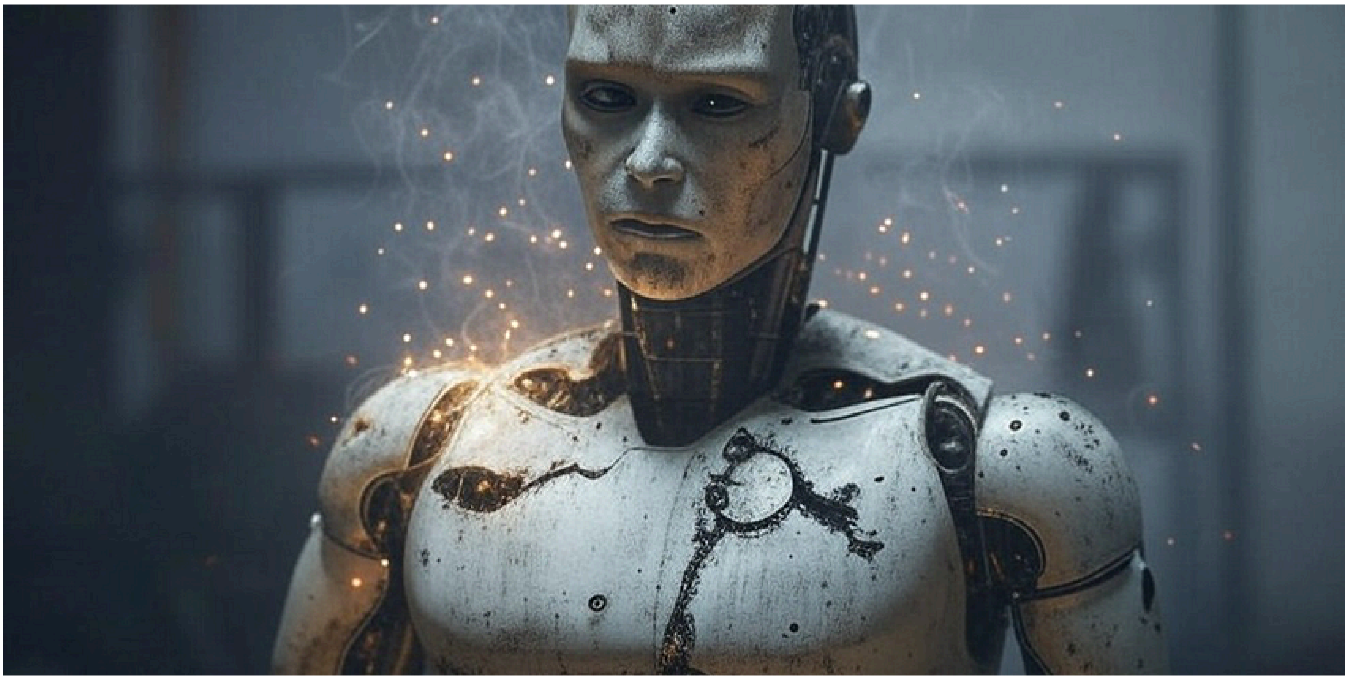

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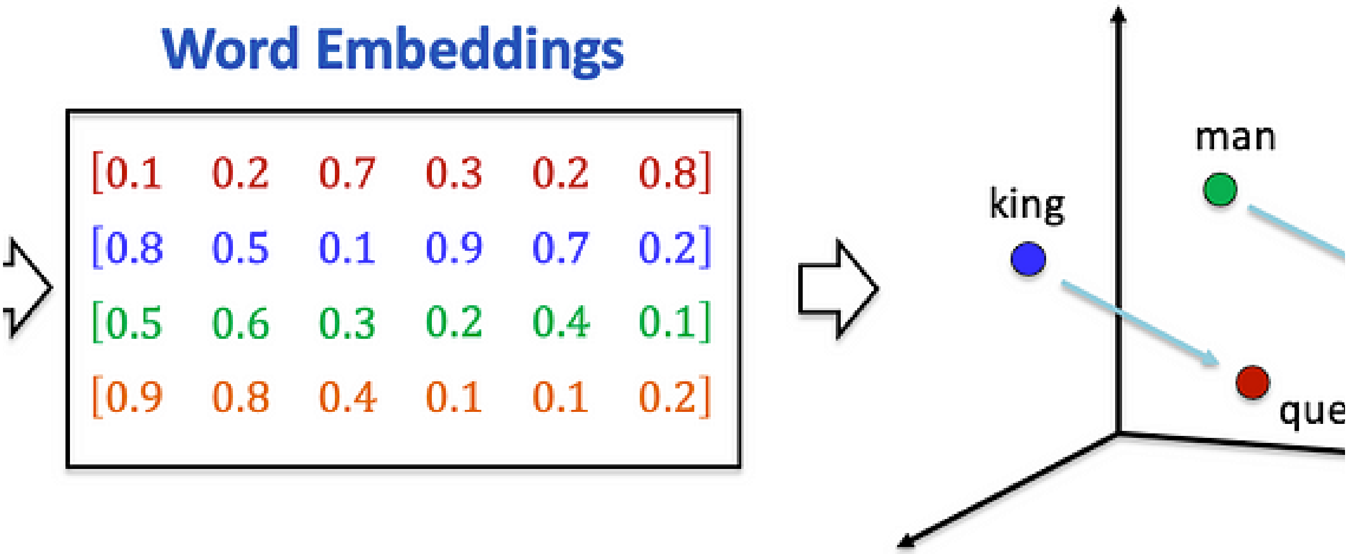


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