

CMPT 125: Introduction to Computing Science and Programming II

Fall 2023

Week 4: Binary encoding, recursion, algorithms

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Joke of the day

BOOLEAN HAIR LOGIC

A



B



AND



OR



XOR

Recap from Last Lecture

- Execution stack of functions – each time a function is called an execution block is created “on top” of the calling function, forming a stack
- Functions in other files – allows better organization of code and functionalities
- Preprocessor directives (Macros) – a step taken during the compilation process
- Global/Local/Static variables – affects the availability of variables to a function
- Memory allocation – allows programs to dynamically request memory for space efficiency, need to be careful about leakage

Review from Last Lecture (I)

- Investigate how to use request memory for structs
 - It uses the same syntax as requesting memory for built-in variables

```
typedef struct {  
    unsigned int capacity;  
    unsigned int used;  
    double* data;  
} doubleArrayWithSize;
```

Using a pointer allows
a variable array size

```
//declaring a normal variable  
doubleArrayWithSize myArray;  
  
//declaring a pointer to a requested memory  
doubleArrayWithSize* dynamicArray = malloc(sizeof(doubleArrayWithSize));  
dynamicArray->capacity = 10;  
dynamicArray->used = 0;  
dynamicArray->data = malloc(sizeof(double) * dynamicArray->capacity);  
  
//...some code  
  
free(dynamicArray->data); //free the fields first  
free(dynamicArray); //free the struct last
```

Free the memory
when done, careful
with the order

Combining Struct Creation with Functions

- **General steps:** dynamically create a complete struct variable → set it up → return its address

```
person* create_person() {  
    person* p = (person*) malloc(sizeof(person));  
    p->name = (char*) malloc(21*sizeof(char));  
    printf("print name (up to 20 chars): ");  
    scanf("%s", p->name);  
  
    printf("print ID: ");  
    scanf("%d", &(p->id));  
  
    return p;  
}
```

```
typedef struct {  
    char* name;  
    int id;  
} person;
```

Inside main()

```
person* people[2];  
  
people[0] = create_person();  
people[1] = create_person();  
  
printf("Person 1 name = %s, id = %d\n", people[0]->name, people[0]->id);  
printf("Person 2 name = %s, id = %d\n", people[1]->name, people[1]->id);  
  
for (int i=0; i<2; i++) {  
    free(people[i]->name);  
    free(people[i]);  
}
```

Note the order:
inside-outside

Review from Last Lecture (2)

- Look up other memory allocation functions: `calloc`, `memcpy`, `memset`

`size_t` is essentially unsigned long long to guarantee all array elements can be indexed

```
void* calloc( size_t num, size_t size );
```

- `calloc` allocates memory for an array of `num` objects of size `size` and initializes all bytes in the allocated storage to 0.

```
void* memcpy( void* dest, const void* src, std::size_t count );
```

- `memcpy` copies `count` bytes from `src` to `dest`. Both objects are reinterpreted as arrays of unsigned char.

```
void* memset( void* dest, int ch, std::size_t count );
```

- `memset` converts the value `ch` to unsigned char and copies it into each of the first `count` characters of `dest`.
- Useful for initializing dynamic memory, be careful with the `values` and `count`

Assignment I

- Read the description file carefully for the questions and submission instructions
- Due on **Sep 29, 11:59p**, submit to CourSys (link can be found from the Canvas assignment description)
- Remember to test your code by compiling and running your programs at a CSIL machine
 - You can do it remotely or in-person
- **DO NOT share your code in any platform** (e.g., Piazza, Discord, Canvas, Replit ...anywhere)
 - Others might use what you post, our similarity report will catch you, both you and copiers get zero for cheating
 - If you used any help (online reference, peer tutor, ...etc.), state them as comment at the top of your files

Assignment I Tips

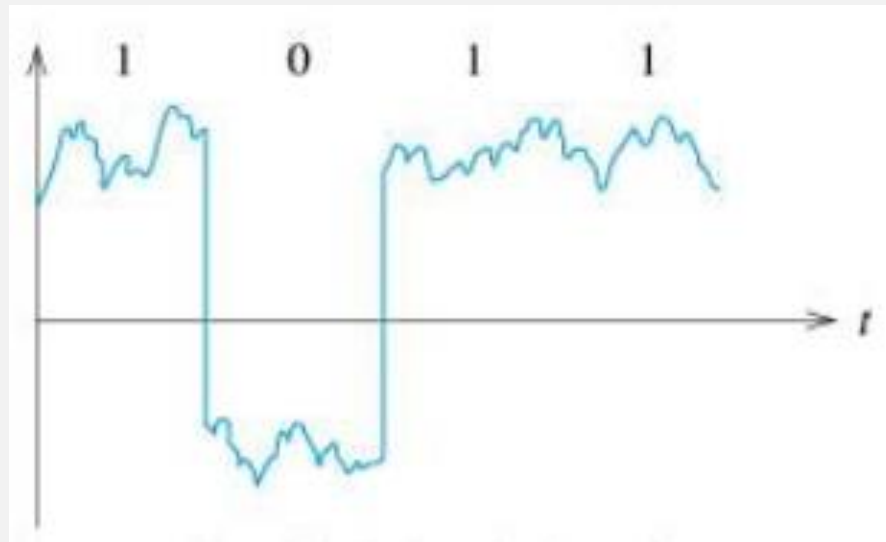
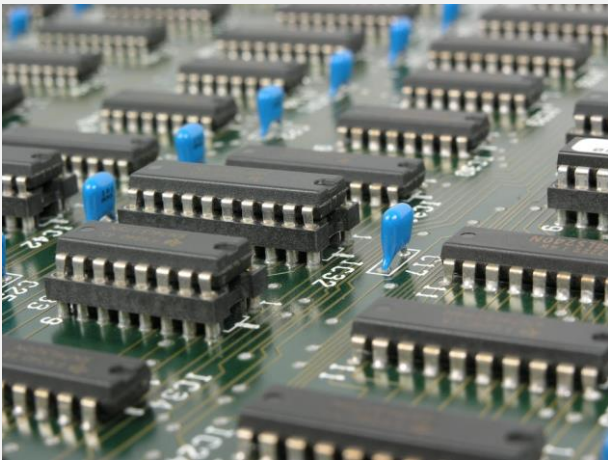
- Do I question at a time
 - Start by reading `al_questionI.h` and `testI.c`
 - Create `al_questionI.c` and start with the directive: `#include "al_questionI.h"`, then implement the function there (you should put your entire answer there, as this is the only file you submit for question I)
 - Put your student information and any help used as comments at the top
 - Use the command `make testI` to compile your code to generate the program `testI`
 - Fix any errors/warnings you see, then repeat `make testI` until you see the program and run it by `./testI`
 - Modify `testI.c` to have more testcases

Today

- Binary encoding
 - and decoding
- Pseudo-code & algorithms
 - syntaxes & uses
- Floating-point encoding
 - and decoding

Fundamental Units in Computers

- Fundamentally, digital computers are machines that convert high and low electrical signals into 0's and 1's
 - All data are stored and transmitted in this format (just very quickly and often simultaneously)
 - Because there are only 2 possible signals, data that are represented (encoded) with this format is considered **binary encoded**



Data in Computers Are Just Sequences of 0's & 1's

This is the “native language” computer speaks, and they are very good at it.

```
0011111100000011111001001100011011100101001111111111101
1110111111000011101011111101100011110111111010101100110
1101111100001110001101010010111010011111000110001000110
11000111011110111111011111111010001111110111011111110
111110011011101111000111101110101110100011111110111110
110011011111101100010010000001011101110111011111111111
111110001111110001111111111111001111001111110010111111
0111111011001110111101111110101111101111111101100111010
11111111100011111110010100101000111110111110110100111101
0001111111111011000101000011110010000000111100100100011
101111100111111111010101111111000101110111000001001111110
0111000011110111011111001111110011111111001100000110111
1011101101000010011001100011101110000110010000001100111
1101100110001010111101101000111110100011010111110010111
1111111110011000111100111101010000110100111000100100010
1110100001011100111111100000111110111111011110101111100
11100100110101111111111100111111111111111111001010111111
0000000001111111100001100111101100100011011011010110001
0100010011111110011111111111110011111000111110011110101
01000111100100111111110000101111011100110111110110011111
```

This is what we call binary digits, which have 2 possible values: **0** and **1**

Compare this with decimal digits, which have 10 possible values: **0** to **9**

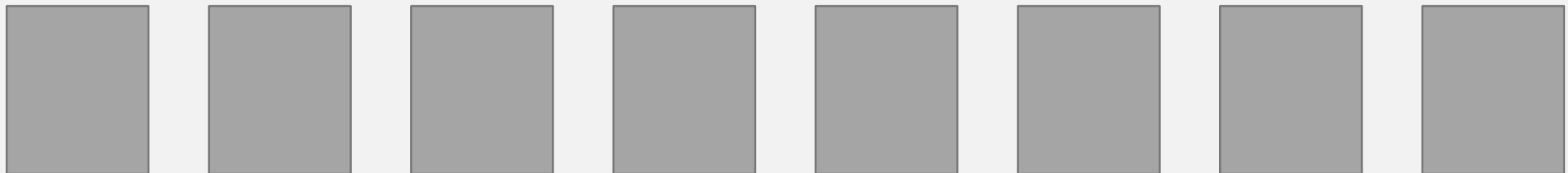
Binary Encoding (I)

- The process of representing a value as a sequence of 0's and 1's

00111111

a bit

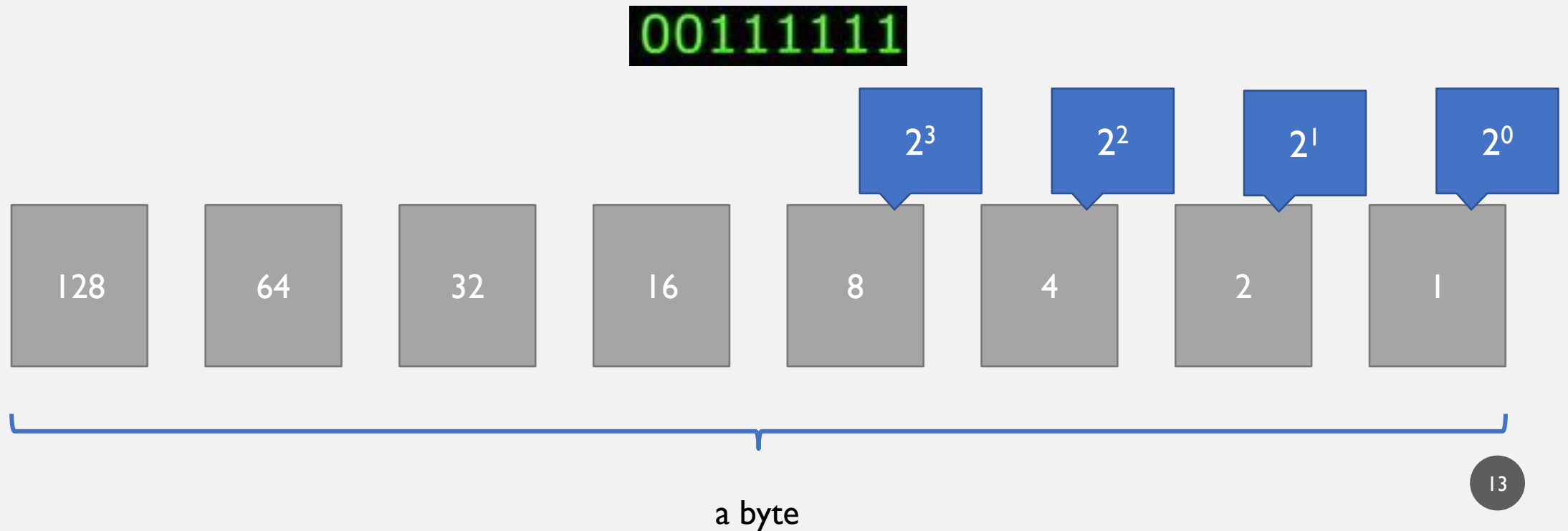
- A **bit** is one value in such sequence (0 or 1), a **byte** is 8 of these values, i.e., 8 bits



a byte

Binary Encoding (2)

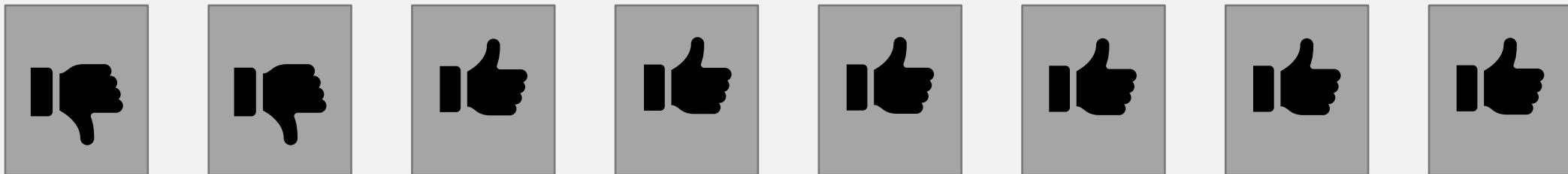
- In binary representation, each bit from right to left represents ascending **powers of 2**: 1, 2, 4, 8, 16, ...etc.



Binary Encoding (3)

- When there is a 1 in that position, it means that the value it represents is there

00111111



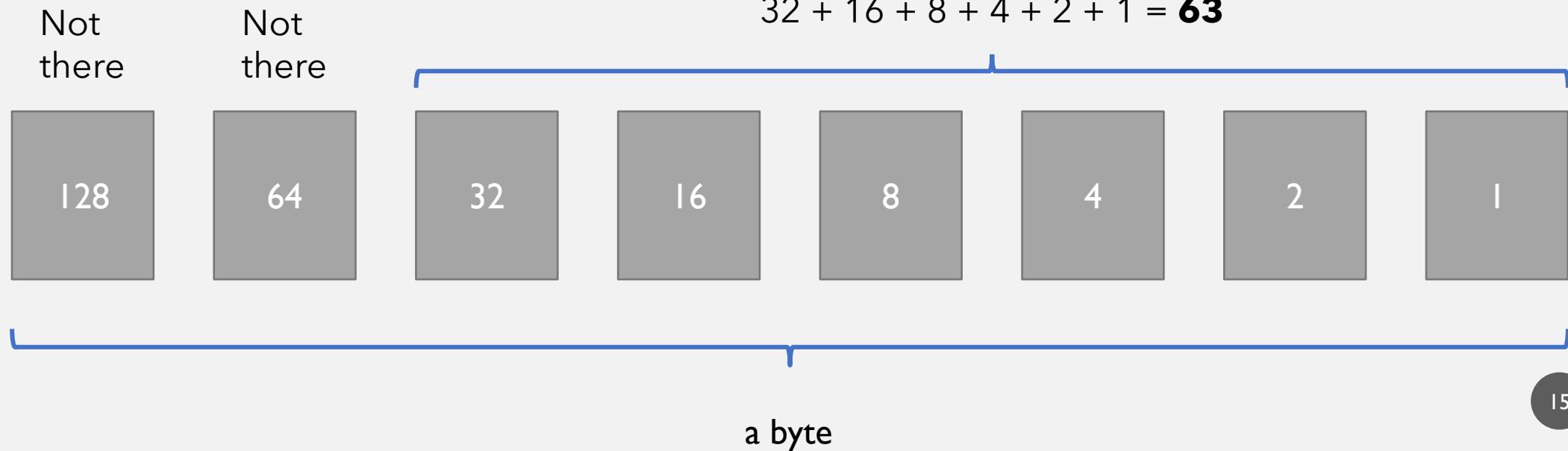
a byte

Binary Encoding (4)

- When there is a 1 in that position, it means that the value it represents is there, and adding them up gives the value the byte is storing

00111111

$$32 + 16 + 8 + 4 + 2 + 1 = \mathbf{63}$$



Most Significant Bit (MSB)

- In our 8-bit example, the largest value represented by a bit is 128
 - We call this bit the “most significant bit” (thus, the smallest value bit is called LSB)
 - This bit can be stored in the lowest memory address, followed by the second largest significant bit, and so on... (our drawings assume memory addresses increase from left to right)
 - Some computer architectures do this in reverse, i.e., the MSB is stored in the highest memory address
- Unless otherwise specified, we assume the MSB is stored in the **lowest memory address**, and we write it as the **leftmost bit** (as shown in our previous examples)

MSB & LSB & Size

- Different computers might use different number of bits to represent the same type, e.g., 32-bit vs 64-bit
- This means the same code using int might not work in some computers because the range is different
 - If the number of bits is not enough, only the LSBs will be stored
- To make sure the size is consistent, we can use `int32_t`, `uint32_t`, `int64_t`, `uint64_t`, ...etc.
 - Definitions found at [inttypes.h](#)

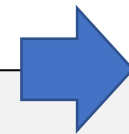
```
// unsigned int with 32 bits representation
uint32_t x = 4000000000;

printf("x= %u \n", x); // %u - unsigned int

// warning! number too large for 4 bytes = 32 bits
uint32_t y = 543210987654321;
// too large [00000000,00000001,11101110,00001100,00101001,11110101,00001100,10110001]

printf("y = %llu\n", y);
// the outputs will be 703925425 [00101001,11110101,00001100,10110001]
// only the least significant 4 bytes are kept

uint64_t z = 543210987654321; // ok
printf("z = %llu\n", z);
```



```
x= 4000000000
y = 703925425
z = 543210987654321
```

Practice Exercise

- Convert the following sequence into decimal numbers one byte at a time

001111110000001111100100110001101110010100111111

63

128

64

32

16

8

4

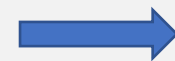
2

1

A Value Can Represent Many Things

- Given a value (e.g., 63 from the previous slide), it can be used to represent many things, including:
 - the number 63 (as an age, count of something, degree, ...etc. in a program)
 - the character/symbol '?' after looking up its position in the ASCII table (<https://en.wikipedia.org/wiki/ASCII>)
 - an identifier of a network port
 - part of another value (e.g., the G value of an RGB colour)
 - ...etc.

00111111



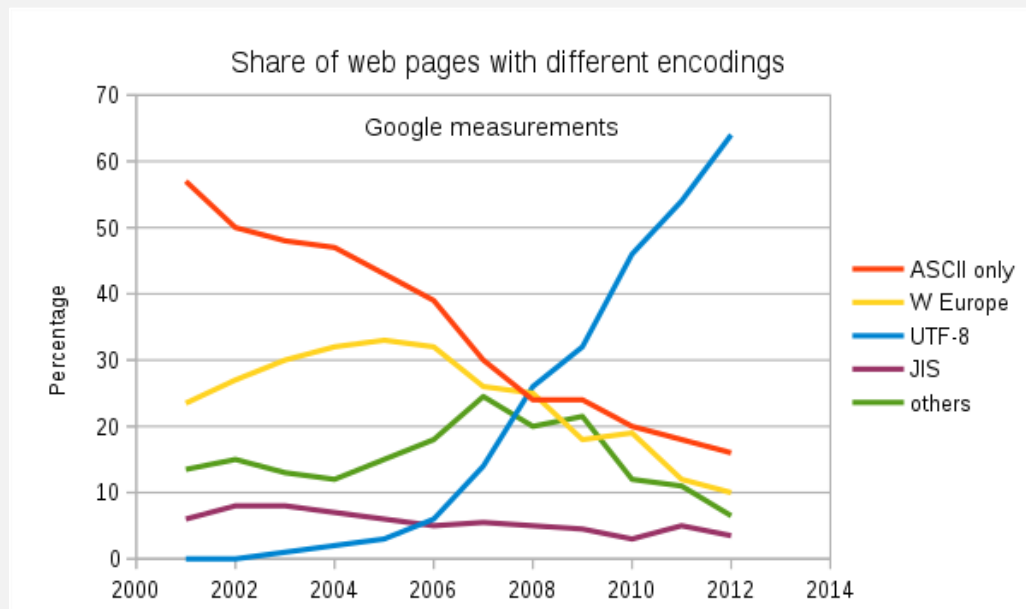
63



Dec	Char	Dec	Char	Dec	Char
32	SPACE	64	@	96	`
33	!	65	A	97	a
34	"	66	B	98	b
35	#	67	C	99	c
36	\$	68	D	100	d
37	%	69	E	101	e
38	&	70	F	102	f
39	'	71	G	103	g
40	(72	H	104	h
41)	73	I	105	i
42	*	74	J	106	j
43	+	75	K	107	k
44	,	76	L	108	l
45	-	77	M	109	m
46	.	78	N	110	n
47	/	79	O	111	o
48	0	80	P	112	p
49	1	81	Q	113	q
50	2	82	R	114	r
51	3	83	S	115	s
52	4	84	T	116	t
53	5	85	U	117	u
54	6	86	V	118	v
55	7	87	W	119	w
56	8	88	X	120	x
57	9	89	Y	121	y
58	:	90	Z	122	z
59	;	91	[123	{
60	<	92	\	124	
61	=	93]	125	}
62	>	94	^	126	~
63	?	95	_	127	DEL

About ASCII Code

- In the past ASCII was used in many places, but people soon find out it's not enough
 - We have many languages, characters, and thus encodings



Dec	Char	Dec	Char	Dec	Char
32	SPACE	64	@	96	`
33	!	65	A	97	a
34	"	66	B	98	b
35	#	67	C	99	c
36	\$	68	D	100	d
37	%	69	E	101	e
38	&	70	F	102	f
39	'	71	G	103	g
40	(72	H	104	h
41)	73	I	105	i
42	*	74	J	106	j
43	+	75	K	107	k
44	,	76	L	108	l
45	-	77	M	109	m
46	.	78	N	110	n
47	/	79	O	111	o
48	0	80	P	112	p
49	1	81	Q	113	q
50	2	82	R	114	r
51	3	83	S	115	s
52	4	84	T	116	t
53	5	85	U	117	u
54	6	86	V	118	v
55	7	87	W	119	w
56	8	88	X	120	x
57	9	89	Y	121	y
58	:	90	Z	122	z
59	;	91	[123	{
60	<	92	\	124	
61	=	93]	125	}
62	>	94	^	126	~
63	?	95	_	127	DEL

Unicode UTF-8

- The maximum number of unique values (i.e., codes) in a byte is 255 (11111111), and ASCII actually uses only 7 bits, so there are only 128 possible values (from 0000000 to 1111111)
- To represent languages other than English, we may need a lot more than 255 types of characters
- **Unicode** uses up to **4 bytes** to represent more characters

? 001111110000001111100100110001101110010100111111111101
111011111100001110101111101100011110111111010101100110
1101111100001110001101010010111010011111000110001000110
? 11000111011110111110111111101000111110111011111110
1111100110111011110001110111010111010001111110111110
1100110111110110001001000000101110111011101111111111
11111000111110001111111111110011110011111100101111111
0111111011001110111110111111010111110111111101100111010
11111111000111111001010010100011111011110110100111101
000111111111011000101000111100100000011100100100011
1011111001111111010101111100010111011100001001111110
0111000011101110111110011111100111111100110000010111
1011101101000010011001100011101110000110010000001100111
110110011000101011101101000111110100011010111110010111
111111111001100011110011110101000010100111000100100010
1110100001011100111111000001111011111101111011111100
1110010011010111111111001111111111111111111001010111111
0000000001111111100001100111101100100011011011010110001
010001001111111001111111111110011111000111110011110101
0100011110010011111110000101111011100110111110110011111

Convert/Translate between Two Systems

- By representing a number using another set of values, we are converting it between different numerical systems
 - what we did was decoding from the binary system (00111111) to the decimal system (63)
 - we can also encode from the decimal system (63) to the binary system (00111111)
- How do we encode a number from the decimal system to the binary system?
 - Let's use 29 as an example
 - do a number of divisions, keep track of the remainders
 - when no more divisions can be done, read the remainders bottom-up: 11101
 - in other words, set the 2^i -th digit to 0 or 1 with increasing i , based on even/odd

$$\begin{array}{r} \underline{29} \\ \underline{14} - 1 \\ \underline{7} - 0 \\ \underline{3} - 1 \\ 1 - 1 \end{array}$$

Pseudo-Code

- A way to systematically describe the sequence of steps to solve a problem (usually computational)
- Similar to code, **but typically uncompileable** due to **lack of syntax rules & variable declarations**
 - high-level description of the steps (aka algorithm)
 - contains essential details needed to implement the steps
 - language independent (uses elements common to most languages, e.g., loops, if-else, assignment, comparison)
 - no syntax rules, but is consistent and readable by humans
 - no language specific elements, like type, memory allocation, ...etc.
- Pseudo-code makes description of algorithms shorter and easier to understand (we'll use it sometimes)

Describing Steps Systematically

- How do we describe the sequence of steps for binary encoding with pseudo-code? Consider this:

Step 1: $i = 0$

$i = 0$: $N = 29$ is odd

set $1 \cdot 2^0 \rightarrow ****1$

set $N = (29-1)/2 = 14$

Step 2: while ($N > 0$)

Step 2.1: if N is even

$i = 1$: $N = 14$ is even

set $0 \cdot 2^1 \rightarrow ***01$

set $N = 14/2 = 7$

Step 2.1.1: set the 2^i -th digit to 0

Step 2.1.2: set N to $N/2$

$i = 2$: $N = 7$ is odd

set $1 \cdot 2^2 \rightarrow **101$

set $N = (7-1)/2 = 3$

Step 2.2: else

Step 2.2.1: set the 2^i -th digit to 1

$i = 3$: $N = 3$ is odd

set $1 \cdot 2^3 \rightarrow *1101$

set $N = (3-1)/2 = 1$

Step 2.2.2: set N to $(N-1)/2$

Step 2.3: $i++$

$i = 4$: $N = 1$ is odd

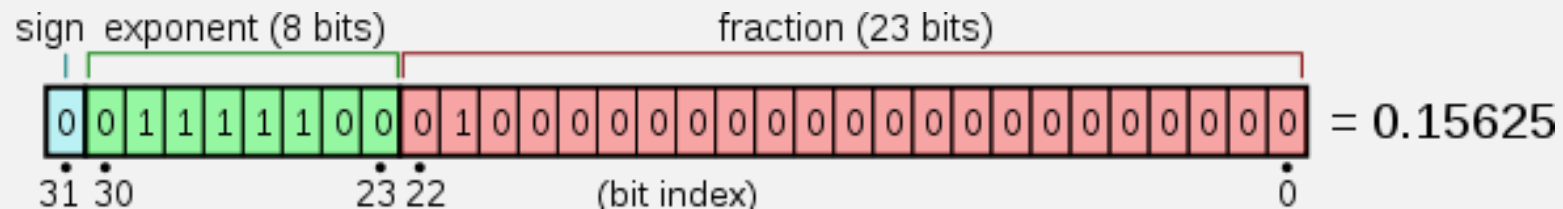
set $1 \cdot 2^4 \rightarrow 11101$

set $N = (1-1)/2 = 0$

Can you
convert the
pseudo-code
to C code?

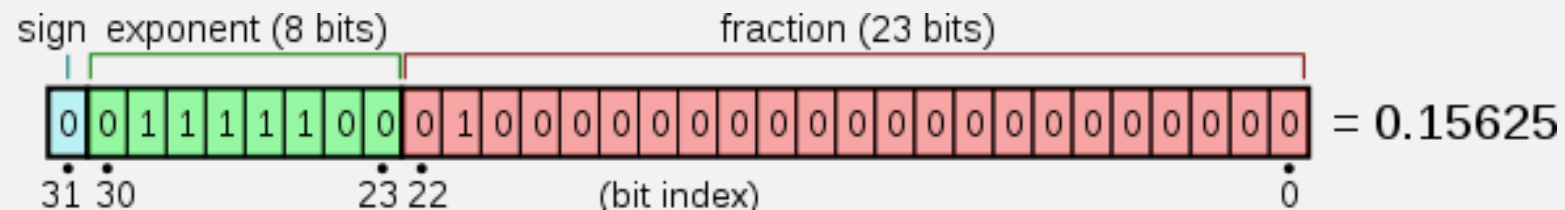
What about Non-Integers?

- With our previous encoding technique, we can only represent integers, how about 0.5? or 1/3?
- We use a total of 32 bits differently to represent numbers with decimal places (in scientific notation)



- 1 bit for sign (0 means positive, 1 means negative)
- 23 bits for the significand (significant digits of the number)
 - $1.b_{22}b_{21}...b_0$ (b_{22} represents the digit of $1/2$, b_{21} represents the digit of $1/4$...)
- 8 bits for the exponent – ranges from -127 to 128

Scientific Notation of Numbers with Decimal Places



$$value = (-1)^{b_{31}} \times (1 + fraction) \times 2^{exponent-127}$$

- $value = (-1)^0 \times (1 + 1/4) \times 2^{124-127} = 1.25 \times 2^{-3} = 0.15625$
- Range of the representation: $1.b_{22}b_{21}...b_0 \times 2^{exponent-127}$
 - smallest number happens when all fraction & exponent bits are 0: $1 \times 2^{-127} = 5.9 \times 10^{-39}$
 - largest number happens when all fraction & exponent bits are 1: $(2-2^{-23}) \times 2^{128} = 6.8 \times 10^{38}$

Floating Point Encoding

- Example: -3.625
- Step 1: split the number at the decimal point, take note of the sign
- Step 2: apply the binary encoding for integers on the integral part ($3 \rightarrow 11$)
- Step 3: apply another version of binary encoding for fraction part ($0.625 \rightarrow 0.101$):
 - $0.625 * 2 = 1.25 \rightarrow 1$
 - $0.25 * 2 = 0.5 \rightarrow 0$
 - $0.5 * 2 = 1$ (stop when there is no more fraction)
 - Get the binary representation of the decimal part by reading from top to bottom: 0.101 (it means $0.625 = 1/2 + 1/8$)
- Step 4: combine the integral part and fraction part (11.101) and normalize it: 1.1101×2^1
- Step 5: apply the binary encoding for the exponent part (exponent - 127 = 1 \rightarrow exponent = 128)

-3.625 \rightarrow

1	1	0	0	0	0	0	0	0	0	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

Another Example of Floating Point Encoding

- Value: 4.1
- Step 1: split the number at the decimal point, take note of the sign
- Step 2: apply the binary encoding for integers on the integral part ($4 \rightarrow 100$)
- Step 3: apply another version of binary encoding for fraction part ($0.1 \rightarrow 0.0$ 0011 0011 0011 0011 ...)
 - Sometimes you get recurring fractions!
- Step 4: combine the integral part and fraction part (100.0 0011 0011 0011 0011 ...) and normalize it:
 1.00 0 0011 0011 0011 0011 ... $\times 2^2$
- Step 5: apply the binary encoding for the exponent part ($\text{exponent} - 127 = 2 \rightarrow \text{exponent} = 129$)
- What is the answer?

Today's Review

- Binary encoding
 - Using only 1's & 0's to represent values
 - conversion between decimal & binary representations (there are others)
- Pseudo-code & algorithms
 - High-level description of an algorithm, language independent, show important steps in a consistent way
- Floating-point encoding (32-bit)
 - Using only 1's & 0's to represent fractional values
 - sign (1-bit), exponent (8-bit), fraction (23-bit)

Homework!

- For practice, perform binary encoding for 1.625, 42, 42.6875, -9.9
- Investigate how double is different from float in C
- Download code files W04-01_Example01.c, W04-01_Example02.c, W04-01_Example03.c from Canvas and run them, take a look at the output and explain them
- Read this for steps & proofs for fraction conversions:
<https://indepth.dev/posts/1019/the-simple-math-behind-decimal-binary-conversion-algorithms>