Erasmus School of Economics

1 IRR: Internal Rate of Return

The Internal rate of return or IRR is a metric which is often used to (statically) assess the profitability of an investment. The IRR is in fact a discount rate, more precise: it is exactly that discount rate which makes the Net Present Value (NPV) of all cash flows from an investment or project equal to zero. Therefore the calculation of the IRR is strongly related to the formula of the NPV. For the formula of the NPV, we know that:

$$\text{NPV} = \sum_{t=1}^{T} \frac{CashFlow_t}{(1 + DiscountFactor)^t} - Invesment, \tag{1.1}$$

that is, the NPV is equal to the sum of all discounted cash flows at times t over a total of T periods, minus the investment amount. Then, to obtain the IRR, we have to solve for IRR in the following equation:

$$0 = \sum_{t=1}^{T} \frac{CashFlow_t}{1 + IRR^t} - Invesment, \tag{1.2}$$

Unfortunately there is often no analytical closed-form solution available to Equation (1). Hence the IRR has to be obtained by trial-and-error or by using programmed functions. In Excel for instance, the function *IRR* can be used to the calculate the IRR. When programming yourself, one of the algorithms to find roots of real-valued functions can be followed, such as Newton's method.

The IRR is also used as proxy for the growth rate that a project is expected to display. In practice an investment is considered profitable when its IRR is higher than its cost of capital. Firms often compare the IRR with the Required Rate of Return (RRR, the minimum return that is accepted as compensation for a certain level of risk). Here the IRR should be higher than the RRR for a project to be considered profitable. A bigger difference between the IRR and RRR of a project is often considered as an indication of a higher likelihood of the project being profitable.

2 Examples

Consider a company with an investment opportunity which requires an immediate investment outlay of \$1m. After 5 years it will receive an one-time cash flow of \$2.4m. What is the IRR of this investment opportunity? Since this is a case with no intermediate varying cash flows, it can be solved analytically:

$$\begin{aligned} 0 =& \frac{CashFlow_5}{(1+IRR)^5} - I = \frac{\$2.4m}{(1+IRR)^5} - \$1m \\ \iff IRR =& \sqrt[5]{2.4m/1m} - 1 \iff IRR \approx 19.136\% \end{aligned}$$

Hence the IRR is approximately 19%, filling this in as discount factor in a NPV calculation, shows:

$$\frac{\$2.4m}{(1+19.136\%)^5} - \$1m \approx 0$$

Now consider a company with an investment opportunity which requires an immediate investment outlay of \$1m at $t = 0$. This project will pay \$100,000 every year, starting from year $t = 1$. What is the IRR of this project? We find the IRR by solving the following equation:

$$\begin{aligned} 0 =& \sum_{t=1}^{\infty} \frac{CashFlow_t}{(1+IRR)^t} - I = \sum_{t=1}^{\infty} \frac{\$100,000}{(1+IRR)^t} - \$1m \\ =& \frac{\$100,000}{IRR} - \$1m = 0 \\ \iff IRR =& \frac{\$100,000}{\$1m} = 10\% \end{aligned}$$

Where we have used the formula for the sum of an infinite geometric series in the second line.

2.1 IRR and Private Equity

The IRR can also be used to determine what maximally can be invested, for instance by a private equity firm.

Consider a private equity firm that wants to acquire a target company at $t = 0$. It will use \$450 million of debt to fund the debt proportion related to the acquisition at the entry (acquisition) date. The private equity investor plans to exit after 5 years (at $t = 5$) and expects to realize an exit EBITA-multiple of $10\times$. Because all intermediate cash flows the company will generate are used to repay the debt, the expected net debt at $t = 5$ will be \$250 million. The EBITA of the company at $t = 5$ equals to \$103 million. What is the maximum enterprise value and equity investment the private equity firm is willing to invest at the entry date when the private equity firm requires an IRR of 25%?

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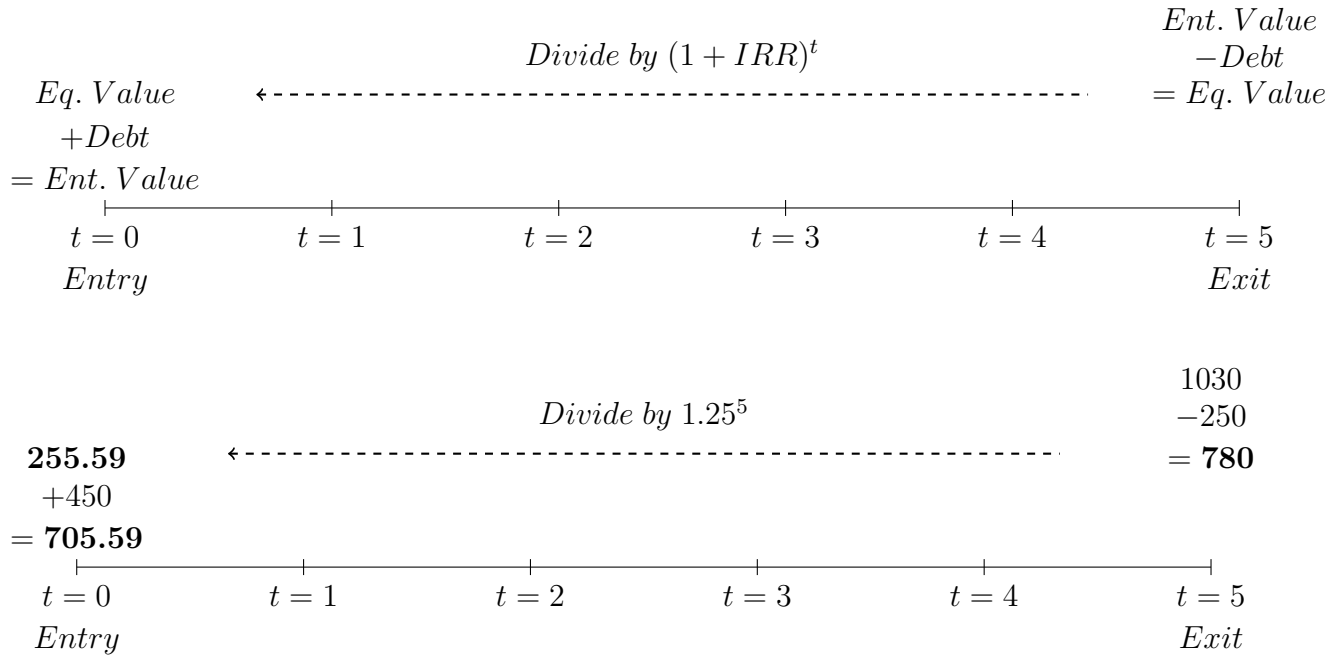
\begin{aligned}
Entry: t &= 0 \\
Exit: t &= 5 \\
Enterprise\,Value_{exit} &= multiple \times EBITA_{exit} \\
&= 10 \times 103 = \mathbf{1030} \\
Debt_{exit} &= 250 \\
Equity\,Value_{exit} &= Enterprise\,Value_{exit} - Debt_{exit} \\
Equity\,Value_{exit} &= 1030 - 250 = \mathbf{780} \\
Equity\,Value_{entry} &= \left.\frac{Equity\,Value_{exit}}{(1+IRR)^t}\right|_{exit:\,t=5} = \frac{780}{(1.25)^5} = \mathbf{255.59} \\
Debt\,Value_{entry} &= 450 \\
Enterprise\,Value_{entry} &= 255.59 + 450 = \mathbf{705.59}
\end{aligned}
$$

Hence the maximum equity value that the private equity firm is willing to invest is equal to \$255.59 million. Schematically, it looks as follows:



3 Factors determining IRR

Several factors determine the height of the IRR, including the size of the cash inflows and outflows, length of the investment period, the timing of the cash flows, and how the project is financed. To consider these factors, as an exercise you can take the above example(s) and use different lengths or cash flows, and then try to calculate the IRR.

A particularly interesting case is to see how a higher leverage ratio (debt to equity ratio) can influence the IRR. Consider again the private equity firm and its investment project. What if the private equity firm is able to finance the project at entry with \$600 million debt. All else equal, what would the IRR be of this project (on equity)? The equity investment at entry would then be $705.59 - 600 = 105.59$. Hence the IRR equals $(780/105.59)^{(1/5)} = 49\%$.

Consider two projects. Project 1 requires an immediate investment outlay of \$1m. After five years it will generate an one time cash inflow of \$2.4m. This project is similar as the one in Section 2, so we know that the IRR is approximately equal to 19%. Now consider Project 2, which requires the same immediate outlay, but generates the cash inflow one year earlier (in four years). The IRR of this second project is approximately equal to 24%. This shows that the IRR is positively influenced when cash inflows can be moved forward in time (obtain cash inflow earlier). Similarly, delaying cash outflows also increases the IRR.

The IRR is not the same as the NPV of a project. Basing your decisions on one of the two metrics can actually lead to different conclusions. This is because the NPV is a monetary value, while the IRR is a percentage return. Consider a company that needs to choose between two projects. Let us assume that the cost of capital of this company is equal to 12%. Project 1 we know: It requires an immediate investment outlay of \$1m. After five years it will generate a one time cash inflow of \$2.4m. The IRR is approximately equal to 19%. The NPV of Project 1 is $(2.4/1.12^5) - 1$, and thus equals \$0.36m. Project 3 requires an immediate investment outlay of \$5m and generates a one time cash inflow of \$10m in five years. The IRR of Project 3 is then approximately equal to 15% and the NPV is equal to \$0.67m. So based on the NPV we would prefer Project 3, but on based on the IRR we would prefer Project 1.