
Which battery model to use?

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Abstract

The use of mobile devices like cell phones, navigation systems, or laptop computers, is limited by the lifetime of the included batteries. This lifetime depends naturally on the rate at which energy is consumed, however, it also depends on the usage pattern of the battery. Continuous drawing of a high current results in an excessive drop of residual capacity. However, during intervals with no or very small currents, batteries do recover to a certain extend. The usage pattern of a device can be well modeled with stochastic workload models. However, one still needs a battery model to describe the effects of the power consumption on the state of the battery. Over the years many different types of battery models have been developed for different application areas. The best type of model to use in the setting of performance modelling are analytical models. In this paper we analyse two well-known analytical models, and show that one is actually an approximation of the other; this was not known previously. Furthermore, we tested the suitability of these models for performance evaluation purpose.

1 Introduction

With the proliferation of cheap wireless access technologies, such as wireless LAN, Bluetooth as well as GSM, the number of wireless devices an average citizen is using has been steadily increasing over the last decade. Such devices do not only add to the flexibility with which we can do our work, but also add to our reachability and our security. Next to these personal wireless devices, an ever growing number of wireless devices is used for surveillance purposes, most notably in sensor-type networks. A common issue to be dealt with in the design of all of these devices is power consumption. Since all of these devices use batteries of some sort, mostly rechargeable, achieving low power consumption for wireless devices has become a key design issue. This fact is witnessed by many recent publications on this topic, and even a special issue of *IEEE Computer* devoted to it [1].

Low-power design is a very broad area in itself, with so-called “battery-driven system design” a special branch of it, that becomes, due to the reasons mentioned, more and more important. A key issue to be addressed is to find the right tradeoff between battery usage and required performance: how can we design a (wireless) system such that with a given battery, good performance

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(throughput, reachability, and so on) is obtained, for a long-enough period. Stated differently, how should the processes in the wireless device be organised such that the battery lifetime (which determines the system lifetime) will be as high as possible. Indeed, it has been observed recently that due to the specific physical nature of batteries, achieving the longest battery lifetime is not always achieved by “just” trying to minimise the power consumption at any point in time. Instead, also the way in which the power is consumed, that is, the current-extraction patterns and the employed current levels play a role in the battery lifetime.

Using an abstract workload model one can model the operation of a system, describing the various states the wireless device can be in, together with the energy consumption rates in those states. Also, the transition possibilities between these states can be represented in the workload model. Such a description can be interpreted as a Markov-reward model in which accumulated reward stands for the amount of energy consumed. The system or battery lifetime then equals the time until a certain level of consumption (the available charge of the battery) is reached. Determining this time, or better, its distribution, could be done with well-known techniques for performability evaluation. However, such an approach does not well take into account the physical aspect of battery operation. Indeed, studies on batteries reveal that the battery depletion rate in general is non-linear in time, and, moreover, also depends on the amount of energy still in the battery. [2, 3] Furthermore, in periods when a battery is not used, subtle but important battery-restoration effects are in place, that apparently refill the battery.

To capture the influence of the power consumption on the battery, a battery model is needed. Over the years, many different types of battery models have been developed for different application areas [3]. For example, the electro-chemical models described in [4, 5, 6] are used in battery design. These models describe the battery in its very detail using a set of six coupled differential equations. Another example are the electrical circuit models used in electrical engineering [7], which focus on the electrical properties of the battery. Although these models describe the battery accurately, they are not suitable to be used in the setting of the performance models because of the detailed description, which would make the combined model unmanageable. What is needed, is an abstract model which focuses on the important battery properties and their effects only. Two analytical models are good candidates: the Kinetic Battery Model (KiBaM) by Manwell and McGowan [8, 9, 10] and the diffusion based model by Rakhmatov and Vrudhula [11]. These two models describe the battery using only two differential equations. Although the equations the two models start from are really different, we show by applying a coordinate transformation on the KiBaM, that the KiBaM is actually a first order approximation of the diffusion model. A further theoretical and practical comparison of the two models is made, which leads to the conclusion which model is best to use in the setting of the performance models.

The rest of the paper is structured as follows. Section 2 gives a short introduction to battery properties that have to be addressed. In Section 3 the two analytical battery models are introduced and Section 4 gives a theoretical and practical comparison of these models, which leads to the conclusion which model is best to use. In Section 5 the limitations of the analytical model are analysed. We end with conclusions and an outlook to future work in Section 6.

2 Batteries

The two most important properties of a battery are its voltage (expressed in volts V) and its capacity (mostly expressed in Ampere-hour, Ah); the product of these two quantities is a measure for the energy stored in the battery. For an ideal battery the voltage stays constant over time until the moment it is completely discharged, then the voltage drops to zero. The capacity in the ideal case is the same for every load for the battery. Reality is different, though: the voltage drops during discharge and the effectively perceived capacity is lower under a higher load. This phenomenon is termed the *rate capacity effect*.

In the ideal case it is easy to calculate the lifetime of a battery. The lifetime (L) in the case of a constant load is the capacity (C) over the load current (I):

$$L = C/I. \quad (1)$$

Due to various nonlinear effects this relation does not hold for real batteries. A simple approximation for the lifetime under constant load can be made with Peukert's law [11]:

$$L = \frac{a}{I^b}, \quad (2)$$

where $a > 0$ and $b > 1$ are constants which depend on the battery. For variable loads ($i(t)$) one can extend this formula by using the average current up until $t = L$:

$$L = \frac{a}{\left(\frac{1}{L} \int_0^L i(t) dt\right)^b} \quad (3)$$

Following (3), all load profiles with the same average would have the same lifetime. Experimentally it can be shown that this is not the case. One of the effects playing an important role here is the *recovery effect* of the battery. This, is the effect that the battery can regain some of its "lost" capacity during idle periods.

3 Battery models

In this section two analytical battery models are discussed. In both models the non-linear effects of the battery are described using two differential equations. In Section 3.1 we present the diffusion model of Rakhmatov and Vrudhula. In Section 3.2 we present the Kinetic Battery Model of Manwell and McGowan. Then, in Section 3.3, we apply a coordinate transformation on the Kinetic Battery Model, which leads the insight that this model is actually an approximation of the diffusion model.

3.1 Rakhmatov and Vrudhula's diffusion model

An analytical battery model based on the diffusion of the ions in the electrolyte has been developed by Rakhmatov and Vrudhula in 2001 [11, 12, 13]. The model describes the evolution of the concentration of the electro-active species in the electrolyte to predict the battery lifetime under a given load. In the

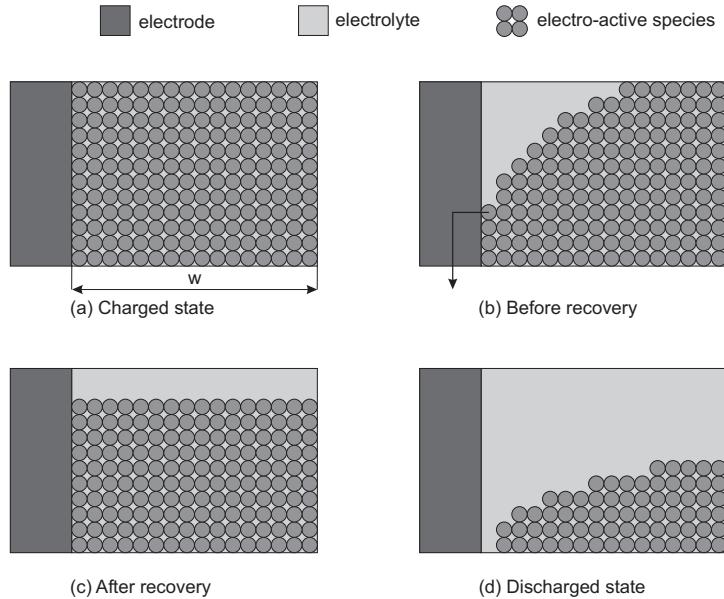


Figure 1: Physical picture of the model by Rakhmatov and Vrudhula

model the processes at both electrodes are assumed identical, thus the battery is assumed symmetric with respect to the electrodes and only one of the electrodes is considered.

Figure 1 shows a simplified view of the battery operation according to the diffusion model. At first, for the full battery, the concentration of the electroactive species is constant over the full width (w) of the electrolyte (Figure 1(a)). When a load is applied to the battery, the electrochemical reaction results in a reduction of the concentration of the species near the electrode. Thus, a gradient is created across the electrolyte (Figure 1(b)). This gradient causes the species to diffuse towards the electrode. Now, when the load is switched off, the concentration of the species at the electrode will increase again (recover) due to the diffusion, and eventually the species will be evenly distributed over the electrolyte again. The concentration, however, will be lower than for the full battery (Figure 1(c)). Finally, when the concentration at the electrode drops below a certain value (C_{cutoff}), the chemical reaction can no longer be maintained and the battery is considered to be empty (Figure 1(d)).

The concentration of the electro-active species at time t and distance $x \in [0, w]$ is denoted by $C(x, t)$. For the full battery the concentration is constant over the length of the electrolyte: $C(x, 0) = C^*$, $x \in [0, w]$. The battery is considered empty when $C(0, t)$ drops below the cutoff level C_{cutoff} . The evolution of the concentration is described by Fick's laws [11]:

$$\begin{cases} -J(x,t) &= D \frac{\partial C(x,t)}{\partial x}, \\ \frac{\partial C(x,t)}{\partial t} &= D \frac{\partial^2 C(x,t)}{\partial x^2}, \end{cases} \quad (4)$$

where $J(x, t)$ is the flux of the electro-active species at time t and distance x from the electrode, and D is the diffusion constant. The flux at the electrode surface ($x = 0$) is proportional to the current ($i(t)$). The flux on the other side of the diffusion region ($x = w$) equals zero. This leads to the following boundary conditions:

$$\begin{cases} D \frac{\partial C(x, t)}{\partial x} \Big|_{x=0} = \frac{i(t)}{\nu F A}, \\ D \frac{\partial C(x, t)}{\partial x} \Big|_{x=w} = 0, \end{cases} \quad (5)$$

where A is the area of the electrode surface, F is Faraday's constant, and ν is the number of electrons involved in the electrochemical reaction at the electrode surface.

It is possible to obtain an analytical solution for this set of partial differential equations (4) together with the initial condition and the boundary conditions 5 using Laplace transforms. From that solution one can obtain an expression for the apparent charge lost from the battery ($\sigma(t)$) [14]:

$$\sigma(t) = \underbrace{\int_0^t i(\tau) d\tau}_{l(t)} + \underbrace{\int_0^t i(\tau) \left(2 \sum_{m=1}^{\infty} e^{-\beta^2 m^2 (t-\tau)} \right) d\tau}_{u(t)}, \quad (6)$$

where $\beta = \pi\sqrt{D}/w$. The apparent charge lost is separated in two parts, the charge lost to the load ($l(t)$) and the unavailable charge ($u(t)$). The first is the charge used by the device. The second is charge which remains in the battery unused. The battery is empty when the apparent charge lost is equal to the battery's capacity.

For a constant current I , (6) can easily be solved. For $l(t)$ one obtains: $l(t) = It$. For the unavailable charge one can interchange the integral and the summation, which leads to:

$$u(t) = 2I \sum_{m=1}^{\infty} \frac{1 - e^{-\beta^2 m^2 t}}{\beta^2 m^2}. \quad (7)$$

During idle periods the unavailable charge will decrease and will be available again for the load. One can compute the function that describes the evolution of the unavailable charge during an idle period after a load I that lasted for a period of length t_l :

$$u(t_i) = 2I \sum_{m=1}^{\infty} \frac{e^{-\beta^2 m^2 t_i} (1 - e^{-\beta^2 m^2 t_l})}{\beta^2 m^2}, \quad (8)$$

where t_i is the idle time.

3.2 Kinetic Battery Model

Another analytical model which can be used for computing battery lifetimes is the Kinetic Battery Model (KiBaM) of Manwell and McGowan [8, 9, 10]. The

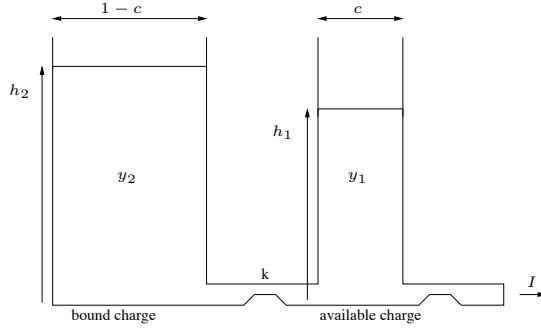


Figure 2: Two-well-model of the Kinetic Battery Model

KiBaM is a very intuitive battery model. In the model the battery charge is distributed over two wells: the available-charge well and the bound-charge well (cf. Figure 2). A fraction c of the total capacity is put in the available charge well, and a fraction $1 - c$ in the bound charge well. The available charge well supplies electrons directly to the load ($i(t)$), whereas the bound-charge well supplies electrons only to the available-charge well. The rate at which charge flows between the wells depends on the height difference between the two wells, and on a parameter k . The heights of the two wells are given by: $h_1 = \frac{y_1}{c}$ and $h_2 = \frac{y_2}{1-c}$. The change of the charge in both wells is given by the following system of differential equations:

$$\begin{cases} \frac{dy_1}{dt} = -i(t) + k(h_2 - h_1), \\ \frac{dy_2}{dt} = -k(h_2 - h_1), \end{cases} \quad (9)$$

with initial conditions $y_1(0) = c \cdot C$ and $y_2(0) = (1 - c) \cdot C$, where C is the total battery capacity. The battery is considered empty when there is no charge left in the available charge well.

When a load is applied to the battery, the available charge reduces, and the height difference between the two wells grows. When the load is removed, charge flows from the bound-charge well to the available-charge well until h_1 and h_2 are equal again. So, during an idle period, more charge becomes available and the battery lasts longer than when the load is applied continuously. In this way the recovery effect is taken into account in the model. Also, the rate capacity effect is covered, since for a higher discharge current the available charge well will be drained faster, less time will be available for the bound charge to flow to the available charge. Therefore, more charge will remain unused, and the effective capacity is lower.

The differential equations (9) can be solved for the case of a constant discharge current ($i(t) = I$) using Laplace transforms, which yields:

$$\begin{cases} y_1 = y_{1,0}e^{-k't} + \frac{(y_{1,0}k'c-I)(1-e^{-k't})}{k'} - \frac{Ic(k't-1+e^{-k't})}{k'}, \\ y_2 = y_{2,0}e^{-k't} + y_0(1-c)(1-e^{-k't}) - \frac{I(1-c)(k't-1+e^{-k't})}{k'}, \end{cases} \quad (10)$$

where k' is defined as $k' = k/c(1 - c)$, $y_{1,0}$ and $y_{2,0}$ are the amount of available and bound charge, respectively, at $t = 0$, and $y_0 = y_{1,0} + y_{2,0}$.

3.3 Coordinate transformation

Although the differential equations (9) nicely describe the discharge process of the battery, and an analytical solution can be obtained for constant discharge currents, the equations can be made more simple when a coordinate transform is applied. In this way even more insight can be obtained in the way the model behaves.

From (9) one can see that the height difference between the two wells ($h_2 - h_1$) plays a major role in the model. This is one of the coordinates after the transformation, the other is the total charge in the battery. So, the transformation changes the coordinates from y_1 and y_2 to $\delta_h = h_2 - h_1$ and $\gamma = y_1 + y_2$. This transformation changes the differential equations to:

$$\begin{cases} \frac{d\delta_h}{dt} = \frac{i(t)}{c} - k'\delta_h, \\ \frac{d\gamma}{dt} = -i(t), \end{cases} \quad (11)$$

with initial conditions $\delta_h(0) = 0$ and $\gamma(0) = C$. In the new coordinate system the condition for the battery to be empty is: $\gamma(t) = (1 - c)\delta_h(t)$. The differential equations are independent and are straightforwardly solved for constant discharge currents:

$$\begin{cases} \delta_h(t) = \frac{I}{c} \cdot \frac{1 - e^{-k't}}{k'}, \\ \gamma(t) = C - It. \end{cases} \quad (12)$$

Like for the diffusion model, one can also compute the evolution of δ_h as a function of the idle time t_i after a load I that lasted for a period t_l :

$$\delta_h(t_i) = \frac{I}{c} \cdot \frac{e^{-k't_r}(1 - e^{-k't_l})}{k'}. \quad (13)$$

The solutions for continuous discharge can be used to obtain a solution for any discharge profile with piecewise constant currents by adapting the initial conditions appropriately. The level of γ and δ_h at the end of a step in the load profile, can be used as initial conditions for the next step.

4 Comparing the analytical models

The interpretation of the diffusion model with its unavailable charge is very similar to the KiBaM with its bound charge. However, it is actually the height difference times $1 - c$ in the KiBaM that plays the same role as the unavailable charge in the diffusion model.

4.1 Continuous discharge

It is possible to write the solution of the transformed KiBaM in the form of the diffusion model with the charge lost split in a load and an unavailable charge part. For constant current discharge this yields:

$$l(t) = C - \gamma(t) = It \quad (14)$$

$$u(t) = (1 - c) \cdot \delta_h(t) = \frac{(1 - c)I}{c} \frac{1 - e^{-k't}}{k'} \quad (15)$$

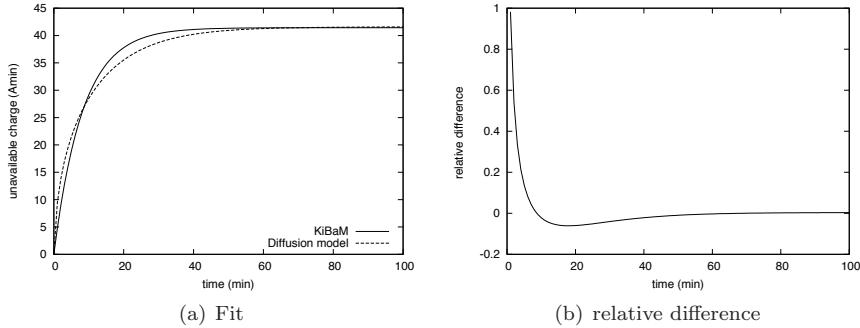


Figure 3: Fit of the KiBaM to the diffusion model. The evolution of the unavailable charge in both the diffusion model and the fitted KiBaM is given in (a). In (b) the relative difference between the two curves is given.

When one compares (15) with (7), one sees that the former has the same form as the first term of the sum of the latter. Setting $c = \frac{1}{3}$ and $k' = \beta^2$ in the KiBaM, results in the first order approximation of the diffusion model. This is of course, a bad approximation of the infinite sum.

One can obtain a much better approximation, when the parameters c and k' are used to fit the KiBaM equation of $u(t)$ to the equation of the diffusion model. Figure 3(a) shows the result of a least squares fitting procedure for the case that $I = 0$. When $\beta = 0.273 \text{ min}^{-\frac{1}{2}}$, the fit results in $c = 0.166$ and $k' = 0.122 \text{ min}^{-1}$. In Figure 3(b) the relative difference between the two curves is shown. This difference is independent of the discharge current. The relative difference is very large, up to 80%, for times smaller than 10 minutes. This implies that the results for battery lifetime computations will differ mainly for high discharge currents.

4.2 Frequency response

Following the method described in [14] an analysis of the frequency response of both the Kinetic Battery Model and Rakhmatov and Vrudhula's diffusion model was done. The results are given in Figure 4. The figure shows that the diffusion model has a higher frequency response for high frequencies. This is due to the high order terms that are included in the diffusion model and not in the KiBaM. However, both models are highly insensitive to high frequency current switching. Currents varying faster than 0.01 Hz can be replaced with an average current without giving significant errors in the battery lifetime computations. The level of the frequency response is mainly determined by the size of the recovery parameter

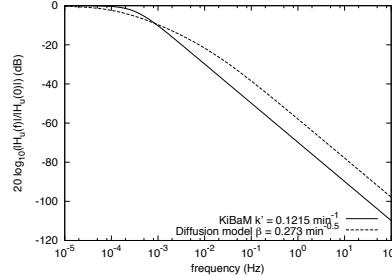


Figure 4: Frequency response for KiBaM and diffusion model

Test	Name	I_{ave} (mA)	DUALFOIL (min)	Diffusion (min)	KiBaM (min)
T1	MPEG	222.7	140.9	139.9	139.9
T2	Dictation	204.5	156.0	156.0	156.0
T3	Talk1	108.3	317.2	331.4	331.4
T4	Talk2	107.5	319.5	334.1	334.1
T5	Talk3	94.9	365.1	384.0	384.0
T6	WAV1	84.3	413.7	437.5	437.5
T7	WAV2	75.5	464.8	493.3	493.3
T8	Idle1	28.0	1278	1400	1401
T9	Idle2	19.5	1852	2029	2029
T10	SleepDC	3.0	12285	13417	13417
T11	IAT	628.0	26	26.6	24.9
T12	IAR	494.7	41.3	41.4	40.5
T13	IST	425.6	54.6	53.9	53.5
T14	ISR	292.3	99.5	96.7	96.7
T15	IAD	265.6	113.1	110.6	110.6
T16	MSD	252.3	120.8	118.6	118.6
T17	DSD	234.1	132.7	131.0	131.0
T18	TSD	137.9	243.6	251.3	251.3
T19	WSD	113.9	300.1	313.0	313.0
T20	ISD	57.6	616.3	659.5	659.5
T21	SSD	32.5	1101	1201	1201
T22	Boot	300.0	96.0	93.2	93.1

Table 1: Lifetimes of continuous current discharge computed with DUALFOIL, the diffusion model (both from [13]) and KiBaM (computed by us). The discharge currents belong to different operational states of the Itsy pocket computer.

(k' or β). An increase of this parameter results in higher frequency response, and thus to a higher sensitivity to fine-grained scheduling.

4.3 Computing lifetimes

Next to the theoretical analysis of the two models, both models were used to compute battery lifetimes for various load profiles.

In [13] Rakhmatov et al. give the battery lifetimes for load profiles of a Compaq Itsy pocket computer, computed both with their diffusion model and the electro-chemical model DUALFOIL [15]. To these results the lifetimes according to the KiBaM model have been added in Table 1 for constant loads and Table 2 for variable-load profiles. Details of the variable-load profiles are given in Table A (in Appendix A).

The lifetimes computed using the KiBaM and diffusion model match very well. The results for continuous discharge only deviate at high discharge currents, as expected from the analysis of the equations, but the difference still is less than 7%. Also, for the variable loads the difference is largest for short battery lifetimes, with a maximum of 5.4% for Case 21.

Figure 5 shows a plot of the lifetimes computed with both models versus the lifetimes computed with the electro-chemical simulation program DUALFOIL. In comparison with DUALFOIL both models overestimate the battery lifetime for the low continuous loads (long lifetimes), with errors growing upto 10%. The results of the variable loads are even better, with a maximum error of 5%.

Case	DUALFOIL (min)	Diffusion (min)	KiBaM (min)	Case	DUALFOIL (min)	Diffusion (min)	KiBaM (min)
C1	36.4	36.2	36.3	C12	159.0	155.4	154.1
C2	57.2	55.8	55.7	C13	133.8	131.7	131.3
C3	74.2	71.9	71.4	C14	132.9	129.7	129.4
C4	128.1	124.9	123.6	C15	207.6	209.2	209.2
C5	178.5	176.7	175.7	C16	202.4	200.7	200.7
C6	41.5	41.0	41.1	C17	253.8	251.2	250.8
C7	30.6	30.8	30.5	C18	204.6	204.6	204.3
C8	37.0	37.4	38.1	C19	209.4	208.7	208.2
C9	35.4	35.2	34.8	C20	31.7	33.2	31.5
C10	135.2	132.6	131.7	C21	55.9	55.9	58.8
C11	108.8	107.4	107.9	C22	97.5	94.5	94.3

Table 2: Lifetimes of variable-load profiles (cf. Appendix A) computed with DUALFOIL, the diffusion model (both from [13]) and KiBaM (computed by us)

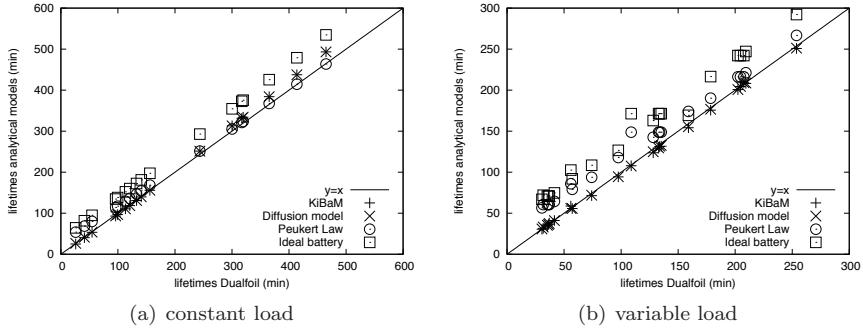


Figure 5: Computed lifetimes according to the Dualfoil simulation program versus the diffusion model and the KiBaM for constant loads (a) and variable loads (b). Next to the two analytical models, the lifetimes according to the formulas of the ideal battery (1) and Peukert's law (3) are shown.

Besides the results of the two models, also the lifetimes according to Peukert's law and the ideal battery model are shown in Figure 5. The ideal battery model always predicts longer lifetimes, since it does not take into account any loss of capacity due to the rate capacity effect. Also, Peukert's law overestimates the battery lifetimes for most cases. Only for the high continuous loads it gives better predictions than the KiBaM and diffusion model.

5 Limitations of analytical battery models

In the previous section we have seen that both models give nearly the same results. In this section, all further results are obtained with the KiBaM, but the conclusions also apply on the diffusion model.

With the KiBaM the effect of a varying load on the charge delivered by the battery was analysed in more detail. A square wave, switching between on (1 A) and off (0 A), was used as load. In Figure 6 the charge delivered is shown as a

function of the frequency of the periodic load. For low frequencies the delivered charge is constant, because the battery is emptied during the first on-period. Therefore, the charge delivered is equal to the case of continuous discharge at 1 A. When the frequency is increased, one sees a sudden discontinuous increase of the charge delivered by the battery. At the point of this jump, the battery is nearly empty at the end of the first on-period, and it has an off-period to recover some of its capacity. The recovered charge can be used in the next on-period, resulting in a considerable increase of the delivered charge. After this increase, the delivered charge slowly decreases when the frequency is further increased. The explanation of this decrease is twofold. First, the off-period is shorter and therefore there is less time for recovery. Second, the first on-period is shorter and less charge is delivered to the load during this time.

Further increase of the frequency results in a discontinuous increase of the charge delivered each time the battery can recover during an extra off-period, followed again by a slow decrease. The increase gets smaller for higher frequencies since the extra recovery-time decreases. When the frequency is $> 10^{-2}$ Hz, the charge delivered is constant again. This is due to the short extra off-time, and the low frequency response at these high frequencies (cf. Section 4.2).

For the chosen load and set of battery parameters the charge delivered is highest for a frequency of $\sim 10^{-4}$ Hz. However, the position of the peaks depends highly on the battery parameters and the level of the on-current, and a slight variation might result in a big change in the charge delivered by the battery. In practice the battery parameters vary even between batteries of the same size and type. Therefore, it does not make sense to do battery lifetime predictions using single traces of a load profile. The used trace could result in a high performance of the battery with one set of the parameters, and a low performance with a slightly different set of parameters.

6 Conclusions & Outlook

The analysis of the KiBaM and diffusion model shows that the KiBaM is actually a first order approximation of the diffusion model. The parameters of the KiBaM can be adapted to make a better approximation of the diffusion model. The performed experiments with both models show that this approximation is very good for most practical loads. Therefore, it is better to use the more simple KiBaM model. However, one has to be careful using this type of model when drawing conclusions from only a few workload traces. A slight change in the battery parameters can change the battery lifetime dramatically especially when the load switching frequencies are low. A good way to avoid this problem is to

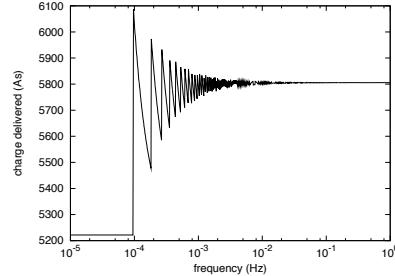


Figure 6: Charge delivered by the battery as a function of the frequency a square wave load. The charge delivered is computed using the KiBaM, with the parameters $c = 0.625$, $k = 4.5 \cdot 10^{-5} \text{ min}^{-1}$ and the capacity of 7200 As.

make use of stochastic workload models. With these models one can capture the full range of different possible workloadtraces. This results in a battery lifetime distribution, which tells us the probability of the battery being empty at time t given the type of workload. Comparing these probabilities one can find the best way to use the battery. Slight changes in the battery parameters, now, will not effect the results dramatically. One approach to do this is the by using Markov reward models, as described in [2]. Another approach is using prized timed automata [16] to describe the workload, and incorporate the Kinetic Battery Model into this model. This approach can help in finding the best scheduling scheme in a multi-battery system.

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A Appendix

Case	Description	Timing (min)
C1	IAT-off-IAT	(0, 19.5, 26.0)
C2	IAR-off-IAR	(0, 31.0, 41.3)
C3	IST-off-IST	(0, 41.0, 54.6)
C4	ISR-off-ISR	(0, 74.6, 99.5)
C5	MPEG-off-MPEG	(0, 105.7, 140.9)
C6	IAT-off-IAT	(0, 19.5, 29.9)
C7	IAT-off-IAT	(0, 19.5, 22.1)
C8	IAT-off-IAT	(0, 23.4, 29.9)
C9	IAT-off-IAT	(0, 15.6, 22.1)
C10	Boot-IAT-IAR-MSD-DSD-TSD-WSD-IAD	(0, 0.5, 5.5, 10.5, 35.5, 60.5, 85.5, 110.5)
C11	Boot-WSD-TSD-DSD-MSD-IAR-IAT-IAD	(0, 0.5, 25.5, 50.5, 75.5, 100.5, 105.5, 110.5)
C12	Boot-WSD-TSD-DSD-MSD-IAR-off...	(0, 0.5, 25.5, 50.5, 75.5, 100.5, 105.5, ...)
	Boot-IAT-IAD	130.5, 131.0, 136.0)
C13	Boot-[IAT-IAR-MSD-DSD-TSD-WSD] ⁵ -IAD	(0, [0.5, 1.5, 2.5, 7.5, 12.5, 17.5] ⁵ _{22.5} , 110.5)
C14	Boot-[WSD-TSD-DSD-MSD-IAR-IAT] ⁵ -IAD	(0, [0.5, 5.5, 10.5, 15.5, 20.5, 21.5] ⁵ _{22.5} , 110.5)
C15	MPEG-Dictation-Talk1-Wav1-MPEG	(0, 50.0, 100.0, 150.0, 200.0)
C16	Wav1-Talk1-Dictation-MPEG-MPEG	(0, 50.0, 100.0, 150.0, 200.0)
C17	Wav1-Talk1-Dictation-off-MPEG-MPEG	(0, 50.0, 100.0, 150.0, 200.0, 250.0)
C18	[Wav1-Talk1-Dictation-MPEG] ¹⁰ -MPEG	(0, 5.0, 10.0, 15.0) ¹⁰ ₀ , 200)
C19	[Wav2-Talk3-Dictation-MPEG] ¹⁰ -MPEG	(0, 5.0, 10.0, 15.0) ¹⁰ _{20.0} , 200)
C20	[IAR-IAT] ∞	([0, 1.0] $_{2.0}^{\infty}$)
C21	[IAR-IAT-ISD] ∞	([0, 1.0, 2.0] $_{3.0}^{\infty}$)
C22	5.0 + (5.0 per min)	(0, 1.0, 2.0, ...)

Table 3: The simulated variable-load profiles [13]