UTM

UNIVERSITI TEKNOLOGI MALAYSIA

UNIVERSITI PENYELIDIKAN

SECI1013

DISCRETE STRUCTURE

SECTION 2

ASSIGNMENT 2

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No.: SECI 1013 : DISCRETE STRUCTURE   Date:

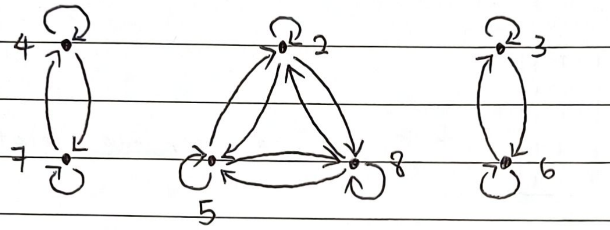
Assignment 2

1. xR_y , $x-y=3n$, $n \in \mathbb{Z}$

$n = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

$3n = \{\ldots, -6, -3, 0, 3, 6, \ldots\}$

$xRy = \{(2,2), (2,5), (2,8), (3,3), (3,6), (4,4), (4,7), (5,2), (5,5), (5,8), (6,3), (6,6), (7,4), (7,7), (8,2), (8,5), (8,8)\}$

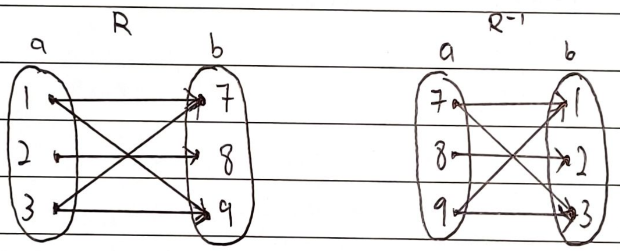


2. $A = \{1, 2, 3\}$ $B = \{9, 8, 7\}$ R : A to B   all $(a,b) \in A \times B$ $aRb \Leftrightarrow a+b = \text{even}$

(a) $R = \{(1,7), (1,9), (2,8), (3,7), (3,9)\}$

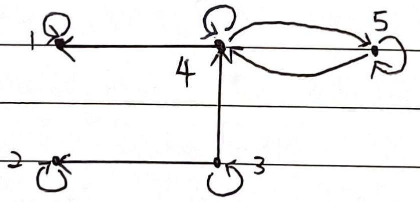
$R^{-1} = \{(7,1), (9,1), (8,2), (7,3), (9,3)\}$ * range become domain

(b)

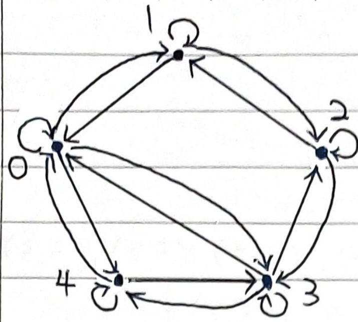


(c) The domain of R becomes the range of R^{-1}.

3. $R = \{(1,1), (2,2), (3,2), (3,3), (3,4), (4,1), (4,4), (4,5), (5,4), (5,5)\}$



vertices	1	2	3	4	5
in-degrees	2	2	1	3	2
out-degrees	1	1	3	3	2

4. 

R is reflexive as $(0,0),(1,1),(2,2),(3,3),(4,4) \in R$

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix}$$

R is symmetric as it have symmetric relation

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

R is not a transitive relation.

5. $R = \{(1,3), (2,6), (3,9), (4,12)\}$

reflexive : R is not reflexive relation, no loop included.

symmetric : R is not symmetric relation, $(1,3) \in R$ but $(3,1) \notin R$.

transitive : R is not transitive relation, $(1,3), (3,9) \in R$ but $(1,9) \notin R$.

6. a) $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

7. The different is a relation can have many outputs for a single input but a function has only one single input for a single output.

8. (i) Yes, it's a function. All variables in A are included and one-to-one relation is a function.

(ii) Yes, it's a function. All variables in A are included and one-to-one relation is a function.

(iii) No, it's not a function. Domain {2} have 2 outputs, many-to-many relation is not a function.

(iv) No, it's not a function. Domain is not equal to A, domain {5} not included.

(v) No, it's not a function. Domain is not equal to A which domain {3} and {5} are not included and domain {2} and {4} have 2 outputs, many-to-many relation is not a function.

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9. $R = \{(1,6), (2,7), (3,8), (4,9), (5,10)\}$

domain $= \{1, 2, 3, 4, 5\}$

range $= \{6, 7, 8, 9, 10\}$

10. (v) $f(x_1) = 1 - 2x$ $\quad f(x_2) = 1 - 2x$

$f(x_1) = f(x_2)$ $\quad$ Let $y = 1 - 2x$ $\quad f\left(\frac{1-y}{2}\right) = 1 - 2\left(\frac{1-y}{2}\right)$

$1 - 2x_1 = 1 - 2x_2$ $\quad x = \frac{1-y}{2}$ $\quad = y$

$-2x_1 = -2x_2$ $\quad f^{-1}(y) = \frac{1-y}{2}$ $\quad f(x) = y$

$2x_1 = 2x_2$

$x_1 = x_2$

* Thus, it's bijective as it's one-to-one and onto.

(vi) $f(x) = 5x^2 - 1$

when $x = 1$, $f(x) = 4$

when $x = -1$, $f(x) = 4$ $\quad$ * It's not one-to-one, onto or bijective.

(vii) $f(x) = x^4$

when $x = 2$, $f(x) = 16$

when $x = -2$, $f(x) = 16$ $\quad$ * It's not one-to-one, onto or bijective

(viii) $f(x_1) = \frac{x_1 - 2}{x_1 - 3}$, $\quad f(x_2) = \frac{x_2 - 2}{x_2 - 3}$

$f(x_1) = f(x_2)$ $\quad$ let $y = \frac{x-2}{x-3}$

$\frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$ $\quad xy - 3y = x - 2$

$(x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$ $\quad xy - x = 3y - 2$

$x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 3x_2 - 2x_1 + 6$ $\quad x(y-1) = 3y - 2$

$2x_1 - 3x_1 = 2x_2 - 3x_2$ $\quad x = \frac{3y-2}{y-1}$

$x_1(2-3) = x_2(2-3)$ $\quad f^{-1}(y) = \frac{3y-2}{y-1}$

$x_1 = x_2$

$$f\left(\frac{3y-2}{y-1}\right) = \frac{\frac{3y-2}{y-1} - 2}{\frac{3y-2}{y-1} - 3}$$

$$= \frac{3y - 2 - 2y + 2}{\cancel{y-1}} \times \frac{\cancel{y-1}}{3y - 2 - 3y + 3}$$

* Thus, it's bijective as it's one-to-one and onto.

$$= y$$

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11. (ix) $f(g(x)) = 3(x^2-1)-1$
$= 3x^2 - 3 - 1$
$= 3x^2 - 4$

$fg(0) = 3(0)^2 - 4 = -4$
$fg(1) = 3(1)^2 - 4 = -1$
$fg(2) = 3(2)^2 - 4 = 8$
$fg(3) = 3(3)^2 - 4 = 23$

(x) $f(g(x)) = (5x-6)^2$
$= (5x-6)(5x-6)$
$= 25x^2 - 60x + 36$

$fg(0) = 25(0)^2 - 60(0) + 36 = 0$
$fg(1) = 25(1)^2 - 60(1) + 36 = 1$
$fg(2) = 25(2)^2 - 60(2) + 36 = 16$
$fg(3) = 25(3)^2 - 60(3) + 36 = 81$

(xi) $f(g(x)) = x^3 + 1 - 1$
$= x^3$

$fg(0) = 0^3 = 0$
$fg(1) = 1^3 = 1$
$fg(2) = 2^3 = 8$
$fg(3) = 3^3 = 27$

12. (xii) $a_2 = 6a_1 - 9a_0 = 6(6) - 9(1) = 27$
$a_3 = 6a_2 - 9a_1 = 6(27) - 9(6) = 108$
$a_4 = 6a_3 - 9a_2 = 6(108) - 9(27) = 405$
$a_5 = 6a_4 - 9a_3 = 6(405) - 9(108) = 1458$

(xiii) $a_3 = 6a_2 - 11a_1 + 6a_0 = 6(15) - 11(5) + 6(2) = 47$
$a_4 = 6a_3 - 11a_2 + 6a_1 = 6(47) - 11(15) + 6(5) = 147$
$a_5 = 6a_4 - 11a_3 + 6a_2 = 6(147) - 11(47) + 6(15) = 455$
$a_6 = 6a_5 - 11a_4 + 6a_3 = 6(455) - 11(147) + 6(47) = 1395$

(xiv) $a_3 = -3a_2 - 3a_1 + a_0 = -3(-1) - 3(-2) + 1 = 10$
$a_4 = -3a_3 - 3a_2 + a_1 = -3(10) - 3(-1) + (-2) = -29$
$a_5 = -3a_4 - 3a_3 + a_2 = -3(-29) - 3(10) + (-1) = 56$
$a_6 = -3a_5 - 3a_4 + a_3 = -3(56) - 3(-29) + 10 = -71$

13. (i) $a_1 = k$
$a_2 = 5a_1 - 3$
$= 5k - 3$

$a_3 = 5a_2 - 3$
$= 5(5k-3) - 3$
$= 25k - 18$

$a_4 = 5a_3 - 3$
$= 5(25k-18) - 3$
$= 125k - 93$

(ii) $a_4 = 7$
$125k - 93 = 7$
$125k = 100$
$k = 0.8$

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