Research Log - Week 07

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June 26, 2016 — Started reading Chapter 2 of [Hartley2004] [1] for information regarding *Homographices.*

Worked on graphics regarding *Epipolar constraint* for inclusion in thesis document.

June 27, 2016 — Continued reading Chapter 2 of [Hartley2004] [1] containing information on *Homographies* for purpose(s) of deriving *Fundamental matrix* formula as well as understanding *Horizontal rectification* used for matching features along scanlines of images.

SUMMARY: Transformations of points in the image plane can be grouped into the following categories:

- **Isometries** (Denoted by $\mathbf{H}_E$): Transfomrations in $\mathbb{P}_2$ including *translation* and *rotation* (including composites of the two) that peserve *Euclidean*-distance. Tranformations are of the form

$$\begin{bmatrix} \epsilon\cos(\theta) & -\sin(\theta) & t_x \\ \epsilon\sin(\theta) & \cos(\theta) & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

  where $\epsilon = \pm 1$. Angles are preserved if $\epsilon = 1$, else if $\epsilon = -1$ angles are reversed (reflection accross an axis).

- **Similarity** (Denoted by $\mathbf{H}_S$): Transformations include *translation*, *rotation*, and *scaling*. Matrices are of the form

$$\begin{bmatrix} s\cos(\theta) & -s\sin(\theta) & t_x \\ s\sin(\theta) & s\cos(\theta) & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

  where s is the scaling factor. While *distances* are not preserved, the *ratio of distances* and *angles* are preserved.

- **Affine** (Denoted by $\mathbf{H}_A$): Transformations include all linear transformations of *translation*, *rotation*, *scaling*, and *shearing*. Matrices are of the form

$$\begin{bmatrix} a_{11} & a_{12} & t_x \\ a_{21} & a_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

- **Projective** (Denoted by $\mathbf{H}_P$): Transformations in $\mathbb{P}_2$ that are linear transformations in $\mathbb{R}_3$. Matrices are of the form

$$\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}$$

June 29, 2016 Continuing to read [Hartley2004] [1] for *affine rectification*. Chapters of [Hartley2004] [1] include:

- **Chapter 2: Projective Geometry**:
  - **Section 2.1: Planar Geometry**:
  - **Section 2.2: The 2D projective plane**:

    Lines in $\mathbb{R}^2$ are detailed by $\mathbf{l} = [a, b, c]^\intercal$ and points as $\mathbf{x} = [x, y, 1]^\intercal$ such that $\mathbf{l}^\intercal \cdot \mathbf{x} = a \cdot x + b \cdot y + 1 = 0$. Coordinates $\mathbf{x} = [x, y, 0]^\intercal$ with a 0 instead of 1 in the last place represent a *point at infinity* since they are the only points where $a \cdot x + b \cdot y + c \cdot 0 = a \cdot x + b \cdot y + c' \cdot 0$ for the two *parallel* lines of $\mathbf{l} = [a, b, c]^\intercal$ and $\mathbf{l}' = [a, b, c']^\intercal$

    Cross product of points $\mathbf{x}$ and $\mathbf{x}'$ result in line $\mathbf{l}$ joining the two points (i.e. $\mathbf{x} \times \mathbf{x}' = \mathbf{l}$). Cross product of lines $\mathbf{l}$ and $\mathbf{l}'$ result in point $\mathbf{x}$ where intersection of two lines (i.e. $\mathbf{l} \times \mathbf{l}' = \mathbf{x}$).

    Circles and ovals can be reprsented by a *conic-matrix* of the form

$$
\begin{aligned}
0 &= \mathbf{x}^\intercal \cdot \mathbf{C} \cdot \mathbf{x} \\
&= \begin{bmatrix} x & y & 1 \end{bmatrix} \cdot \begin{bmatrix} a & b/2 & d/2 \\ b/2 & c & e/2 \\ d/2 & e/2 & f \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \\
&= a \cdot x^2 + b \cdot xy + c \cdot y^2 + d \cdot x + e \cdot y + f \cdot 1
\end{aligned}
$$

  - **Section 2.3: Projective transformations**:

    Point $\mathbf{x}$ on an image is mapped to point $\mathbf{x}'$ via a homography $\mathbf{H}$, such that $\mathbf{x}' = \mathbf{H} \cdot \mathbf{x}$. Because a point $\mathbf{x}$ lies on line $\mathbf{l}$ if $\mathbf{l}^\intercal \cdot \mathbf{x} = 0$, then because

$$
\begin{aligned}
0 &= \mathbf{l}^\intercal \cdot \mathbf{x} \\
&= \mathbf{l}^\intercal \cdot \mathbf{H}^{-1} \cdot \mathbf{H} \cdot \mathbf{x} \\
&= \mathbf{l}^\intercal \cdot \mathbf{H}^{-1} \cdot \mathbf{x}'
\end{aligned}
$$

    the point $\mathbf{x}'$ lies on the line $\mathbf{l}'$ defined by $\mathbf{l}'^\intercal = \mathbf{l}^\intercal \cdot \mathbf{H}^{-1}$, or $\mathbf{l}' = \mathbf{H}^{-\intercal} \cdot \mathbf{l}$. Therefore a homograhy that gives a *point-mapping* of $\mathbf{x}' = \mathbf{H} \cdot x$ has a corresponding *line-mapping* of $\mathbf{l}' = \mathbf{H}^{-\intercal} \cdot \mathbf{l}$.

    Similarly, for a homography given by $\mathbf{x}' = \mathbf{H} \cdot \mathbf{x}$, the conic under the homography is given by

$$
\begin{aligned}
0 &= \mathbf{x}^\intercal \cdot \mathbf{C} \cdot \mathbf{x} \\
&= (\mathbf{H}^{-1} \cdot \mathbf{x}')^\intercal \cdot \mathbf{C} \cdot (\mathbf{H}^{-1} \cdot \mathbf{x}') \\
&= \mathbf{x}'^\intercal \cdot \mathbf{H}^{-\intercal} \cdot \mathbf{C} \cdot \mathbf{H}^{-1} \cdot \mathbf{x}' \\
&= \mathbf{x}'^\intercal \cdot \mathbf{C}' \cdot \mathbf{x}'
\end{aligned}
$$

    where $\mathbf{C}' = \mathbf{H}^{-\intercal} \cdot \mathbf{C} \cdot \mathbf{H}^{-1}$.

  - **Section 2.4: A hierarchy of transformations**:

    See entry from June 27, 2016.

June 29, 2016 *...Continued*

- **Chapter 6: Camera Models**:
  - **Section 6.1: Finite cameras**:

Transformation from *world-coordinate* system $\mathbf{x}$ to *camera-coordinate* system ${}^C\mathbf{x}$ is given by ${}^C\mathbf{x} = \mathbf{R} \cdot (\mathbf{x} - \mathbf{c})$. The Camera in *world-space* occurs at $\mathbf{x} = \mathbf{c}$. *Camera-space* has the camera located at ${}^C\mathbf{x} = 0$ and includes an *image-plane* at $z = f$. All rays intersect the *image plane* at $z = f$ and converge on the origin ${}^C\mathbf{x} = 0$ which is known as the *camera center*. This results in points ${}^C\mathbf{x}$ in *camera space* being projected to points $\tilde{\mathbf{y}}$ in the *image plane* by means of the *projection matrix* $\mathbf{P}$ such that

$$\mathbf{P} \cdot {}^C\tilde{\mathbf{x}} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} {}^Cx_1 \\ {}^Cx_2 \\ {}^Cx_3 \\ 1 \end{bmatrix} = \begin{bmatrix} f \cdot {}^Cx_1 \\ f \cdot {}^Cx_2 \\ {}^Cx_3 \end{bmatrix}$$

$$= {}^Cx_3 \cdot \begin{bmatrix} f \cdot {}^Cx_1 / {}^Cx_3 \\ f \cdot {}^Cx_2 / {}^Cx_3 \\ 1 \end{bmatrix} = {}^Cx_3 \cdot \tilde{\mathbf{y}}$$

This results in points containing infinitley large values of x_3 being mapped to the same *principal point* of $\mathbf{y} = 0$ in the *image plane*. This assumes the *princpal point* is always located in the *image plane* at $\mathbf{y} = 0$. Projecting point $\tilde{\mathbf{x}}$ to the *image plane* with arbitrary *principal point* $\mathbf{p} = [p_x, p_y]$ requires modifying the *projection matrix* to include *camera-specific* parameters. The *camera calibration matrix* $\mathbf{K}$ is given as

$$\mathbf{P} \cdot {}^C\tilde{\mathbf{x}} = \begin{bmatrix} f & 0 & p_x & 0 \\ 0 & f & p_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} {}^Cx_1 \\ {}^Cx_2 \\ {}^Cx_3 \\ 1 \end{bmatrix} = \begin{bmatrix} f \cdot {}^Cx_1 + p_x \cdot {}^Cx_3 \\ f \cdot {}^Cx_2 + p_y \cdot {}^Cx_3 \\ {}^Cx_3 \end{bmatrix}$$

$$= {}^Cx_3 \cdot \begin{bmatrix} f \cdot {}^Cx_1 / {}^Cx_3 + p_x \\ f \cdot {}^Cx_2 / {}^Cx_3 + p_y \\ 1 \end{bmatrix} = {}^Cx_3 \cdot \tilde{\mathbf{y}}$$

June 30, 2016 **Question for Kamangar:** On pages 162 and 244, how is the ray back-projected from $\mathbf{x}$ by $\mathbf{P}$ (where $\mathbf{x} = \mathbf{PX}$ and $\mathbf{P} = \mathbf{K}[\mathbf{R}|\mathbf{t}]$) given by the formula $\mathbf{X}(\lambda) = \mathbf{P}^+\mathbf{x} + \lambda\mathbf{C}$? How is the formula derived?

July 1, 2016 Added section called **Points and Lines in the Image Plane** in the **Background** section.

References

[1] R. I. Hartley and A. Zisserman. *Multiple View Geometry in Computer Vision.* Cambridge University Press, ISBN: 0521540518, second edition, 2004.