

glFrustum

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1 Introduction

http://www.songho.ca/opengl/gl_projectionmatrix.html

<https://www.opengl.org/sdk/docs/man2/xhtml/glFrustum.xml>

`glFrustum(GLdouble left, GLdouble right, GLdouble bottom, GLdouble top, GLdouble near, GLdouble far);`

$${}_E^P M = \begin{bmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

$$X_P = \frac{n \cdot X_E}{-Z_E} \quad \dots \quad Y_P = \frac{n \cdot Y_E}{-Z_E}$$

$$\begin{aligned} \frac{A_Z \cdot n + B_Z}{-n} &= -1 & \dots & \quad \frac{A_Z \cdot f + B_Z}{-f} = 1 \\ A_Z \cdot n + B_Z &= n & \dots & \quad A_Z \cdot f + B_Z = -f \end{aligned}$$

$$\begin{aligned} &= \begin{bmatrix} \frac{2 \cdot \text{near}}{\text{right} - \text{left}} & 0 & \frac{\text{right} + \text{left}}{\text{right} - \text{left}} & 0 \\ 0 & \frac{2 \cdot \text{near}}{\text{top} - \text{bottom}} & \frac{\text{top} + \text{bottom}}{\text{top} - \text{bottom}} & 0 \\ 0 & 0 & -\frac{\text{far} + \text{near}}{\text{far} - \text{near}} & -\frac{2 \cdot \text{far} \cdot \text{near}}{\text{far} - \text{near}} \\ 0 & 0 & -1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} \frac{2 \cdot \text{near}}{\text{right} - \text{left}} & 0 & \left(\frac{2 \cdot \text{near}}{\text{right} - \text{left}} \right) \cdot \left(\frac{\text{right} + \text{left}}{2 \cdot \text{near}} \right) & 0 \\ 0 & \frac{2 \cdot \text{near}}{\text{top} - \text{bottom}} & \left(\frac{2 \cdot \text{near}}{\text{top} - \text{bottom}} \right) \cdot \left(\frac{\text{top} + \text{bottom}}{2 \cdot \text{near}} \right) & 0 \\ 0 & 0 & \left(\frac{-2 \cdot \text{near}}{\text{far} - \text{near}} \right) \cdot \left(\frac{\text{far} + \text{near}}{2 \cdot \text{near}} \right) & \left(\frac{2 \cdot \text{near}}{\text{far} - \text{near}} \right) \cdot (-\text{far}) \\ 0 & 0 & -1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} \frac{2 \cdot \text{near}}{\text{right} - \text{left}} & 0 & 0 & 0 \\ 0 & \frac{2 \cdot \text{near}}{\text{top} - \text{bottom}} & 0 & 0 \\ 0 & 0 & \frac{-2 \cdot \text{near}}{\text{far} - \text{near}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & \frac{\text{right} + \text{left}}{2 \cdot \text{near}} & 0 \\ 0 & 1 & \frac{\text{top} + \text{bottom}}{2 \cdot \text{near}} & 0 \\ 0 & 0 & \frac{\text{far} + \text{near}}{2 \cdot \text{near}} & \text{far} \\ 0 & 0 & -1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} \frac{2 \cdot \text{near}}{\text{right} - \text{left}} & 0 & 0 & 0 \\ 0 & \frac{2 \cdot \text{near}}{\text{top} - \text{bottom}} & 0 & 0 \\ 0 & 0 & \frac{-2 \cdot \text{near}}{\text{far} - \text{near}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & \frac{\text{right} + \text{left}}{2 \cdot \text{near}} & 0 \\ 0 & 1 & \frac{\text{top} + \text{bottom}}{2 \cdot \text{near}} & 0 \\ 0 & 0 & \frac{\text{far} + \text{near}}{2 \cdot \text{near}} & \text{far} \\ 0 & 0 & -1 & 0 \end{bmatrix} \end{aligned}$$