Clarification on image rectification, Result 11.3

The rectification algorithm given in Result 11.3 starts from the factorization of the fundamental matrix as $\mathtt{F} = \mathtt{SM}$, where $\mathtt{S}$ is skew-symmetric and $\mathtt{M}$ is the matrix representing the required homography.

This should not be done using the usual factorization of the fundamental matrix as $\mathtt{F} = [\mathbf{e}]_\times([\mathbf{e}]_\times\mathtt{F})$, so that $\mathtt{S} = [\mathbf{e}]_\times$ and $\mathtt{M} = [\mathbf{e}]_\times\mathtt{F}$, for then $\mathtt{M}$ is singular. The algorithm will not work in this case, since we have a singular homography, namely $\mathtt{M}$.

What should be done is to use the SVD to give

$$\mathtt{F} = \mathtt{UDV}^\mathsf{T} = \mathtt{UWZD'V}^\mathsf{T} = (\mathtt{UWU}^\mathsf{T})\left(\mathtt{UZD'V}^\mathsf{T}\right) = \mathtt{SM}$$

where

$$\mathtt{W} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \ ; \ \mathtt{Z} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and if $\mathtt{D} = \mathrm{diag}(s, t, 0)$, then $\mathtt{D}' = \mathrm{diag}(s, t, (s+t)/2)$.

It is easily verified that $\mathtt{WZ} = \mathrm{diag}(1, 0, 0)$, so $\mathtt{WZD}' = \mathtt{D}$. In addition, the choice of matrix $\mathtt{D}'$, and hence matrix $\mathtt{M}$ is about as far from singular as possible for these equations to hold.