# Number of linearly independent trilinear relations, Section 17.2

On page 416, in the paragraph starting at line 8, the number of linearly independent trilinear relations is discussed. The discussion is too superficial, and the conclusions are wrong. The argument confuses two different concepts. Consider a relationship of the form given in the book by equation (17.11), namely

$$x^i x'^j x''^k \epsilon_{jqu} \epsilon_{krv} \mathcal{T}_i^{qr} = 0_{uv} \ . \quad (17.11)$$

This may be thought of in two ways.

First, given a point correspondence $\mathbf{x} \leftrightarrow \mathbf{x}' \leftrightarrow \mathbf{x}''$ across three views, this formula gives linear relationships between the entries of the trifocal tensor. These relationships may be written as

$$\sum_{i,q,r=1}^{3} \alpha^i_{qr} \mathcal{T}_i^{qr} = 0 \ ,$$

where the coefficients $\alpha^i_{qr}$ are determined by the coordinates of the corresponding points. From the different choices of the free indices $u$ and $v$ we obtain $9$ different linear relationships of this type between the entries of the trifocal tensor. It turns out that only $4$ of these relationships are linearly independent, however. It does not make sense to talk about the number of independent linear relationships for all $3$ tensors, since each such linear relationship involves the entries of only one tensor.

Secondly, consider a specific trifocal tensor $\mathcal{T}_i^{qr}$, corresponding to a configuration of three cameras. Equation (17.11) may be thought of as a linear relationship between the $27$ monomials $x^i x'^j x''^k$ involving the coefficients of matching image points $\mathbf{x} \leftrightarrow \mathbf{x}' \leftrightarrow \mathbf{x}''$. Such a relation, written as

$$\sum_{i,j,k=1}^{3} \beta_{ijk} \left(x^i x'^j x''^k\right)$$

is called a trilinear relation between the coordinates of the matching points. Once again, from different choices of the free indices $u$ and $v$, a total of $9$ different trilinear relations are obtained. In the book it was wrongly stated that only $4$ of these relations are linearly independent, and that from the three different trifocal tensors derived from the same camera geometry a total of $12$ trilinear relations are obtained. The truth is as follows:

1. From a single trifocal tensor, there are $8$ linearly independent relations among the $9$ available.

2. From two of the three trifocal tensors, there are $9$ linearly independent relations among the $18$ available.

3. From all three trifocal tensors, there are $10$ linearly independent relations among the total of $27$ relations.

We do not offer a proof of this fact here, but it may be easily verified using symbolic (or numeric) computation in Mathematica or Matlab. To verify this using Mathematica, you may download the Mathematica notebook file TrifocalRelations2.nb from this web site.