Quasi-Euclidean Uncalibrated Epipolar Rectification[*]

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Abstract

This paper deals with the problem of epipolar rectification in the uncalibrated case. First the calibrated (Euclidean) case is recognized as the ideal one, then we observe that in that case images are transformed with a collineation induced by the plane at infinity, which has a special structure. Hence, that structure is imposed to the sought transformation while minimizing a rectification error. Experiments show that this method yields images that are close to the ones produced by Euclidean rectification.

1 Introduction

Epipolar rectification is an important stage in dense stereo matching, as almost any stereo algorithm requires rectified images, i.e., images where epipolar lines are parallel and horizontal and corresponding points have the same vertical coordinates. In the case of calibrated cameras the *Euclidean* epipolar rectification is unique up to trivial transformations. The problem is addressed in [2].

On the contrary, in the case of uncalibrated cameras, there are more degrees of freedom in choosing the rectifying transformation [4] and a few competing methods are present in the literature [10, 11]. Each aims at producing a "good" rectification by minimizing a measure of distortion, but none is clearly superior to the others, not to mention the fact that there is no agreement on what the distortion criterion should be. Above all, none of them achieves results comparable to the calibrated case, which can be undoubtedly taken as the target result. This paper aims at approaching this target: given the lack of knowledge of the intrinsic camera parameters, we will achieve an approximation of the Euclidean epipolar rectification, which we refer to as *quasi-Euclidean* epipolar rectification.

Geometrically, in the Euclidean frame, rectification is achieved by a suitable rotation of both image planes. The correspondent image transformation is the collineation induced by the plane at infinity. As a result, the plane at infinity is the locus of zero-disparity in the rectified stereo pair. This is signified by saying that Euclidean rectification is done *with respect to* the plane at infinity. In the uncalibrated case the reference plane is generic, as any plane can play the role of the infinity plane in the projective space. Our quasi-Euclidean rectification can be seen as referred to a plane that approximates the plane at infinity.

1.1 Previous work

The first work on uncalibrated rectification (called "matched-epipolar projection") is [5], followed by [4], where the author tidies up the theory. He uses the condition that one of the two collineations should be close to a rigid transformation in the neighborhood of a selected point, while the remaining degrees of freedom are fixed by minimizing the distance between corresponding points (disparity). Along the same line, [13] also proposes a distortion criterion based on simple geometric heuristics.

The state-of-the-art papers in uncalibrated rectification [11, 10] dates back to 1999. Loop and Zhang [11] decompose each collineation into similarity, shearing and projective factors and attempt to make the projective component "as affine as possible". Isgrò and Trucco [10] build upon [4] and propose a method that avoids computation of the fundamental matrix, using the same distortion criterion as in [4]. The practice has shown that the rectification produced by these methods is not always satisfactory, if compared to results obtained in the calibrated case. Wu and Yu [9] argue that minimizing the disparity might be the cause of the problem, and propose a technique which is similar to [10] but uses a different distortion criterion derived from [11]. Rectification ends up in a non-linear minimization with six degrees of freedom. In [3] the transformation that best preserve the sampling of the original images is selected, by penalizing minification and magnification effects.

A different approach is followed in [1]: they design the collineations so as to minimize the relative distortion between the rectified images (instead of the distortion of each rectified image with respect to the original one), and the re-

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[*]This is the extended version of the paper presented at ICPR 2008.

maining degrees of freedom are fixed by choosing the reference plane in the scene, that will have zero disparity in the rectified pair. This choice, which affects sensibly the quality of rectification, is left to the user. The merit of this approach, however, is to make the role of the reference plane explicit.

2 Background

In this section we shall briefly recapitulate the theory of calibrated (or Euclidean) epipolar rectification; the reader is referred to [2] for more details.

Given two camera matrices P_{or} and $P_{o\ell}$, the idea behind rectification is to define two new *virtual* cameras P_{nr} and $P_{n\ell}$ obtained by rotating the actual ones around their optical centres until focal planes become coplanar. The rectification method describes how to compute the new cameras. Then (we concentrate on the right camera, but the same reasoning applies to the left one), the rectifying transformation that is to be applied to images is given by the 3×3 matrix:

$$H_r = P_{nr_{1:3}} P_{or_{1:3}}^{-1} \qquad (1)$$

where the subscript denotes a range of columns.

It is easy to see that H_r is the collineation induced by the plane at infinity between the old and the new cameras, hence it can be written as:

$$H_r = K_{nr} R_r K_{or}^{-1} \qquad (2)$$

where K_{or} and K_{nr} are the intrinsic parameters of the old and new camera respectively, and R_r is the rotation that is applied to the old camera in order to rectify it.

The rectified images are as if they were taken by a pair of cameras related by a translation along the baseline. Hence, the zero-disparity plane (i.e., the reference plane) is at infinity.

3 Method

We shall henceforth concentrate on the uncalibrated case. We assume that intrinsic parameters are unknown and that a number of corresponding points $\mathbf{m}_\ell^j \leftrightarrow \mathbf{m}_r^j$ are available. The method follows the same line as in [10]: it seek the collineations that make the original points satisfy the epipolar geometry of a rectified image pair.

The fundamental matrix of a rectified pair has a very specific form, namely it is the skew-symmetric matrix associated with the cross-product by the vector $\mathbf{u}_1 = (1, 0, 0)$:

$$[\mathbf{u}_1]_\times = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \qquad (3)$$

Let H_r and H_ℓ be the unknown rectifying collineations. The transformed corresponding points must satisfy the epipolar geometry of a rectified pair, hence

$$(H_r \mathbf{m}_r^j)^\mathsf{T} [\mathbf{u}_1]_\times (H_\ell \mathbf{m}_\ell^j) = 0, \qquad (4)$$

As this equation must hold for any correspondence, one obtains a system of non-linear equations in the unknown H_r and H_ℓ. The left-hand side of Equation (4) is an algebraic error, i.e., it has no geometrical meaning, so we used instead the Sampson error [6], that is a first order approximation of the geometric reprojection error. The matrix $F = H_r^\mathsf{T} [\mathbf{u}_1]_\times H_l$ can be considered as the fundamental matrix between the original images, therefore, in our case, the Sampson error for the j-th correspondence is defined as:

$$E_S^j = \frac{(\mathbf{m}_r^{j\mathsf{T}} F \mathbf{m}_\ell^j)^2}{||[\mathbf{u}_3]_\times F \mathbf{m}_\ell^j||^2 + ||\mathbf{m}_r^{j\mathsf{T}} F [\mathbf{u}_3]_\times||^2}$$

where $\mathbf{u}_3 = (0, 0, 1)$

A least-squares solution to the system of equations $\{E_S^j = 0\}$ is sought. The way in which H_r and H_ℓ are parametrized is crucial, and characterizes our approach with respect to the previous ones. We force the rectifying collineations to have the same structure as in the calibrated (Euclidean) case, i.e., to be collineations induced by the plane at infinity, namely

$$H_r = K_{nr} R_r K_{or}^{-1} \qquad H_\ell = K_{n\ell} R_\ell K_{o\ell}^{-1}. \qquad (5)$$

The old intrinsic parameters ($K_{o\ell}, K_{or}$) and the rotation matrices (R_ℓ, R_r) are unknown, whereas the new intrinsic parameters ($K_{n\ell}, K_{nr}$) can be set arbitrarily, provided that vertical focal length and vertical coordinate of the principal point are the same. Indeed, it is easy to verify that the matrix $K_{nr}^\mathsf{T} [\mathbf{u}_1]_\times K_{n\ell}$ is equal (up to scale) to $[\mathbf{u}_1]_\times$, provided that the second and third row of K_{nr} and $K_{n\ell}$ are the same. Hence it is not necessary to include the matrices K_{nr} and $K_{n\ell}$ in the parametrization.

Each collineation depends in principle on five (intrinsic) plus three (rotation) unknown parameters. The rotation of one camera along its X-axis, however, can be eliminated. Consider the matrix

$$F = K_{or}^{-1} R_r^\mathsf{T} [\mathbf{u}_1]_\times R_\ell K_{o\ell}^{-1}. \qquad (6)$$

Let R_r' and R_ℓ' be the same matrices as R_r and R_ℓ after pre-multiplying with an arbitrary (but the same for both) rotation matrix about the X-axis. It is easy to verify that $R_r^\mathsf{T} [\mathbf{u}_1]_\times R_\ell = R_r'^\mathsf{T} [\mathbf{u}_1]_\times R_\ell'$. Geometrically, this coincide with rotating a rectified pair around the baseline, which do not alter the rectification, but, in a real camera, it affects the portion of the scene that is imaged. Accordingly, we set to zero the rotation around the X-axis of the left camera.

We further reduce the number of parameters by making an educated guess on the old intrinsic parameters: no skew, principal point in the centre of the image, aspect ratio equal to one. The only remaining unknowns are the focal lengths of both cameras. Assuming that they are identical and equal to α, we get:

$$K_{or} = K_{o\ell} = \begin{bmatrix} \alpha & 0 & w/2 \\ 0 & \alpha & h/2 \\ 0 & 0 & 1 \end{bmatrix} \quad (7)$$

where w and h are width and height (in pixel) of the image.

In summary, the two collineations are parametrized by six unknowns: five angles and the focal length α. As suggested in [8], the focal length is expected to vary in the interval $[1/3(w+h), 3(w+h)]$, so we consider instead the variable $\alpha' = \log_3(\alpha/(w+h))$ which varies in $[-1, 1]$. The minimization is carried out using Levenberg-Marquardt, starting with all the unknown variables set to zero. The derivatives are computed analytically with the MATLAB "Symbolic Math Toolbox". The MATLAB code of the cost function is reported below.

```
function err  = costRectif(a,w,h,ml,mr) ;
%costRectif  Compute rectification cost
%
% a is a vector of six elements containing the
% independent variables of the cost function,
% i.e, five rotation angles  (Y-left, Z-left,
% X-right, Y-right, Z-right) and the focal length.
% w,h are the image width and height, respectively.
% ml,mr contain the corresponding image points.

yl=a(1); zl=a(2); xr=a(3);
yr=a(4); zr=a(5); f=3^a(6)*(w+h);

% estimate of the intrinsic parameters
% of the old cameras
Kol = [f, 0, w/2;  0, f, h/2; 0, 0, 1];
Kor = Kol;

% eulR applies rotations in the order Y-Z-X
Rl = eulR([0,yl,zl]);
Rr = eulR([xr,yr,zr]);

% fundamental matrix btw original points
F = inv(Kor)'*Rr'*hat([1 0 0])*Rl*inv(Kol);

% compute Sampson error
err = sampson(F,ml,mr);
```

Finally, the the rectifying collineations are computed with Eq. (5). The new intrinsic parameters (K_{nr} and $K_{n\ell}$) are set equal to the old ones: $K_{nr} = K_{n\ell} = K_{o\ell}$, modulo a shift of the principal point, that might be necessary to center the rectified images in the customary image coordinate frame. Horizontal translation has no effect on the rectification, whereas vertical translation must be the same for both images.

4 Results

We present first some results of our rectification algorithm applied to the SYNTIM images[1]. A few (10-20) corresponding points were selected manually, then the algorithm automatically produced the rectified images. It always converged to a solution, in our experiments, without special initialization.

Calibration data (provided with the images) were used for computing the ground truth Euclidean rectification using [2][2]. Unfortunately, a quantitative measure of distortion that captures the desired behaviour of rectification in every respect does not exist. Therefore, we are forced to leave the assessment of our quasi-Euclidean rectification to visual comparison with the ground truth: As the reader can appreciate in Fig. 1 and 2, our results are remarkably close to the Euclidean rectification, with the exeption of possibly *Angle* and *Poly*.

In [10] the rectified images are in some cases very distorted and distinctly dissimilar from the Euclidean case. In the experiments shown in [9] the distortion is considerably reduced, but our results are still closer to the Euclidean rectification. The only one example reported in [11] does not allow for a meaningful comparison: Our quasi-Euclidean rectification, however, produces a very similar result, shown in Fig. 3.

Table 1 reports the rectification error, computed with

$$e_{\text{rec}} = \frac{1}{N} \sqrt{\sum_j E_S^j} \quad (8)$$

where N is the number of corresponding pairs. It can be interpreted as the root mean squared distance (in pixel) from each point to its (horizontal) epipolar line.

Table 1. Rectification errors for the SYNTIM pairs.

Image pair	e_{rec} [pixel]	
	quasi-Euclid.	Euclid.
Angle	0.0919	0.1298
Aout	0.1063	0.3685
BalMire	0.1104	0.1143
BalMouss	0.0753	0.7046
BatInria	0.0886	0.2096
Color	0.1371	0.1992
Poly	0.5402	0.5133
Rubik	0.1059	0.5524
Sport	0.1073	0.1262
Tot	0.0623	0.0946

These figures, beside confirming that the pairs are indeed rectified, reveal that the error achieved by the quasi-Euclidean rectification are consistently smaller than in the

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[1] Available from www-rocq.inria.fr/~tarel/syntim/paires.html

[2] MATLAB code by A. Fusiello (profs.sci.univr.it/~fusiello/demo/rect/)

Euclidean rectification. This makes sense, because the former explicitly minimizes this error, whereas the latter derives the rectifying collineations directly from the camera matrices. This suggests that our technique can be used also in the calibrated case to refine the Euclidean rectification.

We noted experimentally that, considering separately the angle parameters and the focal length as the independent variables, the location of the minimum of the cost function is fairly insensitive to the value of the focal length (i.e., the correct angles can be obtained even when α is far away from the ground truth), whereas, in certain cases, small perturbations of the angles can drift the minimum away from the ground truth for the focal length (the quality of the rectification is unaffected, however). This seems to be related to the fact that computing the focal length from two uncalibrated views [7, 14] is ill conditioned when the cameras are verging.

A few other examples of quasi-Euclidean rectification of pictures taken by the authors is reported in (Fig. 4). The good quality of these images allows to completely automate the process: first SIFT features are extracted in both images and the descriptors are matched as described in [12][3]; then outliers are discarded by fitting a fundamental matrix to the matching points with RANSAC[4]; the inliers are used as input to the rectification. Please note that, apart from the first pair, the others present a notable change of the point of view and, consequently, a significant verging angle.

5 Discussion

We presented a new method for the epipolar rectification of uncalibrated stereo pairs which approximates the Euclidean (calibrated) case by enforcing the rectifying transformation to be a collineation induced by the plane at infinity. The method is based on the minimization of a cost function that has only six degrees of freedom and does not need any specific initialization. The results are close to the target Euclidean rectification and compares favorably with state-of-the-art uncalibrated methods, in terms of distortion applied to the rectified images.

Given the general utility of rectification, a MATLAB toolkit is available on the web[5].

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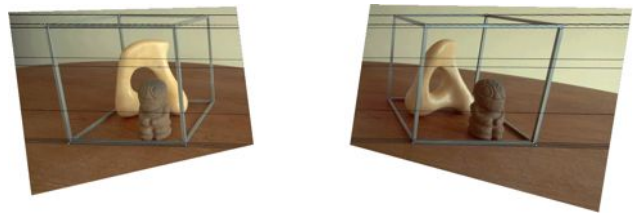

Figure 3. Quasi-Euclidean rectification of the stereo pair used by [11].

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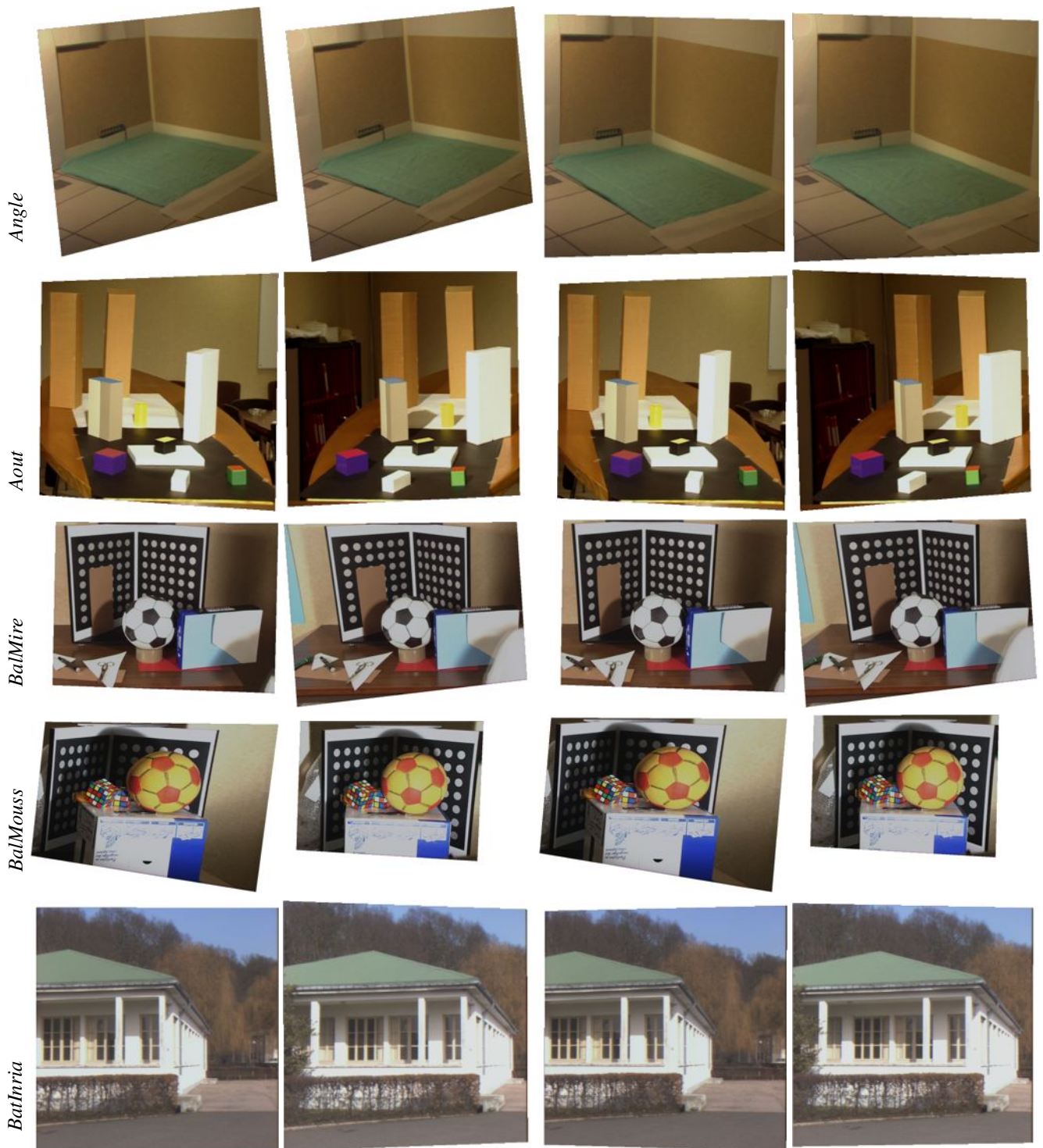
Angle

Aout

BalMire

BalMouss

BatInria

Figure 1. Euclidean (left) and quasi-Euclidean rectification (right) of the SYNTIM pairs.

Figure 2. Euclidean (left) and quasi-Euclidean rectification (right) of the SYNTIM pairs.

Figure 4. Original uncalibrated pairs (left) and Quasi-Euclidean rectification (right).