

Prob Exercises

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The HAND-IN details:

- deadline is **begin 9.00 lecture Thu Sep 26th**
- I am perfectly happy for you to *write answers by hand on paper* – I know I would. However I am going to set up a Blackboard submission and besides giving me your handiwork on paper, I would ask you to make (a legible scan) and to submit that via blackboard. This is just in cases of piece of paper going astray.
- If you want to create your answer purely by digital means that is also absolutely fine.

the questions:

1. Consider the following equations (alternative conditions for independence)
 - (i) $P(A \wedge B) = P(A) \times P(B)$
 - (ii) $P(A|B) = P(A)$show that (i) implies (ii), and also that (ii) implies (i)

2. Suppose *Harry Potter* plays for the *Gryffindor* team, and suppose he ‘often’ catches the so-called *Golden Snitch*, and also that ‘often’ *Gryffindor* win. Suppose the numbers actually are as follows

	<i>gw</i>	$\neg gw$	
<i>ps</i>	28	2	<i>ps</i> means ‘Harry Potter caught the Golden Snitch’
$\neg ps$	140	30	<i>gw</i> means ‘Gryffindor won’

- (a) Calculate $P(gw | ps)$, the conditional prob that *Gryffindor* won given that Harry Potter caught the Golden Snitch and also indicate which counts are irrelevant to this calculation
 - (b) Calculate $P(ps | gw)$, the conditional prob that Harry Potter caught the Golden Snitch given that *Gryffindor* won and also indicate which counts are irrelevant to this calculation
3. A sound clip from YouTube may or may not have been produced by Victor Meldrew. A clip may or may not contain the phrase ‘*I don’t believe it*’.
You hear a clip coming from YouTube containing the phrase ‘*I don’t believe it*’ and want to work out the probability that the speaker was Victor Meldrew.

Formalize with 2 discrete variables

- discrete *Speaker*, values in {'Victor Meldrew', 'Other'}
- discrete *DBI*, values in {true, false}, for whether a sound clip contains the phrase 'I don't believe it'

Let *vmel* stand for *Speaker* = 'Victor Meldrew', *dbi* stands for *DBI* = true

- Work out which of *vmel* or $\neg vmel$ is likelier, given *dbi*, supposing the probabilities $p(vmel) = 0.01$, $p(dbi|vmel) = 0.95$, $p(dbi|\neg vmel) = 0.01$
 - Do the same assuming $p(vmel) = 0.15$, $p(dbi|vmel) = 0.95$, $p(dbi|\neg vmel) = 0.01$
 - Do the same assuming $p(vmel) = 0.01$, $p(dbi|vmel) = 0.95$, $p(dbi|\neg vmel) = 0.001$
4. Consider someone who lives in a basement flat. Sometimes it is quite noisy in the flat, and sometimes not. Sometimes it is rather cool in the flat, and sometimes not. Let *noisy* be a variable indicating whether it is rather noisy or not, on a given day, and let *cool* be a variable indicating whether it is rather cool or not.

Consider the frequency table

	<i>noisy</i> : +	<i>noisy</i> : -
<i>cool</i> : +	62	108
<i>cool</i> : -	38	292

find $p(cool : +)$ and $p(cool : + | noisy : +)$

and conclude from this whether or not *cool* : + is independent of *noisy* : +

5. Unknown to the occupant of the flat there is ventilator fixture in the wall which can be opened and shut to let air from the street outside in, or keep it out. Unknown to the occupant a pet cat plays about with this at night-time, sometimes leaving it open and sometimes leaving it shut. The table (1) concerns 500 days. The two tables below split these into a group of 100 days where the cat has left the ventilator open (2), and 400 days where the cat has left it shut (3)

<i>open</i> : +	<i>noisy</i> : +	<i>noisy</i> : -
<i>cool</i> : +	54	36
<i>cool</i> : -	6	4

<i>open</i> : -	<i>noisy</i> : +	<i>noisy</i> : -
<i>cool</i> : +	8	72
<i>cool</i> : -	32	288

With reference to the table (2), find $p(cool : + | open : +)$ and $p(cool : + | open : +, noisy : +)$ and conclude from this whether or not *cool* : + is conditionally independent of *noisy* : + given *open* : +.

With reference to the table (3), find $p(cool : + | open : -)$ and $p(cool : + | open : -, noisy : +)$ and conclude from this whether or not *cool* : + is conditionally independent of *noisy* : + given *open* : -