CS3081 Assignment 3

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Question 1 (Problem 4.26)

(i) (a) = 4, (b)= 7
(ii) (a)=2.2, (b)=7
(iii) (a)=4, (b)=2.2
(iv) (a)=7, (b)=4

Your Answer ((i)-(iv)): (i) a= 4, b = 7

Question 2 (Problem 6.13)

(i) 420W
(ii) 420KW
(iii) 530W
(iv) 580KW

Your Answer ((i)-(iv)): (iii) 530W

Question 3 (Problem 8.7)

The truncation error is:

(i) O(h)
(ii) O(h^2)
(iii) O(h^3)
(iv) O(h^4)

Your Answer ((i)-(iv)) (ii) O(h^2)

Question 4 (Problem 8.9)

(i) f’_male(2006)=4965;
f’_female(2006)=10681;
Predicted_Males(2008)=673601;
Error_Males=0.62%;
Predicted_Females(2008)=277990;
Error_Females=0.58%

(ii) f’_male(2006)=4940;
f’_female(2006)=10681;
Predicted_Males(2008)=673601;
Error_Males=0.62%;
Predicted_Females(2008)=277987;
Error_Females=0.57%

(iii) f’_male(2006)=4940;
f’_female(2006)=10681;
Predicted_Males(2008)=673601;
Error_Males=0.68%;
Predicted_Females(2008)=277987;
Error_Females=0.42%

(iv) f’_male(2006)=4965;
f’_female(2006)=10670;
Predicted_Males(2008)=673601;
Error_Males=0.68%;
Predicted_Females(2008)=277987;
Error_Females=0.52%

Your Answer ((i)-(iv)): (ii)

SOLUTIONS BELOW ON NEXT PAGE

Question 1 (Problem 4.26)

```
function N = InfinityNorm(A)
% Getting dimensions of matrix..
[m, n] = size(A);
% If matrix is not square, output error message and return..
if (m ~= n)
      disp("The matrix must be square");
      return;
end
% Setting up some variables
max = 0;
temp = 0;

% Get sum of all values of all rows and keep the max..
for i = 1:m
      for j = 1:n
      temp = temp + abs(A(i,j));
      end
      if (temp > max)
      max = temp;
      end
      temp = 0;
end
disp(max);
N = max;
end

% Answer = (i)
```

Question 2 (Problem 6.13)

Wind Speed (Mph)...   14, 22, 30, 38, 46.
Electric Power (W)... 320, 490, 540, 500, 480.

Lagrange Polynomials...

First-order polynomial: f(x) = ((x-x2)/(x1-x2))*y1 + ((x-x1)/(x2-x1))*y2

Second-order polynomial: f(x) = ((x-x2)*(x-x3)/(x1-x2)*(x1-x3))*y1 + .. + ((x-x1)*(x-x2)/(x3-x1)*(x3-x2))*y3

General formula:
f(x) = sumof(n)(i=1) yi * Li(x).. where Li(x) = productof(n)(j=1)(j!=i) (x-xj)/(xi-xj)

QUESTION:

Determine fourth-order polynomial in the Lagrange form that passes through the points..
use this polynomial to calculate the power at a wind speed of 26mph..

Fourth-order polynomial:
f(x) = ((x-x2)*(x-x3)*(x-x4)*(x-x5))/((x1-x2)*(x1-x3)*(x1-x4)*(x1-x5)) * y1
+ ((x-x1)*(x-x3)*(x-x4)*(x-x5))/((x2-x1)*(x2-x3)*(x2-x4)*(x2-x5)) * y2
+ ((x-x1)*(x-x2)*(x-x4)*(x-x5))/((x3-x1)*(x3-x2)*(x3-x4)*(x3-x5)) * y3
+ ((x-x1)*(x-x2)*(x-x3)*(x-x5))/((x4-x1)*(x4-x2)*(x4-x3)*(x4-x5)) * y4
+ ((x-x1)*(x-x2)*(x-x3)*(x-x4))/((x5-x1)*(x5-x2)*(x5-x3)*(x5-x4)) * y5

f(26) = ((26-22)*(26-30)*(26-38)*(26-46))/((14-22)*(14-30)*(14-38)*(14-46)) * 320
+ ((26-14)*(26-30)*(26-38)*(26-46))/((22-14)*(22-30)*(22-38)*(22-46)) * 490
+ ((26-14)*(26-22)*(26-38)*(26-46))/((30-14)*(30-22)*(30-38)*(30-46)) * 540
+ ((26-14)*(26-22)*(26-30)*(26-46))/((38-14)*(38-22)*(38-30)*(38-46)) * 500
+ ((26-14)*(26-22)*(26-30)*(26-38))/((46-14)*(46-22)*(46-30)*(46-38)) * 480

f(26) = (-12.5)
+ (229.6875)
+ (379.6875)
+ (-78.125)
+ (11.25)

f(26) = 530W

Answer = (iii)

Question 3 (Problem 8.7)

Derive a finite difference approximation formula for f''(xi) using three points:
1. xi-1
2. xi
3. xi+1,

Where the spacing is such that:
1. xi-(xi-1) = 2h
2. (xi+1)-xi = h

Taylor series:
f(x) = f(a) + (f'(a)/1!)(x-a) + (f''(a)/2!)(x-a)^2 + (f'''(a)/3!)(x-a)^3

Taylor series expansion for point xi+1:
f(xi+1) = f(xi) + f'(xi)((xi+1)-xi) + (f''(xi)/2!)((xi+1)-xi)^2

Sub h for (xi+1)-xi...

$f(xi+1) = f(xi) + f'(xi)(h) + (f''(xi)/2!)(h)^2$

Taylor series expansion for point xi-1:
$f(xi-1) = f(xi) - f'(xi)(xi-(xi-1)) + (f''(xi)/2!)(xi-(xi-1))^2$

Sub 2h for xi-(xi-1)...
$f(xi-1) = f(xi) - f'(xi)(2h) + (f''(xi)/2!)(2h)^2$

Add both equations...
$f(xi+1) + f(xi-1) = f(xi) + f'(xi)(h) + (f''(xi)/2!)(h)^2 +$
$f(xi) - f'(xi)(2h) + (f''(xi)/2!)(2h)^2$

$f(xi+1) + f(xi-1) = 2f(xi) - f'(xi)(h) + (5)(f''(xi)/2!)(h)^2$

Solve for f''(xi) and introduce truncation error...

$f(xi+1) + f(xi-1) = 2f(xi) - f'(xi)(h) + (5)(f''(xi)/2!)(h)^2$

$(5)(f''(xi)/2!)(h)^2 = f(xi+1) + f(xi-1) - 2f(xi) + f'(xi)(h)$

$(5)f''(xi)(h)^2 = 2(f(xi+1) + f(xi-1) - 2f(xi) + f'(xi)(h))$

$f''(xi)(h)^2 = 2(f(xi+1) + f(xi-1) - 2f(xi) + f'(xi)(h)) / 5$

$f''(xi) = (2(f(xi+1) + f(xi-1) - 2f(xi) + f'(xi)(h)) / 5) + O(h^2)$

Question 4 (Problem 8.9)

Year	1980	1990	2000	2002	2003	2006	2008
#Males	413,395	511,227	618,182	638,182	646,493	665,647	677,807
#Females	54,284	104,194	195,537	215,005	225,042	256,257	276,417

(A) Calculate rate of change in # of male and female physicians in 2006 by using 3-point backward difference formula for the derivative, with unequally spaced points.

$f'(xi+2) = ((xi+2)-(xi+1) / (xi-xi+1)(xi-xi+2))*yi +$

((xi+2)-(xi) / (xi+1-xi)(xi+1-xi+2))*yi+1 +
((2xi+2)-(xi)-(xi+1) / (xi+2-xi)(xi+2-xi+1))*yi+2

f'_male(2006) = ((2006-2003) / ((2002-2003)*(2002-2006)))*(638,182) +
((2006-2002) / ((2003-2002)*(2003-2006)))*(646,493) +
(((2*2006)-2002-2003) / ((2006-2002)*(2006-2003)))*(665,647)

f'_male(2006) = (478636.5) +
(-861990.666667) +
(388294.083333)

f'_male(2006) = 4939.916666

f'_female(2006) = ((2006-2003) / ((2002-2003)*(2002-2006)))*(215,005) +
((2006-2002) / ((2003-2002)*(2003-2006)))*(225,042) +
(((2*2006)-2002-2003) / ((2006-2002)*(2006-2003)))*(256,257)

f'_female(2006) = (161253.75) +
(-300056) +
(149483.25)

f'_female(2006) = 10,681

(B) Use result from part (A) and three-point central difference formula for the derivative with unequally spaced points to predict the number of male and female physicians in 2008.

f'(xi+1) = ((xi+1)-(xi+2) / (xi-xi+1)(xi-xi+2))*yi +
((2xi+1)-(xi)-(xi+2) / (xi+1-xi)(xi+1-xi+2))*yi+1 +
((xi+1)-(xi) / (xi+2-xi)(xi+2-xi+1))*yi+2

Male:
4939.916666 = ((2006-2008) / ((2003-2006)*(2003-2008)))*(646,493) +
(((2*2006)-2003-2008) / ((2006-2003)*(2006-2008)))*(665,647) +
((2006-2003) / ((2008-2003)*(2008-2006)))*(X)

4939.916666 = (-86199.0666667) + (-110941.166667) + (0.3X)

X = ((4949.916666) + (86199.0666667) + (110941.166667)) / 0.3

X = 673600.499979

ERROR: $|1 - (677{,}807 / 673600.499979)| = 0.006244 = 0.6244\%$

Female:

$$
\begin{aligned}
10{,}681 = {} & ((2006-2008) / ((2003-2006)*(2003-2008)))*(225{,}042) + \\
& (((2*2006)-2003-2008) / ((2006-2003)*(2006-2008)))*(256{,}257) + \\
& ((2006-2003) / ((2008-2003)*(2008-2006)))*(X)
\end{aligned}
$$

$$10{,}681 = (-30005.6) + (-42709.5) + (0.3X)$$

$$X = ((10{,}681) + (30005.6) + (42709.5)) / 0.3$$

$X = 277{,}987$

ERROR: $|1 - (276{,}419 / 277{,}987)| = 0.00564 = 0.564\%$