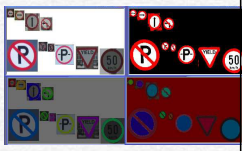
Binary

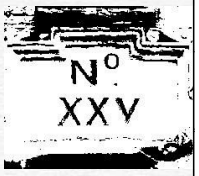
- Thresholding
- Threshold detection
- Variations
- Mathematical Morphology



1

Thresholding

- Binary thresholding
  - for all pixels
    - g(i,j) = 1 for f(i,j) ≥ T
    - = 0 for f(i,j) < T
  - Simple scenes?
  - LUT
    - for all grey levels
      - LUT(k) = 1 for k ≥ T
      - = 0 for k < T
    - for all pixels
      - g(i,j) = LUT(f(i,j))
  - Objects of interest vs. background

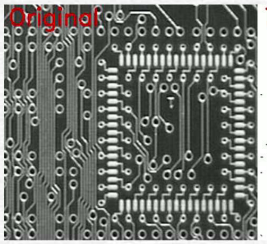



```
threshold(gray_image,binary_image,threshold,
                                    255,THRESH_BINARY);
```


2

Thresholding

- Distinct foreground & background needed



- How do we determine the best threshold?

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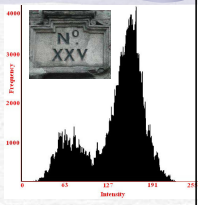
Threshold Detection

- Manual Setting
  - Issue of changing lighting
- Need to determine automatically



- For the techniques which follow:
  - Image – $f(i,j)$
  - Histogram – $h(g)$
  - Probability Distribution – $p(g) = h(g) / \Sigma_g h(g)$

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Threshold Detection – Otsu Thresholding

- Minimize the spread of the pixels…
  - Smallest within class variance

$$\sigma_W^2(T) = w_f(T)\sigma_f^2(T) + w_b(T)\sigma_b^2(T)$$

$$w_f(T) = \sum_{g=T}^{255} p(g) \qquad \sigma_f^2(T) = \sum_{g=T}^{255} p(g).(g-\mu_f(T))^2 \Big/ w_f(T)$$

$$w_b(T) = \sum_{a=0}^{T-1} p(g) \qquad \sigma_b^2(T) = \sum_{g=0}^{T-1} p(g).(g-\mu_b(T))^2 \Big/ w_b(T)$$

$$\mu_f(T) = \sum_{g=T}^{255} p(g).g \Big/ w_f(T) \qquad \mu_b(T) = \sum_{g=0}^{T-1} p(g).g \Big/ w_b(T)$$

  - Largest between class variance

$$\sigma_B^2(T) = w_f(T)w_b(T)\left(\mu_f(T) - \mu_b(T)\right)^2$$

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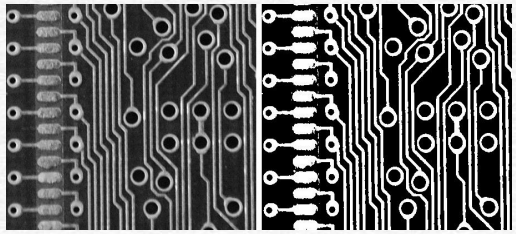
Threshold Detection – Otsu Thresholding



```
threshold( gray_image, binary_image, threshold,
           255, THRESH_BINARY | THRESH_OTSU );
```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Variations – Adaptive Thresholding

- The adaptive thresholding algorithm is
  - Divide the image into sub-images,
  - Compute thresholds for all sub-images,
  - Interpolate thresholds for every point using bilinear interpolation.

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Variations – Adaptive Thresholding

- *OpenCV version:*

if $((f(i,j) - (\Sigma_{a=-m..m,\ b=-m..m} f(i+a,j+b) / (2m+1)^2)) > offset)$
g(i,j) = 255
else g(i,j) = 0


```
adaptiveThreshold( gray_image,binary_image,output_value,
   ADAPTIVE_THRESH_MEAN_C,THRESH_BINARY,
   block_size,offset );
```


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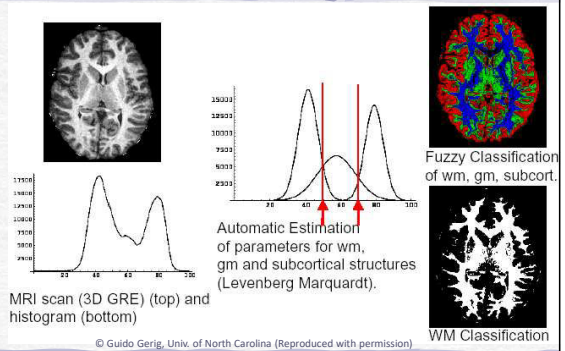
Variations – Multi-Spectral Thresholding

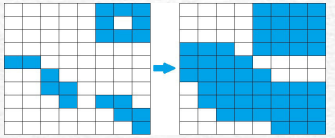


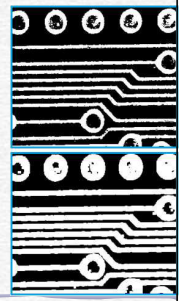
Cleaning Binary Images

- Cannot use normal smoothing operations
- Need to remove noisy points and smooth binary region boundaries.
- Mostly we use operations originally defined as part of "mathematical morphology" (which treats images as sets).
- Most common operations:
  - Erosion
  - Dilation
  - Opening
  - Closing

Mathematical Morphology – Dilation

- Minkowski set addition:
  - $X \oplus B = \{ p \in \varepsilon^2; p = x+b, x \in X \text{ and } b \in B \}$
  - B is the *structuring element*
  - B is typically isotropic
- Adds pixels around borders

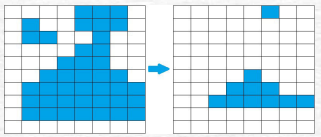


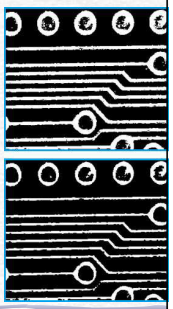


- Effects:
  - Makes 'regions' bigger
  - Fills small holes
  - Joins close 'regions'

Mathematical Morphology – Erosion

- Minkowski set subtraction:
  - $X \ominus B = \{ p \in \varepsilon^2; p+b \in X \text{ for every } b \in B \}$
  - Again B is typically isotropic
- Removes border pixels

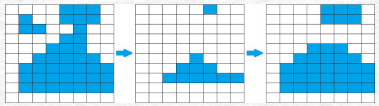




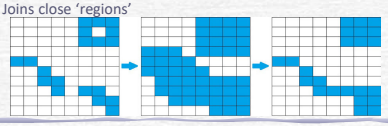
- Effects:
  - Makes 'regions' smaller
  - Removes noise
  - Removes narrow bridges

Mathematical Morphology –

- Opening: $X \circ D = (X \ominus D) \oplus D$
  - Removes noise
  - Removes narrow bridges
  - Roughly maintains 'region' size
  - Smooths shape



- Closing: $X \bullet D = (X \oplus D) \ominus D$
  - Fills small holes
  - Joins close 'regions'
  - Roughly maintains 'region' size



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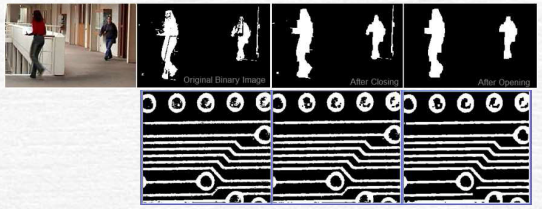
Mathematical Morphology –

- Opening:
  - $X \circ D = (X \ominus D) \oplus D$
- Closing:
  - $X \bullet D = (X \oplus D) \ominus D$
- Isotropic structuring element:
  - Eliminates small image details
- Properties
  - $X \circ D = (X \circ D) \circ D$ and $X \bullet D = (X \bullet D) \bullet D$



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Mathematical Morphology – OpenCV Code

```
dilate( binary_image, dilated_image, Mat());;

Mat structuring_element( 5, 5, CV_8U, Scalar(1) );
dilate( binary_image, dilated_image, structuring_element);

erode( binary_image, eroded_image, Mat());

Mat structuring_element( 5, 5, CV_8U, Scalar(1) );
erode( binary_image, eroded_image, structuring_element);

Mat five_by_five_element( 5, 5, CV_8U, Scalar(1) );
morphologyEx( binary_image, opened_image,
                        MORPH_OPEN,  five_by_five_element );
 morphologyEx( binary_image, closed_image,
                        MORPH_CLOSE, five_by_five_element );
```


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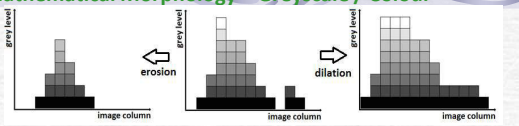
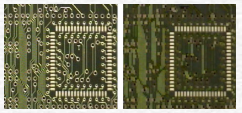
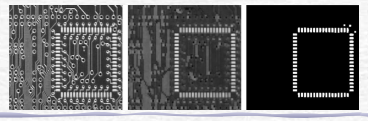
Mathematical Morphology – Greyscale / Colour



- One set per grey level (g)
  - All points >= g...





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Mathematical Morphology – Local maxima

- Can be used to locate local maxima and minima

```
Mat dilated, thresholded_input, local_maxima, thresholded_8bit;
dilate( input, dilated, Mat());
compare( input, dilated, local_maxima, CMP_EQ );
threshold( input, thresholded_input, threshold, 255,
                                                THRESH_BINARY );
thresholded_input.convertTo( thresholded_8bit, CV_8U );
bitwise_and( local_maxima, thresholded_8bit, local_maxima );
```


Edges

- Edge Detection
- Contour Segmentation
- Hough Transform
- Least Squared Error
- Random Sample Consensus (RANSAC)

Edges

- An approach to segmentation.
- The analysis of the discontinuities in an image.

- No correct answer?
- An alternative to region based processing.

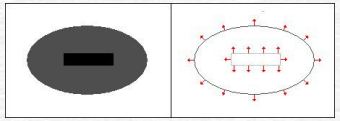
Edge Detection – Topics

- 1st derivative edge detection
- 2nd derivative edge detection
- Multispectral edge detection
- Image sharpening

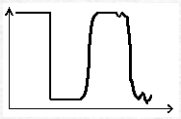
3

Edge Detection – What is an edge?

- Where brightness changes abruptly
- Edges have
  - Magnitude (Gradient)
  - Direction (Orientation)



- Edge Profiles
  - Step
  - Real
  - Noisy



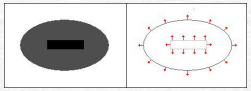
4

Edge Detection – 1st derivative definitions

- Calculus...
  - Rate of change in two directions
  - Vector variable:
    - Gradient Magnitude
    - Orientation (0^o is East)

$$\nabla f(i,j) = \sqrt{\left(\frac{\delta f(i,j)}{\delta i}\right)^2 + \left(\frac{\delta f(i,j)}{\delta j}\right)^2}$$

$$\phi(i,j) = \arctan\left(\frac{\delta f(i,j)}{\delta j}, \frac{\delta f(i,j)}{\delta i}\right)$$



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Edge detection – Digital images

- Derivatives work on continuous functions
  - Map every point in the input image to the output
- Discrete domain
  - Differences
  - Orthogonal

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Edge detection – 1st derivative – Roberts

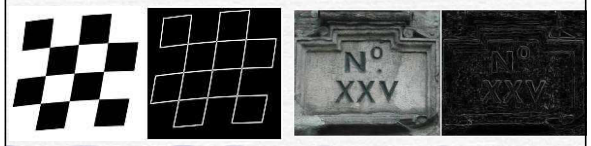
$$\delta_1(i,j) = f(i,j) - f(i+1,j+1)$$

$$\delta_2(i,j) = f(i,j+1) - f(i+1,j)$$

- Convolution Masks

$$h_1(i,j) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad h_2(i,j) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

- Sensitivity to Noise
- Binary Images



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Edge detection – Digital images

- Need to define the partial derivates so that they:
  1. Cross at a single middle point
  2. Preferably cross at the centre of a pixel
  3. Evaluate points which are not too close together
  4. Deal with some degree of image noise

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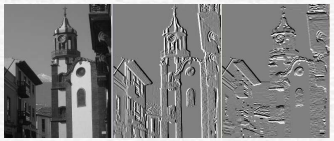
Edge detection – 1st derivative – Compass

- Compass edge detectors:
  - Partial derivatives defined for a number of orientations (typically 8)
  - Prewitt:

$$h_1(i,j) = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix} \quad h_2(i,j) = \begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ -1 & -1 & 0 \end{bmatrix} \quad h_3(i,j) = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \quad h_4(i,j) = \begin{bmatrix} -1 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$h_5(i,j) = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \quad h_6(i,j) = \begin{bmatrix} 0 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix} \quad h_7(i,j) = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix} \quad h_8(i,j) = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & -1 \end{bmatrix}$$

  - Use $h_3(i,j)$ and $h_1(i,j)$ to derive gradient and orientation



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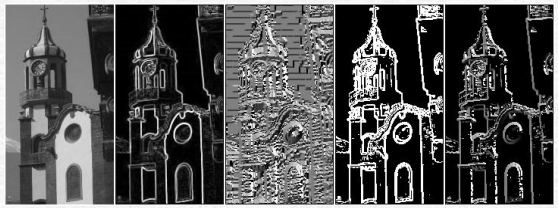
Edge detection – 1st derivative – Compass

- Sobel

$$h_1(i,j) = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix} \quad h_3(i,j) = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$



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Edge detection – 1st derivative – Compass

$$\nabla f(i,j) = \sqrt{\left(\frac{\delta f(i,j)}{\delta i}\right)^2 + \left(\frac{\delta f(i,j)}{\delta j}\right)^2}$$

$$\nabla f(i,j) = \left|\frac{\delta f(i,j)}{\delta i}\right| + \left|\frac{\delta f(i,j)}{\delta j}\right|$$

$$\phi(i,j) = \arctan\left(\frac{\delta f(i,j)}{\delta j}, \frac{\delta f(i,j)}{\delta i}\right)$$

```
Mat horizontal_derivative, vertical_derivative;
Sobel( gray_image, horizontal_derivative, CV_32F,1,0 );
Sobel( gray_image, vertical_derivative, CV_32F,0,1 );
Mat abs_gradient, l2norm_gradient, orientation;
abs_gradient = abs(horizontal_derivative) +
                                        abs(vertical_derivative);
cartToPolar(horizontal_derivative,vertical_derivative,
                                        l2norm_gradient,orientation);
```


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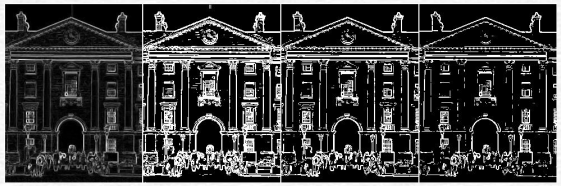
Edge detection – 1st derivative – Thresholding

- Simple thresholding
  - Too many points
  - Too few points



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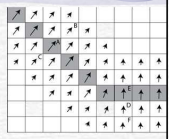
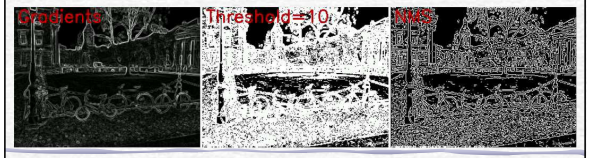
Edge detection – 1st derivative – Non maxima suppression

- Use orientation information
- Algorithm:
  - Quantise edge orientations
  - For all points (i,j)
    - Look at the 2 points orthogonal to edge
    - if gradient(i,j) < gradient(either of these 2 points)
      - output(i,j) = 0
      - else output(i,j) = gradient(i,j)
- Now thresholding can be used… or hysteresis thresholding (detailed later…)





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Edge detection – 1st derivative – NMS

```
nms_result = gradients.clone();
for (int row=1; row < gradients.rows-1; row++)
 for (int column=1; column < gradients.cols-1; column++)
 {
  float curr_gradient = gradients.at<float>(row,column);
  float curr_orientation = orientations.at<float>(row,column);
  // Determine which neighbours to check
  int direction = (((int) (16.0*(curr_orientation)/(2.0*PI))+15)%8)/2;
  float gradient1 = 0.0, gradient2 = 0.0;
  switch(direction)
  {
  case 0:
   gradient1 = gradients.at<float>(row-1,column-1);
   gradient2 = gradients.at<float>(row+1,column+1);
   break;
```


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Edge detection – 1st derivative – NMS

```
  case 1:
    gradient1 = gradients.at<float>(row-1,column);
    gradient2 = gradients.at<float>(row+1,column);
    break;
  case 2:
    gradient1 = gradients.at<float>(row-1,column+1);
    gradient2 = gradients.at<float>(row+1,column-1);
    break;
  case 3:
    gradient1 = gradients.at<float>(row,column+1);
    gradient2 = gradients.at<float>(row,column-1);
    break;
  }
  if ((gradient1 > curr_gradient) || (gradient2 > curr_gradient))
    nms_result.at<float>(row,column) = 0.0;
}
```


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Edge detection –Canny

- Combination of first and second derivative detectors
- Second derivative intended to improve localisation
- Thresholding incorporated so result is binary
- (Optional?) Analysis at multiple scales

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Edge detection – Canny algorithm

1. Convolve image with Gaussian
2. Compute the first derivative gradients and orientations
3. Apply non-maxima suppression
4. Apply a double threshold to find strong & weak edge points.
5. Threshold edges with hysteresis
   - Remove any weak edges points not connected to strong edge points.

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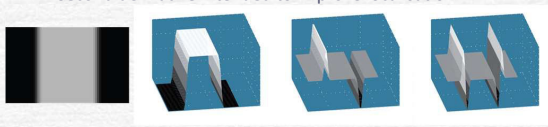
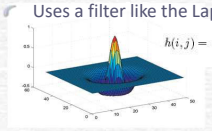
Edge detection – Canny – uses 2nd derivative?

- According to the original paper Canny is supposed to be a combination of first and second derivative detectors
  - Second derivative intended to improve localisation



- Uses a filter like the Laplacian (of Gaussian)



$$h(i,j) = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

- Need to fine zero-crossings.

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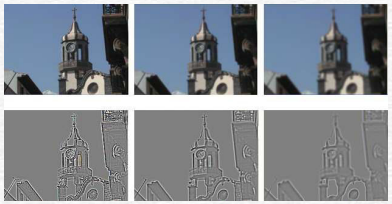
Edge detection –Canny

- Also according to the original paper the image is supposed to be analysed at multiple scales
  - Smooth image using different Gaussians
  - Look for correspondences across scale to identify significant discontinuities



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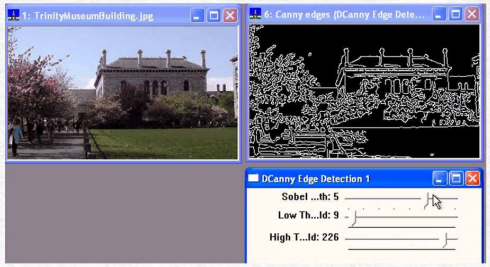
Edge detection – advanced – Canny




```
Canny( gray_image, binary_edges, 100, 200 );
```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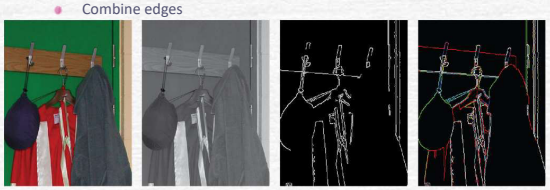
Multispectral edge detection

- Grey levels are ordered, colours are not.
- Multispectral edge detection is addressed in 3 ways:
  1. Output fusion
     - Combine edges



  2. Multidimensional gradient methods
     - Combine gradients
  3. Vector methods
     - Distance between colours in 3D

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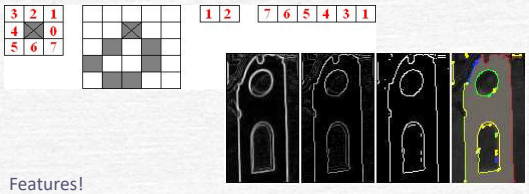
Contour Segmentation – Topics

- Edge data representations
- Border Detection
- Line segment extraction

22

Contour Segmentation – Boundary Chain Codes

- Each chain contains
  - Start point
  - A list of orientations



- Features!
  - Orientation & Scale dependent
  - Slightly position dependent
  - Can be smoothed to reduce boundary noise
  - Difficulties obtaining consistent representation from image.

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Contour Segmentation – Boundary Chain Codes

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Contour Segmentation – Boundary Chain Codes

```
vector<vector<Point>> contours;
vector<Vec4i> hierarchy;
findContours( binary_edge_image, contours, hierarchy,
              CV_RETR_CCOMP, CV_CHAIN_APPROX_NONE );
```

```
for (int contour_number=0;
     (contour_number<contours.size()); contour_number++)
{
  Scalar colour( rand()&0xFF, rand()&0xFF, rand()&0xFF );
  drawContours( display_image, contours, contour_number,
                colour, 1, 8, hierarchy );
}
```


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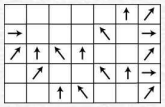
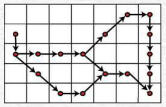
Contour Segmentation

- Represent an edge image as a **graph**:
  - Node n_i
    - Corresponds to pixel x_i
    - Associated cost $s(x_i)$ and orientation $\phi(x_i)$
  - n_i, n_j are connected by an **arc** if
    - Pixels x_i, x_j are 8-connected neighbours
    - $\phi(x_i)$ and $\phi(x_j)$ match the local border:
      - x_j must be one of 3 neighbours in the direction $[\phi(x_i)-\pi/4, \phi(x_i)+\pi/4]$
      - $| \phi(x_i) - \phi(x_j) | < \pi/2$



- When expanding edge chains consider
  - Strength of edges
    - E.g. $(\max_{image} s(x_k)) - s(x_i)$
  - Border curvature
    - diff[$\phi(x_i) - \phi(x_j)$]



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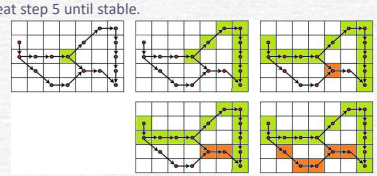
Contour Segmentation

- Purpose: Segment all edge chains in an image
- Algorithm:
  1. Search for the strongest (unused) node in the graph. If none go to step 5.
  2. Expand all the edges in front of the specified edge
  3. Expand all the edges behind of the specified edge
  4. If the edge chain consists of > 3 pixels store it. Go to step 1.
  5. Modify edge chains to fill small breaks and to remove duplicate contours.
  6. Repeat step 5 until stable.



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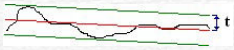
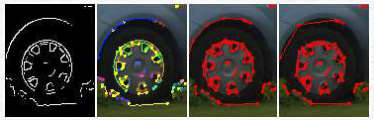
Contour Segmentation – Segment sequences

- Boundary
  - Sequence of segments
- Segment
  - Start and end points: x_1 and x_2
  - Type: straight line, some type of curve, etc.
  - Other parameters
- Accurate polygonal representation
  - Must define acceptable tolerances
  - Vertices – where?





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Contour Segmentation – Straight line extraction example

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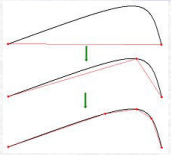
Contour Segmentation – Obtaining polygonal segments

- Recursive boundary splitting
  - Split at furthest point
  - Keep going until with tolerance



```
vector<vector<Point>> approx_contours(contours.size());
for (int contour_number=0;
        (contour_number<contours.size()); contour_number++)
    approxPolyDP( Mat(contours[contour_number]),
                  approx_cont ours[contour_number], 3, true );
```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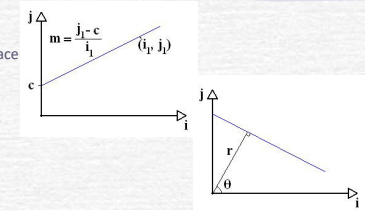
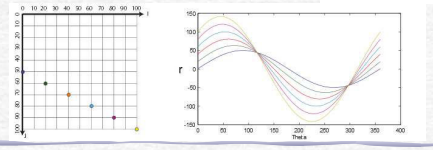
Hough transform

- Direct transformation from image space to probability of the existence of some feature…
  - Lines
  - Circles
  - Generalised shapes

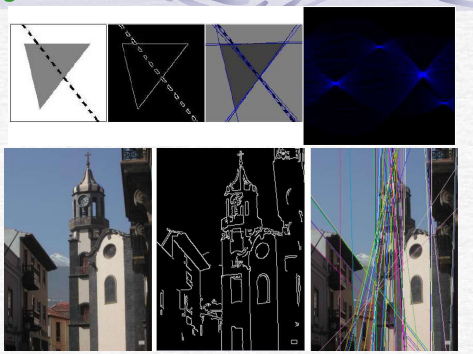
Hough Transform – Line detection

- Line equation:
  - j = m.i + c
    - Lines in Hough space
    - What about i = c
  - r = i.cos θ + j.sin θ
    - Sinusoidal curves



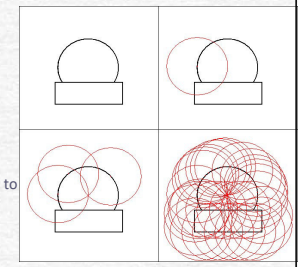


Hough Transform – Line detection

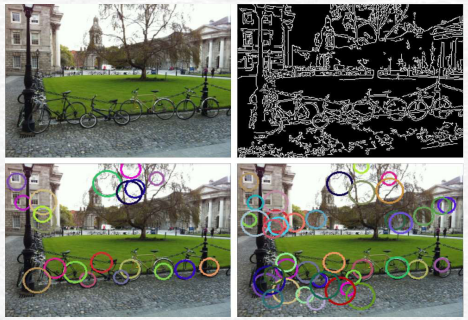


Hough Transform – Circle detection

- Equation of a circle $(i-a)^2 + (j-b)^2 = r^2$
  - Assume constant radius *r*
- Transform
  - From Image space (x, y)
  - To Hough space (a, b)
- Algorithm
  - Initialise accumulator to 0
  - For every edge point
    - Increment cells in accumulator corresponding to all possible circle centers
  - Search for Maximums



Hough Transform – Circle detection



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Hough transform – in OpenCV

- Hough for lines:

```
vector<Vec2f> hough_lines;
HoughLines( binary_edge_image, hough_lines, 1, PI/200.0, 60);
```

- Probabilistic Hough for line segments:

```
vector<Vec4i> line_segments;
HoughLinesP( binary_edge_image, line_segments, 1.0,
                                                PI/200.0, 20, 20, 5);
```

- Hough for circles:

```
vector<Vec3f> circles;
HoughCircles( gray_image, circles, CV_HOUGH_GRADIENT,
                                        2,20,300,20,5,15);
```


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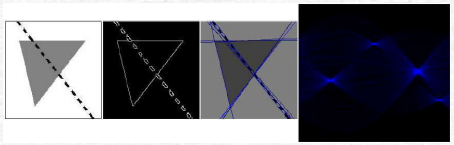
Hough Transform – Resolution of Hough Space

- Circle
  - (a, b) could be anywhere up to *r* outside the image space
  - Can detect the circle centre to higher precision than the image points
- Lines
  - $-\text{dist}(0,0,M,N) \leq r \leq +\text{dist}(0,0,M,N)$
  - $-\pi \leq \theta \leq \pi$
  - Precision of s and θ application dependent



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Hough Transform – Efficient Implementation

- Use edge orientations
  - Restrict the mapping into space
- Use half the accumulator
  - For lines only
- Multi-resolution
  - Process at a small resolution
  - Higher resolution to find accurate data
- Problem
  - What if the size of the circle is unknown
  - 3-D Accumulator??

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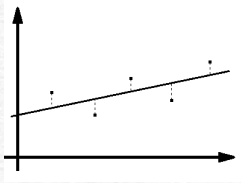
Other techniques for estimating pose (lines ++)

- Common to have a number of data points which may
  - form part of a feature (such as a straight line) OR
  - represent possible matches with some known object features
- We need to determine the best possible position or pose of this feature or object
- Techniques:
  - Least squared error solution
  - RANSAC

Least squared error solution

- Linear fit which best matches the data.
- For a straight line it minimises the sum of the vertical residuals



- Line equation: $y = m.x + c$
- First compute the slope m
- Then the intercept c

Least squared error solution

Given (x_i, y_i) where $i = 1..N$

$\mu_x = \frac{1}{N}\sum_{i=1}^{N} x_i$ $\mu_y = \frac{1}{N}\sum_{i=1}^{N} y_i$

$\sigma_x = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \mu_x)^2}$ $\sigma_y = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(y_i - \mu_y)^2}$

Pearson's correlation coefficient: $\rho_{xy} = \frac{COV_{xy}}{\sigma_x \sigma_y} = \frac{\sum_{i=1}^{N}(x_i-\mu_x)(y_i-\mu_y)}{\sigma_x \sigma_y}$

$m = \rho_{xy}\frac{\sigma_y}{\sigma_x}$ $c = \mu_y - m.\mu_x$

Least squared error solution – Problems

- Assumes line is not vertical
- Assumes that error distribution is normal
- Assumes that the points which should be included in the regression are known (i.e. segmented)
- Assumes no significant outliers

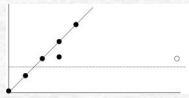


Figure 10.7: Influence of an outlier in least squares line fitting. With 6 valid data points and 1 gross outlier (white), the best line is shown in solid. Least squares, followed by discarding the worst outlier, reaches the dotted line after 3 discards [Fischler and Bolles, 1981]. © *Cengage Learning 2015.*

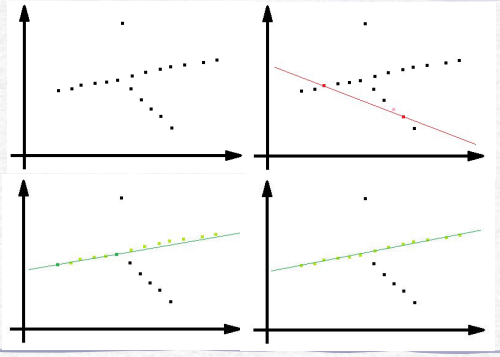
Random Sample Consensus (RANSAC)

- Uses the minimum number of data points (m) to determine the model
  - For a straight line $m=2$

Technique

1. Randomly select the minimum number (m) of data points from the N data points (x_i, y_i) where $i = 1..N$
2. Determine the model from the selected data points
3. Determine how many data points are within some tolerance of the model – the consensus set
4. If the consensus set is not big enough (i.e. is smaller than some pre-set threshold) go back to step 1 (OR fail if tried too often).
5. If the consensus set was big enough, re-compute the model using all points in the consensus set.

RANSAC Example



RANSAC Example

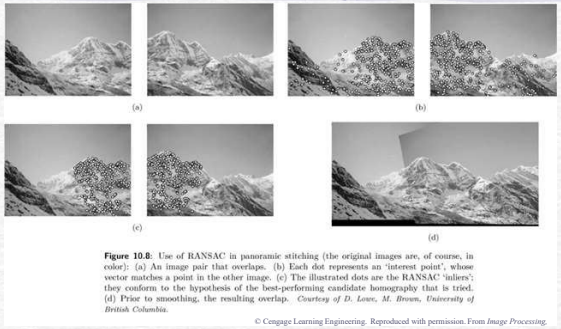


Figure 10.8: Use of RANSAC in panoramic stitching (the original images are, of course, in color): (a) An image pair that overlaps. (b) Each dot represents an 'interest point', whose vector matches a point in the other image. (c) The illustrated dots are the RANSAC 'inliers': they conform to the hypothesis of the best-performing candidate homography that is tried. (d) Prior to smoothing, the resulting overlap. *Courtesy of D. Lowe, M. Brown, University of British Columbia.*

RANSAC Examples

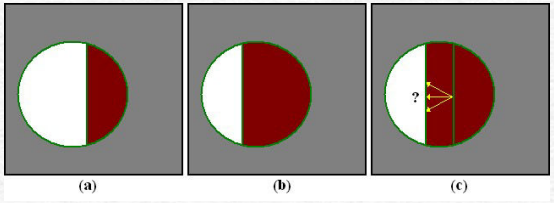
Features

- Introduction
- Moravec Corner Detection
- Harris/Plessey Corner Detection
- Scale Invariant Feature Transform (SIFT)

1

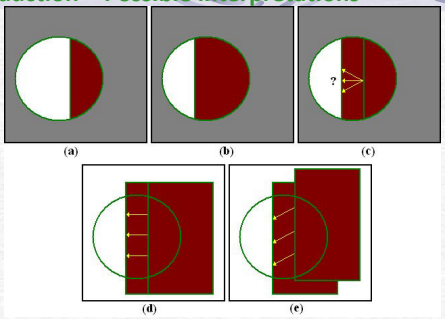
Introduction – Where have the edges gone?

- Given two images (a) and (b) taken at different times determine the movement of edge points from frame to frame…



2

Introduction – Possible interpretations

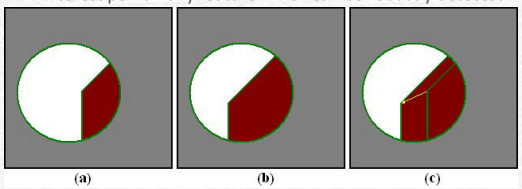


- The Aperture Problem.

3

Introduction – Using Features / Corners instead

- Use corners / image features / interest points
  - Corner = intersection of two edges
  - Interest point = any feature which can be robustly detected

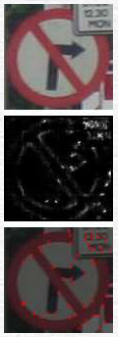


- Reduces number of points
- Easier to establish correspondences
- Spurious features

4

Introduction – Steps for corner detection

1. Determine cornerness values.
   - For each pixel
   - Main difference
   - Produces a Cornerness map.
2. Non-maxima suppression.
   - Multiple responses
   - Compare to local neighbours
3. Threshold the cornerness map.
   - Significant corners.



Moravec corner detection

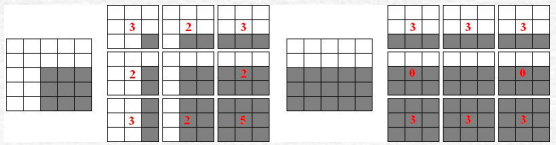
- Looks at the local variation around a point
  - Compares local image patches

$$V_{u,v}(i,j) = \sum_{\forall a,b \in Window} (f(i+u+a, j+v+b) - f(i+a, j+b))^2$$

  - where *(u,v)* $\in \{ (-1,-1), (-1,0), (-1,1), (0,-1), (0,1) (1,-1), (1, 0), (1,1,) \}$ and the Window is typically 3x3, 5x5 or 7x7
  - Select the Minimum value of $V_{u,v}(i,j)$

Moravec – Binary Example

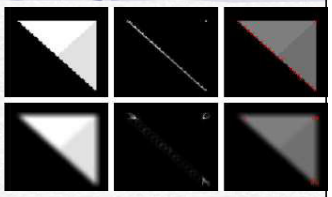
Corner: Edge:



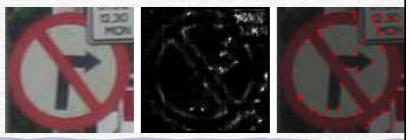
Minimum difference: 2 Minimum difference: 0

Moravec – Flaws.

- Anisotropic response
  - Diagonal lines
  - Smoothing



- Noisy response
  - Larger area
  - Smoothing



Harris / Plessey corner detection

- Difference – Cornerness determination
- Uses
  - Partial derivatives
  - Gaussian weighting
  - Matrix Eigenvalues

```
Ptr<FeatureDetector> harris_detector =
                GFTTDetector::create( 1000, 0.01, 10, 3, true );
vector<KeyPoint> keypoints;
harris_detector->detect( gray_image, keypoints );
```


9

Harris / Plessey corner detection

- Consider the intensity variation for an arbitrary shift (Δi, Δj) as

$$SSD_W(\Delta i, \Delta j) = \sum_{(i,j)\in W} (f(i,j) - f(i-\Delta i, j-\Delta j))^2$$

- If $f(i-\Delta i, j-\Delta j) \approx f(i,j) + \begin{bmatrix} \frac{\delta f(i,j)}{\delta i} & \frac{\delta f(i,j)}{\delta j} \end{bmatrix} \begin{bmatrix} \Delta i \\ \Delta j \end{bmatrix}$
- Then $SSD_W(\Delta i, \Delta j) = \sum_{(i,j)\in W} \left(f(i,j) - f(i,j) - \begin{bmatrix} \frac{\delta f(i,j)}{\delta i} & \frac{\delta f(i,j)}{\delta j} \end{bmatrix} \begin{bmatrix} \Delta i \\ \Delta j \end{bmatrix} \right)^2$

$$SSD_W(\Delta i, \Delta j) = \sum_{(i,j)\in W} \left(\begin{bmatrix} \frac{\delta f(i,j)}{\delta i} & \frac{\delta f(i,j)}{\delta j} \end{bmatrix} \begin{bmatrix} \Delta i \\ \Delta j \end{bmatrix} \right)^2$$

$$SSD_W(\Delta i, \Delta j) = \sum_{(i,j)\in W} \left(\begin{bmatrix} \Delta i & \Delta j \end{bmatrix} \left(\begin{bmatrix} \frac{\delta f(i,j)}{\delta i} \\ \frac{\delta f(i,j)}{\delta j} \end{bmatrix} \begin{bmatrix} \frac{\delta f(i,j)}{\delta i} & \frac{\delta f(i,j)}{\delta j} \end{bmatrix} \right) \begin{bmatrix} \Delta i \\ \Delta j \end{bmatrix} \right)$$

$$SSD_W(\Delta i, \Delta j) = \begin{bmatrix} \Delta i & \Delta j \end{bmatrix} \begin{bmatrix} \sum_{(i,j)\in W} \left(\frac{\delta f(i,j)}{\delta i} \right)^2 & \sum_{(i,j)\in W} \frac{\delta f(i,j)}{\delta i} \frac{\delta f(i,j)}{\delta j} \\ \sum_{(i,j)\in W} \frac{\delta f(i,j)}{\delta i} \frac{\delta f(i,j)}{\delta j} & \sum_{(i,j)\in W} \left(\frac{\delta f(i,j)}{\delta j} \right)^2 \end{bmatrix} \begin{bmatrix} \Delta i \\ \Delta j \end{bmatrix}$$

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Harris / Plessey corner detection

- From the matrix we can compute the Eigenvalues
  - Both high => corner
  - One high => edge
  - None high => constant region
- Harris & Stephens proposed the following cornerness measure:

$$M = \begin{bmatrix} \sum_{(i,j)\in W} \left(\frac{\delta f(i,j)}{\delta i} \right)^2 & \sum_{(i,j)\in W} \frac{\delta f(i,j)}{\delta i} \frac{\delta f(i,j)}{\delta j} \\ \sum_{(i,j)\in W} \frac{\delta f(i,j)}{\delta i} \frac{\delta f(i,j)}{\delta j} & \sum_{(i,j)\in W} \left(\frac{\delta f(i,j)}{\delta j} \right)^2 \end{bmatrix} = \begin{bmatrix} A & B \\ B & C \end{bmatrix}$$

$$\det(M) = \lambda_1 \lambda_2 = AC + B^2$$

$$\text{trace}(M) = \lambda_1 + \lambda_2 = A + C$$

$$C(i,j) = \det(M) - k(trace(M))^2$$

0.04	**0.12**	**0.04**
0.12	**0.36**	**0.12**
0.04	**0.12**	**0.04**

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Harris / Plessey corner detection

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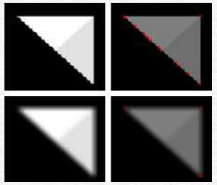
Harris / Plessey – Pros and Cons

- Cons:
  - More expensive computationally
  - Sensitive to noise
  - Somewhat anisotropic
- Pros:
  - Very repeatable response
  - Better detection rate



```
Mat display_image;
drawKeypoints( image, keypoints, display_image,
                                    Scalar( 0, 0, 255 ) );
```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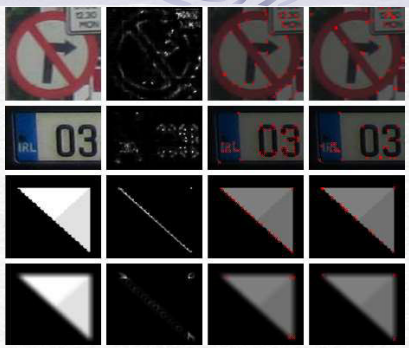
Moravec & Harris/Plessey



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Scale Invariant Feature Transform (SIFT)

- FROM "Distinctive Image Features from Scale-Invariant Keypoints",
- by David G. Lowe in International Journal of Computer Vision, 60, 2 (2004), pp.91-110
- Motivation:
  - Providing repeatable robust features for
    - Tracking,
    - Recognition,
    - Panorama Stitching, etc.

- Features:
  - Invariant to scaling, & rotation.
  - Partly invariant to illumination and viewpoint changes

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SIFT – Overview & Contents

1. Scale Space Extrema Detection
   - Scale Space
   - Difference of Gaussian
   - Locate Extrema
2. Accurate Keypoint Location
   - Sub-pixel locate
   - Filter response – remove low contrast and features primarily along an edge
3. Keypoint Orientation assignment
4. Keypoint Descriptors

- Matching Descriptors – including dropping poor ones
- Applications

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SIFT – OpenCV code

```
// The following code is valid up to v2.4.11:
Ptr<FeatureDetector> feature_detector =
                                    FeatureDetector::create("SIFT");
vector<KeyPoint> keypoints;
feature_detector->detect( gray_image, keypoints );
```


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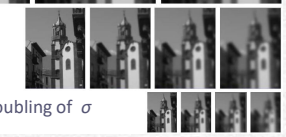
SIFT – Scale Space Extrema Detection

- For scale invariance, consider the image at multiple scales
- $L(x,y,\sigma)$ $L(x,y,k\sigma)$ $L(x,y,k^2\sigma)$ $L(x,y,k^3\sigma)$

- $L(x,y,\sigma) = G(x,y,\sigma) * I(x,y)$
- Applied in different octaves of scale space
- Each octave corresponds to a doubling of σ



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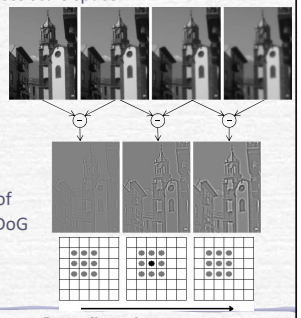
SIFT – Scale Space Extrema Detection

- Stable keypoint locations defined to be at extrema in the Difference of Gaussian (DoG) functions across scale space…
- $D(x,y,\sigma) = L(x,y,k\sigma) - L(x,y,\sigma)$



- Extrema..
  - Centre point is Min or Max of
  - Local 3x3 region in current DoG
  - and in adjacent scales
  - IN any octave

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SIFT – Accurate Keypoint Location

- Originally location and scale taken from central point
- Locate keypoints more precisely
  - Model data locally using a 3D quadratic
  - Locate interpolated maximum/minimum

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SIFT – Accurate Keypoint Location

- Discard low contrast keypoints
  - If the local contrast is too low discard the keypoint
  - Evaluated from the curvature of the 3D quadratic…

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SIFT – Accurate Keypoint Location

- Discard poorly localised keypoints (e.g. along an edge)

$$\mathbf{H} = \begin{bmatrix} D_{xx} & D_{xy} \\ D_{xy} & D_{yy} \end{bmatrix}$$

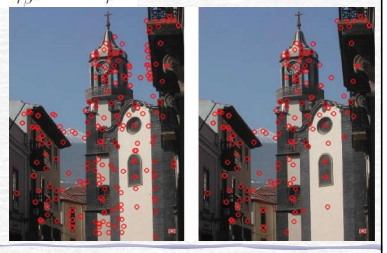
$$\mathrm{Tr}(\mathbf{H}) = D_{xx} + D_{yy} = \alpha + \beta$$

$$\mathrm{Det}(\mathbf{H}) = D_{xx}D_{yy} - (D_{xy})^2 = \alpha\beta$$

$$\alpha = r\beta$$

$$\frac{\mathrm{Tr}(\mathbf{H})^2}{\mathrm{Det}(\mathbf{H})} = \frac{(\alpha+\beta)^2}{\alpha\beta} = \frac{(r\beta+\beta)^2}{r\beta^2} = \frac{(r+1)^2}{r}$$

$$\frac{\mathrm{Tr}(\mathbf{H})^2}{\mathrm{Det}(\mathbf{H})} < \frac{(r+1)^2}{r}$$



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SIFT – Keypoint Orientation

- For scale invariance, the keypoint scale is used to select the smoothed image with the closet scale
- For orientation invariance we describe the keypoint wrt. the principal orientation
  - Create an orientation histogram (36 bins)

$$m(x,y) = \sqrt{(L(x+1,y) - L(x-1,y))^2 + (L(x,y+1) - L(x,y-1))^2}$$

$$\theta(x,y) = \tan^{-1}((L(x,y+1) - L(x,y-1))/(L(x+1,y) - L(x-1,y)))$$

  - Weight by gradient magnitude
  - Sample points around the keypoint
  - Highest peak + peaks within 80%
    - Oriented keypoint(s)
  - Stable results….

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SIFT – Keypoint Description

- Could sample image intensity at relevant scale
  - Match using normalized cross correlation
  - Sensitive to
    - affine transformations,
    - 3D viewpoint changes and
    - non-rigid deformations
- A better approach (Edelman et al. 1997)
  - Based on a model of biological vision
  - Consider gradients at particular orientations and spatial frequencies
  - Location not required to be precise

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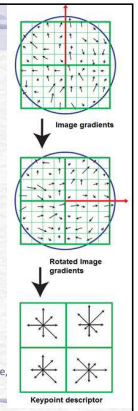
SIFT – Keypoint Description

- Use blurred image at closest scale
- Sample points around the keypoint
- Compute gradients and orientations
- Rotate by keypoint orientation
- Divide region into subregions
- Create histograms (8 bins) for subregions
  - Weight by gradient
  - Weight by location (Gaussian)
  - Distribute into bins (trilinear interpolation)



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SIFT – Matching Keypoints

- Nearest neighbour matching
  - Euclidean distance between keypoints
  - Use a global distance threshold?
  - Better to…
    - Compare distance to closest neighbour to distance to 2nd closest neighbour (from a different object)
    - Distance ratio > 0.8 eliminates
      - 90% false matches
      - 5% correct matches

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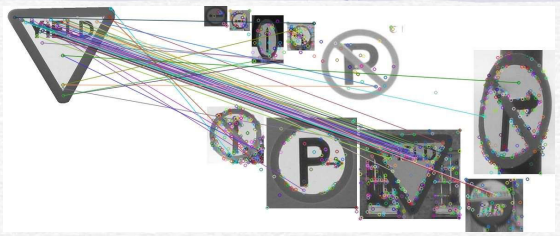
SIFT – Matching Keypoints



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SIFT – OpenCV code

```
// The following code is valid up to v2.4.11:
SiftFeatureDetector sift_detector;
vector<KeyPoint> keypoints1, keypoints2;
sift_detector.detect( gray_image1, keypoints1 );
sift_detector.detect( gray_image2, keypoints2 );
// Extract feature descriptors
SiftDescriptorExtractor sift_extractor;
Mat descriptors1, descriptors2;
sift_extractor.compute( gray_image1, keypoints1, descriptors1 );
sift_extractor.compute( gray_image2, keypoints2, descriptors2 );

…
```


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SIFT – OpenCV code

```
…
// Match descriptors.
BFMatcher sift_matcher(NORM_L2);
vector< DMatch > matches;
matcher.match( descriptors1, descriptors2, matches );
// Display SIFT matches
Mat display_image;
drawMatches( gray_image1, keypoints1, gray_image2,
                          keypoints2, matches, display_image );
```


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SIFT – Recognition

- Match at least 3 features
- Cluster matches
  - Hough transform
    - Location (2D)
    - Scale
    - Orientation
  - Really a 6D problem
    - Use broad bins
      - 30^O for orientation
      - Factor of 2 for scale
      - 0.25 times image dimension for location
  - Consider all bins with at least 3 entries
    - Determine affine transformation

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SIFT – Application: Recognition

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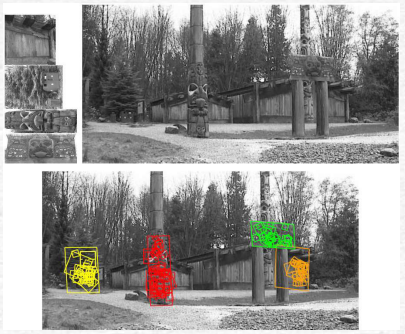
SIFT – Application: Recognition



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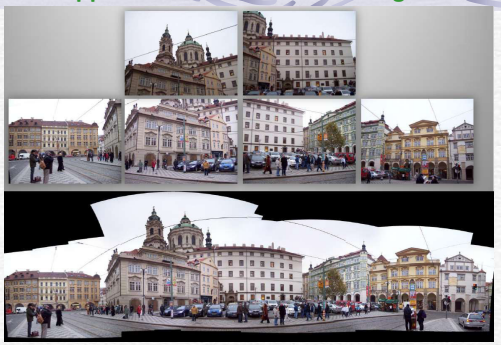
SIFT – Application: Panorama Stitching



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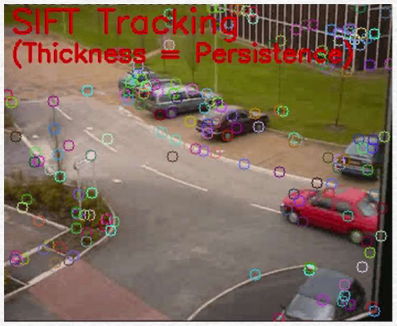
SIFT – Application: Tracking



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Geometric

- Geometric transformations
- Pixel co-ordinate transformationns
- Brightness interpolation

1

Geometric transformations

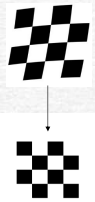
- Computer graphics
  - Introduce distortion
- Image processing
  - Image mosaicing
  - Matching/Registering image
  - Eliminating distortion
  - Simplifying further processing
    - e.g. OCR

Examples

2

Geometric transformations - Specification

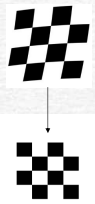
- Problem:
  - Distorted image $f(i,j)$
  - Corrected image $f'(i',j')$
- Mapping: $i = T_i(i',j')$ $j = T_j(i',j')$



3

Geometric transformations - Specification

- Application steps:
  - Define the transformation
    - Known in advance
    - Determine through correspondences
      - Image to known
      - Image to image
  - Apply the transformation
    - For every point in the output image
      - Determine where it came from using T
      - Interpolate a value for the output point



4

Output ➔ Input

- A backwards transformation?
  - Why aren't we mapping from the input image to the output?
    - For each point maintain the new continuous coordinates
  - For each output point
    - Find the nearest transformed point(s)
    - Interpolate a value from this point(s)
- We would have to
  - Store all real coordinates
  - Search for nearest values for each output pixel
  - Interpolate in corrected image space

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Affine transformations

- Definition

$$\begin{bmatrix} i \\ j \end{bmatrix} = \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \end{bmatrix} \begin{bmatrix} i' \\ j' \\ 1 \end{bmatrix}$$

- Known transformations...
  - E.g. Translation

$$\begin{bmatrix} i \\ j \end{bmatrix} = \begin{bmatrix} 1 & 0 & m \\ 0 & 1 & n \end{bmatrix} \begin{bmatrix} i' \\ j' \\ 1 \end{bmatrix}$$

- Unknown transformations...
  - Requires at least 3 observations

```
Mat affine_matrix( 2, 3, CV_32FC1 );
…
warpAffine( image, result, affine_matrix, image.size() );
```


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Simple affine transformations

- Rotation

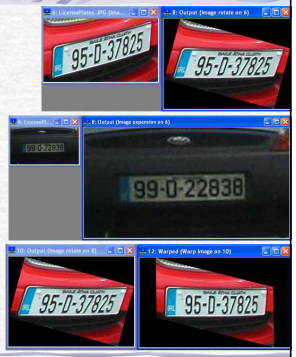
$$\begin{bmatrix} i \\ j \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} i' \\ j' \\ 1 \end{bmatrix}$$

- Change of scale

$$\begin{bmatrix} i \\ j \end{bmatrix} = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \end{bmatrix} \begin{bmatrix} i' \\ j' \\ 1 \end{bmatrix}$$

- Skewing

$$\begin{bmatrix} i \\ j \end{bmatrix} = \begin{bmatrix} 1 & \tan\phi & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} i' \\ j' \\ 1 \end{bmatrix}$$



More affine transformations

- Panoramic distortion

$$\begin{bmatrix} i \\ j \end{bmatrix} = \begin{bmatrix} a & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} i' \\ j' \\ 1 \end{bmatrix}$$

Unknown affine transformations

- Given...

$$\begin{bmatrix} i \\ j \end{bmatrix} = \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \end{bmatrix} \begin{bmatrix} i' \\ j' \\ 1 \end{bmatrix}$$

- If we have 3 observations...

$$(i_1, j_1) \leftrightarrow (i'_1, j'_1)$$
$$(i_2, j_2) \leftrightarrow (i'_2, j'_2)$$
$$(i_3, j_3) \leftrightarrow (i'_3, j'_3)$$

- We can reorganise..

$$\begin{bmatrix} i_1 \\ j_1 \\ i_2 \\ j_2 \\ i_3 \\ j_3 \end{bmatrix} = \begin{bmatrix} i'_1 & j'_1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & i'_1 & j'_1 & 1 \\ i'_2 & j'_2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & i'_2 & j'_2 & 1 \\ i'_3 & j'_3 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & i'_3 & j'_3 & 1 \end{bmatrix} \begin{bmatrix} a_{00} \\ a_{01} \\ a_{02} \\ a_{10} \\ a_{11} \\ a_{12} \end{bmatrix}$$

- And take the inverse to compute the a coefficients

Unknown affine transformations


```
Point2f source [3], destination [3];
…
affine_matrix = getAffineTransform(source,destination);
```

- More observations
  - Better estimate of the a coefficients
  - Must use the psuedo inverse

Perspective transformations

- Perspective projection
  - Planar surface
  - Not parallel to the image plane.
    - Cannot be corrected with the affine transformation
  - Need a perspective transformation

$$\begin{bmatrix} i.w \\ j.w \\ w \end{bmatrix} = \begin{bmatrix} p_{00} & p_{01} & p_{02} \\ p_{10} & p_{11} & p_{12} \\ p_{20} & p_{21} & 1 \end{bmatrix} \begin{bmatrix} i' \\ j' \\ 1 \end{bmatrix}$$

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Perspective transformations

- From $\begin{bmatrix} i.w \\ j.w \\ w \end{bmatrix} = \begin{bmatrix} p_{00} & p_{01} & p_{02} \\ p_{10} & p_{11} & p_{12} \\ p_{20} & p_{21} & 1 \end{bmatrix} \begin{bmatrix} i' \\ j' \\ 1 \end{bmatrix}$

- We know $i.w = p_{00}.i' + p_{01}.j' + p_{02}$

  $w = p_{20}.i' + p_{21}.j' + 1$
- Hence $i = p_{00}.i' + p_{01}.j' + p_{02} - p_{20}.i.i' - p_{21}.i.j'$
- And similarly $j = p_{10}.i' + p_{11}.j' + p_{12} - p_{20}.j.i' - p_{21}.j.j'$
- If we observe 4 mappings...

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Perspective transformations

- Then

$$\begin{bmatrix} i_1 \\ j_1 \\ i_2 \\ j_2 \\ i_3 \\ j_3 \\ i_4 \\ j_4 \end{bmatrix} = \begin{bmatrix} i'_1 & j'_1 & 1 & 0 & 0 & 0 & -i_1 i'_1 & -i_1 j'_1 \\ 0 & 0 & 0 & i'_1 & j'_1 & 1 & -j_1 i'_1 & -j_1 j'_1 \\ i'_2 & j'_2 & 1 & 0 & 0 & 0 & -i_2 i'_2 & -i_2 j'_2 \\ 0 & 0 & 0 & i'_2 & j'_2 & 1 & -j_2 i'_2 & -j_2 j'_2 \\ i'_3 & j'_3 & 1 & 0 & 0 & 0 & -i_3 i'_3 & -i_3 j'_3 \\ 0 & 0 & 0 & i'_3 & j'_3 & 1 & -j_3 i'_3 & -j_3 j'_3 \\ i'_4 & j'_4 & 1 & 0 & 0 & 0 & -i_4 i'_4 & -i_4 j'_4 \\ 0 & 0 & 0 & i'_4 & j'_4 & 1 & -j_4 i'_4 & -j_4 j'_4 \end{bmatrix} \begin{bmatrix} p_{00} \\ p_{01} \\ p_{02} \\ p_{10} \\ p_{11} \\ p_{12} \\ p_{20} \\ p_{21} \end{bmatrix}$$

- Multiply by the inverse of the square matirx...
- More observations gives
  - A more accurate transformation
  - Matrix becomes non-square...
  - Pseudo inverse required

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Perspective transformations

```
Point2f source [4], destination [4];
// Assign values to source and destination points.
perspective_matrix = getPerspectiveTransform( source, destination );
warpPerspective( image, result, perspective_matrix, result.size() );
```


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More complex transformations

- Approximation by a polynomial

$$i = T_i(i', j') = a_{00} + a_{10}i' + a_{01}j' + a_{11}i'j' + a_{02}(j')^2 + a_{20}(i')^2 + a_{12}i'(j')^2 + a_{21}(i')^2j' + a_{22}(i')^2(j')^2 + \ldots$$

$$j = T_j(i', j') = b_{00} + b_{10}i' + b_{01}j' + b_{11}i'j' + b_{02}(j')^2 + b_{20}(i')^2 + b_{12}i'(j')^2 + b_{21}(i')^2j' + b_{22}(i')^2(j')^2 + \ldots$$

  - No. of correspondences.
  - Distribution of points
  - Solve simultaneous linear equations
  - Least squared error solution

- Even more complex
  - Partition the image.

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Brightness interpolation

- Locations are not integer coordinates
- Interpolate output pixel value from
  - The nearby pixels in the original (uncorrected) image
- Loss of accuracy
  - Complexity of interpolation scheme
- Interpolation schemes:
  - Nearest neighbour
  - Bilinear interpolation
  - Bicubic interpolation

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Nearest neighborhood interpolation

- Simple formulation: $f'(i', j') = f\big(round(i), round(j)\big)$
- Error is perceptible
  - Step edges

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Bi-linear interpolation

$$f'(i', j') = (trunc(i) + 1 - i)(trunc(j) + 1 - j)f\big(trunc(i), trunc(j)\big) + \big(i - trunc(i)\big)(trunc(j) + 1 - j)f\big(trunc(i) + 1, trunc(j)\big) + (trunc(i) + 1 - i)\big(j - trunc(j)\big)f\big(trunc(i), trunc(j) + 1\big) + \big(i - trunc(i)\big)\big(j - trunc(j)\big)f\big(trunc(i) + 1, trunc(j) + 1\big)$$

- Brightness function is bilinear
- Neighbours contribute
- Error is perceptible
  - Blurring

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Bi-cubic interpolation

- Approximate using a bi-cubic polynomial surface
- No step-like boundary problems
- Copes with linear interpolation blurring
- Often used in raster displays…

Brightness interpolation

```
int interpolation_scheme = INTER_LINEAR;
// Nearest neighbour interpolation: INTER_NEAREST
// Bilinear interpolation: INTER_LINEAR
// Bicubic interpolation: INTER_CUBIC
warpAffine( image, result, affine_matrix, result_size,
                                        interpolation_scheme );
warpPerspective( image, result, perspective_matrix,
                                result_size, interpolation_scheme );
```


Camera models – Distortion

- Radial distortion
  - Radially symmetric from the optical axis
  - Change in magnification
    - Barrel distortion
    - Pincushion distortion
  - Caused by the lenses

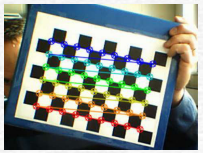
$$i' = i(1 + k_1 r^2 + k_2 r^4 + k_3 r^6)$$
$$j' = j(1 + k_1 r^2 + k_2 r^4 + k_3 r^6)$$
$$r = \sqrt{i^2 + j^2}$$

- Tangential distortion
  - Uneven magnification from one side to the other
  - Lenses not parallel to the image plane

$$i' = i + (2p_1 ij + p_2(r^2 + 2i^2))$$
$$j' = j + (2p_2 ij + p_1(r^2 + 2j^2))$$

Camera models – Removing distortion

- Calibrate using multiple images
  - Known object
  - Different positions
  - Computes camera matrix
  - & distortion parameters
- Remove distortion
  - Mainly using the distortion parameters

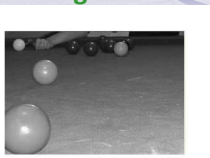


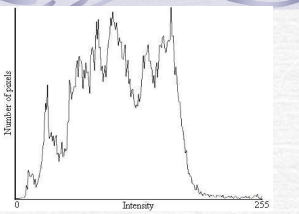
```
calibrateCamera( object_points, image_points,
    image_size, camera_matrix, distortion_coefficients,
    rotation_vectors, translation_vectors );
…
undistort( camera_image, corrected_image,
        camera_matrix, distortion_coefficients );
```

Histograms

- 1D Histograms
- 3D Histograms
- Equalisation
- Histogram Comparison
- Back Projection

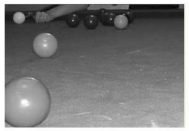
1D Histograms

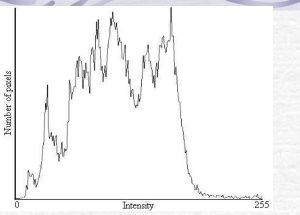




- Determine the frequency of brightness values
  - Initialise:
    - h(z) = 0 for all values of z
  - Compute:
    - For all pixels (i,j): h(f(i,j))++

1D Histograms





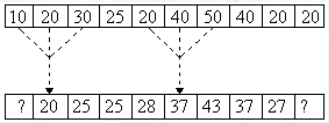
- Is this useful?
  - Global information
  - Useful for classification ?
  - Not unique

1D Histograms

```
Mat  display_image;
MatND  gray_histogram;
const int* channel_numbers = { 0 };
float channel_range[] = { 0.0, 255.0 };
const float* channel_ranges = channel_range;
int number_bins = 64;
calcHist( &gray_image, 1, channel_numbers, Mat(),
          gray_histogram, 1, &number_bins, &channel_ranges );
OneDHistogram::Draw1DHistogram( &gray_histogram, 1,
                                display_image );
imshow("Greyscale histogram", display_image);
```

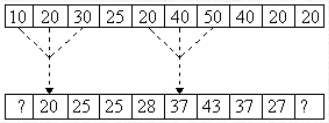
1D Histograms – Smoothing

- Local minima and maxima are useful.
- But how can we deal with noise though?



- Smooth the histogram…
  - For all values v: $h_{new}(v) = (h(v-1) + h(v) + h(v+1)) / 3$
- What do we do at the ends?
  - Do not compute OR Wraparound OR Duplicate values OR Reflect values OR Constant values ??

1D Histograms – Smoothing

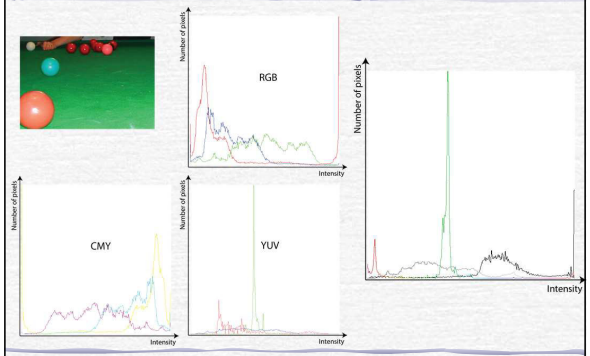



```
MatND smoothed_histogram = histogram[channel].clone();
for(int i = 1; i < histogram[channel].rows - 1; ++i)
  smoothed_histogram[channel].at<float>(i) =
    (histogram.at<float>(i-1) + histogram.at<float>(i) +
                  histogram.at<float>(i+1)) / 3;
```


1D Histograms – Colour Histograms

- Determine histograms for each channel independently…
- Choice of colour space…

1D Histograms – Colour Histograms

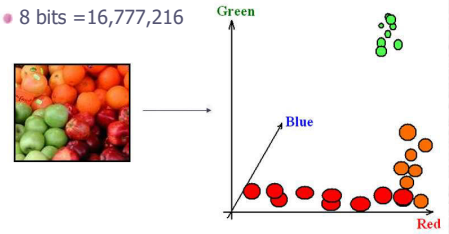


1D Histograms – Colour Histograms

```
MatND* histogram = new MatND[image.channels() ];
vector<Mat> channels(image.channels() );
split( image, channels );
const int* channel_numbers = { 0 };
float channel_range[] = { 0.0, 255.0 };
const float* channel_ranges = channel_range;
int number_bins = 64;
for (int chan=0; chan < image.channels(); chan++)
        calcHist( &(channels[chan]), 1, channel_numbers, Mat(),
          histogram[chan], 1, &number_bins, &channel_ranges );
OneDHistogram::Draw1DHistogram(histogram,
                                image.channels(), display_image );
```

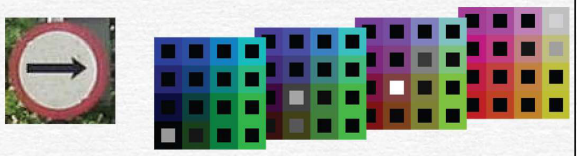

3D Histograms

- Channels are not independent
- Better discrimination comes from considering all channels simultaneously
- Number of cells?
  - 8 bits =16,777,216



3D Histograms

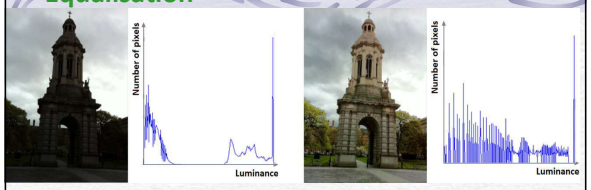
- Reduce quantisation
  - 6 bits = 262,144
  - 4 bits = 4,096
  - 2 bits = 64



3D Histograms

```
MatND histogram;
int channel_numbers[] = { 0, 1, 2 };
int* number_bins = new int[image.channels()];
for (ch=0; ch < image.channels(); ch++)
        number_bins[ch] = 64;
float ch_range[] = { 0.0, 255.0 };
const float* channel_ranges[] = {ch_range, ch_range, ch_range};
calcHist( &image, 1, channel_numbers, Mat(), histogram,
            image.channels(), a_number_bins, channel_ranges );
```

Equalisation

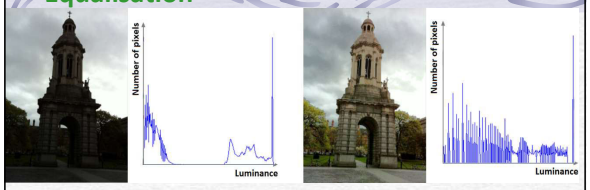


- If an image has insufficient contrast
- Human can distinguish 700-900 greyscales
- Evenly distribute the greyscales…
  - Result has missing greyscales
- Normally equalise only the greyscales / luminance

Equalisation

```
// Create a lookup table to map the luminances
// h[x] = histogram of luminance values image f(i,j).
pixels_so_far = 0
num_pixels = image.rows * image.cols
output = 0
for  input = 0 to 255
  pixels_so_far = pixels_so_far + h[ input ]
  new_output = (pixels_so_far*256) / (num_pixels+1)
  LUT[ input ] = (output+1+new_output) / 2
  output = new_output
// Apply the lookup table LUT(x) to the image:
for  every pixel f(i,j)
  f'(i,j) = LUT[ f(i,j) ]
```


Equalisation




```
vector<Mat> channels( hls_image.channels() );
split(hls_image, channels);
equalizeHist( channels[1], channels[1] );
merge( channels, hls_image );
```


Histogram Comparison

- To find similar images…
  - Use metadata
  - Compare images
- Compare the colour distributions
- Need a metric for comparisons…

Histogram Comparison

Histogram Comparison – Metrics

- $D_{Correlation}(h_1, h_2) = \frac{\sum_i (h_1(i)-\overline{h_1})(h_2(i)-\overline{h_2})}{\sqrt{\sum_i (h_1(i)-\overline{h_1})^2 \sum_i (h_2(i)-\overline{h_2})^2}}$
- $D_{Chi-Square}(h_1, h_2) = \sum_i \frac{(h_1(i)-h_2(i))^2}{(h_1(i)+h_2(i))}$
- $D_{Intersection}(h_1, h_2) = \sum_i min(h_1(i), h_2(i))$
- $D_{Bhattacharyya}(h_1, h_2) = \sqrt{1 - \frac{1}{\sqrt{\overline{h_1}.\overline{h_2}.N^2}} \sum_i \sqrt{h_1(i).h_2(i)}}$
- where
  - N is the number of bins in the histograms,
  - $\overline{h_k} = \sum_i (h_k(i)) / N$

Histogram Comparison – Earth Mover

- An alternative to the metrics is to compute the Earth Mover's Distance…
  - Minimum cost for turning a distribution into another distribution
  - Compare images
- 1D solution:
  - EMD(-1) = 0
  - EMD(i) = $h_1(i)$ + EMD(i-1) - $h_2(i)$
  - Earth Mover's Distance = Σ_i | EMD(i) |
- Colour EMD is harder to compute…

Histogram Comparison

```
normalize( histogram1, histogram1, 1.0);
normalize( histogram2, histogram2, 1.0);
double matching_score = compareHist( histogram1,
                        histogram2, CV_COMP_CORREL);
```

We can also use Chi-Square (CV_COMP_CHISQR), Intersection (CV_COMP_INTERSECT) or Bhattacharyya (CV_COMP_BHATTACHARYYA) metrics or alternatively can use the Earth Mover's Distance function (EMD())

Histogram Back Projection

- A better approach to selecting colours (based on samples):

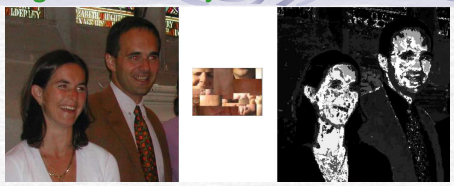
1. Obtain a representative sample set of the colours.
2. Histogram those samples.
3. Normalize that histogram so that the maximum value is 1.0.
4. Back project the normalized histogram onto any image f(i,j).
5. This will provide a 'probability' image p(i,j) which indicates the similarity between f(i,j) and the sample set.

Images

- Camera models
- Digital images
- Colour images
- Noise
- Smoothing

1

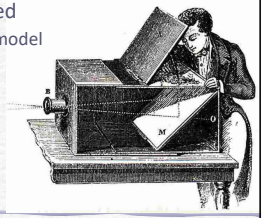
Histogram Back Projection



```
calcHist( &hls_samples_image, 1, channel_numbers, Mat(),
  histogram,image.channels(),number_bins,channel_ranges);
normalize( histogram, histogram, 1.0);
Mat probabilities = histogram.BackProject( hls_image );
```

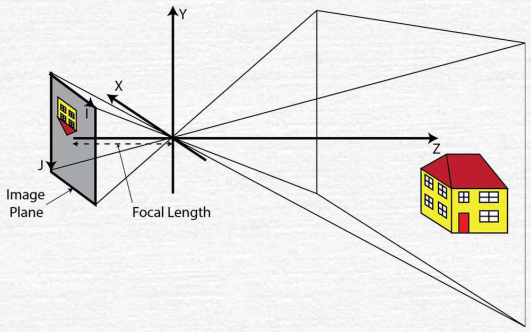

Camera models

- Components:
  - A photosensitive image plane
  - A housing
  - A lenses
- Mathematical model needed
  - The simple pinhole camera model
  - Distortions



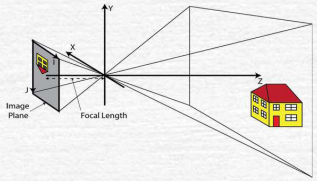
2

Camera models – Simple Pinhole Model



Camera models – Simple Pinhole Model

$$\begin{bmatrix} i.w \\ j.w \\ w \end{bmatrix} = \begin{bmatrix} f_i & 0 & c_i \\ 0 & f_j & c_j \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$



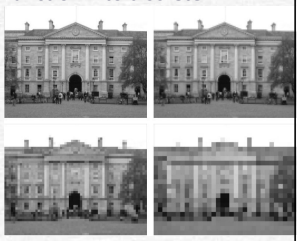
- 3-D point
- 2-D image point
- Scaling factor
- Combination of focal length and image coordinate system
- Location of the optical centre

Digital Images

- Theoretically images are continuous 2D functions of reflected scene brightness.
  - (i, j) or (column, row) or (x, y)
- To process on a computer we need a discrete representation
  - Sample
  - Quantise

Digital Images – Sampling

- Sample the continuous 2D function into discrete elements.
- Sensor
  - 2D array
  - Photosensitive elements
  - Non photosensitive gaps
- Issues
  - Elements have a fixed area
  - Gaps



Digital Images – Sampling

- How many samples do we need ?
  - Wasted space and computation time
  - Enough for the objects of interest


```
Mat image, smaller_image;
resize( image, smaller_image,
                    Size( image1.cols/2, image.rows/2 ));
```


7

Digital Images – Quantisation

- Represent the individual image points as digital values.
  - Typically 8 bits

8

Digital Images – Quantisation

- How many bits do we need?
  - Wasted space ?
  - Losing the ability to distinguish objects


```
void ChangeQuantisationGrey( Mat &image, int num_bits )
{
  CV_Assert( (image.type() == CV_8UC1) && (num_bits >= 1) &&
                                          (num_bits <= 8) );
  uchar mask = 0xFF << (8-num_bits);
  for (int row=0; row < image.rows; row++)
    for (int col=0; col < image.cols; col++)
      image.at<uchar>(row,col) = image.at<uchar>(row,col) & mask;
}
```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Colour Images

- Luminance only
  - Simple representation
  - Humans can understand
- Colour images (luminance + chrominance)
  - Multiple channels (typically 3)
  - Around 16.8 million colours
  - More complex to process
  - Facilitate certain operations

10

Colour Images – Processing

```
void InvertColour( Mat& input_image, Mat& output_image )
{
 CV_Assert( input_image.type() == CV_8UC3 );`
 output_image = input_image.clone();
 for (int row=0; row < input_image.rows; row++)
  for (int col=0; col < input_image.cols; col++)
   for (int channel=0; channel <
              input_image.channels(); channel++)
        output_image.at<Vec3b>(row,col)[channel] = 255 –
          input_image.at<Vec3b>(row,col)[channel];
}
```


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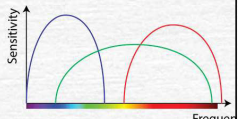
Colour Images – Efficient processing

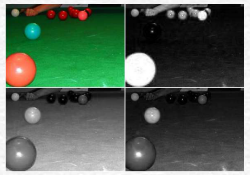
```
int image_rows = image.rows;
int image_columns = image.cols;
for (int row=0; row < image_rows; row++) {
 uchar* value = image.ptr<uchar>(row);
 uchar* result_value = result_image.ptr<uchar>(row);
 for (int column=0; column < image_columns; column++)
 {
  *result_value++ = *value++ ^ 0xFF;
  *result_value++ = *value++ ^ 0xFF;
  *result_value++ = *value++ ^ 0xFF;
 }
}
```


12

Colour Images – RGB Images

- Red-Green-Blue images
  - Most common
  - Channels correspond roughly to
    - Red (700nm)
    - Green (546nm)
    - Blue (436nm)
  - Colours combined in display





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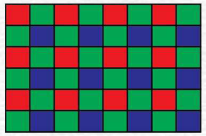
Colour Images – RGB Images

- Converting to Greyscale
  - *Y = 0.299R + 0.587G + 0.114B*
- Camera photosensitive elements
  - Separate Red, Green & Blue elements
  - Sometimes sensitive to all visible wavelengths
  - Bayer pattern:



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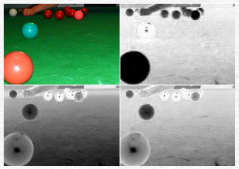
Colour Images – RGB Images

```
Mat bgr_image, grey_image;
cvtColor(bgr_image, grey_image, CV_BGR2GRAY);
vector<Mat> bgr_images(3);
split(bgr_image, bgr_images);
Mat& blue_image = bgr_images[0];
```


15

Colour Images – CMY Images

- Cyan-Magenta-Yellow images
  - Secondary colours
  - Subtractive colour scheme
    - *C = 255 – R*
    - *M = 255 – G*
    - *Y = 255 – B*
  - Often used in printers



CMY is not directly supported in OpenCV.

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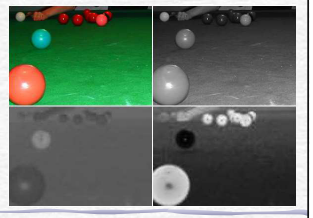
Colour Images – YUV Images

- Used for analogue television signals
  - PAL, NTSC
  - 4 Y to 1 U to 1 V
- Conversion from RGB
  - Y = 0.299R + 0.587G + 0.114B
  - U = 0.492 * (B-Y)
  - V = 0.877 * (R-Y)



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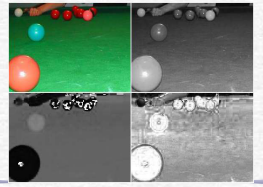
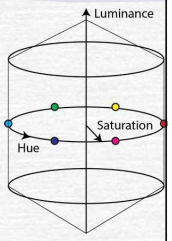
Colour Images – HLS Images

- Hue-Luminance-Saturation images
  - Separates Luminance & Chrominance
  - Values we humans can relate to…
    - Hue 0°..360°
    - Luminance 0..1
    - Saturation 0..1
  - Watch out for circular Hue…





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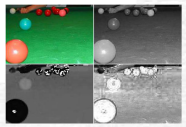
Colour Images – HLS Images

- Conversion from RGB

$$L = {Max(R,G,B) + Min(R,G,B)}/{2}$$

$$S = \begin{cases} {Max(R,G,B) - Min(R,G,B)}/{Max(R,G,B) + Min(R,G,B)} & \text{if } L < 0.5 \\ {Max(R,G,B) - Min(R,G,B)}/{2 - (Max(R,G,B) + Min(R,G,B))} & \text{if } L \geq 0.5 \end{cases}$$

$$H = \begin{cases} {60.(G-B)}/{S} & \text{if R} = Max(R,G,B) \\ 120 + {60.(B-R)}/{S} & \text{if G} = Max(R,G,B) \\ 240 + {60.(R-G)}/{S} & \text{if B} = Max(R,G,B) \end{cases}$$



```
cvtColor(bgr_image, hls_image, CV_BGR2HLS);
// Hue ranges from 0 to 179.
```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Colour Images – Other colour spaces

- HSV
- YCrCb
- CIE XYZ
- CIE L*u*v*
- CIE L*a*b*
- Bayer

20

Noise

- Affects most images
- Degrades the image
- Can cause problems with processing
- Causes?
- Measuring noise: $S/N\ ratio = {\sum_{(i,j)} f^2(i,j)}/{\sum_{(i,j)} v^2(i,j)}$
- Correcting noise…
- Types
  - Gaussian
  - Salt and Pepper

21

Noise – Salt and Pepper Noise

- Impulse noise
- Noise is maximum or minimum values

22

Noise – Gaussian Noise

- Good approximation to real noise
- Distribution is Gaussian (mean & s.d.)

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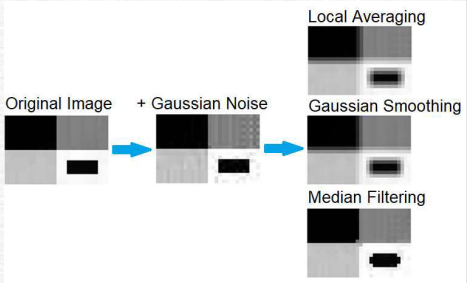
Smoothing

- Removing or reducing noise…
- Linear & non-linear transformations



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Smoothing – Averaging Filters (linear)

- Linear transformation (convolution)
- Local neighbourhood
  - $f(i,j) = \sum_{(m,n) \in O} \sum h(i-m, j-n).g(m,n)$
  - Different masks…
    - Local Average
    - Gaussian
- Acceptable results?
  - Suppression of (small) image noise
  - Blurring of edges

$$h = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

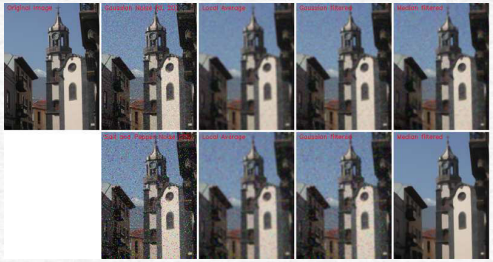
$$h = \frac{1}{10}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$h = \frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

```
blur(image,smoothed_image,Size(3,3));
GaussianBlur(image,smoothed_image,Size(5,5),1.5);
```


25

Smoothing examples



26

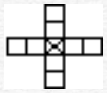
Smoothing – Median Filter (non-linear)

- Use the median value…

11 18 20 21 23 25 25 30 250

Median = 23 Average = 47

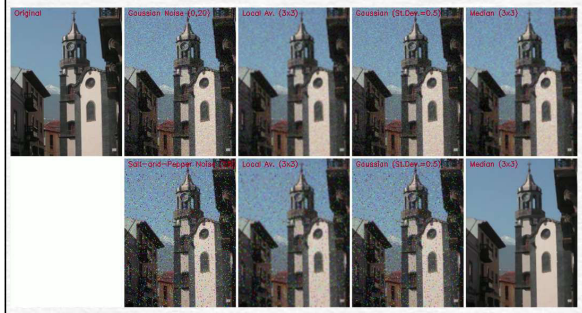
- Not affected by noise
- Doesn't blur edges much
- Can be applied iteratively
- Damages thin lines and sharp corners
  - Change region shape
- Computational expensive
  - Standard – $O(r^2 \log r)$
  - Huang – $O(r)$
  - Perreault (2007) – $O(1)$



```
medianBlur(image, smoothed_image, 5);
```


27

Smoothing – Effects of mask size



28

Image Pyramids

- To process images
  - At multiple scales
  - Efficiently
- Technique
  - Smooth image (often Gaussian)
  - Subsample (usually by a factor of 2)

```
pyrDown( image, smaller_image,
  Size( (image1.cols+1)/2, (image.rows+1)/2 ));
```


29

Learning & Evaluation

- Data / Experience
  - Unsupervised
  - Supervised
- Performance Evaluation
  - Ground Truth
  - Metrics
  - Training vs. Validation

1

Machine learning

- An algorithm that can learn from data.
  - Data / Experience
  - Some task
    - Often classification
  - Performance improved by data
    - Measure accuracy or error rate
- Data / Experience
  - Unsupervised / Unlabelled.
  - Supervised / Labelled.

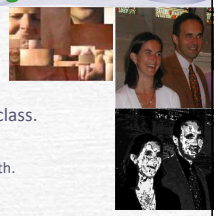
2

Unsupervised machine learning

- k-means.
  - Data – Unlabelled image data
  - Task – Grouping pixels to reduce the number of colours
  - Performance – Very application/image dependent
    - Measure accuracy or error rate
- Gaussian Mixture Model
  - Data – Unlabelled video data.
  - Task – Identification of moving objects.
  - Performance
    - Accuracy based on hand labelled group truth.

3

Supervised machine learning



- Back-projection.
  - Data – Labelled sample pixel values
  - Task – Classifying pixels as a particular class.
  - Performance
    - Accuracy based on hand labelled group truth.
- Statistical Pattern Recognition / Support Vector Machines
  - Data – Labelled samples (feature vectors).
  - Task – Recognition of objects/shapes.
  - Performance
    - Accuracy based on hand labelled group truth.

4

Evaluating Computer Vision Performance

- Does it work in real time?
  - What is real time?
  - Does it matter?
- What is the correct answer?
  - Ground truth.
  - How do we generate ground truth?
- How do we assess how well we have done?
  - Success/failure metrics?
  - Overlap?
  - What about time?

5

Performance – Ground Truth

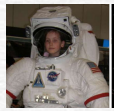
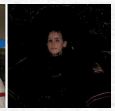
- What should any computer vision system achieve?
  - The correct answer is easy for some tasks. e.g.
    - Is this picture me?
    - Determine the state of the traffic lights.
  - For some tasks it is harder.
    - Classify the object in this image.
    - Count the people.
    - Drive from A to B.




6

Performance – Ground Truth

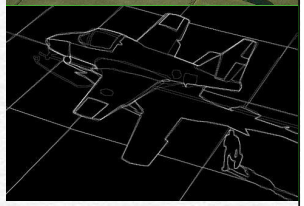
- What should any computer vision technique achieve?
  - The correct answer is easy for some techniques. e.g.
    - Find skin pixels
  - For some techniques it is harder.
    - Find the edges.
  - The usefulness of a technique
    - depends on the application…

7

Ground Truth – Example

- How should an image be segmented?
  - Significant edges/regions?
- Impossible to get agreement?
- Berkeley segmentation dataset
  - 1,000 images
  - 12,000 segmentations (colour & grayscale)
  - Hand-labelled (30 subjects)
- Is this a good way to evaluate a segmentation technique?

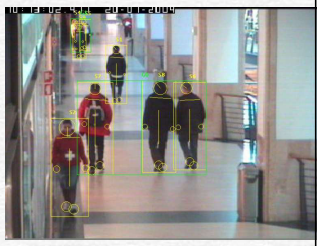
From the public "Berkeley Segmentation dataset"

8

Ground Truth – Bounding boxes

- Boxes incorporate parts of other objects/background.
- Labour intensive for video.

- CAVIAR:
  - Shopping centre (Lisbon)
  - Bounding boxes for all people.
  - Some have head, hands, and feet annotated.
  - Manually annotated.



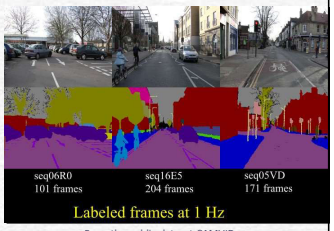
From the public dataset CAVIAR

9

Ground Truth – Pixel based

- Better validity.
- Very expensive to create.

- CAMVID:
  - 32 classes
  - 10 minutes @30/15Hz
  - 700 frames
  - Manually annotated.
  - 2nd person inspection.



From the public dataset CAMVID

10

Performance – Simple Metrics

- Compute
  - TP
  - TN
  - FP
  - FN
- and then compute…

$$Recall = \frac{TP}{TP + FN}$$

$$Precision = \frac{TP}{TP + FP}$$

$$Accuracy = \frac{TP + TN}{Total\ Samples}$$

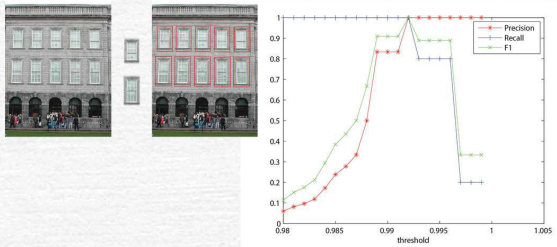
$$Specificity = \frac{TN}{FP + TN}$$

$$F_\beta = (1 + \beta^2).\frac{Precision.Recall}{(\beta^2.Precision) + Recall}$$

11

Performance – Tuning

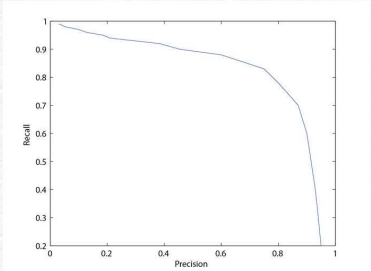
- We can tune performance by altering parameters
  - For example altering the threshold used for accepting a match in a template matching example



12

Performance – Precision-Recall Curves

- Alternative (common) visualisation
- Select the parameters which result in the point on the PR curve which is closest to P=1.0, R=1.0

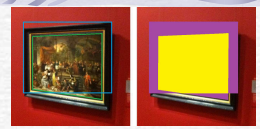


13

Performance – Overlap & mAP

- Intersection over Union (IoU)

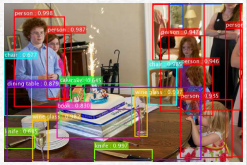
$$IoU(Detected, GroundTruth) = \frac{Detected \cap GroundTruth}{Detected \cup GroundTruth}$$



  - Values of IoU > 0.5 are often considered good matches.

- When considering multiple classes…
  - Mean Average Precision (mAP)

$$mAP = \frac{1}{|classes|} \sum_{c \in classes} \frac{\#TP(c)}{\#TP(c) + \#FP(c)}$$



ResNet result on COCO dataset (2015)

14

Performance – Confusion Matrix

- It is helpful to know how things are misclassified
  - Actual class
  - Determined class

Result \ Ground Truth	Walking	Running	Standing	Dancing	Hopping
Walking	0.9	0.0	0.0	0.0	0.0
Running	0.0	0.7	0.0	0.1	0.1
Standing	0.1	0.0	1.0	0.0	0.0
Dancing	0.0	0.2	0.0	0.4	0.0
Hopping	0.0	0.1	0.0	0.5	0.9

- If considering a binary classification
  - Confusion matrix shows TP, FP, FN, TN

	A	Not A
A	TP	FP
Not A	FN	TN

	Face	Not Face
Face	3	0
Not Face	1	65532

15

Training & Validation

- Typically dataset are divided into two parts:
  - Training set.
  - Validation/Testing set.

- This can be
  - Static
  - Random
  - Leave p-out cross validation
    - Leave p samples out – for testing. Train on the rest.
    - Do multiple rounds of training and testing using different partitions.
    - Average results.
    - Intended to determine how the results will generalise.

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Outcome

- So what does testing tell me?
  - How well the system/technique works on the test data (images/videos)…
  - Actually, how close the system/technique gets to the manually created ground truth…
- How representative is the test set?
  - What conditions matter?
  - Can I ever be sure that a system will work in general?

Recognition

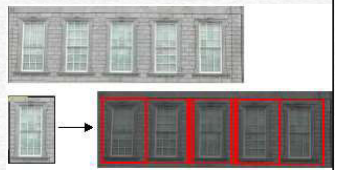
- Template Matching
  - Chamfer Matching
- Statistical Pattern Recognition (SPR)
- Support Vector Machines (SVM)
- Cascade of Haar classifiers (Haar)
- Principal Components Analysis (PCA)

Summary

- Learning (Data – Task – Performance)
  - Unsupervised
  - Supervised
- Ground Truth
  - Expensive to obtain.
  - Often unclear what is the right answer.
- Metrics
  - Precision, Recall, Accuracy
  - Tuning
  - IoU, mAP
- Training vs. Validation
  - Leave p-out cross validation

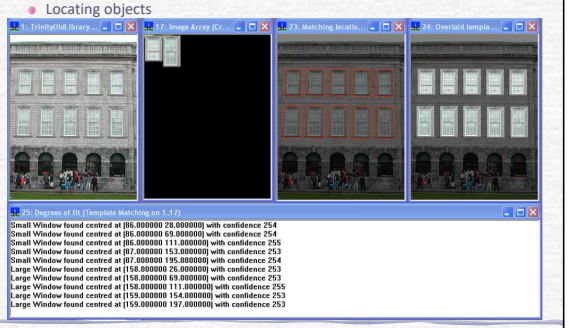
Template Matching - Topics

- Applications
- Matching criteria
- Control strategies
- Chamfer Matching



Template – Applications for template matching

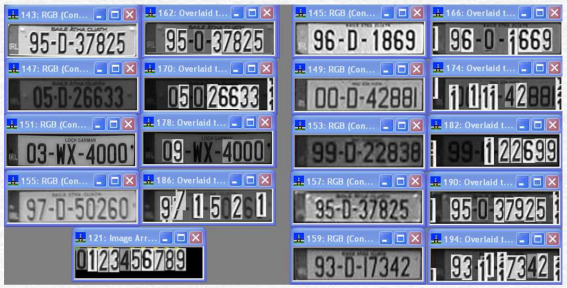
- Searching
  - Locating objects



3

Template – Applications for template matching

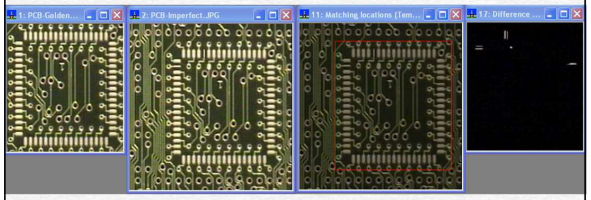
- Recognition



4

Template – Applications for template matching

- Visual inspection
  - Golden template matching



- Matching
  - Stereo Vision
  - Tracking

5

Template – Matching Algorithm

- Basic Algorithm
  - Inputs – Image & Object
  - For every possible position of the object in the image
    - Evaluate a match criterion
  - Search for local maxima of the match criterion above some threshold
- Problems
  - 'Every possible position'?
  - 'Match criterion'
  - 'Local maxima above some threshold'

6

Template – Matching criteria

$$D_{SquareDifferences}(i,j) = \sum_{(m,n)} (f(i+m,j+n) - t(m,n))^2$$

$$D_{NormalisedSquareDifferences}(i,j) = \frac{\sum_{(m,n)}(f(i+m,j+n) - t(m,n))^2}{\sqrt{\sum_{(m,n)} f(i+m,j+n)^2 \sum_{(m,n)} t(m,n)^2}}$$

$$D_{CrossCorrelation}(i,j) = \sum_{(m,n)} f(i+m,j+n).t(m,n)$$

$$D_{NormalisedCrossCorrelation}(i,j) = \frac{\sum_{(m,n)} f(i+m,j+n).t(m,n)}{\sqrt{\sum_{(m,n)} f(i+m,j+n)^2 \sum_{(m,n)} t(m,n)^2}}$$

7

Template – Matching criteria example

- Simple example
  - Matching criteria
  - Boundaries…

0	0	0	0	0	1	0	0
0	0	0	0	0	1	0	0
0	0	0	1	1	1	1	0
0	0	0	1	1	0	1	0
0	0	0	1	0	0	1	0
0	0	0	1	0	0	1	0
0	0	0	1	1	1	1	0
0	0	0	0	0	0	0	0

Image

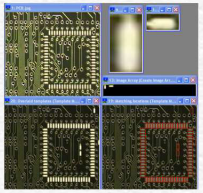
1	1	1	1
1	0	0	1
1	0	0	1
1	0	0	1
1	1	1	1

Template

11	13	14	11	14
10	14	16	8	14
9	12	12	1	10
10	14	15	7	15

Matching Space

- Real example
  - Uses normalised cross correlation.



8

Template Matching in OpenCV

```
Mat matching_space;
matching_space.create(
   search_image.cols–template_image.cols+1,
   search_image.rows–template_image.rows+1, CV_32FC1 );
matchTemplate( search_image, template_image,
                matching_space, CV_TM_CCORR_NORMED );
// Other measures: CV_TM_CCORR, CV_TM_SQDIFF,
//                 CV_TM_SQDIFF_NORMED
```


9

Template – Finding Local Maxima in OpenCV

- Local maxima – Dilate + Look for unchanged values + Threshold
- Local minima – Erode + Look for unchanged values + Threshold

```
Mat dilated, thresholded_matching_space, local_maxima,
                                    thresholded_8bit;
dilate( matching_space, dilated, Mat());
compare( matching_space, dilated, local_maxima, CMP_EQ );
threshold( matching_space, thresholded_matching_space,
                    threshold, 255, THRESH_BINARY );
thresholded_matching_space.convertTo( thresholded_8bit,
                                    CV_8U );
bitwise_and( local_maxima,thresholded_8bit,local_maxima );
```


10

Template – Control Strategies for Matching

- Goal: Localise close copies
  - Size, orientation
  - Geometric distortion
- Use an image hierarchy
  - Low resolution first
  - Limit higher resolution search
- Search higher probability locations first
  - Known / learnt likelihood
  - From lower resolution

11

Template – Chamfer matching

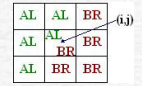
- Template matching requires very close matches
- Objects often appear very slightly different
  - Orientation
  - Noise
  - Sampling
- We want a more flexible approach

12

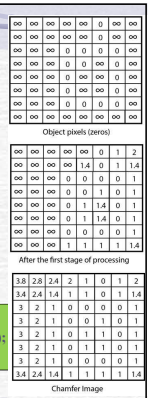
Template – Chamfering

- Compute chamfered image for a binary edge image.

For every point
  If (edge point)
    Set c(i, j) = 0
    Else Set c(i, j) = ∞
For j = min to max
  For i = min to max
    $c(i, j) = \min_{q \in AL}$ [distance((i, j), q) + f(q)]
For j = max to min
  For i = max to min
    $c(i, j) = \min_{q \in BR}$ [distance((i, j), q) + f(q)]

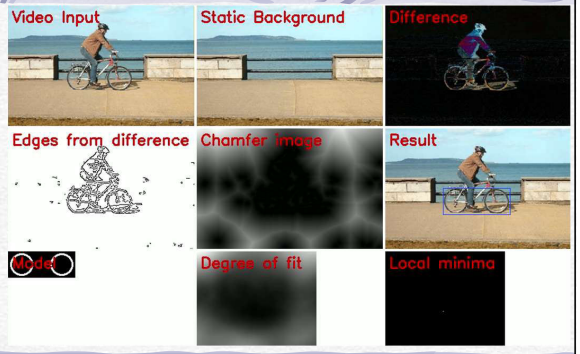



```
Canny( gray_image, edge_image, 100, 200, 3);
threshold(edge_image,edge_image,127,255,THRESH_BINARY_INV);
distanceTransform(edge_image,chamfer_image,CV_DIST_L2,3);
```



13

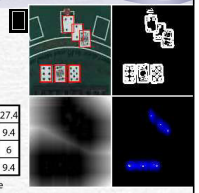
Template – Chamfer matching



14

Template – Chamfering for matching

- Compare with boundary template
  - Sum of chamfer values for boundary points
  - Low values best



3.8	2.8	2.4	2	1	0	1	2
3.4	2.4	1.4	1	1	0	1	1.4
3	2	1	0	0	0	0	1
3	2	1	0	0	1	0	1
3	2	1	0	1	1	0	1
3	2	1	0	1	1	0	1
3	2	1	0	0	0	0	1
3.4	2.4	1.4	1	1	1	1	1.4

Chamfer Image

1	1	1	1	
1				1
1				1
1				1
1	1	1	1	

Template

27.4	19.6	12.8	8	27.4
23.2	16.8	10.4	5	9.4
21	14	8	0	6
23.2	16.8	10.4	5	9.4

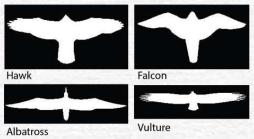
Matching Space

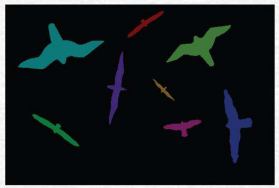
Chamfer matching is a simple routine to write (provided in the text);

15

SPR – Statistical Pattern Recognition

- Probability Review
- Features
- Classification





	Rectangularity	Elongatedness
Albatross	0.28	3.63
Vulture	0.45	5.56
Hawk	0.39	2.39
Falcon	0.33	2.11

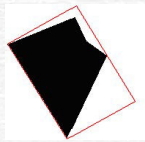
0.33	2.11	Falcon
0.49	5.29	Vulture
0.30	3.67	Albatross
0.48	5.45	Vulture

0.33	2.10	Falcon
0.31	3.65	Albatross
0.41	2.38	Hawk
0.37	2.72	Hawk

16

SPR – Features: Minimum Bounding Rectangle

- Turn rectangle through discrete steps
- Only one quadrant
- Metrics:
  - Length to Width ratio
    - Length / Width
  - Rectangularity
    - Area / (Length * Width)
  - Convex Hull to Minimum Bounding Rectangle area ratio
    - Area inside convex hull / (Length * Width)



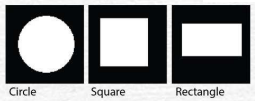
```
RotatedRect min_bounding_rectangle =
                    minAreaRect(contours[contour_number]);
```


17

SPR – Features: Elongatedness

- CANNOT BE the ratio of the length and width of the minimum bounding rectangle
- Ratio of region area divided by the square of it's thickness
  - Erosion

$$elongatedness = \frac{area}{(2d)^2}$$



	Rectangularity	Elongatedness
Circle	0.79	1.00
Square	1.00	1.00
Rectangle	1.00	1.85

1.00	1.82	Rectangle
0.99	1.81	Rectangle
0.39	1.82	Rectangle

0.78	1.00	Circle
0.40	1.00	Circle
1.00	1.00	Square

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SPR – Probabitity Review – Basics

- $P(A) = lim_{n \to \infty}\ N(A) / n$
- Probability of two events *A* and *B*:
  - Independent:
    $P(AB) = P(A)P(B)$
  - Dependent:
    $P(AB) = P(A|B)P(B)$
    - Conditional Probability $P(A|B)$
- Typical problem:
  - Given some evidence *x* from an unknown object.
  - What class W_i is the object?
  - Training
    - *A-priori* probability – $p(x \mid W_j)$
    - Relative probability – $p(W_j)$
  - *A-posteriori* probability – $p(W_i \mid x)$

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SPR – Probabitity Review – Bayes Theorem

- For two classes *A* and *B* the *a-posteriori* probability is:

$$P(B|A) = P(A|B)P(B) / P(A)$$

- Where W_i forms a partitioning of the event space:

$$p(W_i \mid x) = \frac{p(x \mid W_i)P(W_i)}{\sum_j p(x \mid W_j)P(W_j)}$$

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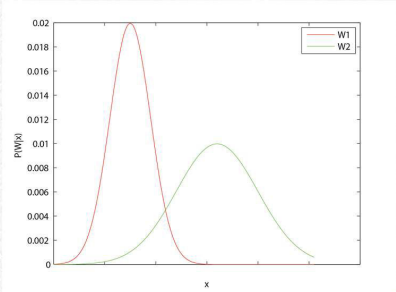
SPR – Probability Density Functions

- Given a class (W_i) and an event (x)
  - Determine the probability of any particular value occurring...



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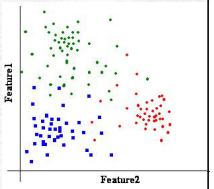
SPR – Classification

- Object recognition
  - Classes ($w_1, w_2, \ldots w_R$)
- Classifier
  - Input Pattern / features ($x_1, x_2, \ldots x_n$)
- Feature space
  - Choosing the features (Example)
  - Clusters in feature space
- Separability
  - Hyper-surfaces
  - Linear separability
  - Inseperable classes
- Probabilistic Classifier



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SPR – Probabilistic classifier

- Optimal classifier
- Based on Probability Density Functions
  - Remove normalisation from Bayes rule... $\sum_j p(x|W_j)P(W_j)$
  - Resultant Discrimination function: $g_r(x) = p(x|W_r)P(W_r)$
  - $g_r(x) \geq g_s(x)$
  - Unknown class: $g_r(x) >$ threshold
  - Mean Loss Function: $J(q) = \int_x \sum_{s=1..R} \lambda[d(x,q)|W_s]p(x|W_s)P(W_s)\,dx$

    where $\lambda[W_r|W_s] = \begin{cases} 0 & if\ r = s \\ 1 & otherwise \end{cases}$
- Advantages / Disadvantages
  - Accuracy
  - Extensive training

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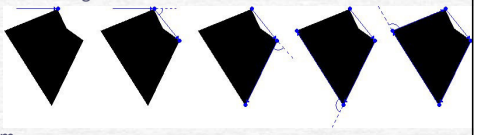
SPR – Features: Convex hull

- Smallest convex region



- Algorithm
  1. Start with
     - Any point on convex hull
     - Previous vector direction
  2. Search all other boundary points
     - Find point with least angle to previous vector
  3. Switch to new point and vector
  4. Go to 2 unless new point = start point.

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SPR – Features: Convex hull

```
vector<vector<Point>> hulls(contours.size());
for (int contour=0; (contour<contours.size()); contour++)
{
 convexHull(contours[contour], hulls[contour]);
}
```


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SPR – Features: Concavities and Holes

- Identify concavities from the convex hull
- Identify holes in the binary shape

	Shape	Height / Width	Hull / Box	# holes	Hole Areas	# concavities	Concavity Details
0	2.46	0.85	1	4.1		0	
1	4.71	0.58	0			2	0.4@243 0.03@261
2	2.83	0.85	0			5	3.6@213 2.5@45
3	2.83	0.82	0			3	5.7@185 0.2@354
4	2.38	0.62	1	0.6		3	0.9@101 0.1@207
5	2.54	0.85	0			4	3.1@140 2.4@302
6	2.13	0.79	1	1.5		3	3.1@317 0.03@94
7	2.36	0.57	0			9	4.3@205 0.1@294
8	2.83	0.80	2	1.8 1.6		2	0.3@205 0.2@355
9	2.62	0.81	1	1.9		4	3.0@143 0.1@240

	Shape	Height / Width	Hull / Box	# holes	Hole Areas	# concavities	Concavity Details
9	2.62	0.81	1	1.8		3	2.9@137 0.03@66
5	2.54	0.85	0			4	3.0@140 2.4@302
0	2.54	0.87	1	4.5		3	0.4@171 0.1@116
3	2.62	0.81	0			3	5.7@184 0.4@343
7	2.36	0.59	0			4	4.2@211 0.4@0
8	2.62	0.81	2	1.6 1.5		3	0.3@354 0.3@203
2	2.43	0.84	0			3	3.9@201 2.8@40
5	2.62	0.83	0			4	3.0@143 2.4@300

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SPR – Features: Concavities and Holes

```
vector<vector<int>> hull_indices(contours.size());
vector<vector<Vec4i>> convexity_defects(contours.size());
for (int contour=0; (contour<contours.size()); contour++)
{
 convexHull( contours[contour], hull_indices[contour] );
 convexityDefects( contours[contour], hull_indices[contour],
                                      convexity_defects[contour]);
}
```


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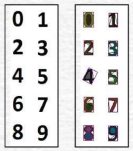
SPR – Features: Perimeter Length and Circularity

- Perimeter length
  - Approximately the number of boundary elements

    `contours[contour].size()`
  - Should really take into account direction
- Circularity = Perimeter length / (4*π*Area)



Shape	Perimeter	Area	Bounding Box	Convex Hull	Hu Moments			Hole Areas	
0	80	406	651	555	0.17	0.00	0.00	175	
1	101	274	522	429	0.47	0.13	0.01		
2	111	347	570	528	0.42	0.07	0.00		
3	121	351	522	530	0.40	0.06	0.00		
4	85	347	592	442	0.20	0.01	0.00	60	
5	122	351	570	504	0.43	0.06	0.00		
6	102	392	620	534	0.23	0.01	0.00	76	
7	94	267	551	394	0.46	0.09	0.04		
8	65	438	651	571	0.20	0.01	0.00	45	62
9	101	389	589	524	0.23	0.01	0.00	74	

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SPR – Classifier learning

- Training set
  - Must be representative
  - Must be inductive
    - Can use Kernel Density Estimation
- Training set size
  - Training set provides the unknown statistical information
  - Size will typically have to be increased several times
- Learning strategies
  - Supervised:
    - Probability density estimation – estimating $p(x|w_i)$ & $P(w_i)$
    - Training set includes class specification for every instance
  - Unsupervised:
    - Cluster Analysis
    - Look for similarities in feature space

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SPR: Real example

Shape	Height / Width	Hull / Box	# holes:	Hole Areas	# concavities	Concavity Details
0	2.46	0.85	1	4.1	0	
1	4.71	0.58	0		2	0.4@243 0.03@261
2	2.83	0.85	0		5	3.6@213 2.5@45
3	2.83	0.82	0		3	5.7@185 0.2@354
4	2.38	0.62	1	0.6	3	0.9@101 0.1@207
5	2.54	0.85	0		4	3.1@140 2.4@302
6	2.13	0.79	1	1.5	3	3.1@317 0.03@94
7	2.36	0.57	0		9	4.3@205 0.1@294
8	2.83	0.80	2	1.8 1.6	2	0.3@205 0.2@355
9	2.62	0.81	1	1.9	4	3.0@143 0.1@240

Shape	Height / Width	Hull / Box	# holes:	Hole Areas	# concavities	Concavity Details
9	2.62	0.81	1	1.8	3	2.9@137 0.03@66
5	2.54	0.85	0		4	3.0@140 2.4@302
0	2.54	0.87	1	4.5	3	0.4@171 0.1@116
3	2.62	0.81	0		3	5.7@184 0.4@343
7	2.36	0.59	0		4	4.2@211 0.4@0
8	2.62	0.81	2	1.6 1.5	3	0.3@354 0.3@203
2	2.43	0.84	0		3	3.9@201 2.8@40
5	2.62	0.83	0		4	3.0@143 2.4@300

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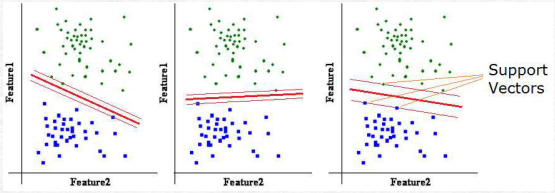
Support Vector Machines

- Popular technique for two-class problem
- Consider two linearly separable classes in *n*-dimensional feature space
- Many hyper-surfaces usually exist
- Optimal classification
  - Maximise the margin between the classes.



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Support Vector Machines – Example

From *Vision-Based Traffic Data Collection Sensor for Automotive Applications* by Llorca DF, Sánchez S, Ocaña M, Sotelo MA - (2010) http://openi.nlm.nih.gov/detailedresult.php?img=3270873_sensors-10-00860f12&req=4

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Support Vector Machines – Formulation

- Given many *n*-dimensional training samples x_i with associated class identifiers $\omega_i = \{-1, 1\}$
- Define the separating hyper-planes $w.x + b = 0$
  $w.x + b = 1$ $w.x + b = -1$
- Constraint: $\omega_i(w.x_i + b) \geq 1$
- If x^+ is a point on $w.x + b = 1$ and x^- is the nearest point on $w.x + b = -1$
  - then $\lambda w = x^+ - x^-$
- Multiply by w $\lambda w.w = x^+.w - x^-.w = (1 - b) - (-1 - b) = 2$
  - Therefore $\lambda w.w = \lambda\|w\|^2 = 2$ as $\|w\| = \sqrt{w.w}$ and...

$$\|x^+ - x^-\| = \|\lambda w\| = \left\|2.w/\|w\|^2\right\| = \|2.w/w.w\| = \|2/w\| = 2/\|w\|$$

- Hence we must minimise $\|w\|$ to maximise the separation, subject to $\omega_i(w.x_i + b) \geq 1$

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Support Vector Machines – Lagrangian Optimisation

- This is a minimisation problem subject to constraints which is amenable to solution by Lagrangian optimisation

$$L(w, b, \alpha) = \|w\|/2 - \sum_{i=1}^{n} \alpha_i[\omega_i.(w.x_i + b) - 1]$$

- To find the minimums we set the partial differentials to 0
- Classification is simply based on
  - entering an unknown feature vector x_i
  - into the original equation $f(x_i) = w.x_i + b$
  - and classifying based on whether the value is positive or negative (preferably >= 1).

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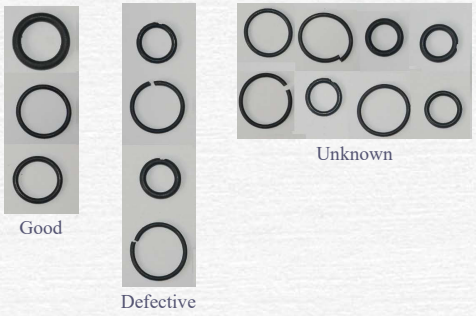
Support Vector Machines – Example



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SVM Example – Training

```
Ptr<SVM> svm = ml::SVM::create();
svm->setType(SVM::C_SVC);
svm->setKernel(SVM::LINEAR);
Mat labelsMat(number_of_samples, 1, CV_32SC1, labels);
Mat trainingDataMat(number_of_samples, 2, CV_32FC1,
                                                    training_data);
Ptr<ml::TrainData> tData = ml::TrainData::create(trainingDataMat,
                        ml::SampleTypes::ROW_SAMPLE, labelsMat);
svm->train(tData);
```


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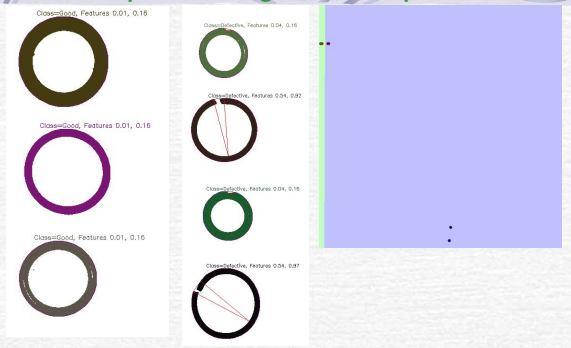
SVM Example – Training & Feature Space



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SVM – Support vectors

```
Mat support_vectors = svm->getSupportVectors();
for (int support_vector_index = 0; support_vector_index <
support_vectors.rows; ++support_vector_index)
{
  const float* v = support_vectors.ptr<float>(support_vector_index);
  circle(feature_space, Point((int) v[0],(int) v[1]), 3,Scalar(0,0,255));
}
```


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SVM Example – Prediction

```
// Try to predict the class
Mat sampleMat = (Mat_<float>(1,2) << feature[0], feature[1]);
float prediction = svm->predict(sampleMat);
class_id = (prediction > 0.0) ? 1 : (prediction < 0.0) ? 2 : 0;
```


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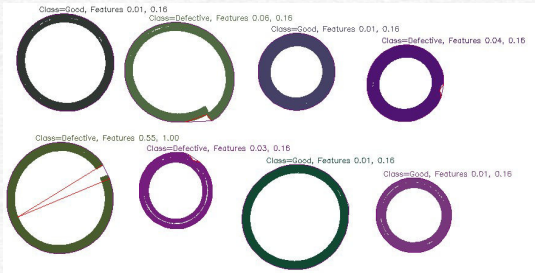
SVM Example – Prediction



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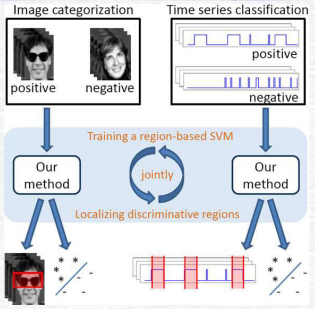
Support Vector Machines – Example



From *Weakly supervised discriminative localization and classification: a joint learning process.* by Nguyen, M.H., Torresani, L., De la Torre, F., & Rother, C. (2009) Proceedings of International Conference on Computer Vision. http://www.robots.ox.ac.uk/~minhhoai/projects/segsvm.html

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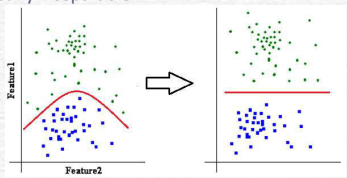
Support Vector Machines – Nonlinear classification

- What if classes are linearly inseparable



- Apply the kernel trick
  - Dot products replaced by some non-linear kernel function
    - E.g. $k(x_i, x_j) = (x_i . x_j)^d$
- Other technique
  - Soft Margin
  - Multiclass SVM

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Support Vector Machines – Example

From *Taxonomic Multi-class Prediction and Person Layout using Efficient Structured Ranking*
by A. Mittal, M. B. Blaschko, A. Zisserman, P. H. S. Torr , in *European Conference on Computer Vision, 2012*
http://www.robots.ox.ac.uk/~vgg/software/struct_rank/

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Haar – Robust Object Detection using a cascade of classifiers.

- Features
  - Efficient calculation
- Training
- Weak & Strong Classifiers
  - Adaboost
- Recognition

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Haar – In OpenCV.

```
CascadeClassifier cascade;
if( !cascade.load(
            "haarcascades/haarcascade_frontalface_alt.xml" )
{
 vector<Rect> faces;
 equalizeHist( gray_image, gray_image );
 cascade.detectMultiScale( gray_image, faces, 1.1, 2,
                    CV_HAAR_SCALE_IMAGE, Size(30, 30) );
}
```


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Haar – Overview

- Training using a number of positive and negative samples
- Uses simple (Haar like) features
  - Efficient calculation…
- Selects a large number of these features during training to create strong classifiers
- Links a number of strong classifiers into a cascade for recognition
  - Efficiency…
- Can work at different scales

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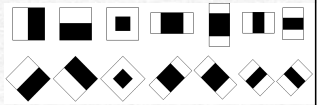
Haar – Features

- Features determined as the difference of the sums of a number of rectangular regions



- Place the mask in a specific location
- and at a specific scale
- Then subtract the sum of the 'white pixels'
- from the sum of the 'black pixels'
- Why does this work?

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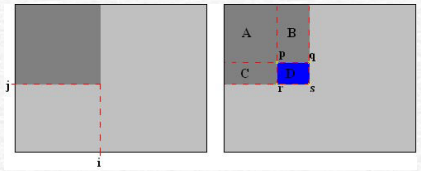
Haar – Efficient Calculation – Integral Image

- Integral Image:
  - Every point ii(i,j) = $\Sigma_{i'=0..i} \Sigma_{j'=0..j}$ image(i',j')
  - Sum of points in rectangle D:
    - sum(D) = ii(p) + ii(s) – ii(q) – ii(r)



- Features can be computed at any scale for the same cost

An **integral** function is provided which computes the integral image both normally and at 45°

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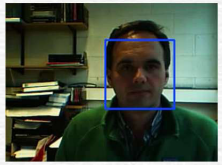
Haar – Training

- Number of possible features.
  - the variety of feature types
  - allowed variations in size and position
- Training must
  - Identify the best features to use at each stage
  - To do this positive and negative samples are needed…



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Haar – Weak & Strong Classifiers

- Weak Classifier
  - $p_j feature_j(x) < p_j \vartheta_j$
  - Tune threshold (ϑ_j)
- Strong Classifiers
  - Combine a number of weak classifiers…
  - E.g. 100% true positives with only 40% false positives using only two face features…

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Haar – Strong Classifiers – Adaboost

Given n example images $x_1..x_n$ together with classifications $y_1..y_n$
where $y_i = 0, 1$ for negative and positive examples respectively.

Initialise weights $w_{1,i} = 1 / (2*(m*(1- y_i)+l*y_i))$
where m and l are the number of negative and positive examples respectively.

For $t=1,...,T$

1. Normalize the weights (i.e. for all i): $w_{t,i} = w_{t,i} / (\Sigma_{j=1..n} w_{t,j})$

2. For each feature, j, train a weak classifier $h_j(x)$ and evaluate the error taking into account the weights: $\varepsilon_j = \Sigma_i \, w_{t,i} \mid h_j(x_i) - y_i \mid$

3. Select the classifier, $h_j(x)$, with the lowest ε_j, save as $c_t(x)$ with error E_t

4. Update the weights (i.e. for all i): $w_{t+1,i} = w_{t,i} \beta_t^{(1-e_i)}$
where $e_i = \mid c_t(x_i) - y_i \mid$ and $\beta_t = E_t / (1-E_t)$

The final strong classifier is: $h(x) = 1$ if $\Sigma_{t=1..T} \alpha_t c_t(x) \geq \frac{1}{2} \Sigma_{t=1..T} \alpha_t$,
0 otherwise
where $\alpha_t = \log 1/\beta_t$

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Haar – Classifier cascade

- Object recognition is possible with a single strong classifier
- To improve detection rates AND to reduce computation time:
  - A cascade of strong classifiers can be used
  - Each stage either accepts or rejects
  - Only those accepted pass to the next stage
    - Efficient computation…
  - Strong classifiers trained using AdaBoost
    - On the remaining set of data

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Haar – Recognition

- Face recognition
  - 38 stages
  - 6000+ features
  - 4916 positive samples
  - 9544 negative samples
  - Scale independence

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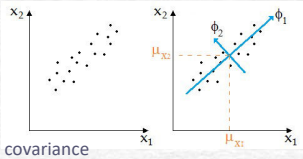
Principal Component Analysis (PCA)

- Statistical Technique for
  - Data analysis
  - Data compression
  - Recognition
- Analyses data covariance
- Identifies principal directions

For example:

- Given *2* dimensional data
- and *N* samples/vectors
- Find
  - the mean sample
  - the direction of maximum covariance
  - the direction orthogonal to this



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PCA – Background – Eigenvalues and Eigenvectors

- Consider a square matrix *A*
- The eigenvalues of *A* are the roots of the *characteristic equation*:

$$determinant(A - \lambda I) = |A - \lambda I| = 0$$

- For each eigenvalue λ there will be an eigenvector x such that

$$Ax = \lambda x$$

  - Many possible values of x

- An *n x n* matrix will have *n* eigenvalues
- Consider *n=2*. There will be 2 eigenvalues: λ_1, λ_2
  - with corresponding eigenvectors x_1, x_2
  - where $Ax_1 = \lambda_1 x_1$ and $Ax_2 = \lambda_2 x_2$

PCA – Background – Eigenvalues and Eigenvectors

- Combining we get $A[x_1 \quad x_2] = [x_1 \quad x_2]\begin{bmatrix}\lambda_1 & 0\\ 0 & \lambda_2\end{bmatrix}$
  - and $A\Phi = \Phi\Lambda$
  - where $\Phi = [x_1 \quad x_2]$ and $\Lambda = \begin{bmatrix}\lambda_1 & 0\\ 0 & \lambda_2\end{bmatrix}$

- If we normalise the eigenvectors then $\Phi\Phi^T = \Phi^T\Phi = I$
  - so $\Phi^T A\Phi = \Phi^T\Phi\Lambda = \Lambda$
  - and $A = A\Phi\Phi^T = \Phi\Lambda\Phi^T$

PCA – Theory – Organise Data

- In PCA we have *N* samples/vectors in some *n*-dimensional space
- Combine into a data matrix *D*
  - Each row is a sample
- Determine the mean of the samples
- Compute mean-centred data *U*

$$D = \begin{bmatrix} x_1^1 & x_2^1 & .. & x_n^1 \\ x_1^2 & x_2^2 & .. & x_n^2 \\ : & : & & : \\ x_1^N & x_2^N & .. & x_n^N \end{bmatrix}$$

$$\mu = \left[\Sigma_{i=1}^N x_1^i/_N \quad \Sigma_{i=1}^N x_2^i/_N \quad .. \quad \Sigma_{i=1}^N x_n^i/_N\right]$$

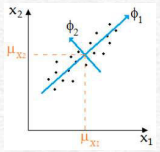
$$U = D - \begin{bmatrix}\mu\\ \mu\\ :\\ \mu\end{bmatrix} = \begin{bmatrix} x_1^1 - \Sigma_{i=1}^N x_1^i/_N & x_2^1 - \Sigma_{i=1}^N x_2^i/_N & .. & x_n^1 - \Sigma_{i=1}^N x_n^i/_N \\ x_1^2 - \Sigma_{i=1}^N x_1^i/_N & x_2^2 - \Sigma_{i=1}^N x_2^i/_N & .. & x_n^2 - \Sigma_{i=1}^N x_n^i/_N \\ : & : & & : \\ x_1^N - \Sigma_{i=1}^N x_1^i/_N & x_2^N - \Sigma_{i=1}^N x_2^i/_N & .. & x_n^N - \Sigma_{i=1}^N x_n^i/_N \end{bmatrix}$$

PCA – Theory – Covariance matrix

- Using the mean-centred data *U*, compute the covariance matrix Σ

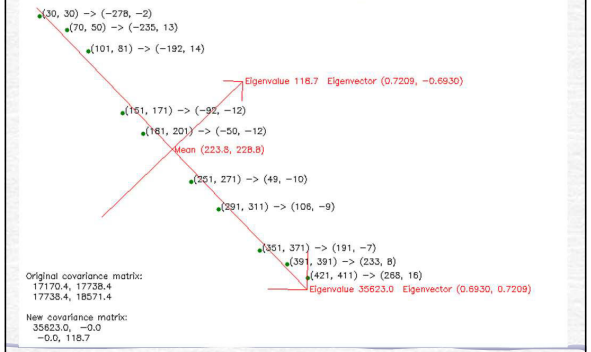
$$\Sigma = {U^T U}/{(N-1)}$$

- We can then determine $\Phi^T\Sigma\Phi = \Lambda$
  - where the eigenvectors are in an orthogonal matrix Φ
  - and the eigenvalues are in an ordered diagonal matrix Λ



- We can consider Φ as a linear transformation
  - e.g. in our example it transforms $[x_1, x_2]$ to $[\phi_1, \phi_2]$
- The amplitudes of the eigenvalues in Λ are proportional to the percentage of overall variance accounted for by the associated eigenvector.

PCA – Example



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PCA – Basics

```
float samples[][2] = { {30,30}, {70,50}, …, {421,411}, {391,391} };
int number_of_samples = sizeof(samples)/sizeof(int[2]);
Mat samples_matrix(number_of_samples, 2, CV_32FC1, samples);
…
PCA pca(samples_matrix, Mat(), 0, 2 );
Mat eigenvalues = pca.eigenvalues;
Mat eigenvectors = pca.eigenvectors;
Mat mean = pca.mean;
```


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PCA – Basics

```
Mat transformed_samples_matrix = samples_matrix.clone();
for (int sample_no=0; sample_no<number_of_samples; sample_no++)
{
  Mat sample = samples_matrix.row(sample_no);
  Mat transformed_sample;
  pca.project(sample, transformed_sample);
  transformed_sample.row(0).copyTo(
                                    transformed_samples_matrix.row(sample_no));
}
Mat covariance,average,new_covariance,new_average;
calcCovarMatrix(samples_matrix,covariance,average,
                                    CV_COVAR_NORMAL | CV_COVAR_ROWS);
calcCovarMatrix(transformed_samples_matrix,new_covariance,
                    new_average,CV_COVAR_NORMAL | CV_COVAR_ROWS);
```


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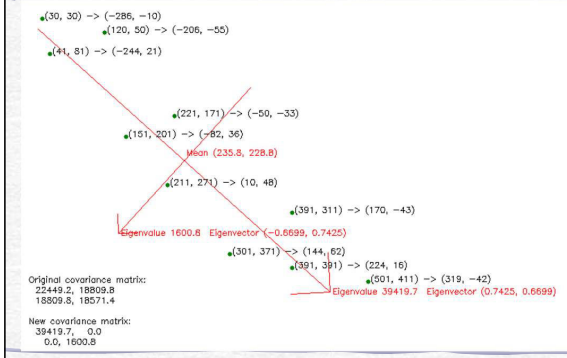
PCA – Example



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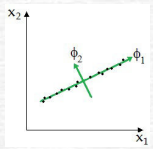
PCA – Compression

- If the variance on an axis is very small
  - Variance caused by noise?
  - Drop this dimension?



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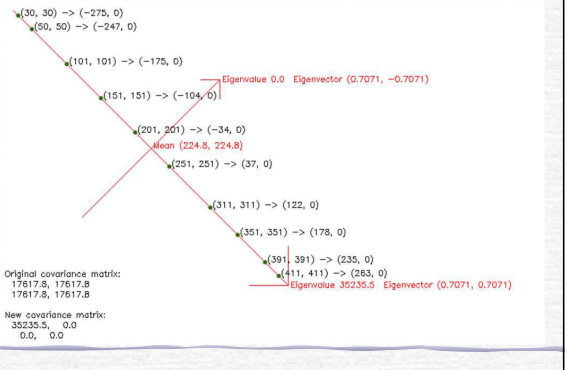
PCA – Example



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PCA – Face Recognition

- Common example application of PCA
- Each face image is treated as a vector
  - with *n=rows*columns* dimensions!

$= [\ 9c, 1a, 97, af, \ldots, 74, 6f\ ]_{900}$

- In the following example… *n = 10304*

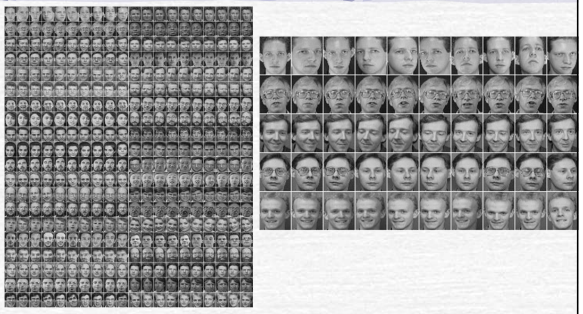
65

PCA – Face Recognition

- Create a matrix *D*
  - where each face image is a row
  - *D* is a *N x n* matrix
- Create mean centred data *U*
  - Determine the mean of the rows (face images)
  - Subtract from all of the rows
- Compute the covariance matrix $\Sigma = {}^{U^T U}/_{(N-1)}$
- Solve for Eigenvectors and Eigenvalues as before.
- Normalise the Eigenvectors
- Select *m* Eigenvectors (with the largest *m* Eigenvalues)
  - Usually 20-50 for face recognition.
- For an unknown image find its location in PCA space
  - Look for the closest match $p_\phi = (p_x - \mu_x).\Phi_{pca}^T$

66

PCA – Face Recognition – Database



Original images provided by AT&T Laboratories Cambridge
http://www.cl.cam.ac.uk/research/dtg/attarchive/facedatabase.html

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PCA – Face Recognition – Some known faces

Original images provided by AT&T Laboratories Cambridge
http://www.cl.cam.ac.uk/research/dtg/attarchive/facedatabase.html

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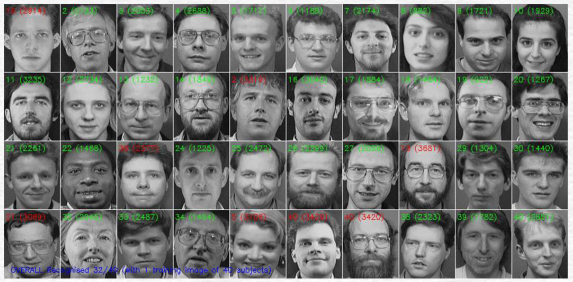
PCA – Face Recognition – Recognised faces



Original images provided by AT&T Laboratories Cambridge (Composite recognised images © Kenneth Dawson-Howe
http://www.cl.cam.ac.uk/research/dtg/attarchive/facedatabase.html

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PCA – Face Recognition – Training & Prediction

```
vector<Mat> known_images;          vector<int> known_labels;
Mat current_image, unknown_image;
int current_label;
...
known_images.push_back(current_image);
known_labels.push_back(current_label);
...
Ptr<FaceRecognizer> recogniser = createEigenFaceRecognizer();
recogniser->train(known_images, known_labels);
...
int predicted_label = 0;          double confidence = 0.0;
recogniser->predict(unknown_image,predicted_label,confidence);
```


70

PCA – Image Reconstruction using Eigenvectors

- We can reconstruct images from PCA space

$$p_x = p_\phi \cdot \Phi_{pca}^T + \mu_x$$

- Using some number (*m*) of principal components
- So that some percentage of variance is accounted for

$$\%\ of\ Total\ Variance = 100. \sum_{i=1}^{m} \lambda_i \Big/ \sum_{j=1}^{n} \lambda_j$$

Figure 3.23: 32 original images of a boy's face, each 321 × 261 pixels. © *Cengage Learning 2015.*

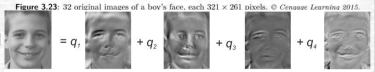


Figure 3.24: Reconstruction of the image from four basis vectors $\mathbf{b}_i$, $i = 1, \ldots, 4$ which can be displayed as images. The linear combination was computed as $q_1\mathbf{b}_1 + q_2\mathbf{b}_2 + q_3\mathbf{b}_3 + q_4\mathbf{b}_4 = 0.078\,\mathbf{b}_1 + 0.062\,\mathbf{b}_2 - 0.182\,\mathbf{b}_3 + 0.179\,\mathbf{b}_4$. © *Cengage Learning 2015.*

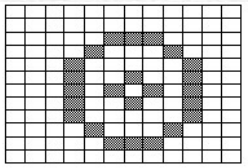
Region Segmentation

- Binary Regions
- k means clustering
- Watershed Segmentation
- Mean Shift Segmentation

Segmentation

- Split images into smaller parts
  - Preferably corresponding to (parts of) objects.
- Two main approaches
  - Region based
  - Edge based
- Video segmentation
  - Breaking video into clips
  - Segmenting objects/regions consistently over time

Connectivity – Paradoxes



- Use pixel Adjacency to build contiguous regions
  - Objects
  - Background
  - Holes

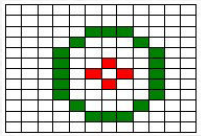
3

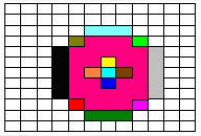
Connectivity – 4 adjacency & 8 adjacency

- Have to use either 4-adjacency or 8-adjacency
  - Label each non-zero pixel…

3	2	1
4	8	0
5	6	7

3	2	1
4	8	0
5	6	7

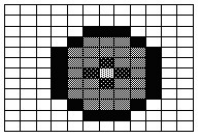




4

Connectivity –

- What do we actually want?

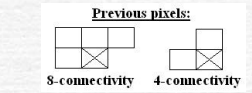


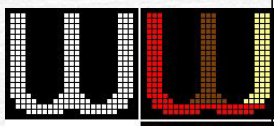
- One possibility
  - Treat background using 4-adjacency
  - Treat object using 8-adjacency
  - Treat holes using 4-adjacency
  - Treat objects in holes using 8-adjacency
  - …

5

Connectivity – Connected Components Analysis

- Search image row by row
  - Label each non-zero pixel
  - If previous pixels are all background
    - Assign New Label
  - Otherwise
    - Pick any label from the previous pixels
    - If any of the other previous pixels have a different label
      - Note equivalence
- Relabel equivalent labels.





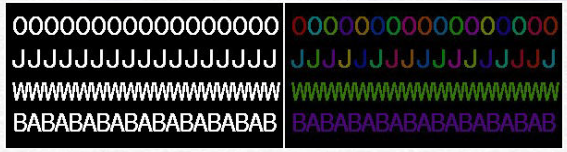
6

Connectivity – Extracting regions

```
vector<vector<Point>> contours;
vector<Vec4i> hierarchy;
findContours( binary_image, contours, hierarchy,
              CV_RETR_TREE, CV_CHAIN_APPROX_NONE );
```


7

Connectivity – Labelling regions



```
for (int contour=0; (contour < contours.size()); contour++)
 {
  Scalar colour( rand()&0xFF,rand()&0xFF,rand()&0xFF );
  drawContours( contours_image, contours, contour, colour,
                                  CV_FILLED, 8, hierarchy );
 }
```


8

k means Clustering

- We'd like to
  - identify significant colours in images
    - Concise descriptions
    - Object tracking
  - reduce the number of colours in any image
    - Compression

- How do we find the best colours?

- k means clustering
  - Creates k clusters of pixels
  - Unsupervised learning

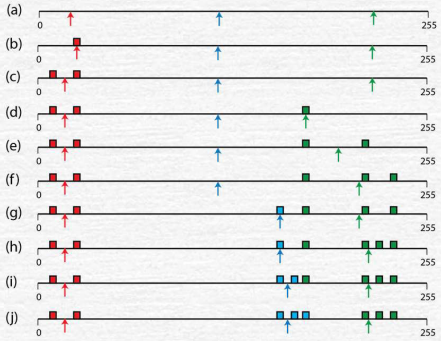
9

k means Clustering – Algorithm

- Number of clusters (k) is known in advance (or determine the k with the maximum confidence)
- Initialise the k cluster exemplars either randomly or use the first k patterns or …
- 1st pass: Allocate patterns to the closest existing cluster exemplar and recompute the exemplar as the centre of gravity
- 2nd pass: Using the final exemplars from the first pass allocate all patterns cluster exemplars.

10

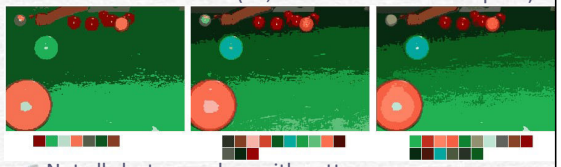
k means Clustering



11

k means Clustering

- Different values of k (10, 15 & 20 random exemplars):



- Not all clusters end up with patterns
- More exemplars generally gives a more faithful representation

12

k means Clustering

- Choosing the best number of exemplars
  - Evaluate the resulting clusters

- Davies-Bouldin index measures cluster separation:
  - $DB = 1/k \ \Sigma_{1..k} \ max_{i \neq j} ((\Delta_i + \Delta_j)/ \delta_{i,j})$
- OR Check that distributions are normal

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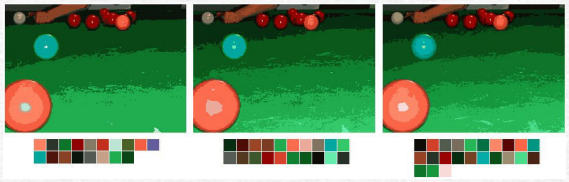
k means Clustering

- Using random exemplars gives non-deterministic results (30 random exemplars each time):



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k means Clustering

```
// Store the image pixels as an array of samples
Mat samples(image.rows*image.cols, 3, CV_32F);
float* sample = samples.ptr<float>(0);
for(int row=0; row<image.rows; row++)
  for(int col=0; col<image.cols; col++)
    for (int channel=0; channel < 3; channel++)
          samples.at<float>(row*image.cols+col,channel) =
                        (uchar) image.at<Vec3b>(row,col)[channel];
// Apply k-means clustering, determining the cluster
// centres and a label for each pixel.
  ....
```


15

k means Clustering

```
Mat labels, centres;
kmeans(samples, k, labels, TermCriteria( CV_TERMCRIT_ITER|
              CV_TERMCRIT_EPS, 0.0001, 10000), iterations,
               KMEANS_PP_CENTERS, centres );
// Use centres and label to populate result image
Mat& result_image = Mat( image.size(), image.type() );
for(int row=0; row<image.rows; row++)
  for(int col=0; col<image.cols; col++)
    for (int channel=0; channel < 3; channel++)
      result_image.at<Vec3b>(row,col)[channel] =
                    (uchar) centres.at<float>( *(labels.ptr<int>(
                                  row*image.cols+col )), channel);
```


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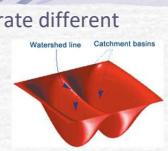
Watershed segmentation

- In geology watersheds separate different catchment basins.
  - Where the rain gets caught
- In computer vision we
  - Identify all minima.
  - Label as different regions.
  - Flood from the minima extending the regions.
  - Where regions meet we have watershed lines.
- Minimum what?
  - Greyscale
  - Gradient
  - Inverse of the chamfer distance



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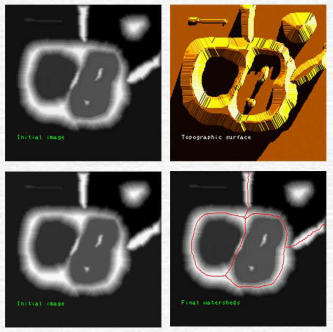
Watershed segmentation example



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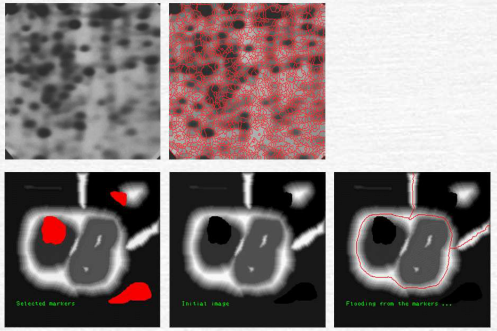
Watershed segmentation with markers



Images from http://cmm.ensmp.fr/~beucher/wtshed.html Reproduced with permission. © 2010, Serge Beucher

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Watershed segmentation with markers

- Generally we get too many regions
  - Use a priori labels to identify 'objects' and expand from these rather than the minima.

20

Mean Shift Segmentation

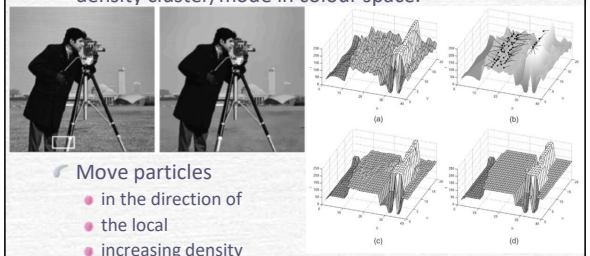
- k-means clustering
  - Requires the number of clusters to be known
  - Takes no account of spatial location
- Mean Shift Segmentation…
  - Do not need to know the number of clusters
  - Can provide spatial as well as colour segmentation
  - Developed by Comaniciu & Meer 2002

Image from "Mean Shift: A Robust Approach Toward Feature Space Analysis" Dorin Comaniciu,, and Peter Meer , IEEE TRANSACTIONS ON PATTERN ANALYSIS AND MACHINE INTELLIGENCE, VOL. 24, NO. 5, MAY 2002;

21

Mean Shift for segmentation– goal

- To associate each point with a particular high-density cluster/mode in colour space.



- Move particles
  - in the direction of
  - the local
  - increasing density

Images from "Mean Shift: A Robust Approach Toward Feature Space Analysis" Dorin Comaniciu,, and Peter Meer , IEEE TRANSACTIONS ON PATTERN ANALYSIS AND MACHINE INTELLIGENCE, VOL. 24, NO. 5, MAY 2002;

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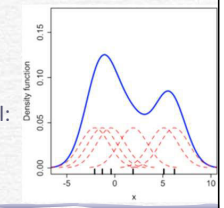
Kernel Density Estimation

- The Problem: Given a sparse dataset determine an estimate of density at each point.

$$\hat{f}_h(x) = \frac{1}{n}\sum_{i=1}^{n} K_h(x - x_i) = \frac{1}{nh}\sum_{i=1}^{n} K\left(\frac{x - x_i}{h}\right)$$

  - Kernel Density K() function n samples x_i
  - h is the bandwidth
- Effectively
  - Smooth all data samples
  - Add them all together
- If the dataset is multidimensional:

$$\hat{f}(\mathbf{x}) = \frac{1}{nh^d}\sum_{i=1}^{n} K\left(\frac{\mathbf{x} - \mathbf{x}_i}{h}\right).$$



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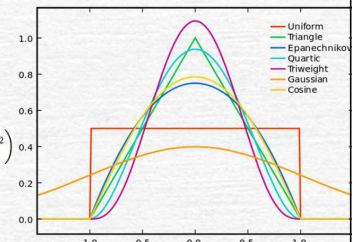
Kernel Density Estimation – Kernels

- Many different kernels.
- Kernel functions must integrate to 1.
- Typically use either
  - Uniform

$$k_E(x) = \begin{cases} 1 - x & 0 \le x \le 1 \\ 0 & x > 1, \end{cases}$$

  - Gaussian

$$K_N(\mathbf{x}) = (2\pi)^{-d/2}\exp\left(-\frac{1}{2}\|\mathbf{x}\|^2\right)$$



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Mean Shift – Basic Algorithm

- For each particle (pixel)
  - Estimate the local kernel density and more importantly the direction of local increasing density (the *mean shift vector)*
  - Shift the particle to the new mean.
  - Re-compute until the location stabilizes.
- Finally identify which pixels ended up in the same location
  - Mark these as members of the same cluster.
  - Determine the local mean of similar particles

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Mean Shift – The local kernel density

- We limit the points included in the kernel density estimate based on distance and on similar to the current point.

$$K_{h_s,h_r}(\mathbf{x}) = \frac{C}{h_s^2 h_r^r} k\left(\left\|\frac{\mathbf{x}^s}{h_s}\right\|^2\right) k\left(\left\|\frac{\mathbf{x}^r}{h_r}\right\|^2\right)$$

  - We must use both a spatial kernel and a colour kernel.
  - Both can be Gaussian
  - Spatial kernel limits/weights the region to consider around the current point
  - Colour/range kernel limits/weights the colour/intensity of the points to be included in the mean.

Maths from "Mean Shift: A Robust Approach Toward Feature Space Analysis" Dorin Comaniciu,, and Peter Meer , IEEE TRANSACTIONS ON PATTERN ANALYSIS AND MACHINE INTELLIGENCE, VOL. 24, NO. 5, MAY 2002;

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Mean Shift – The mean shift vector

- For radially symmetric kernels $K(\mathbf{x}) = c_{k,d}k(\|\mathbf{x}\|^2)$
- So the KDE becomes $\hat{f}_{h,K}(\mathbf{x}) = \frac{c_{k,d}}{nh^d}\sum_{i=1}^{n} k\left(\left\|\frac{\mathbf{x}-\mathbf{x}_i}{h}\right\|^2\right)$ as $\hat{f}(\mathbf{x}) = \frac{1}{nh^d}\sum_{i=1}^{n} K\left(\frac{\mathbf{x}-\mathbf{x}_i}{h}\right)$
- We are interested in the rate of change…

$$\hat{\nabla} f_{h,K}(\mathbf{x}) \equiv \nabla \hat{f}_{h,K}(\mathbf{x}) = \frac{2c_{k,d}}{nh^{d+2}}\sum_{i=1}^{n}(\mathbf{x}-\mathbf{x}_i)k'\left(\left\|\frac{\mathbf{x}-\mathbf{x}_i}{h}\right\|^2\right)$$

- Set $g(x) = -k'(x)$ so

$$= \frac{2c_{k,d}}{nh^{d+2}}\sum_{i=1}^{n}(\mathbf{x}_i-\mathbf{x})g\left(\left\|\frac{\mathbf{x}-\mathbf{x}_i}{h}\right\|^2\right)$$

$$= \frac{2c_{k,d}}{nh^{d+2}}\left[\sum_{i=1}^{n} g\left(\left\|\frac{\mathbf{x}-\mathbf{x}_i}{h}\right\|^2\right)\right]\left[\frac{\sum_{i=1}^{n}\mathbf{x}_i g\left(\left\|\frac{\mathbf{x}-\mathbf{x}_i}{h}\right\|^2\right)}{\sum_{i=1}^{n} g\left(\left\|\frac{\mathbf{x}-\mathbf{x}_i}{h}\right\|^2\right)} - \mathbf{x}\right]$$

- Define the mean shift vector as $\mathbf{m}_{h,G}(\mathbf{x}) = \frac{\sum_{i=1}^{n}\mathbf{x}_i g\left(\left\|\frac{\mathbf{x}-\mathbf{x}_i}{h}\right\|^2\right)}{\sum_{i=1}^{n} g\left(\left\|\frac{\mathbf{x}-\mathbf{x}_i}{h}\right\|^2\right)} - \mathbf{x}$

Maths from "Mean Shift: A Robust Approach Toward Feature Space Analysis" Dorin Comaniciu,, and Peter Meer ,
IEEE TRANSACTIONS ON PATTERN ANALYSIS AND MACHINE INTELLIGENCE, VOL. 24, NO. 5, MAY 2002;

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Mean Shift – Spatial kernel functions

Varying the Spatial radius and varying the colour radius

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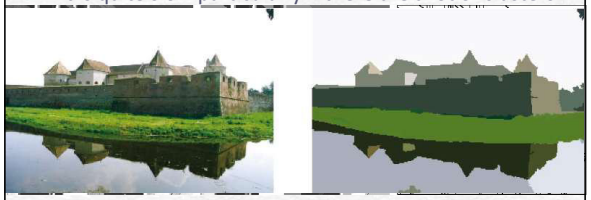
Mean Shift – Pros and cons

- + Do not need to know the number of clusters *a priori*.
- + Provides spatial as well as colour segmentation.
- - Selection of kernel widths can be very hard.
- - It is quite slow particularly if there are a lot of clusters.



Images from "Mean Shift: A Robust Approach Toward Feature Space Analysis" Dorin Comaniciu,, and Peter Meer ,
IEEE TRANSACTIONS ON PATTERN ANALYSIS AND MACHINE INTELLIGENCE, VOL. 24, NO. 5, MAY 2002;
From http://www.comaniciu.net/Papers/MsRobustApproach.pdf

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Video

- Introduction
  - Static Background
- Background Models
  - Median
  - Gaussian Mixture Model
  - Shadow Detection
- Tracking
  - Exhaustive Search, Mean Shift, Optical Flow, Feature Point Tracking

1

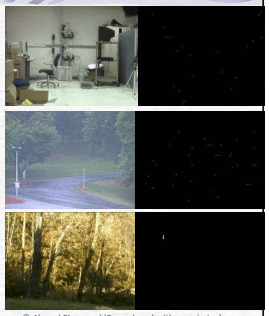
Introduction – Objects of interest

- Reliably detecting moving objects of interest in a scene.
  1. Motion detection
  2. Moving object detection & location
  3. Derivation of 3D object properties
- When is an object of interest?
  - Size
  - Max and min velocity and acceleration
  - Assumptions:
    - Mutual correspondence
    - Common motion

2

Introduction – Common problems

- Illumination & appearance changes
  - Gradual (e.g. time of day)
  - Sudden (e.g. clouds, lights)
  - Shadows
  - Weather (e.g. rain, snow)
- Background changes
  - Objects becoming part of the background
  - Objects leaving the background
  - Background objects oscillating slightly
- Setup
  - Camera motion
  - Frame rate
  - Field of view
  - Distance to objects
  - Location of camera



3

Introduction – Static background

- Image subtraction $d(i,j) = |f_k(i,j) - b(i,j)|$
  - Difference of current frame and background image..

```
absdiff( frame, background, difference );
```

  - Background b(i,j) = first frame?

4

Introduction – Thresholding

- Threshold result

$$d(i,j) = \begin{cases} 0 & \text{if } |f_k(i,j) - b(i,j)| < T \\ 1 & \text{otherwise} \end{cases}$$

- Threshold T sensitivity?
  - Too high ➔ False negatives
  - Too low ➔ False positives
  - Just right??
- High contrast required.

Introduction – Colour

- But how did we deal with colour?
  1. Average difference of all channels?
  2. Just process hue channel?
     - What about white/grey/black?
  3. Threshold channels separately
     - Any one channel or all channels?
     - Which colour model?

Introduction – Static Background

```
absdiff( current_frame, first_frame, difference );
cvtColor( difference, moving_points, CV_BGR2GRAY );
threshold( moving_points, moving_points,  30, 255, THRESH_BINARY );
Mat display_image = Mat::zeros( moving_points.size(), CV_8UC3 );
current_frame.copyTo( display_image, moving_points );
```

Background Models – Median

- Median: Middle value (from an ordered list)

$$h_n(i,j,p) = \sum_{k=(n-m+1)..n} \begin{cases} 1 & \text{... if } (f_k(i,j) = p) \\ 0 & \text{... otherwise} \end{cases}$$

  - No. of frames (m)
  - Histogram quantisation?
  - Computational expense
    - Adding, storing and removing frames
    - Change in median can be tracked inexpensively from frame to frame
    - Can be approximated using aging

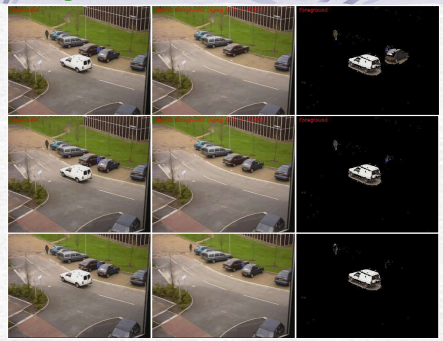
$$h_n(i,j,p) = \sum_{k=1..n} \begin{cases} w_k & \text{... if } (f_k(i,j) = p) \\ 0 & \text{... otherwise} \end{cases}$$

where $w_1 = 1$ and $w_k = w_{k-1} * 1.001$

  - Can also use selective update
  - Could use the **Mode** instead... (Most common value)

$$b_n(i,j) = p \quad \text{where } h_n(i,j,p) \geq h_n(i,j,q) \text{ for all } q \neq p$$

Background Models – Median Results



Learning Rate = 1.001

Learning Rate = 1.005

Learning Rate = 1.02

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Background Models – Median Algorithm

First frame:

$total = 1$

for all pixels (i,j)

$median = f_1(i,j)$

$less\_than\_median(i,j) = 0$

Current frame (n):

$total = total + w_n$

for all pixels (i,j)

if $(median(i,j) > f_n(i,j))$

$less\_than\_median(i,j) = less\_than\_median(i,j) + w_n$

while $(less\_than\_median(i,j) + h_n(i,j,median(i,j)) < {}^{total}/_{2})$

$less\_than\_median(i,j) = less\_than\_median(i,j) + h_n(i,j,median(i,j))$

$median(i,j) = median(i,j) + 1$

while $(less\_than\_median(i,j) > {}^{total}/_{2})$

$median(i,j) = median(i,j) - 1$

$less\_than\_median(i,j) = less\_than\_median(i,j) - h_n(i,j,median(i,j))$

Background Models – Gaussian Mixture Model

- How to deal with multi-modal background pixels
  - e.g. from trees, water
  - *Stauffer & Grimson, 2000*
  - *Algorithm presented is based on that in Sonka (3rd edition) pp.777-780*
- Model multiple values (3-5) at each point.
- Unsupervised learning…
- Most popular method for background modelling

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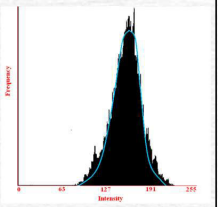
Background Models – GMM Model details

- Model each pixel $f_n(i,j)$ using k Gaussian distributions each with
  - $\pi_n(i,j,m)$
  - $\mu_n(i,j,m)$
  - $\sigma^2_n(i,j,m)$
    - Must assume different components independent and of equal variance
- Set a learning constant α
  - in the range 0.01 … 0.1



Background Models – GMM Model Update

- For a new sample $f_n(i,j)$
  - Select **the best close Gaussian distribution**
    - **close** = within *2.5* $\sigma_n(i,j,m)$ of $\mu_n(i,j,m)$
  - If there is a best close Gaussian *I*

$$\pi_{n+1}(i,j,m) = \alpha * O_n(i,j,m) + (1-\alpha) * \pi_n(i,j,m)$$

where $O_n(i,j,m) = 1$ for **the close Gaussian distribution** and 0 otherwise

$$\mu_{n+1}(i,j,m) = \mu_n(i,j,m) + O_n(i,j,m) * (\alpha / \pi_{n+1}(i,j,m)) * (f_n(i,j)-\mu_n(i,j,m))$$

$$\sigma^2_{n+1}(i,j,m) = \sigma^2_{n+1}(i,j,m) + O_n(i,j,m) * (\alpha / \pi_{n+1}(i,j,m)) * ((f_n(i,j)-\mu_n(i,j,m))^2 - \sigma^2_n(i,j,m))$$

  - If there is no close Gaussian (replace one…)
    - $x = argmin_m(\pi_n(i,j,m))$
    - $\mu_{n+1}(i,j,x) = f_n(i,j)$
    - $\sigma^2_{n+1}(i,j,x) = 2.max_m\, \sigma^2_n(i,j,m)$
    - $\pi_{n+1}(i,j,x) = ½.min_m\, \pi_n(i,j,m)$

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Background Models – GMM Moving Points

- Identifying background distributions
  - Define *T*, a proportion of the frames in which background pixels should be visible.
  - Order the Gaussians by $\pi_{n+1}(i,j,m) / \sigma_{n+1}(i,j,m)$
  - Gaussians *1..B* are considered background where
    - $B = argmin_b((\Sigma_{b=1..m}\, \pi_{n+1}(i,j,m)) > T)$
- Just check if best close Gaussian (or the new Gaussian distribution) is a background distribution
- Finally use morphological dilations and erosions to remove small regions and fill in holes.

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Background Models – GMM issues

- GMM issues:
  - may fail under fast variations,
  - low sensitivity around Gaussian tails,
  - less frequent events produce low probability & high variance,
  - needs to compute floating point probabilities.

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Background Models – Shadow Detection

$$SP_k(i,j) = \begin{cases} 1 \ldots \text{if } \left(\alpha < \frac{f_k^V(i,j)}{B_k^V(i,j)} < \beta\right) \text{ and } \left(\left(f_k^S(i,j) - B_k^S(i,j)\right) < \tau_S\right) \text{ and } \left(\left|f_k^H(i,j) - B_k^H(i,j)\right| < \tau_H\right) \\ 0 \ldots \text{otherwise} \end{cases}$$

- Compare current frame with background image… (Prati 2003)

  - Intensity / luminance / value drops

  - Intensity / luminance / value has not dropped too much

  - Saturation does not increase too much
  - Hue does not change too much
- Hue unpredictable & Change in Luminance can be small (Tattersall 2003)

$$SP_k(i,j) = \begin{cases} 1 \ldots \text{if } \left(\lambda < \frac{f_k^V(i,j)}{B_k^V(i,j)} < 1.0\right) \text{ and } \left(\left|f_k^S(i,j) - B_k^S(i,j)\right| < \tau_S\right) \\ 0 \ldots \text{otherwise} \end{cases}$$

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Background Models – Shadow Detection

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Background Models – Shadow Detection – Code

```
Ptr<BackgroundSubtractorMOG2> gmm =
                                createBackgroundSubtractorMOG2();
gmm->apply(current_frame, foreground_mask);
gmm( current_frame, foreground_mask );
threshold( foreground_mask, moving_points, 150, 255, THRESH_BINARY );
…
threshold( foreground_mask, changing_points, 50, 255, THRESH_BINARY );
absdiff( moving_points, changing_points, shadow_points );
…
Mat mean_background_image;
gmm->getBackgroundImage( mean_background_image );
```


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Tracking – Introduction

- Used in video surveillance, sports video analysis, vehicle guidance systems, etc.
- A hard task because objects
  - may be undergoing complex motion
  - may change shape
  - may be occluded
  - may change appearance due to lighting/weather
  - may physically change appearance

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Tracking – Topics

- Exhaustive search
- Mean Shift
- Optical Flow
- Feature based tracking

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Tracking – Exhaustive Search

- Extract object to be tracked from frame
- Compare in all possible positions in future frame(s)
  - Use a similarity metric
    - E.g. normalised cross correlation
  - Just pick the best match?
- Need extra degrees of freedom for scale and orientation.
- May fail if object motion is too complex.
- Template matching and chamfer matching support this type of tracking.

Tracking – Mean Shift

- Tracks "objects" in video by
  - Searching *locally* for the most similar region.
  - Using a histogram to represent the "object".
  - Using (iterative) gradient ascent to locate the best match.
- Mean shift applied to tracking in 2002 by Comaniciu and Meer.
  - Also used for segmentation.
- Histogram of
  - Colours
  - Oriented gradients
  - Texture
  - etc.

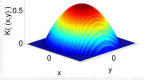
Tracking – Mean Shift

- Model the "object"

$$\hat{q}_u \triangleq C \sum_{i=1}^{n} k\left(\left\|\frac{\mathbf{x}_i}{h_q}\right\|^2\right) \delta\left[b\left(\mathbf{x}_i\right) - u\right]$$

  - Probability Density Function (histogram) of colours.
    - Limit the number of bins
  - Typically an elliptical region is used.

  - Weight the values relative to their location.
    - Epanechnikov kernel



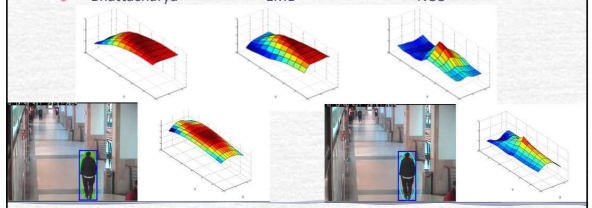
Tracking – Mean Shift

- Model candidates regions…

$$\hat{p}_u(\mathbf{y}) \triangleq C_h \sum_{i=1}^{n_h} k\left(\left\|\frac{\mathbf{y} - \mathbf{x}_i}{h_p}\right\|^2\right) \delta\left[b\left(\mathbf{x}_i\right) - u\right]$$

- Matching to find the best new location…
  - Compare distributions directly.
  - Move to the mode in matching space.

Bhattacharya EMD NCC



Tracking – Mean Shift

- Mean shift approach
  - Consider the gradient of the similarity function (the Bhattacharyya coefficient)…
  - Gradient of superposition of kernels, centered at each data point is equivalent to convolving the data points with the gradient of the kernel

$$w_i = \sum_{u=1}^{m} \sqrt{\frac{\hat{q}_u}{\hat{p}_u(\mathbf{y_0})}} \delta\left[b\left(\mathbf{x}_i\right) - u\right] \qquad \mathbf{y_1} = \frac{\sum_{i=1}^{n_h} \mathbf{x}_i w_i}{\sum_{i=1}^{n_h} w_i}$$

    - Derived from the Bhattacharyya similarity measure
    - Assumes Epanechnikov kernels
    - Move in the direction of the highest gradient
  - Iterate until convergence
    - Separation between y_0 and y_1 less than ε

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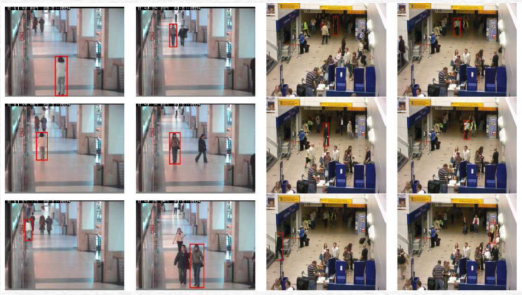
Tracking – Mean Shift – Test dataset

- CAVIAR (14) and PETS (7) tracking scenarios



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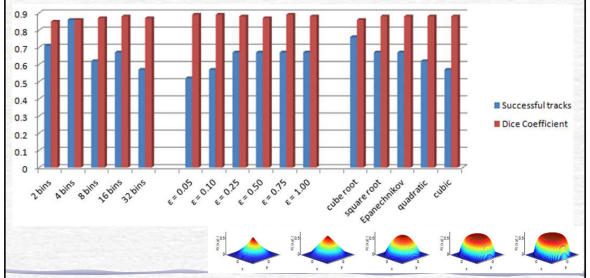
Tracking – Mean Shift – Changing parameters

- Number of bins per channel
- Convergence parameter (ε)
- Kernel type



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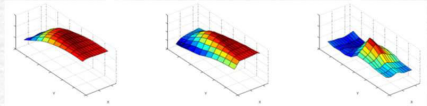
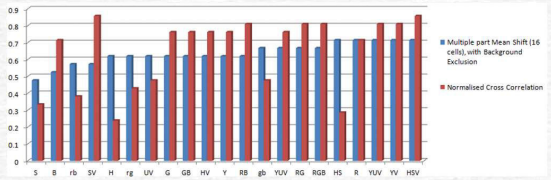
Tracking – Mean Shift – The best technique?

- Bhattacharya
- EMD
- NCC



- Colour Space



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Tracking – Mean Shift – Background Exclusion

- IF a background model of the scene is available
  - Favour image regions which are similar to the object model
  - AND dissimilar to the corresponding background region.

- Analogous to background subtraction
  - No image differencing.

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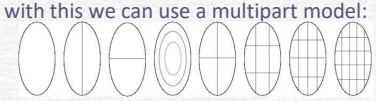
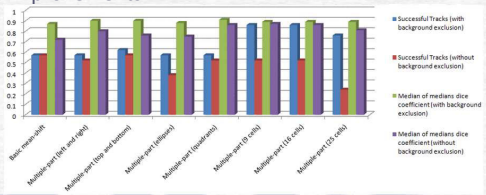
Tracking – Mean Shift – Multipart models

- Histograms have a lack of spatial structure
- To deal with this we can use a multipart model:



- Improvements:



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Tracking – Mean Shift – In OpenCV

- Back-projects a histogram of the object into the current frame
- Searches for a region of the same size within the back projection looking for the highest (weighted) sum.

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Tracking – Mean Shift – In OpenCV

```
Rect position(starting_position);
TermCriteria criteria( cv::TermCriteria::MAX_ITER, 5, 0.01);
meanShift( back_projection_probabilities, position, criteria);
```

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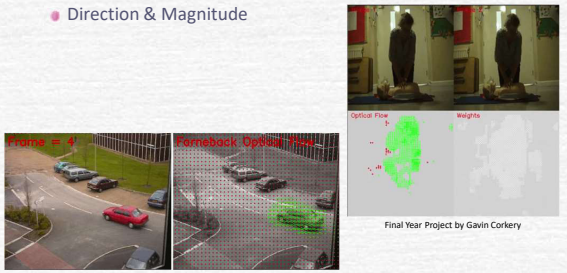
Tracking – Dense Optical Flow

- Compute a motion field (known as optical flow) for the entire image
  - Direction & Magnitude



Final Year Project by Gavin Corkery

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Tracking – Dense Optical Flow

- Based on the brightness constancy constraint
  - Object points will have the same brightness over a short period of time $f_t(i,j) = f_{t+\Delta t}(i+\Delta i, j+\Delta j)$
- Need to find the displacement $(\Delta i, \Delta j)$ which will minimise the residual error

$$\varepsilon(\Delta i, \Delta j) = \sum_{i=i_{current}-w}^{i_{current}+w} \sum_{j=j_{current}-w}^{j_{current}+w} f_t(i,j) - f_{t+\Delta t}(i+\Delta i, j+\Delta j)$$

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Tracking – Dense Optical Flow

- To compute the optical flow $\left(\frac{\Delta i}{\Delta t}, \frac{\Delta j}{\Delta t}\right)$, assuming that the displacement is small…

$$f_{t+\Delta t}(i+\Delta i, j+\Delta j) = f_t(i,j) + \frac{\partial f}{\partial i}\Delta i + \frac{\partial f}{\partial j}\Delta j + \frac{\partial f}{\partial t}\Delta t$$

- Hence (given our previous equation

$$\frac{\partial f}{\partial i}\Delta i + \frac{\partial f}{\partial j}\Delta j + \frac{\partial f}{\partial t}\Delta t = 0$$

- So

$$\frac{\partial f}{\partial i}\frac{\Delta i}{\Delta t} + \frac{\partial f}{\partial j}\frac{\Delta j}{\Delta t} + \frac{\partial f}{\partial t} = 0$$

- And reorganising

$$\begin{bmatrix}\frac{\partial f}{\partial i} & \frac{\partial f}{\partial j}\end{bmatrix}\begin{bmatrix}\frac{\Delta i}{\Delta t} \\ \frac{\Delta j}{\Delta t}\end{bmatrix} = -\frac{\partial f}{\partial t}$$

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Tracking – Dense Optical Flow – Code

```
calcOpticalFlowFarneback (previous_gray_frame, gray_frame,
                optical_flow, 0.5, 3, 15, 3, 5, 1.2, 0);
cvtColor (previous_gray_frame, display, CV_GRAY2BGR);
for (int row = 4; row < display.rows; row+=8)
 for(int column = 4; column < display.cols; column+8)
 {
  Point2f& flow = optical_flow.at<Point2f>(row,column);
  line (display, Point(column,row), Point(
          cvRound(column+flow.x), cvRound(row+flow.y)),
          passed_colour);
 }
gmm.getBackgroundImage( mean_background_image );
```


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Tracking – Dense Optical Flow – Problems

- Optical flow is the apparent motion of points within a scene.
  - Based on brightness patterns
  - Needs texture

Q. What happens if the brightness changes?
- i.e. brightness constancy does not hold.

A. Perhaps look at optical flow in gradient space.

Q. What if a point moves differently to all its neighbours?

A. Use Region based optical flow

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Tracking – Dense Optical Flow – Problems

Q. What happens to a rotating sphere? What about a "barber pole"?

A. We get the wrong motion. i.e. it fails.

Q. What happens if the motion is too large?

A. Use Iterative Refinement (Lucas-Kanade)

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Tracking – Feature-based Optical Flow

- We cannot accurately compute optical flow for constant regions or along edges.
- Often better to compute optical flow just for features… (e.g. Lucas Kanade feature tracker)…

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