ST3009 Weekly Questions 2

Séamus Woods
15317173

08/02/2019

0.1 Question 1

A 6-sided die is rolled three times.

(a) How many elements are there in the same space?
Sample space S is the set of all possible outcomes of an experiment. If we roll a 6-sided dice 3 times we have 6^3 possible outcomes, or 216.

(b) Out of the possible sets of outcomes, calculate in how many at one least 2 rolled. Using this, calculate what the probability is that at least one 2 is rolled.
The way I calculated this was by finding out how many times tere is exactly one 2, exactly two 2's and then all 2's and adding those together. The sum for this is $\binom{3}{1} * 6^2 + \binom{3}{2} * 6 + \binom{3}{3}$, which adds up to be 92. To calculate the probability that there will be at least one 2, we simply divide this by the total amount of possibilities. $\frac{92}{216} = 0.42$.

(c) Write a small Matlab simulation of this experiment and confirm that the observed probability that at least one 2 is rolled matches your calculation in (a).
I wrote a small simulation of this in Matlab as seen below.

```
count = 0;
rolls = 10000;
roll1 = randi([1 6], 1, rolls);
roll2 = randi([1 6], 1, rolls);
roll3 = randi([1 6], 1, rolls);

for i = 1:numel(roll1)
   if (roll1(i) == 2) || (roll2(i) == 2) || (roll3(i) == 2)
       count = count + 1;
   end
end

prob = count / rolls;
disp(prob);
```

Basically I create a count variable, which will be used to count the amount of rolls that have at least one 2 rolled, I set the amount of rolls to 10,000 just for a bit of consistency. I then create three arrays of length 10,000, with number between 1 and 6 inclusive. I then just created a for loop to check for each roll if there was at least a 2 rolled, and if there was a incremented the count, after the loop was done I calculated the probability. I ran this program several times and got 0.4280, 0.4307, 0.4151, 0.4195, 0.4129.. so you can see that the numbers match up.

(d) What is the probability that the sum of the die rolls is 17?
The only combination from 3 dice that will add up to 17 is two 6's and a 5. This combination can be arranged in three different ways.. [6,6,5], [6,5,6] and [5,6,6]. With only 3 possibilities for 3 dice to sum to 17, the probability of this happening is $\frac{3}{216} = 0.0138$

(e) What is the probability that the sum of the three die rolls is 12 given that the first roll was a 1?
The way I thought about this is that our roll will be [1,x,y], where 1 is constant here and the sum must be 12. Therefore x and y must sum to 11. This can only be done with a 5 and a 6, leaving us with 2 possibilities. [1,5,6] and [1,6,5]. The probability of this happening is $\frac{2}{36} = 0.055$. We divide by 36 here because the chance relied on the rolling of 2 dice.

0.2 Question 2

I roll a 6-sided die. If it comes up a 1 then I throw a six-sided die and otherwise a 20-sided die.

(a) What is the probability that the second throw comes up a 5?
So there are two ways in which the second roll can be 5. The first way is that we roll a 1 on the first die ($\frac{1}{6}$) and then roll a 5 on the second die ($\frac{1}{6}$) which has the probability of $\frac{1}{36}$ ($\frac{1}{6} * \frac{1}{6}$), or we don't roll a 1 on the first die ($\frac{5}{6}$) and have to roll the 20-sided die, where we have a $\frac{1}{20}$ chance of rolling a 5, giving us $\frac{1}{24}$ ($\frac{5}{6} * \frac{1}{20}$). Summing these together $\frac{1}{36} + \frac{1}{24}$ we have a 0.0694 chance of rolling a 5 on the second die.

(b) What is the probability that the second throw comes up a 15?
The only possible way we can roll a 15 on the second throw is if we don't roll a 1 on the first. So we have the chance we don't roll a 1 $\frac{5}{6}$ multiplied by the chance we roll a 15 on the 20 sided die $\frac{1}{20}$ giving us a probability of $\frac{1}{24}$ or 0.0417.

0.3 Question 3

At a certain stage of a criminal investigation, the inspector in charge is 60 percent convinced of the guilt of a certain suspect. Suppose, however that a new piece of evidence which shows that the criminal has a certain characteristic (such as left-handedness, baldness, or brown hair) is uncovered. If 20 percent of the population possesses this characteristic, use Bayes Rule to calculate how certain of the guilt of the suspect should the inspector now be if it turns out that the suspect has the characteristic.

$$\text{Bayes Rule: } P(E|F) = \frac{P(F|E)P(E)}{P(F)}$$

$$P(F) = P(F|E)P(E) + P(F|E^c)P(E^c)$$

In this text we are given two bits of information. How certain the investigator is that the suspect is guilty (0.6) and the probability that an individual in the population has the same characteristic that the criminal has (0.2). We will make $P(E)$ 0.6, since it is our prior belief that the suspect is guilty. The chance that the suspect has this characterisitc given that they are guilty must be 100% therefore we can make $P(F|E) = 1$. In Bayes Rule we do not know what $P(F)$ is so we have to use the above equation. Here we can make $P(F|E^c) = 0.2$ since it is the probability that somebody has the trait but is not the criminal. No we have all variables needed to solve the certainty the detective will be that the suspect is guilty if they have this characteristic ($P(E|F)$). Subbing our values into the following formula:

$$P(E|F) = \frac{P(F|E)P(E)}{P(F|E)P(E)+P(F|E^c)P(E^c)}$$

We get that $P(E|F)$ is 0.8823.

0.4 Question 4

Your cell phone is constantly trying to keep track of where you are. At any given point in time, for all nearby locations, your phone stores a probability that you are in that location. Right now your phone believes that you are in one of 16 different locations arranged in a grid with the following probabilities (see the figure on the left):

Prior Belief of Location

$$\begin{bmatrix} 0.05 & 0.10 & 0.05 & 0.05 \\ 0.05 & 0.10 & 0.05 & 0.05 \\ 0.05 & 0.05 & 0.10 & 0.05 \\ 0.05 & 0.05 & 0.10 & 0.05 \end{bmatrix}$$

P(Observe two bars signal | Location)

$$\begin{bmatrix} 0.75 & 0.95 & 0.75 & 0.05 \\ 0.05 & 0.75 & 0.95 & 0.75 \\ 0.01 & 0.05 & 0.75 & 0.95 \\ 0.01 & 0.01 & 0.05 & 0.75 \end{bmatrix}$$

our phone connects to a known cell tower and records two bars of signal. For each grid location L you know the probability of observing two bars from this particular tower, given that the cell phone is in location L (see the figure on the right). Example: the highlighted cell on the left figure means that you believed there was a 0.05 probability that the user was in the bottom right grid cell prior to observing the cell tower signal. The highlighted cell on the right figure means that you think the probability of observing two bars, given the user was in the bottom right grid cell, is 0.75. For each of the 16 location positions, calculate the new probability that the user is in each location given the cell tower observation. Write a program to calculate the probabilities. Report the probabilities of all 16 cells and write a short explanation of your program.

To understand how we're going to write a program to solve the entire matrix, lets solve one cell by hand, lets take the bottom right for instance. The easiest way I found of doing this was to lay out what I knew. I know that $P(E)$ = 0.05 (Prior Belief of Location). We don't know what $P(F)$ is, which is the probability of observing two bars of signal. We know that $P(F|E)$ is 0.75. We can intuitively tell that $P(F|E^c)$ is 0.25, since it is the probability that we have two bars signal given we **do not** know the location, and we know that probability of having two bars of signal given we **do** know the location is 0.75. Given all this we have enough to solve for $P(E|F)$, proability of location given two bars signal, using the same equation as the last question. Subbing our values into the following formula:

$$P(E|F) = \frac{P(F|E)P(E)}{P(F|E)P(E)+P(F|E^c)P(E^c)}$$

We get that $P(E|F)$ is 0.136. I wrote a Matlab program to solve this for every cell of the matrix, which can be seen below.

```
a = [0.05 0.10 0.05 0.05;
     0.05 0.10 0.05 0.05;
     0.05 0.05 0.10 0.05;
     0.05 0.05 0.10 0.05]

ba = [0.75 0.95 0.75 0.05;
      0.05 0.75 0.95 0.75;
      0.01 0.05 0.75 0.95;
      0.01 0.01 0.05 0.75]

ab = zeros(size(a));

for i = 1:numel(a)
    part1 = ba(i) * a(i)
    part2 = 1 - ba(i)
    part3 = 1 - a(i)
    ab(i) = part1 / (part1 + part2*part3)
end

disp(ab)
```

The output that I got from this is below.

$$\text{P(Location} \mid \text{Observing two bars signal)}$$
$$\begin{bmatrix} 0.1363 & 0.6785 & 0.1363 & 0.0027 \\ 0.0027 & 0.2500 & 0.5000 & 0.1363 \\ 0.0005 & 0.0027 & 0.2500 & 0.5000 \\ 0.0005 & 0.0005 & 0.0050 & 0.1363 \end{bmatrix}$$