ST3009 Weekly Questions 5

Séamus Woods
15317173

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0.1 Question 1

A box contains 5 red and 5 blue marbles. Two marbles are withdrawn randomly. If they are the same color, then you win $1.10; if they are different colors, then you lose $1.00. Calculate:

(a) The expected value of the amount you win.
The expected value of a discrete random variable X taking values in $\{x_1, x_2, ..., x_n\}$ is defined to be:

$$E[X] = \sum_{i=1}^{n} x_i P(X = x_i)$$

First lets calculate the chance of winning. The chance we pick two balls of the same color: $\frac{5}{10} * \frac{4}{9} = 0.2222$. Now we can calculate the expected value as $(0.2222 * 0.1) + ((1 - 0.2222) * -0.1) = -0.05556$.

(b) The variance of the amount you win.
Let X be a random variable with mean μ. The variance of X is

$$Var(X) = E[(X - \mu)^2].$$

Using this formula we can calculate the variance of the amount we win, using the mean we calculated in the last question. $Var(X) = (1 - (-0.05556))^2 * (-0.05556) + (0 - (-0.05556))^2 * -0.05556$

0.2 Question 2

Suppose you carry out a poll following an election. You do this by selecting n people uniformly at random and asking whether they voted or not, letting $X_i = 1$ if persion i voted and $X_i = 0$ otherwise. Suppose the probability that a person voted is 0.6.

(a) Calculate $E[X_i] and Var(X_i)$.
Answer here..

Let $Y = \sum_{i=1}^{n} X_i$.
(b) What is $E[Y]$? Is it the same as $E[X]$ or different, and why?
Answer here..

(c) What is $E[\frac{1}{n}Y]$?
Answer here..

(d) What is the variance of $\frac{1}{n}Y$ (express in terms of $Var(X)$) ?
Answer here..

Hints: Use linearity of the expecation and the fact that people are sampled independently.

0.3 Question 3

Suppose that 2 balls are chosen without replacement from an urn consisting of 5 white and 8 red balls. Let X_i equal 1 if the i'th ball selected is white, and let it equal 0 otherwise.

(a) Give the joint probability mass function of X_1 and X_2.
Answer here..

(b) Are X_1 and X_2 independent? (Use the formal definition of independence to determine this).
Answer here..

(c) Calculate $E[X_2]$.
Answer here..

(d) Calculate $E[X_2]X_1 = 1$.