# Chapter 7: Thin Wall Pressure Vessels

G. Jeon

April 6, 2024

**Coverage:** Hoop stress, longitudinal, spherical, cylinder, combinations of both

## 1 Geometries

More often than not, we're working with pretty simple geometries. This list is commonly made up of spheres, cylinders, and mixes of both (like compressed air canisters). Well what makes a vessel's walls thin? We simply check that the ratio of the vessel radius and thickness is greater than a factor of 10.

$$\frac{r}{t} > 10 \tag{1}$$

## 2 Hoop and Longitudinal Stress

It's exactly what it sounds like. We're working with two axes on a tank. You can probably guess that it's in tension and throughout the tank, the hoops are all feeling the same stress. For hoop stresses, there are two formulas we are working with, one for a cylinder (Eqn. 1) and one for a sphere (Eqn. 2).

$$\sigma_\theta = \frac{Pr}{t} \tag{2}$$

$$\sigma_\theta = \frac{Pr}{2t} \tag{3}$$

Similarly, in the longitudinal axis, the stress a vessel will experience in said axis will be akin to that of a sphere.

$$\sigma_z = \frac{Pr}{2t} \tag{4}$$

There also exists a radial stress which acts perpendicular to the surface of the vessel.

$$\sigma_r = -P \approx 0 \tag{5}$$

But for now, we will ignore the radial stress because with the thin-wall assumption, the ratio, $\frac{r}{t}$, is so large that we are considering it negligible.

# 3 Principal Stresses

As you can imagine we can just use mohr's circle for finding the principal stresses. However for the sake of time and energy I'll give you guys some neat tricks and tips on how all of these variables will stack.

$$\sigma_1 = \sigma_h = \frac{Pr}{t} \tag{6}$$
$$\sigma_2 = \sigma_l = \frac{Pr}{2t} \tag{7}$$

Here, we denote the hoop and longitudinal stress. On a stress element, these stresses are always in tension with zero shear on mohr's circle. Now what if we add a third dimension to the stress element? Well the overall max shear the element will experience will simply be the radius of the third circle that encompasses the overall circle of the original mohr's circle. Let me draw it for you.

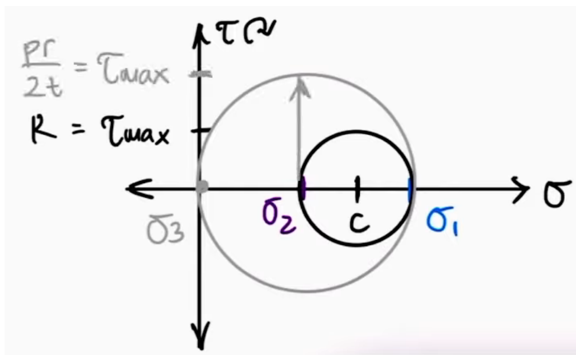


Figure 1: A 3-D mohr's circle

And finally to summarize for the cyclinder

$$\sigma_1 = \sigma_h = \frac{Pr}{t} \tag{8}$$
$$\sigma_2 = \sigma_l = \frac{Pr}{2t} \tag{9}$$
$$\sigma_3 = 0 \tag{10}$$
$$\therefore \tau_{max} = \frac{Pr}{2t} \tag{11}$$

and circle

$$\sigma_1 = \frac{Pr}{2t} \tag{12}$$
$$\sigma_2 = \frac{Pr}{2t} \tag{13}$$
$$\sigma_3 = 0 \tag{14}$$

# 4 Strains

Strains are pretty cringe but they're derived from the formulas we learned earlier in the course. Why do we need to know this? I don't know.

$$\varepsilon_1 = \frac{Pr}{Et}(1 - \frac{v}{2}) \tag{15}$$

$$\varepsilon_2 = \frac{Pr}{Et}(\frac{1}{2} - v) \tag{16}$$

$$\varepsilon_3 = -\frac{3vPr}{2Et} \tag{17}$$