Unit Conversions:

1 slug = 14.594 kg
1 ft = 0.305 m
1 K = 1.8 R
°C + 273 = K
1 lbf = 1 sl · ft/s² = 4.448 N
1 Pa = 1 N/m²
1 N = 1 kg·m/s²
1 atm = 101350 Pa
1 gal = 3.785 L
$1\,L_{H_2O} = 0.001\ m^3$
1 rpm = 2π/60 rad/s = ω

Buckingham π thm.:

Step 1: List the parameters in the problem and count their total number *n*.

Step 2: List the primary dimensions of each of the *n* parameters.

Step 3: Set the *reduction j* as the number of primary dimensions. Calculate *k*, the expected number of Π's,

$k = n - j$

Step 4: Choose *j repeating parameters*.

Step 5: Construct the *k* Π's, and manipulate as necessary.

Step 6: Write the final functional relationship and check your algebra.

Reynold's TT:

General:

$$\frac{dB}{dt} = \frac{d}{dt}\int_{cv} b\rho\, d\forall + \int_{cs} \rho b(\vec{V}\cdot\cancel{\vec{n}})\, dA = 0$$

Cons. of mass ($B = m$, $b = 1$):

Incompressible:

$\Sigma Q_{in} = \Sigma Q_{out}$

$\Sigma (A_i V_i)_{in} = \Sigma (A_i V_i)_{out}$

Fixed, steady:

$\Sigma (\dot{m}_i)_{in} = \Sigma (\dot{m}_i)_{out}$

$\Sigma (\rho_i A_i V_i)_{in} = \Sigma (\rho_i A_i V_i)_{out}$

Cons. of lin. mom. ($B = m\vec{V}$, $b = \vec{V}$)

1D in/out-lets:

$\Sigma F = -F + \Sigma(\dot{m}_i V_i)_{out} - \Sigma(\dot{m}_i V_i)_{in}$

↳ $\dot{m} = \rho A V = \frac{\dot{w}}{g}$, $Q = AV = \frac{\dot{w}}{\rho g}$

usually gauge

↳ $\Sigma F = -F + P_1 A_1 + P_2 A_2 = \dot{m}_2 V_2 - \dot{m}_1 V_1$

Bernoulli eqn:

$$\frac{P_1}{\rho} + \frac{1}{2}V_1^2 + g z_1 = \frac{P_2}{\rho} + \frac{1}{2}V_2^2 + g z_2$$

Cons. of ang. mom. ($B = H = \int \vec{r}\times\vec{v}\, dm$, $b = \vec{r}\times\vec{v}$):

General:

$$\Sigma M_o = \frac{d}{dt}\left[\int_{cv} (\vec{r}\times\vec{v})\rho\, dV\right] + \int_{cs} (\vec{r}\times\vec{v})\rho(\vec{V}\cdot n)\, dA$$

1D in/out-lets:

$$\int_{cs} (\vec{r}\times\vec{v})(\vec{V}\cdot n)\, dA = \Sigma(\vec{r}\times\vec{v})_{out}\dot{m}_{out} - \Sigma(\vec{r}\times\vec{v})_{in}\dot{m}_{in}$$

Energy eqn ($B = E$, $b = e = \frac{dE}{dM}$):

General:

$$\dot{Q} - \dot{W}_s - \dot{W}_v = \frac{d}{dt}\left[\int_{cv}\left(\hat{u} + \frac{1}{2}\vec{v}^2 + gz\right)\rho\, dV\right] + \int_{cs}\left(\hat{h} + \frac{1}{2}\vec{V}^2 + gz\right)\rho(\vec{V}\cdot n)\, dA$$

Steady flow:

$$\left(\frac{P}{\gamma} + \frac{V^2}{2g} + z\right)_{in} = \left(\frac{P}{\gamma} + \frac{V^2}{2g} + z\right)_{out} + \underbrace{h_{fric} - h_{pump} + h_{turbine}}_{\text{head (length, m)}}$$

$\gamma = \rho g$

or:

$$\dot{Q} - \dot{W}_s - \dot{W}_v = \dot{m}_{out}\left(\frac{P_{out}}{\rho_{out}} + \frac{V_{out}^2}{2}\right) - \dot{m}_{in}\left(\frac{P_{in}}{\rho_{in}} + \frac{V_{in}^2}{2}\right)$$

heat; shaft / turbine; vol change / piston

Turbine P: $P = \gamma Q h$

Ex π-theorem

Want $\tau = f(U, \delta, u', \rho, dp/dx)$ → n = 6 (1 2 3 4 5 6)

Setup MTL:

τ	U	δ	u'	ρ	dp/dx
$ML^{-1}T^{-2}$	LT^{-1}	L	LT^{-1}	ML^{-3}	$ML^{-2}T^{-2}$

→ M, L, T → j = 3

$\frac{P}{L} = \frac{ML^{-1}T^{-2}}{L}$

Now find k = n − j = 6 − 3 possible π_k's given (ρ, U, δ) as rep. var. If not given, choose a length, velocity, and a mass/density. Remember our fxn will be $\pi_1 = f(\pi_2, \pi_3)$

$\pi_1 = \rho^a U^b \delta^c \tau = M^0 L^0 T^0 = (ML^{-3})^a (LT^{-1})^b (L)^c (ML^{-1}T^{-2})$

M: 0 = a + 1, a = −1
L: 0 = −3a + b + c − 1, c = 0
T: 0 = −b − 2, b = −2

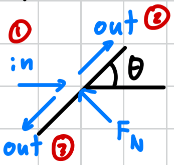
$\pi_1 = \rho^{-1} U^{-2} \delta^0 \tau = \frac{\tau}{\rho U^2}$

Repeat for $\pi_2 = \rho^a U^b \delta^c u'$ and $\pi_3 = \rho^a U^b \delta^c \frac{dp}{dx}$

to write final answer: $\boxed{\frac{\tau}{\rho U^2} = f\left(\frac{u'}{U}, \frac{\delta\, dp}{\rho U^2 dx}\right)}$

Ex Cons. of lin. mom.

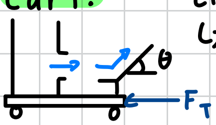
Plate:



$\Sigma F = -F + \dot{m}_2 V_2 + \dot{m}_3(-V_3) - \dot{m}_1 V_1$

$F_N = \dot{m}_2 V_2 + \dot{m}_3(-V_3) - \dot{m}_1 V_1 \cos\theta$

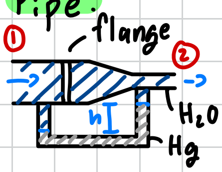
Cart:



Closed sys.

↳ $\Sigma F_x = F_T = \dot{m}_{out} V_{out}$
$= \rho A V_{out}^2 \cos\theta$

Pipe:

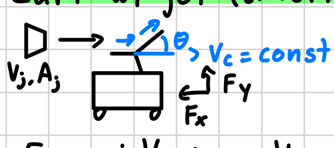


$P_1 - P_2 = \Delta\gamma h = (\gamma_{Hg} - \gamma_{H_2O})h = P_{gage}$ or P_1

$Q_1 = Q_2 = A_1 V_1 = A_2 V_2$ since $\dot{m}_1 = \dot{m}_2$

$\Sigma F = -F_{flange} + P_1 A_1 = \dot{m} V_{out} - \dot{m} V_{in}$
$= \dot{m}(V_{out} - V_{in})$

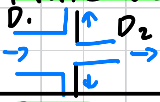
Cart w/ jet (external):



$-F_x = \dot{m} V_{out} - \dot{m} V_{in}$
$= \rho A_j (V_j - V_c)(V_j - V_c)\cos\theta - \rho A_j (V_j - V_c)^2$

Lin. mom.:

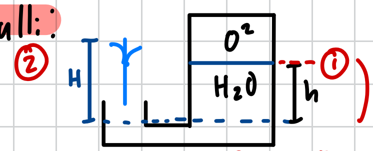
Plate V2:



$\sum F_x = -F_{plate} = \dot m_{hole} V_{hole} + \cancel{\dot m_{up} V_{up}}^{\,0} + \cancel{\dot m_{down} V_{down}}^{\,0} - \dot m_{in} V_{in}$

$= \dot m_{hole} V_{hole} - \dot m_{in} V_{in}$

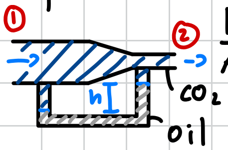
Bernoulli:



$\frac{P_1}{\rho} + \cancel{\frac{1}{2} V_1^2}^{\,0} + g z_1 = \cancel{\frac{P_2}{\rho}}^{\,0} + \cancel{\frac{1}{2} V_2^2}^{\,0} + g z_2$

$\frac{P_1}{\rho} + gh = gH$

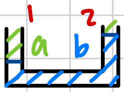
Pipe:



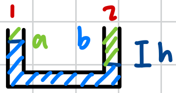
$P_1 - P_2 = (\rho_{oil} - \rho_{CO_2}) g h$

$\frac{P_1}{\rho} + \frac{1}{2} V_1^2 + \cancel{g z_1} = \frac{P_2}{\rho} + \frac{1}{2} V_2^2 + \cancel{g z_2}$

→ if



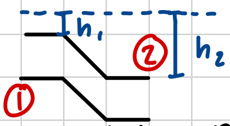
$P_1 - P_2 = (\rho_b - \rho_a) g h$



$P_2 - P_1 = (\rho_b - \rho_a) g h$

Ang. mom.:

Turbomachies:



$T = P_1 A_1 h_1 - P_2 A_2 h_2$

$= \dot m (h_2 V_2 - h_1 V_1)$

Euler turbine formula:

$T = \rho Q (r_2 V_{t2} - r_1 V_{t1})$

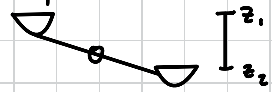
Spinning vel:

1 rpm = $2\pi/60$ rad/s = ω

$V = \omega R$

Energy:

Pump-turbine:



h_f: friction head loss

$\cancel{\frac{P_1}{\rho g}} + \cancel{\frac{V_1^2}{2g}} + z_1 = \cancel{\frac{P_2}{\rho g}} + \cancel{\frac{V_2^2}{2g}} + z_2 + h_f + h_t \rightarrow P = \gamma Q h$

$= \rho g Q h$