Chapter 5: Stress Transformations

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Coverage: Stress Elements, Stress Transformations, Mohr's Circle, Min-Max Equations

1 Stress Elements

To understand stress transformations, we need to understand the use of stress elements. The best way to think of them is just infinitesimally small pieces of material that we can blow up to visualize applied loads.

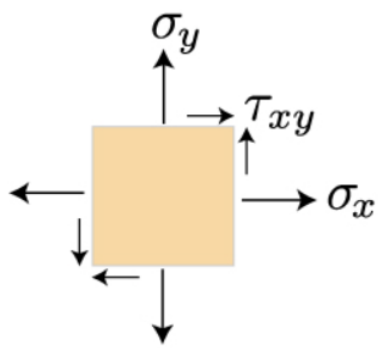

Figure 1: Imagine the little pieces of material but blown up

As seen on Figure 1, stress elements will typically have three pieces of information to it: a stress in the x-direction, σ_x, a stress in the y-direction, σ_y, and a perpendicular shear, τ_{xy}. Recall that the "heads" of the arrows on the top-right and bottom-left corners are positive and the "tails" of the arrows are negative. This is due to equilibrium of moments.

1.1 Stress Transformations

Recall that stress is directional. What if our stress element is rotated by some degree? We can probably guess that the stresses in the x'- and y'-directions won't be the same as those in the upright orientation. However, we can infer that these reoriented stresses can be compartmentalized into their x'- and y'-components. The transformed stresses are derived as follows:

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \tag{1}$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta \tag{2}$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \tag{3}$$

Note 1. CCW is positive (+), CW is negative (−), so adjust accordingly; i.e. if a question states 60° CW, $\theta = -120°$.

2 Mohr's Circle

At some point we'll get asked questions that fall within the purview of finding stresses that may be difficult to find let alone visualize. To do this, we need a visual guide such as a circle. This makes sense since the magnitude of stress should be independent of the element's orientation.

2.1 Construction

Mohr's Circle is a 2-D representation of the stress element and can help us to easily perform calculations and find safe estimations. It's made from just using the information from the stress element but the information it yields can beat any formula we have to memorize. As long as we understand how a circle works, we can find most if not everything else. Here are the steps:

1. Draw a 2-axis plane. Label the horizontal, σ, and the vertical, τ, and remember that up is negative and right is positive

2. We need 2 points of form (σ, τ). We get these points from looking at the 2 axes of our stress element, x and y. In general, if σ is in tension, σ is positive. If σ is in compression, then σ is negative. To find positive τ, check for CCW. For negative τ check for CW. Plot these two points on our plane

3. Using these two points as guides, get a compass and draw a circle about a location on the σ-axis that will pass through these two points.

From the following construction we are able to immediately produce the following properties via the following methods:

σ_{avg}	average stress, middle of the circle
τ_{max}	max shear, bottom of the circle
σ_1, σ_2	principal stresses: right and leftmost points of the circle, respectively

Note 2. Occasionally you may be asked to find the angle, θ, needed to reach the principle stresses. Do **NOT** use the angle on the circle. Instead, find the angle between the horizontal and axis across the two points and divide by 2. Furthermore if you're asked a question requesting the aforementioned values but on a different orientation, multiply the new angle by 2 and add that angle to the orientation in the original "circle domain". I know this can get confusing.

2.2 Min-Max Equations

You can also be a lazy bum and plug and chug the given stress element values into the following equations:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \tag{4}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \tag{5}$$

$$\tau_{\text{max, min}} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \tag{6}$$

$$\tan 2\theta_s = -\left(\frac{\sigma_x - \sigma_y}{2\tau_{xy}}\right) \tag{7}$$

Note 3. The equations above will more often then not produce a value very close to the actual solution. Whether or not it's the right solution is on the discretion of the instructor.