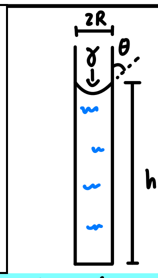
Unit Conversions: 1 ft = 0.305 m °C + 273 = K 1 Pa = 1 N/m² 1 atm = 101350 Pa
1 slug = 14.594 kg 1 K = 1.8 °R 1 lbf = 1 sl · ft/s² = 4.448 N 1 N = 1 kg · m/s² 1 gal = 3.785 L

Examination Aid Sheet 1 lb = 0.454 kg
Faculty of Applied Science & Engineering
Both sides of the sheet may be used; must be printed on 8.5" x 11" paper. 1 in = 0.0254 m

$1 L_{H_2O} = 0.001\ m^3$ $\omega = 1\ rpm = \frac{2\pi}{60}\ rad/s$

Subject: MIE 312
Candidate's name: Gordon Jeon
Candidate's signature: [signature]

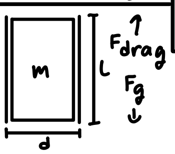


Spec. Grav.:
$SG_{gas} = \frac{\rho_{gas}}{\rho_{air}}$
$SG_{liq} = \frac{\rho_{liq}}{\rho_{H_2O}}$

Reynolds no.:
$Re = \frac{\rho V L}{\mu} = \frac{VL}{\nu}$
V, velocity
L, length
μ, viscosity

Long. shear:
$\tau = \mu \frac{V}{h}$
Rot. shear:
$\tau = \mu \frac{\Omega r}{H}$
flow between plates

h, H, length
Ω, ang. vel.
r, radius

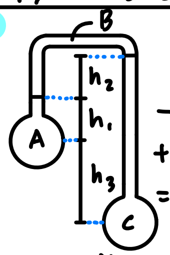


Force Balance: $F_{drag} = \tau_{wall} A_{wall} = F_g = \mu \frac{V}{c_r} \cdot \pi D L$
c_r, radius clearance

Hydrostatic pressure:
liquid: $P_2 - P_1 = -\rho g (z_2 - z_1)$ linear
gas: $P(z) = P_a \left(1 - \frac{B \cdot z}{T_0}\right)^{5.26}$
T_0, 288.16 K
B, 0.0065 K

Meniscus and capillary (< 1mm):
$\rho g \pi R^2 \cdot h = 2\pi R \gamma \cdot \cos\theta$
$h = \frac{2\gamma \cos\theta}{\rho g R}$
γ, surface tension
θ, meniscus angle

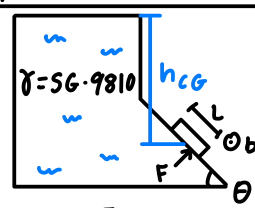
Barometer Qs:
$P_{Tot} = P_a + P_1 + P_2 + P_3 + P_4 + P_5$



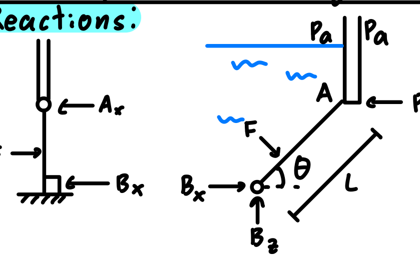
$P_A - P_A(h_1) - P_B(h_2) + P_C(h_1 + h_2 + h_3) = P_C$

Note: $P = \rho g h$ Pa $\gamma = \rho g$ N/m³

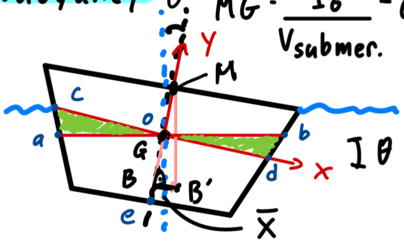
Force on surface:
$F = P_{CG} A = \gamma h_{CG} A$
$I_{xx} = \frac{bL^3}{12}$
$X = \frac{L}{2} - Y_{cp}$
$Y_{cp} = -\frac{I_{xx} \sin\theta}{h_{cg} A}$ $X_{cp} = -\frac{I_{xy} \sin\theta}{h_{cg} A}$



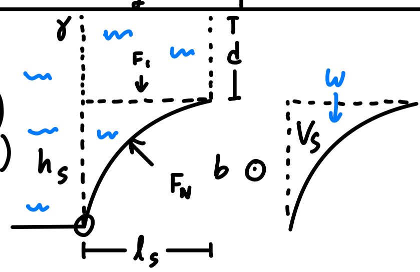
Reactions:



Buoyancy: θ, $\overline{MG} = \frac{I_o}{V_{submer.}} - \overline{GB}$



Force on curved surface:
$F_N = \sqrt{F_x^2 + F_y^2}$
$F_x = \gamma h_c A = \gamma (d + \frac{1}{2} h_s)(b \cdot h_s)$
$F_y = F_1 + W = \gamma (d b \ell_s) + \gamma (V_s)$
$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$



Solid mechanics:
Thin-wall cyl: $\frac{Pr}{t}$
Long: $\frac{Pr}{2t}$
Shear: $\tau = \frac{F}{A}$
Torque: $T = F \cdot r$

Reynold's TT:
General:
$\frac{dB}{dt} = \frac{d}{dt} \int_{cv} b \rho\, d\forall + \int_{cs} \rho b (\vec{V} \cdot \cancel{\vec{n}})\, dA = 0$

Cons. of mass (B = m, b = 1):
Incompressible:
$\Sigma Q_{in} = \Sigma Q_{out}$
$\Sigma (A_i V_i)_{in} = \Sigma (A_i V_i)_{out}$
Fixed, steady:
$\Sigma (\dot{m}_i)_{in} = \Sigma (\dot{m}_i)_{out}$
$\Sigma (\rho_i A_i V_i)_{in} = \Sigma (\rho_i A_i V_i)_{out}$

Cons. of lin. mom. ($B = m\vec{V}$, $b = \vec{V}$):
1D in/out-lets:
$\Sigma F = -F + \Sigma (\dot{m}_i V_i)_{out} - \Sigma (\dot{m}_i V_i)_{in}$
↳ $\dot{m} = \rho A V = \frac{\dot{w}}{g}$, $Q = AV = \frac{\dot{w}}{\rho g}$
usually gauge
↳ $\Sigma F = -F + P_1 A_1 + P_2 A_2 = \dot{m}_2 V_2 - \dot{m}_1 V_1$

Bernoulli eqn:
$\frac{P_1}{\rho} + \frac{1}{2} V_1^2 + g z_1 = \frac{P_2}{\rho} + \frac{1}{2} V_2^2 + g z_2$

Cons. of ang. mom. ($B = H = \int \vec{r} \times \vec{v}\, dm$, $b = \vec{r} \times \vec{v}$):
General:
$\Sigma M_o = \frac{d}{dt} \left[\int_{cv} (\vec{r} \times \vec{v}) \rho\, d\forall \right] + \int_{cs} (\vec{r} \times \vec{v}) \rho (\vec{V} \cdot n)\, dA$
1D in/out-lets:
$\int_{cs} (\vec{r} \times \vec{v})(\vec{v} \cdot n)\, dA = \Sigma (\vec{r} \times \vec{v})_{out} \dot{m}_{out} - \Sigma (\vec{r} \times \vec{v})_{in} \dot{m}_{in}$

Energy eqn ($B = E$, $b = e = \frac{dE}{dM}$):
General:
$\dot{Q} - \dot{W}_s - \dot{W}_v = \frac{d}{dt} \left[\int_{cv} (\hat{u} + \frac{1}{2} \vec{v}^2 + gz) \rho\, d\forall \right] + \int_{cs} (\hat{h} + \frac{1}{2} \vec{V}^2 + gz) \rho (\vec{V} \cdot n)\, dA$

Steady flow:
$\left(\frac{P}{\gamma} + \frac{V^2}{2g} + z \right)_{in} = \left(\frac{P}{\gamma} + \frac{V^2}{2g} + z \right)_{out} + \underbrace{h_{fric} - h_{pump} + h_{turbine}}_{\text{head (length, m)}}$
$\gamma = \rho g$

or: $\dot{Q} - \dot{W}_s - \dot{W}_v = \dot{m}_{out} \left(\frac{P_{out}}{\rho_{out}} + \frac{V_{out}^2}{2} \right) - \dot{m}_{in} \left(\frac{P_{in}}{\rho_{in}} + \frac{V_{in}^2}{2} \right)$
heat; shaft turbine; vol change piston

Turbine P: $P = \gamma Q h$

3. $F_D = \frac{C_D \rho V^2 A}{2}$ (x 2) $P = F_D V$
lam → Re < 10^6
turb → Re > 10^6

Drag Force Qs:
1. $Re = \frac{\rho V L}{\mu} = \frac{VL}{\nu}$
2. $C_D = \frac{1.328}{Re^{1/2}}$, $\frac{\delta}{x} = \frac{5}{Re^{1/2}}$ → lam or $= \frac{0.031}{Re^{1/7}}$, $\frac{\delta}{x} = \frac{0.16}{Re^{1/7}}$ → turb

Pipes:

$$\left(\frac{P}{\gamma}+\frac{V^2}{2g}+z\right)_{in}=\left(\frac{P}{\gamma}+\frac{V^2}{2g}+z\right)_{out}+h_f+h_m+h_{turb}-h_{pump}\rightarrow h_f=\frac{fLV^2}{2Dg}$$, major

$\hookrightarrow \frac{1}{\sqrt{f}}=-1.8\log_2\left(\frac{6.9}{Re_d}+\left(\frac{\varepsilon/d}{3.7}\right)^{1.11}\right)$

Redef. Re_d: $Re_d=\frac{4Q}{\pi d\nu}=\frac{4\rho Q}{\pi\mu d}=\frac{\rho VD}{\mu}=\frac{VD}{\nu}$ (ν: visc.)

$h_m=K_L\left(\frac{V^2}{2g}\right)\rightarrow K_L=\sum K_n$, minor

1. Find V ($Q=VA$) 2. Find Re 3. Find f, a) moody b) haaland
4. Find h_f + h_m 5. Plug into energy eqn.

Ex π-theorem

Want $\tau=f(U,\delta,u',\rho,dp/dx)$ → $n=6$ (1 2 3 4 5 6)

Setup MTL:

τ	U	δ	u'	ρ	dp/dx
$ML^{-1}T^{-2}$	LT^{-1}	L	LT^{-1}	ML^{-3}	$ML^{-2}T^{-2}$

$\frac{P}{L}=\frac{ML^{-1}T^{-2}}{L}$ → M, L, T → $j=3$

Now find $k=n-j=6-3$ possible π_k's given (ρ,U,δ) as rep. var. If not given, choose a length, velocity, and a mass/density. Remember our fxn will be $\pi_1=f(\pi_2,\pi_3)$

$\pi_1=\rho^aU^b\delta^c\tau=M^0L^0T^0=(ML^{-3})^a(LT^{-1})^b(L)^c(ML^{-1}T^{-2})$

M: $0=a+1$, $a=-1$
L: $0=-3a+b+c-1$, $c=0$
T: $0=-b-2$, $b=-2$

Repeat for $\pi_2=\rho^aU^b\delta^cu'$ and $\pi_3=\rho^aU^b\delta^c\frac{dp}{dx}$

$\pi_1=\rho^{-1}U^{-2}\delta^0\tau=\frac{\tau}{\rho U^2}$

to write final answer: $\boxed{\frac{\tau}{\rho U^2}=f\left(\frac{u'}{U},\frac{\delta\, dp}{\rho U^2dx}\right)}$

Ex Cons. of lin. mom.

Plate: $\sum F=-F+\dot m_2V_2+\dot m_3(-V_3)-\dot m_1V_1$

$F_N=\dot m_2V_2+\dot m_3(-V_3)-\dot m_1V_1\cos\theta$

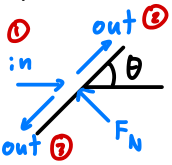
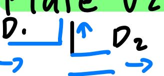


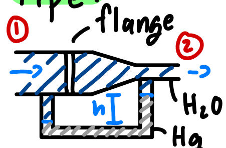
Plate V2: $\sum F_x=-F_{plate}=\dot m_{hole}V_{hole}+\cancel{\dot m_{up}V_{up}}+\cancel{\dot m_{down}V_{down}}-\dot m_{in}V_{in}$ (0)

$=\dot m_{hole}V_{hole}-\dot m_{in}V_{in}$



$F=\dot mV$
$Q=AV$
$\dot m=\rho Q=\rho VA$,

Pipe:

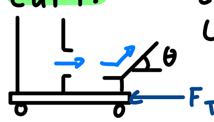


$P_1-P_2=\Delta\gamma h=(\gamma_{Hg}-\gamma_{H_2O})h=P_{gage}$ or P_1

$Q_1=Q_2=A_1V_1=A_2V_2$ since $\dot m_1=\dot m_2$

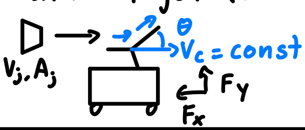
$\sum F=-F_{flange}+P_1A_1=\dot mV_{out}-\dot mV_{in}$
$=\dot m(V_{out}-V_{in})$

Cart:



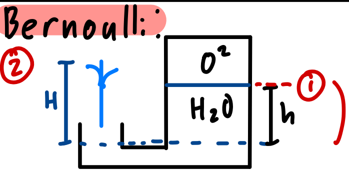
Closed sys.
$\hookrightarrow\sum F_x=F_T=\dot m_{out}V_{out}$
$=\rho AV_{out}^2\cos\theta$

Cart w/ jet (external):



$-F_x=\dot mV_{out}-\dot mV_{in}$
$=\rho A_j(V_j-V_c)(V_j-V_c)\cos\theta-\rho A_j(V_j-V_c)^2$

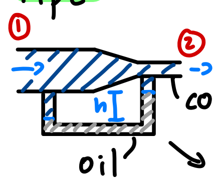
Bernoulli:



$\frac{P_1}{\rho}+\cancel{\frac{1}{2}V_1^2}+gz_1=\cancel{\frac{P_2}{\rho}}+\cancel{\frac{1}{2}V_2^2}+gz_2$ (0, 0, 0)

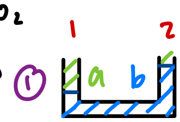
$\frac{P_1}{\rho}+gh=gH$

Pipe:

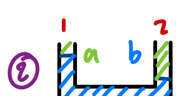


$P_1-P_2=(\rho_{oil}-\rho_{CO_2})gh$

$\frac{P_1}{\rho}+\frac{1}{2}V_1^2+\cancel{gz_1}=\frac{P_2}{\rho}+\frac{1}{2}V_2^2+\cancel{gz_2}$



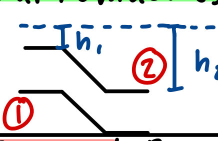
① $P_1-P_2=(\rho_b-\rho_a)gh$



② $P_2-P_1=(\rho_b-\rho_a)gh$

Ang. mom.:

Turbomachies:



$T=P_1A_1h_1-P_2A_2h_2$
$=\dot m(h_2V_2-h_1V_1)$

Euler turbine formula:

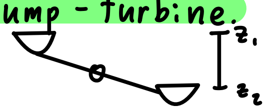
$T=\rho Q(r_2V_{t2}-r_1V_{t1})$

Spinning vel.:

1 rpm = $2\pi/60$ rad/s = ω
$V=\omega R$

Energy:

Pump-turbine:

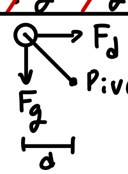


h_f: friction head loss

$\cancel{\frac{P_1}{\rho g}}+\cancel{\frac{V_1^2}{2g}}+z_1=\cancel{\frac{P_2}{\rho g}}+\cancel{\frac{V_2^2}{2g}}+z_2+h_f+h_t\rightarrow P=\gamma Qh$
$=\rho gQh$

Drag force equilibrium:

V to push object (radius r)



$M_D=M_g$
$F_D\cdot R=F_g\cdot d$
$\frac{C_D\rho AV^2}{2}\cdot R=mgd$