Chapter 6: Design Against Failure

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Coverage: Designing against failure, Factor of Safety (FOS), Yield Criterion, Tresca, Von Mises

1 Defining Failure

For the sake of consistency, we define failure as simply exceeding the yield strength, σ_y. For now we'll only cover Brittle Fracture and Ductile Failure (Yielding).

2 Failure Theories

2.1 Rankine Theory/Maximum Normal Stress Theory (Brittle)

Rankine states that when the **maximum normal stress** at any point reaches a value equal to or greater than the fracture strength, σ_{fracture}, the specimen will fail.

$$\sigma_{\max} \leq \sigma_{\text{fracture}} \tag{1}$$
$$|\sigma_1| \leq \sigma_{\text{ult}} \tag{2}$$
$$|\sigma_2| \leq \sigma_{\text{ult}} \tag{3}$$

2.2 Tresca Failure Theory/Maximum Shear Stress Theory (Ductile)

Tresca states that a specimen will fail if the **maximum shear**, $\tau_{\max}$, exceeds the shear yield, K.

$$\frac{\sigma_{\max} - \sigma_{\min}}{2} \leq \frac{\sigma_y}{2} = \tau_y \tag{4}$$
$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} \geq K \tag{5}$$
$$|\sigma_1| \leq \sigma_y \tag{6}$$
$$|\sigma_2| \leq \sigma_y \tag{7}$$
$$|\sigma_1 - \sigma_2| \leq \sigma_y \tag{8}$$

Note 1. Apply eqns. 6, 7 when σ_1, σ_2 have same signs, i.e. $++/--$. For different signs, use eqn. 8.

2.3 Von Mises/Maximum Distortion Energy Theory (Ductile)

Von Mises' theory revolves around the application of torsion.

$$\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right] \leq 2\sigma_y^2 \tag{9}$$
$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 \leq \sigma_y^2 \tag{10}$$

Note 2. Eqn. 10 is for biaxial stress.