Chapter 10: Statically Indeterminate Elastic Beams

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Coverage: Singularity functions, Beam Analysis

1 Preface

In past chapters, stresses and deflections were determined for beams having various conditions of loading and support. In the cases treated it was always possible to completely determine the reactions exerted upon the beam merely by checking the equilibrium. In these cases the beams are said to be "statically determinate".

But you can probably guess that sometimes the number of unknown reactions exceeds the number of equilibrium equations available for the system. In these cases the beams are said to be "statically indeterminate". For this chapter, I want to emphasize focusing on working with singularity functions.

2 Types of Statically Indeterminate Beams

We'll be exclusively working with beams supported by simple loads, i.e. no springs. This means we'll be facing beams such as cantilevers supported on 1 end, 2 ends, and a continuous beam.

In terms of loads we recall the supports such as ball and pin, UDLs, and point loads and moments. From here I think the best way to learn how to solve question is via an example.

3 Example

The beam of flexural rigidity EI shown in Figure 1 is clamped at both ends and subject to a UDL extending along the region BC of length *0.6L*. Let's find all the reactions.

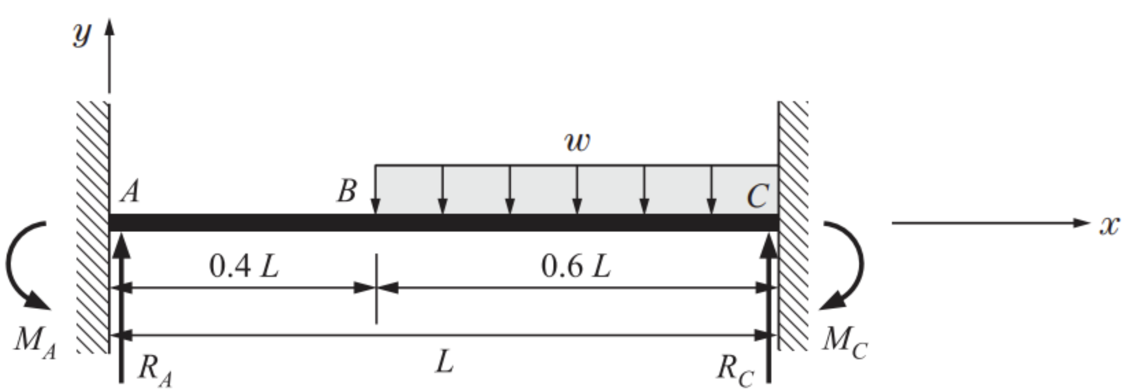

Figure 1: For Section 3. Example

Between A and B we are given the bending moments M_A and M_B as well as the point loads R_A and R_C. Let's use the relevant singularity functions given in our formula sheet and find the moment, M.

$$
\begin{aligned}
EI\frac{d^2y}{dx^2} = M &= -M_A\langle x-0\rangle^0 + R_A\langle x-0\rangle - \frac{w}{2}\langle x-0.4L\rangle^2 \\
&= -M_A\langle x\rangle^0 + R_A\langle x\rangle - \frac{w}{2}\langle x-0.4L\rangle^2
\end{aligned}
$$

Let's integrate, finding the slope.

$$
EI\frac{dy}{dx} = -M_A\langle x\rangle^1 + \frac{R_A}{2}\langle x\rangle^2 - \frac{w}{6}\langle x-0.4L\rangle^3 + C_1
$$

And integrate again, finding the deflection.

$$
EIy = \frac{-M_A}{2}\langle x\rangle^2 + \frac{R_A}{6}\langle x\rangle^3 - \frac{w}{24}\langle x-0.4L\rangle^4 + C_2
$$

Looks like we got our boundary conditions, C_1 and C_2. How do we get rid of them? Consider that when $x=0$, $\frac{dy}{dx}$ and $y=0$. Furthermore, when $x=L$, $\frac{dy}{dx}$ and $y=0$. Sub these values in, and we get the following equations.

$$
\begin{aligned}
0 &= -M_AL + \frac{R_AL^3}{2} - \frac{w}{6}(0.6L)^3 \\
0 &= \frac{-M_AL^2}{2} + \frac{R_AL^3}{6} - \frac{w}{24}\langle 0.6L\rangle^4
\end{aligned}
$$

From here, we let both of those equations equal each other to obtain a new equation that solves for R_A. Now it's just a matter of solving it and finding the other reactions that have R_A.

$$
\begin{aligned}
R_A &= wL\left\{(0.6)^3 - \frac{0.6^4}{2}\right\} = 0.1512wL \\
M_A &= 0.0396wL^2
\end{aligned}
$$

And from the sum of forces and moments.

$$
\begin{aligned}
\sum F_y &= 0 = 0.1512wL - w(0.6L) + R_C \\
\therefore R_C &= 0.4488wL \\
\sum M_A &= 0 = -0.0396wL^2 - M_C + (0.4488wL)(L) - w(0.6L)(0.7L) \\
\therefore M_C &= 0.0684wL^2
\end{aligned}
$$