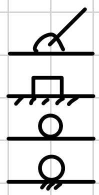
# of links, n:

↳ Binary - 2 connection , n = 2
Ternary - 3 connection , n = 3
n-element , n = n

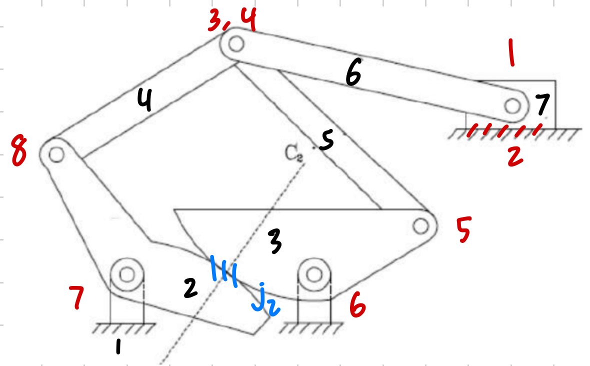
joint, $j_n$:

↳ turning pair, $j_1$
sliding pair, $j_1$
roll-no slip, $j_1$
roll-slip, $j_2$



} $j_1$ = 1 DOF joints
$j_2$ = 2 DOF joints

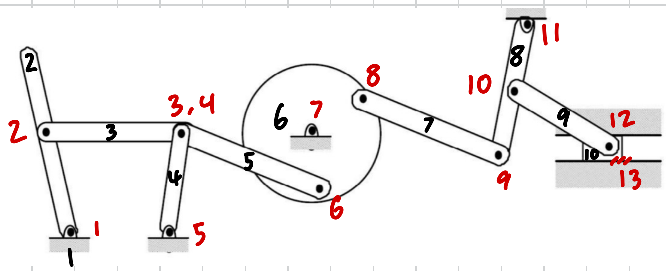
$m = 3(n-1) - 2j_1 - j_2$




n = 7
$j_1 = 8$
$j_2 = 1$

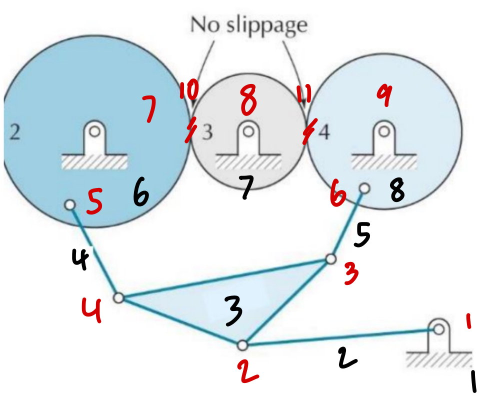
Binary : 4, 5, 6, 7
Ternary: 1, 2, 3

Dof = m = 3(6) - 2(8) - 1
= 1



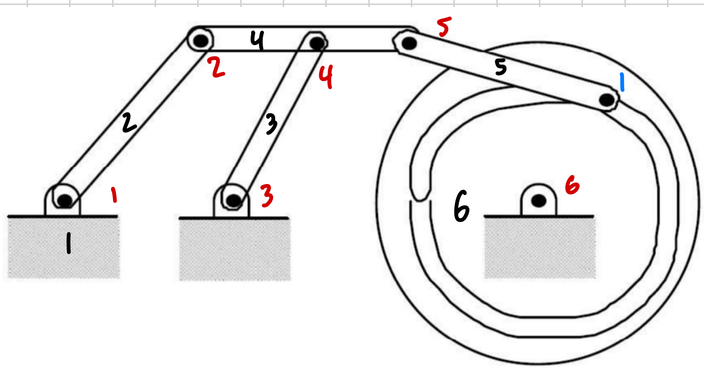

n = 10
$j_1 = 13$
$j_2 = 0$

m = 3(9) - 2(13) - 0
= 1




n = 8
$j_1 = 11$
$j_2 = 0$

m = 21 - 22 - 0
= -1 → redundancy



n = 6
$j_1 = 6$
$j_2 = 1$

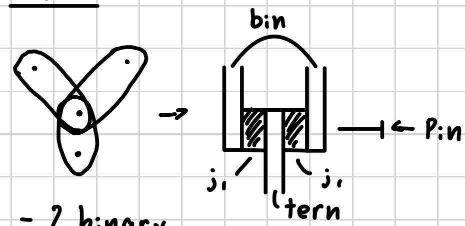
m = 15 - 12 - 1
= 2

4-bar mechanism:

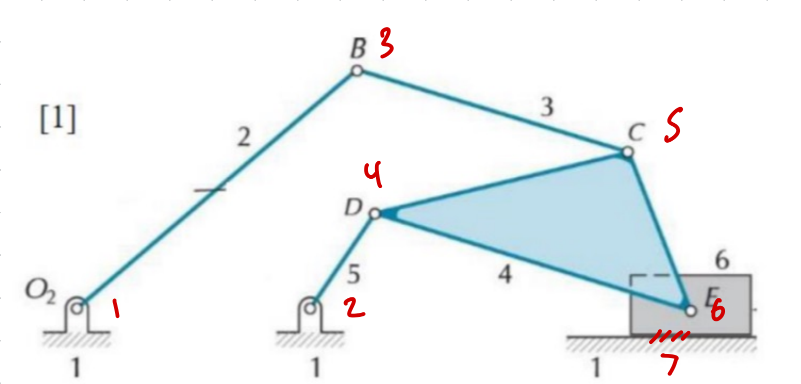
Either (s+ℓ) < (P+q)
or (s+ℓ) > (P+q)

If s input → crank rocker
" " base link → drag link
Else → rocker - rocker

Note:




- 2 binary
- 1 ternary

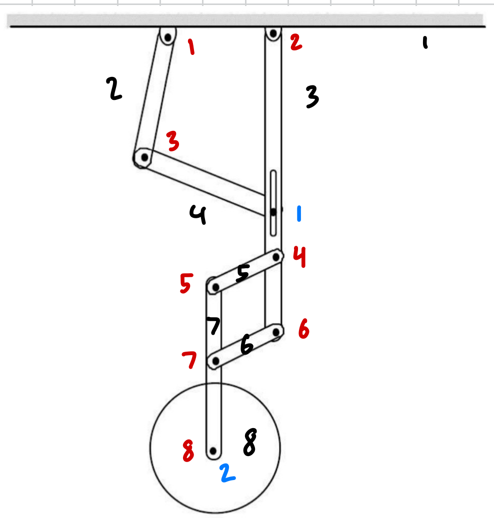



$n = 6$

$j_1 = 7$

$j_2 = 0$

$m = 3(5) - 2(7) - 0$
$= 1$

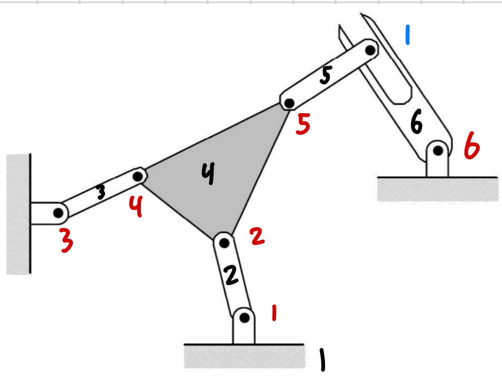


$n = 8$

$j_1 = 8$

$j_2 = 2$

$m = 3(7) - 2(8) - 2$
$= 3$

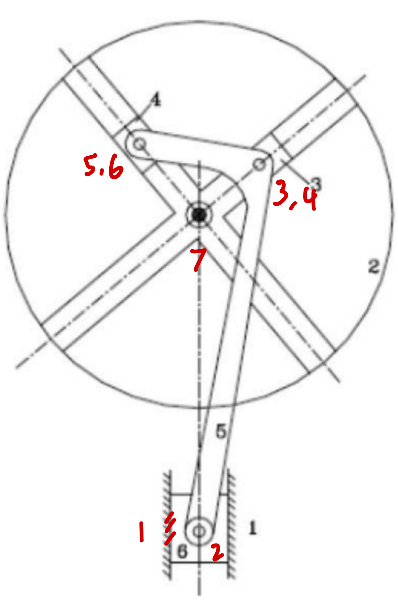


$n = 6$

$j_1 = 6$

$j_2 = 1$

$m = 3(5) - 2(6) - 1$
$= 2$



$n = 6$

$j_1 = 7$

$j_2 = 0$

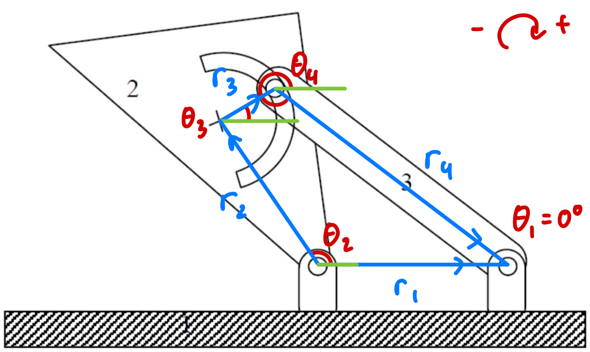
$m = 3(5) - 2(7) - 0$
$= 1$

# Vector loops:

1. Connect pins of binary links
2. If there's a slider, isolate sliding component by drawing 2-axis parallels
3. If there's a ternary link, draw a fixed reference point with arrows pointing to pins
4. Draw arrows to establish positive direction
5. Angles are taken from tail end of vector, rotating CCW from x-axis

For finding knowns and unknowns:

- 2 grounded pins
  - r = constant ✓
  - theta = constant ✓
- Pinned input member
  - r = constant ✓
  - theta = input ✓
- Pinned free member
  - r = constant ✓
  - theta = pin ✗
- Slider
  - r = slider ✗
  - theta = fixed ✓

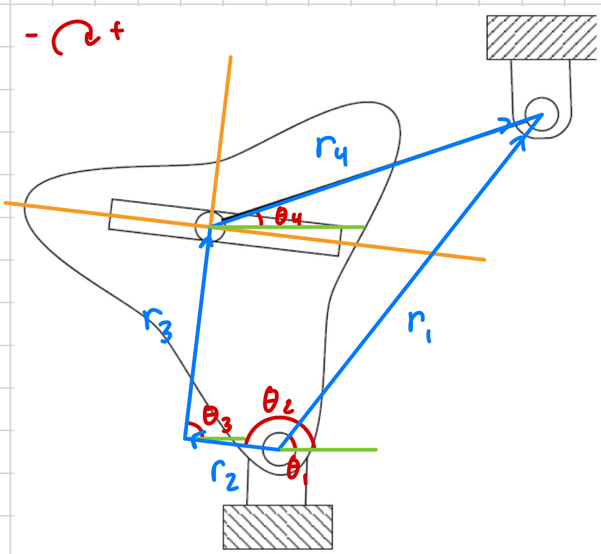



$$\vec{r}_2+\vec{r}_3+\vec{r}_4-\vec{r}_1=0$$

| | | | | |
|---|---|---|---|---|
| $r_1$, rigid | ✓ | $\theta_1$, fixed | ✓ | Unknowns: $\theta_3, \theta_4$ |
| $r_2$, | ✓ | $\theta_2$, input | ✓ | |
| $r_3$, | ✓ | $\theta_3$, pin | ✗ | |
| $r_4$, | ✓ | $\theta_4$, | ✗ | |

$$x = r_2\cos\theta_2 + r_3\cos\theta_3 + r_4\cos\theta_4 - r_1\cos\cancel{\theta_1}^{\,1} = 0$$
$$= r_2\cos\theta_2 + r_3\cos\theta_3 + r_4\cos\theta_4 - r_1 = 0$$
$$y = r_2\sin\theta_2 + r_3\sin\theta_3 + r_4\sin\theta_4 - r_1\sin\cancel{\theta_1}^{\,0} = 0$$
$$= r_2\sin\theta_2 + r_3\sin\theta_3 + r_4\sin\theta_4 = 0$$

∴ 2 eqn. 2 unkns. → solvable

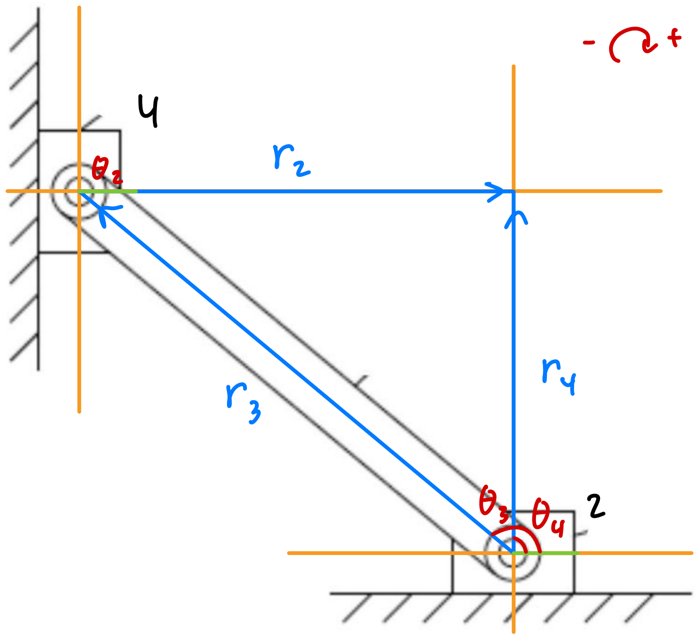



$$\vec{r}_2+\vec{r}_3+\vec{r}_4-\vec{r}_1=0$$

| | | | |
|---|---|---|---|
| $r_1$, const | ✓ | $\theta_1$, fixed | ✓ |
| $r_2$, slider | ✗ | $\theta_2$, input | ✓ |
| $r_3$, const | ✓ | $\theta_3$, relative $\theta_2 - 90$ | ✓ |
| $r_4$, const | ✓ | $\theta_4$, pin | ✗ |

$$x = r_2\cos\theta_2 + r_3\cos\theta_3 + r_4\cos\theta_4 - r_1\cos\theta_1 = 0$$
$$= r_2\cos\theta_2 + r_3\cos(\theta_2 - 90) + r_4\cos\theta_4 - r_1\cos\theta_1 = 0$$
$$y = r_2\sin\theta_2 + r_3\sin\theta_3 + r_4\sin\theta_4 - r_1\sin\theta_1 = 0$$
$$= r_2\sin\theta_2 + r_3\sin(\theta_2 - 90) + r_4\sin\theta_4 - r_1\sin\theta_1 = 0$$

2 eqns, 2 unkns → solvable




$\vec{r}_2 + \vec{r}_3 - \vec{r}_4 = 0$

$r_2$, input ✓ $\theta_2$, fixed (0°) ✓
$r_3$, const ✓ $\theta_3$, pin ✗
$r_4$, slider ✗ $\theta_4$, fixed (90°) ✓

$$x = r_2\cancel{\cos\theta_2}^{1} + r_3\cos\theta_3 - r_4\cancel{\cos\theta_4}^{0} = 0$$
$$= r_2 + r_3\cos\theta_3 = 0$$
$$y = r_2\cancel{\sin\theta_2}^{0} + r_3\sin\theta_3 - r_4\cancel{\sin\theta_4}^{1} = 0$$
$$= r_3\sin\theta_3 - r_4 = 0$$

2 eqns. 2 unkwns → solvable

## Loop A

Relative:

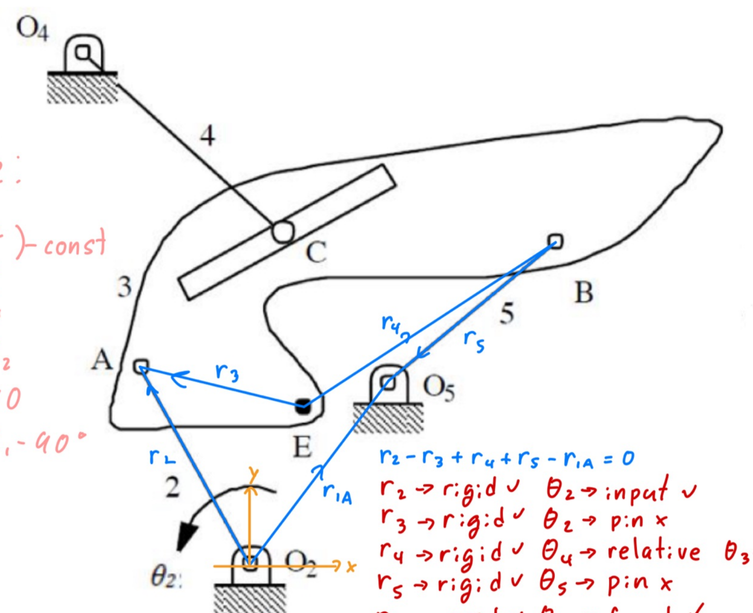
$\left.\begin{array}{l}\theta_2 = \theta_1 - C_2 \\ \theta_3 = \theta_1 + C_2\end{array}\right\}$ const

$\theta_6 = \theta_3 - C_1$

$\theta_4 = \theta_3 - C_2$

$\theta_7 = \theta_6 - 90$

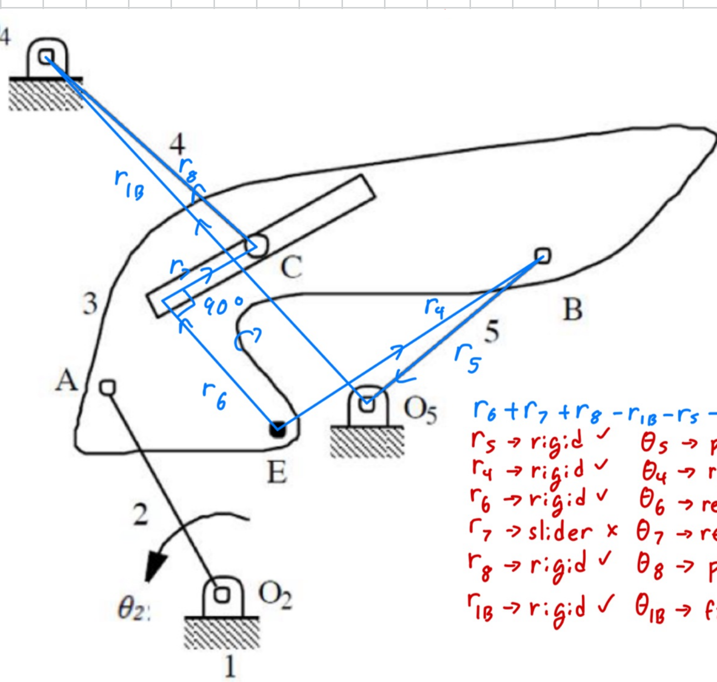
$= \theta_3 - C_1 - 90°$




$r_2 - r_3 + r_4 + r_5 - r_{1A} = 0$

$r_2$ → rigid ✓ $\theta_2$ → input ✓
$r_3$ → rigid ✓ $\theta_2$ → pin ×
$r_4$ → rigid ✓ $\theta_4$ → relative $\theta_3 - C_2 = \theta_4$ ✓
$r_5$ → rigid ✓ $\theta_5$ → pin ×
$r_{1A}$ → rigid ✓ $\theta_{1A}$ → fixed ✓

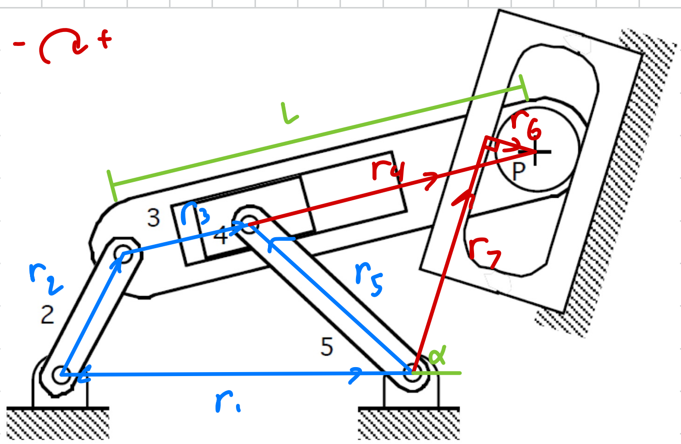
## Loop B




$r_6 + r_7 + r_8 - r_{1B} - r_5 - r_4 = 0$

$r_5$ → rigid ✓ $\theta_5$ → pin
$r_4$ → rigid ✓ $\theta_4$ → relative
$r_6$ → rigid ✓ $\theta_6$ → relative, $\theta_3 - C_1$
$r_7$ → slider × $\theta_7$ → relative, $\theta_3 - C_1 - 90$
$r_8$ → rigid ✓ $\theta_8$ → pin ×
$r_{1B}$ → rigid ✓ $\theta_{1B}$ → fixed ✓

# Kinematic Coefficients:




**loop 1:** $\vec{r}_2 + \vec{r}_3 - \vec{r}_5 - \vec{r}_1 = 0$

$r_2$, const ✓ — $\theta_2$, pin ✓
$r_3$, slider × — $\theta_3$, pin ×
$r_5$, const ✓ — $\theta_5$, pin ×
$r_1$, const ✓ — $\theta_1$, fixed (0°) ✓

$$x = r_2\cos\theta_2 + r_3\cos\theta_3 - r_5\cos\theta_5 - r_1\cancelto{1}{\cos\theta_1} = 0$$
$$= r_2\cos\theta_2 + r_3\cos\theta_3 - r_5\cos\theta_5 - r_1 = 0$$
$$y = r_2\sin\theta_2 + r_3\sin\theta_3 - r_5\sin\theta_5 - r_1\cancelto{0}{\sin\theta_1} = 0$$
$$= r_2\sin\theta_2 + r_3\sin\theta_3 - r_5\sin\theta_5 = 0$$

Relations: $L = r_3 + r_4 \rightarrow r_4 = L - r_3$

3 unkwns, 2 eqns

**loop 2:** $\vec{r}_5 + \vec{r}_4 - \vec{r}_6 - \vec{r}_7 = 0$

$r_5$, const ✓ — $\theta_5$, pin ✓
$L - r_3 \leftarrow$ $r_4$, slider ✓ — $\theta_4$, fixed, $\alpha$ ✓
$r_6$, const ✓ — $\theta_6$, fixed ✓
$r_7$, slider × — $\theta_7$, $\alpha$ ✓

$$x = r_5\cos\theta_5 + (L - r_3)\cos\theta_4 - r_6\cos\theta_6 - r_7\cos\theta_7 = 0$$
$$y = r_5\sin\theta_5 + (L - r_3)\sin\theta_4 - r_6\sin\theta_6 - r_7\sin\theta_7 = 0$$

1 unkwn, 2 eqns

## Coefficients:

$$h_3 = \frac{dr_3}{d\theta_2} \longrightarrow$$
$$h_{\theta_3} = \frac{d\theta_3}{d\theta_2}$$
$$h_{\theta_5} = \frac{d\theta_5}{d\theta_2}$$
$$h_7 = \frac{dr_7}{d\theta_2}$$

**Loop 1:**

$$x': -r_2\sin\theta_2 + h_3\cos\theta_3 - r_3 h_{\theta_3}\sin\theta_3 + r_5 h_5 \sin\theta_5 - 0 = 0$$
$$y': r_2\cos\theta_2 - h_3\sin\theta_3 - r_3 h_{\theta_3}\cos\theta_3 - r_5 h_5\cos\theta_5 = 0$$
$$x'': -r_2\cos\theta_2 - \frac{dh_3}{d\theta_2}\cos\theta_3 - h_3 h_{\theta_3}\sin\theta_3 - h_3 h_{\theta_3}\sin\theta_3 - r_3\frac{dh_{\theta_3}}{d\theta_2}\sin\theta_3$$
$$- r_3(h_{\theta_3})h_{\theta_3}\cos\theta_3 + r_5\frac{dh_5}{d\theta_2}\sin\theta_5 + r_5(h_5)h_5\cos\theta_5 = 0$$
$$y'': -r_2\sin\theta_2 + \frac{dh_3}{d\theta_2}\sin\theta_3 + h_3 h_{\theta_3}\cos\theta_3 + h_3 h_{\theta_3}\cos\theta_3 + r_3\frac{dh_{\theta_3}}{d\theta_2}\cos\theta_3$$
$$- r_3(h_{\theta_3})h_{\theta_3}\sin\theta_3 - r_5\frac{dh_5}{d\theta_2}\cos\theta_5 + r_5(h_5)h_5\sin\theta_5 = 0$$

**Loop 2:**

$$x': -r_5 h_5\sin\theta_5 - (L - r_3)h_{\theta_3}\sin\theta_3 - h_3\cos\theta_3 - 0 - h_7\cos\alpha = 0$$
$$y': r_5 h_5\cos\theta_5 + (L - r_3)h_{\theta_3}\cos\theta_3 - h_3\sin\theta_3 - 0 - h_7\sin\alpha$$
$$x'': -r_5\frac{dh_5}{d\theta_2}\sin\theta_5 - r_5(h_5)h_5\cos\theta_5 - (L - r_3)(h_{\theta_3})h_{\theta_3}\cos\theta_3 + h_3 h_{\theta_3}\sin\theta_3$$
$$- \frac{dh_3}{d\theta_2}\cos\theta_3 + h_3 h_{\theta_3}\sin\theta_3 - \frac{dh_7}{d\theta_2}\cos\alpha = 0$$
$$y'': r_5\frac{dh_5}{d\theta_2}\cos\theta_5 - r_5(h_5)h_5\sin\theta_5 - (L - r_3)(h_{\theta_3})h_{\theta_3}\sin\theta_3 - h_3 h_{\theta_3}\cos\theta_3$$
$$- \frac{dh_3}{d\theta_2}\sin\theta_3 - h_3 h_{\theta_3}\cos\theta_3 - \frac{dh_7}{d\theta_2}\sin\alpha = 0$$

1. In a mechanism link 2 is known to be rotating with angular velocity -10 rad/sec and angular acceleration of 1 rad/sec$^2$. The velocity and acceleration of point P are measured to be

$$\bar{V}_P = -20\,\bar{i} - 40\,\bar{j}$$
$$\bar{A}_P = 10\,\bar{i} + 50\,\bar{j}$$

What would be the velocity and acceleration of point P if link 2 is rotating with constant angular velocity of 100 rad/sec?

lin vel: $V_x = f_x \omega_x$, $V_y = f_y \omega_y$ where $f_x = \frac{dv_x}{d\theta_2}$, $f_y = \frac{dv_y}{d\theta_2}$

W/ chain rule: $a_x = \frac{dv_x}{dt} = \left(\frac{df_x}{d\theta_2} \cdot \frac{d\theta_2}{dt}\right)\omega_2 + f_x \dot{\omega}_2$

$$= f_x' \omega_2^2 + f_x \alpha_2$$
$$a_y = f_y' \omega_2^2 + f_y \alpha_2$$

Given $v_x, v_y, a_x, a_y, \omega_2, \alpha_2$, find $f_x, f_x', f_y, f_y'$:

$V_x = f_x \omega_2 \rightarrow f_x = 2$
$-20 = f_x(-10)$

$V_y = f_y \omega_2 \rightarrow f_y = 4$
$-40 = f_y(-10)$

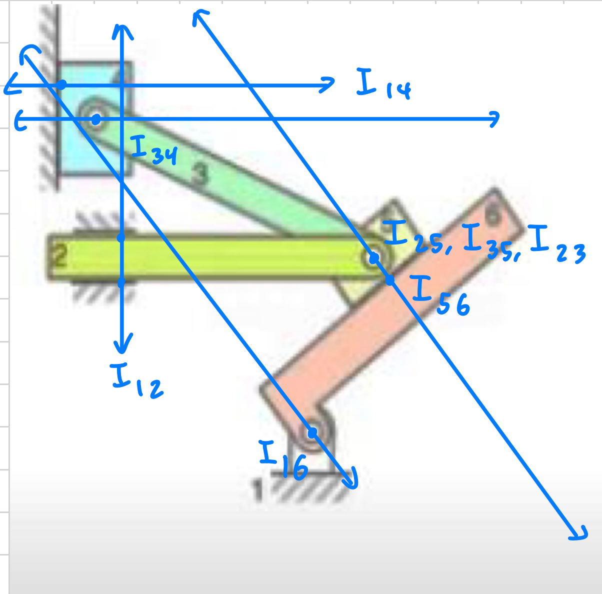
$a_x = f_x' \omega_2^2 + f_x \alpha_2 \rightarrow f_x' = 0.08$
$10 = f_x'(-10)^2 + 2(1)$

$a_y = f_y' \omega_2^2 + f_y \alpha_2 \rightarrow f_y' = 0.46$
$50 = f_y'(-10)^2 + 4(1)$

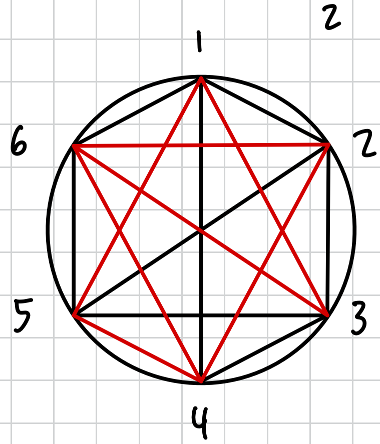
Plug and find coef:

$$V_x = f_x \omega_2 = 2 \cdot 100 = 200$$
$$V_y = f_y \omega_2 = 4 \cdot 100 = 400$$
$$a_x = f_x' \omega_2^2 + \cancel{f_x \alpha_2}^{0} = 0.08(10000) = 800$$
$$a_y = f_y' \omega_2^2 + \cancel{f_y \alpha_2} = 0.46(10000) = 4600$$

## Instantaneous centers:




$\#$ of ICs. $IC = \frac{n(n-1)}{2} = \frac{6(5)}{2} = 15$




$I_{15} = I_{16,65}$
$\quad\;\; I_{12,25}$

$I_{13} = I_{12,23}$
$\quad\;\; I_{14,43}$

$I_{62} = I_{65,52}$
$\quad\;\; I_{61,12}$

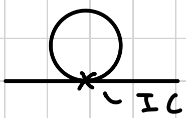
$I_{63} = I_{65,53}$
$\quad\;\; I_{62,23}$

$I_{64} = I_{65,54}$
$\quad\;\; I_{61,14}$

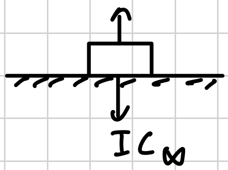
$I_{54} = I_{53,34}$
$\quad\;\; I_{15,14}$

$I_{42} = I_{43,32}$
$\quad\;\; I_{41,12}$

## Note:

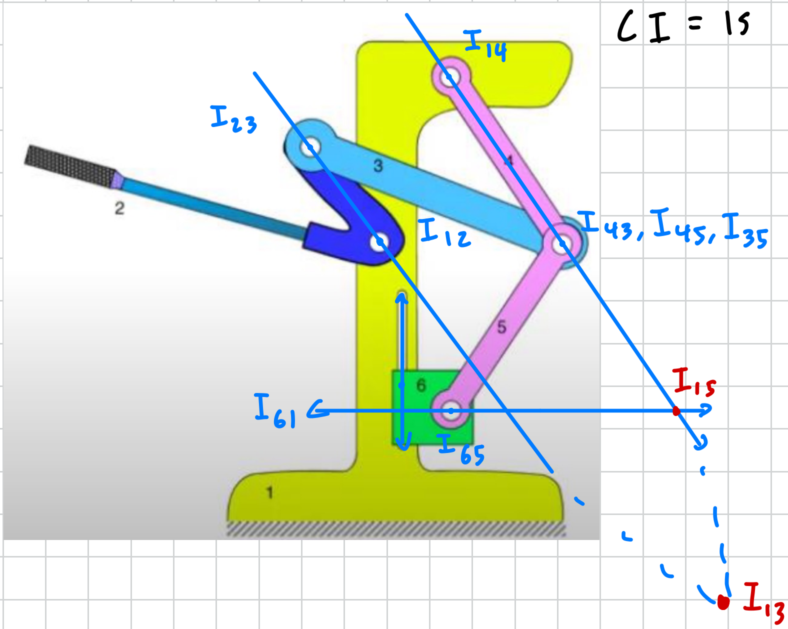


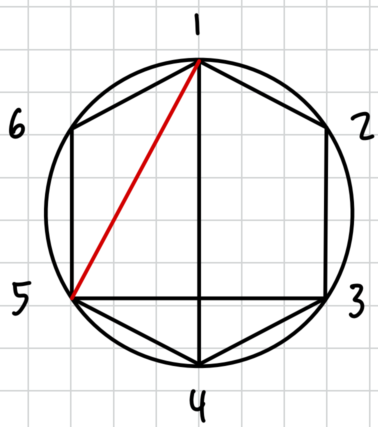

Roller




Slider

$CI = 15$




$I_{15} = I_{16,65}$
$\quad\;\; I_{14,45}$

$I_{13} = I_{12,23}$
$\quad\;\; I_{14,43}$

## Mechanical Advantage:

Since $F_i d_i \omega_i = F_o d_o \omega_o$, $M_A = \frac{F_o}{F_i}$

$$= \frac{d_i \omega_i}{d_o \omega_o}$$

$$= \frac{d_i}{d_o}\left(\frac{I_{1o} I_{io}}{I_{1i} I_{io}}\right)$$

If denom $= 0$, $M_A = \lim_{t \to 0} \frac{d_i}{d_o}\left(\frac{I_{1o} I_{io}}{t}\right)$, $M_A$ can be $\infty$

$h_j = \frac{I_{1i} I_{ij}}{I_{1j} I_{ij}}$ (rad/cm)

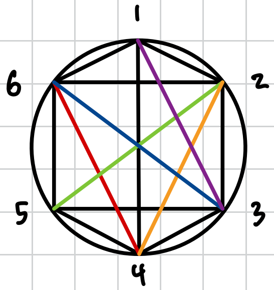
$f_j = I_{1i} I_{ij}$

$\omega_j = \frac{\omega_i I_{ij}}{I_{jk}}$ (rad/s)

$\frac{\omega_i}{\omega_j} = \frac{I_{jc}}{I_{ic}}$

$\omega = \frac{V_{slider}}{r}$

Find $\omega_6$ . $\dot{\theta}_2 = 10$ rpm CW



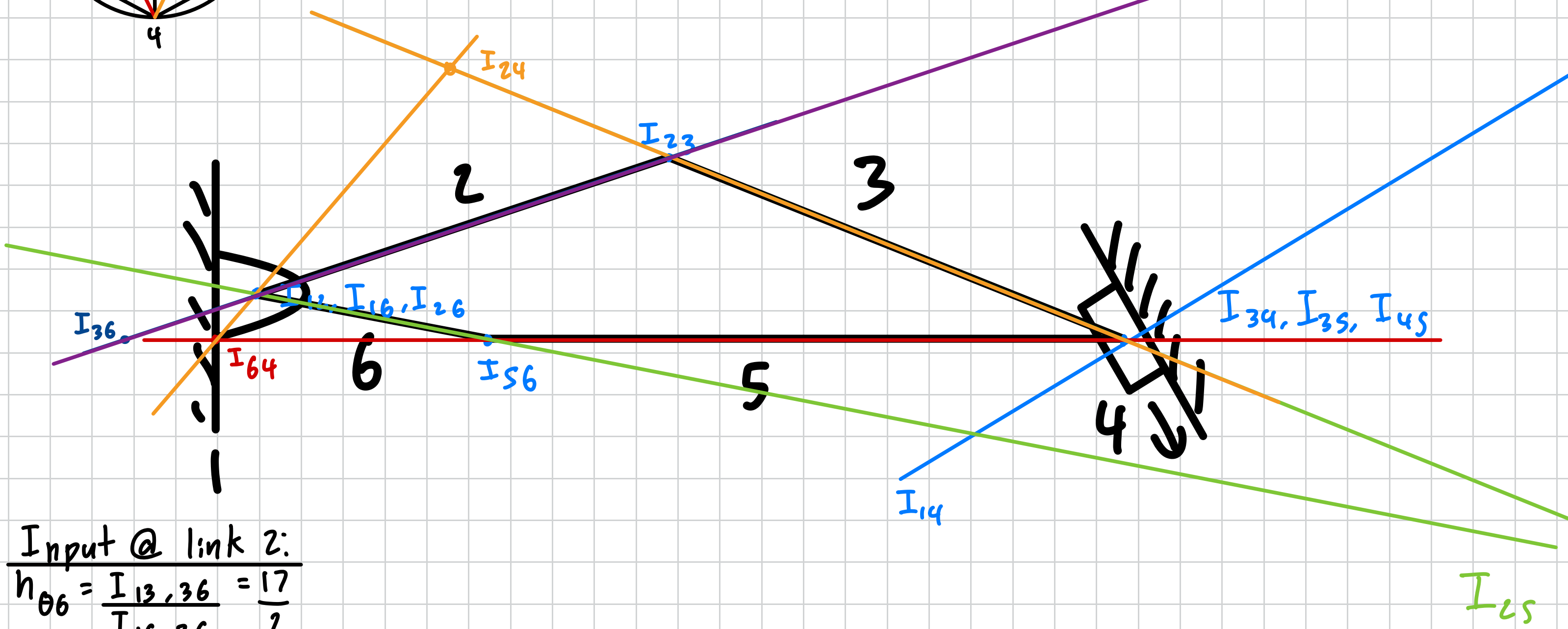

$I_{64} = I_{61,14}$
$\quad\quad I_{65,54}$

$I_{25} = I_{26,65}$
$\quad\quad I_{23,35}$

$I_{24} = I_{23,34}$
$\quad\quad I_{26,64}$

$I_{36} = I_{32,26}$
$\quad\quad I_{35,56}$

$I_{31} = I_{32,21}$
$\quad\quad I_{34,41}$




Input @ link 2:

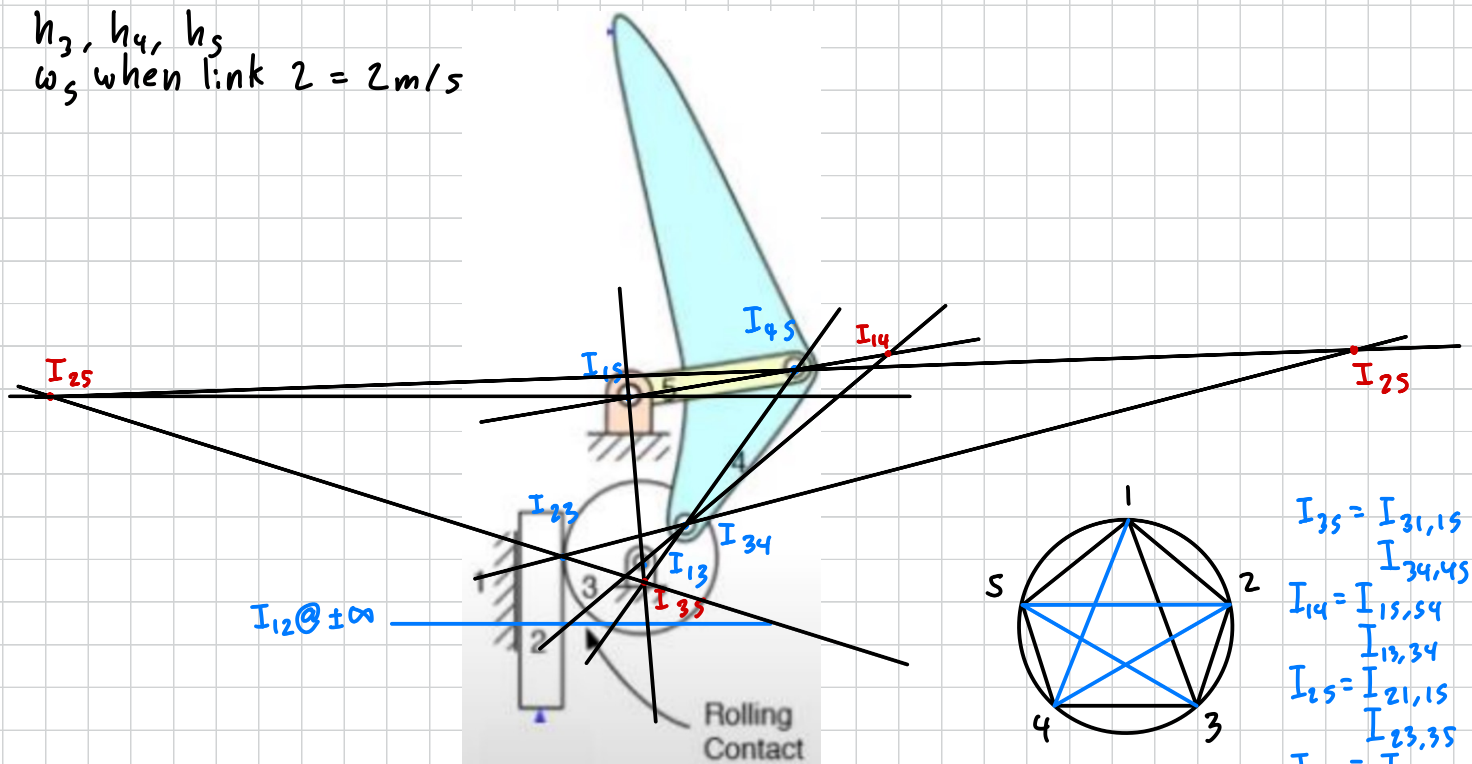
$h_{\theta 6} = \frac{I_{13,36}}{I_{16,36}} = \frac{17}{2}$

$\omega_6 = h_{\theta 6}\,\omega_3 = 26.15$ rpm CCW

$h_{\theta 3} = \frac{I_{12}\,I_{23}}{I_{13}\,I_{23}} = -0.308$

$\omega_3 = 3.077$

Find $h_3, h_4, h_5$
$\omega_5$ when link 2 = 2 m/s



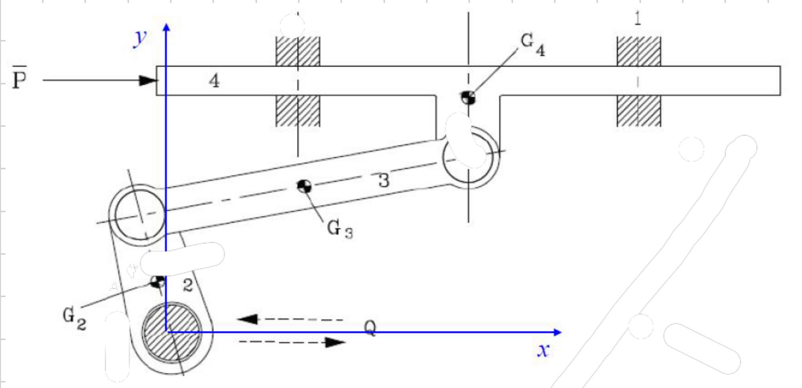

$I_{35} = I_{31,15}$
$\quad\quad I_{34,45}$

$I_{14} = I_{15,54}$
$\quad\quad I_{13,34}$

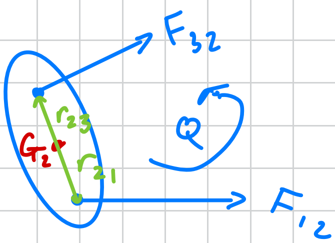
$I_{25} = I_{21,15}$
$\quad\quad I_{23,35}$

$I_{24} = I_{23,34}$
$\quad\quad I_{25,54}$

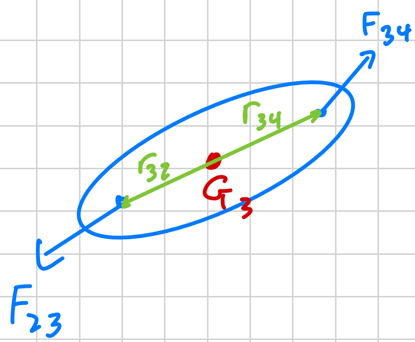
# Force analysis:




link 2:

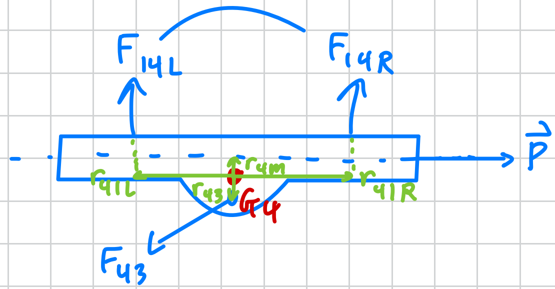



link 3:




link 4:

can't combine




link 2:
$$\sum F_x = F_{32x} + F_{12x} = m_2 a_{G_2x} \quad (1)$$
$$\sum F_y = F_{32y} + F_{12y} = m_2 a_{G_2y} \quad (2)$$
$$\sum M = (r_{21x}F_{12y} - r_{21y}F_{12x}) + (r_{23x}F_{32y} - r_{23y}F_{32x}) = I_2\alpha_2 - Q \quad (3)$$

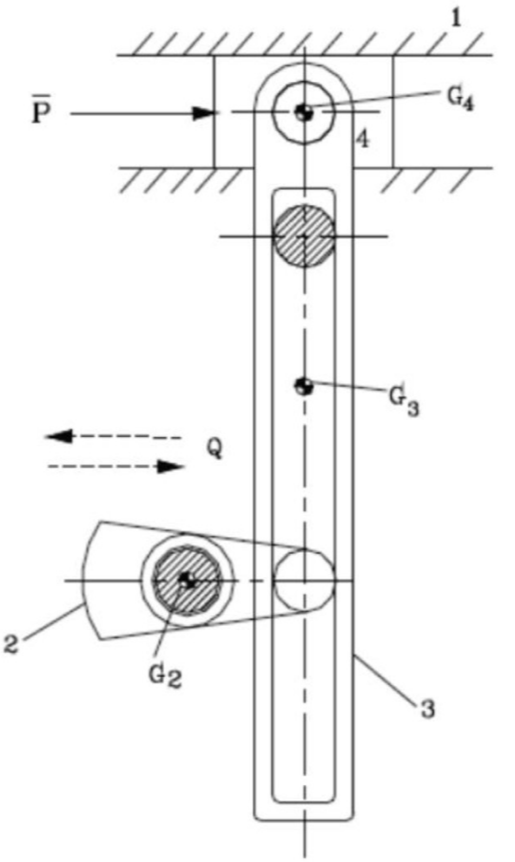
link 3:
$$\sum F_x = -F_{32x} + F_{34x} = m_3 a_{G_3x} \quad (4)$$
$$\sum F_y = -F_{32y} + F_{34y} = m_3 a_{G_3y} \quad (5)$$
$$\sum M = (r_{32x}(-F_{32y})) - r_{32y}(-F_{32x})) + (r_{34x}F_{34y} - r_{34y}F_{34x}) = I_3\alpha_3 \quad (6)$$

link 4:
$$\sum F_x = -F_{34x} + P = m_4 a_{G_4x} \quad (7)$$
$$\sum F_y = -F_{34y} + F_{14L} + F_{14R} = m_4 \overset{0}{\cancel{a_{G_4y}}} = 0 \quad (8)$$
$$\sum M = -r_{34y}(-F_{34x}) - r_{14L}F_{14L} + r_{14R}F_{14R} - r_{4m}P = I_4\overset{0}{\cancel{\alpha_4}} = 0 \quad (9)$$

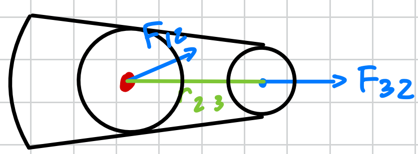
Note: $-F_{34} = F_{43}$, $-F_{32} = F_{23}$

Unknowns: $F_{12x}, F_{12y}, F_{32x}, F_{32y}, F_{34x}, F_{34y}, F_{14L}, F_{14R}, P$

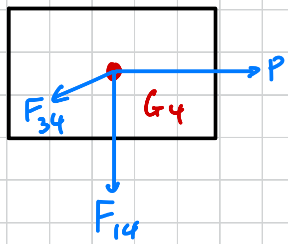
$$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -r_{21y} & r_{21x} & -r_{23y} & r_{23x} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & r_{32y} & -r_{32x} & -r_{34y} & r_{34x} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & r_{43y} & -r_{41L} & r_{41R} & 0 & -r_{4m} \end{pmatrix} \begin{pmatrix} F_{12x} \\ F_{12y} \\ F_{32x} \\ F_{32y} \\ F_{34x} \\ F_{34y} \\ F_{14L} \\ F_{14R} \\ P \end{pmatrix} = \begin{pmatrix} m_2 a_{G_2x} \\ m_2 a_{G_2y} \\ I_2\alpha_2 - Q \\ m_3 a_{G_3x} \\ m_3 a_{G_3y} \\ I_3\alpha_3 \\ m_4 a_{G_4x} \\ 0 \\ 0 \end{pmatrix}$$



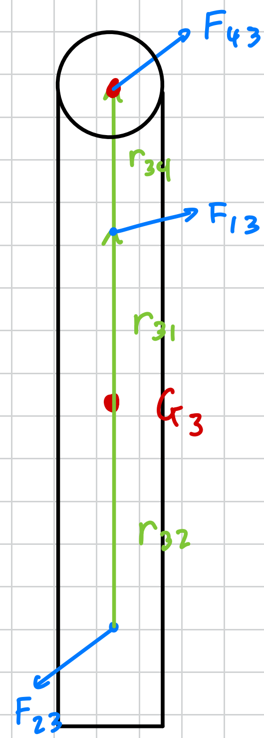

link 2:




link 4:




link 3:




link 2: $\sum F_x = F_{12x} + F_{32x} = m_2 \mathring{\alpha}_{G_2 x} = 0$ (1)

$\sum F_y = F_{12y} + F_{32y} = m_2 \mathring{\alpha}_{G_2 y} = 0$ (2)

$\sum M = r_{23x} F_{32y} - r_{23y} F_{32x} = I_2 \alpha_2 - Q$ (3)

link 3: $\sum F_x = -F_{32x} + F_{13x} + F_{43x} = m_3 a_{G_3 x}$ (4)

$\sum F_y = -F_{32y} + F_{13y} + F_{43y} = m_3 a_{G_3 y}$ (5)

$\sum M = (r_{32x}(-F_{32y}) - r_{32y}(-F_{32x})) + (r_{31x} F_{13y} - r_{31y} F_{13x}) + (r_{34x} F_{43y} - r_{34y} F_{43x}) = I_3 \alpha_3$ (6)

$F_{32} \cdot r_{32} = 0 \rightarrow r_{32x} F_{32x} + r_{32y} F_{32y} = 0$ (7)

$F_{13} \cdot r_{31} = 0 \rightarrow r_{31x} F_{13x} + r_{31y} F_{13y} = 0$ (8)

link 4: $-F_{43x} + P = m_4 a_{G_4 x}$ (9)

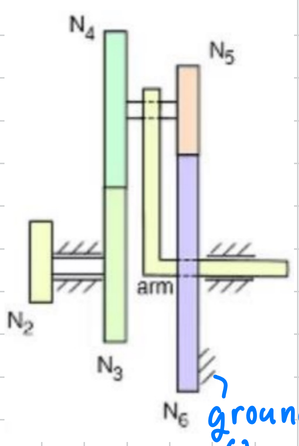
$-F_{43y} + F_{14} = m_4 a_{G_4 y} = 0$ (10)

Unknowns: $F_{12x}, F_{12y}, F_{32x}, F_{32y}, F_{13x}, F_{13y}, F_{43x}, F_{43y}, F_{14}, P$

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -r_{23y} & r_{23x} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & r_{32y} & -r_{32x} & -r_{31y} & r_{31x} & -r_{34y} & r_{34x} & 0 & 0 \\ 0 & 0 & r_{32x} & r_{32y} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & r_{31x} & r_{31y} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \end{pmatrix} \begin{pmatrix} F_{12x} \\ F_{12y} \\ F_{32x} \\ F_{32y} \\ F_{13x} \\ F_{13y} \\ F_{43x} \\ F_{43y} \\ F_{14} \\ P \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ I_2 \alpha_2 - Q \\ m_3 a_{G_3 x} \\ m_3 a_{G_3 y} \\ I_3 \alpha_3 \\ 0 \\ 0 \\ m_4 a_{G_4 x} \\ 0 \end{pmatrix}$$

# Gears

General formula: $\omega_a = -\frac{N_b}{N_a}\omega_b$ where $N$ = # of teeth, $1\ rpm = \frac{2\pi}{60}\ rad/s$




grounded $\omega_6 = 0$

| Gear | 2 | 3 | 4 | 5 | 6 | arm |
|---|---|---|---|---|---|---|
| arm fixed | $\omega_{2r}$ | $\omega_{2r}$ | $-\frac{N_3}{N_4}\omega_{2r}$ | $-\frac{N_3}{N_4}\omega_{2r}$ | $\frac{N_5}{N_6}\frac{N_3}{N_4}\omega_{2r}$ | 0 |
| gear locked (arm rot.) | $\omega_{arm}$ | $\omega_{arm}$ | $\omega_{arm}$ | $\omega_{arm}$ | $\omega_{arm}$ | $\omega_{arm}$ |
| total | $\omega_2$ | $\omega_3$ | $\omega_4$ | $\omega_5$ | 0 | $\omega_{arm}$ |

$\omega_{2r} + \omega_{arm} = \omega_2$

$\omega_{2r} = \omega_2 - \omega_{arm}$

$\frac{N_5}{N_6}\frac{N_3}{N_4}\omega_{2r} + \omega_{arm} = 0$

$\frac{N_5 N_3}{N_6 N_4}(\omega_2 - \omega_{arm}) + \omega_{arm} = 0$

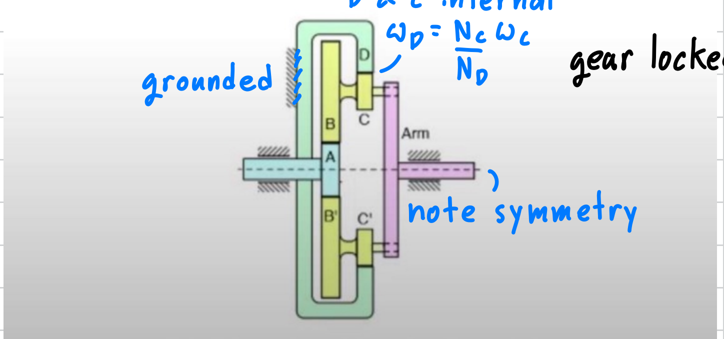
$\omega_{arm} = \frac{N_5 N_3}{N_3 N_5 - N_6 N_4}\omega_2$

EXAMPLE 1 Consider the gear train in Fig 19. Gear A drives the gear train and the arm is the output. Obtain angular velocity of each gear in terms of the input angular velocity. Find kinematic coefficient for each gear.

D & C internal

$\omega_D = \frac{N_C}{N_D}\omega_C$

grounded




note symmetry

Fig. 19 Gear train with input A and output arm

| Gear | A | B(B') | C(C') | D | arm |
|---|---|---|---|---|---|
| arm fixed | $\omega_{Ar}$ | $-\frac{N_A}{N_B}\omega_{Ar}$ | $-\frac{N_A}{N_B}\omega_{Ar}$ | $-\frac{N_C}{N_D}\frac{N_A}{N_B}\omega_{Ar}$ | 0 |
| gear locked (arm rot.) | $\omega_{arm}$ | $\omega_{arm}$ | $\omega_{arm}$ | arm | $\omega_{arm}$ |
| total | $\omega_A$ | $\omega_{B/B'}$ | $\omega_{C/C'}$ | 0 | $\omega_{arm}$ |

$\omega_{Br} = -\frac{N_A}{N_B}\omega_{Ar}$

$\omega_{Dr} = -\frac{N_C}{N_D}\frac{N_A}{N_B}\omega_{Ar}$

$\omega_A = \omega_{arm} + \omega_{Ar}$

$\omega_{Ar} = \omega_A - \omega_{arm}$

$-\frac{N_C N_A}{N_D N_B}\omega_{Ar} + \omega_{arm} = 0$

$-\frac{N_C N_A}{N_D N_B}(\omega_A - \omega_{arm}) + \omega_{arm} = 0$

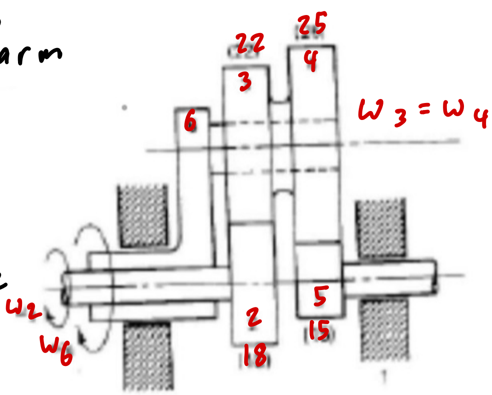
$\omega_{arm} = \frac{N_A N_C}{N_A N_C + N_B N_D}\omega_A$

In the gear train shown below obtain the output shaft speed in terms of the angular speeds of gear 2 and arm 6. What are the kinematic coefficients for the output?

$\omega_2$ $\quad$ $\omega_6 = \omega_{arm}$

$\omega_3 = \omega_4$

$\omega_{2r} + \omega_6 = \omega_2$




$\omega_{3r} = -\frac{N_2}{N_3}\omega_{2r}$

$\omega_2 = \omega_{arm} + \omega_{2r}$

$\omega_2 - \omega_{arm} = \omega_{2r}$

$\omega_2 - \omega_6 = \omega_{2r}$

$\omega_5 = \frac{N_4}{N_5}\frac{N_2}{N_3}\omega_{2r} + \omega_{arm}$

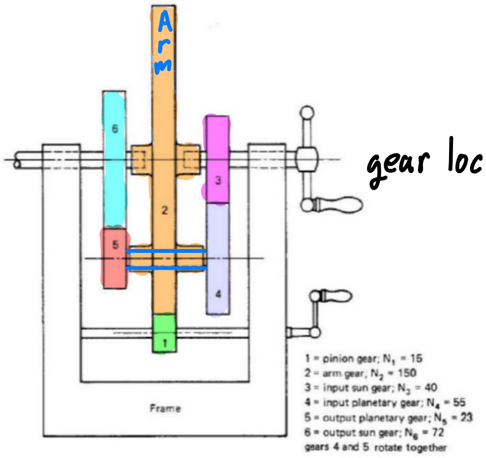
$= \frac{N_4}{N_5}\frac{N_2}{N_3}(\omega_2 - \omega_6) + \omega_6$

$h_5 = \frac{\omega_5}{\omega_2}$

| Gear | $\omega_6$ = arm | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| arm fixed | 0 | $\omega_{2r}$ | $-\frac{N_2}{N_3}\omega_{2r}$ | $-\frac{N_2}{N_3}\omega_{2r}$ | $\frac{N_4}{N_5}\frac{N_2}{N_3}\omega_{2r}$ |
| gear locked (arm rot.) | $\omega_{arm}$ | $\omega_{arm}$ | $\omega_{arm}$ | $\omega_{arm}$ | $\omega_{arm}$ |
| total | $\omega_6$ | $\omega_2$ | $\omega_3$ | $\omega_4$ | $\omega_5$ |

In the gear train shown below gear 3 rotates at 100 rpm clockwise and gear 1 rotates at 200 rpm counterclockwise (both viewed from the right). Determine the angular velocity of gear 6. How would you define kinematic coefficients for gear 6.

$\omega_3 = 100$ CW $\rightarrow (-100)$
$\omega_1 = 200$ CCW $\rightarrow (200)$




| Gear | arm | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| arm fixed | 0 | $\omega_{3r}$ | $-\frac{N_3 \omega_{3r}}{N_4}$ | $-\frac{N_3 \omega_{3r}}{N_4}$ | $\frac{N_5}{N_6}\frac{N_3 \omega_{3r}}{N_4}$ |
| gear locked (arm rot.) | $\omega_{arm}$ | $\omega_{arm}$ | $\omega_{arm}$ | $\omega_{arm}$ | $\omega_{arm}$ |
| total | $\omega_2$ | $\omega_3$ | $\omega_4$ | $\omega_5$ | $\omega_6$ |

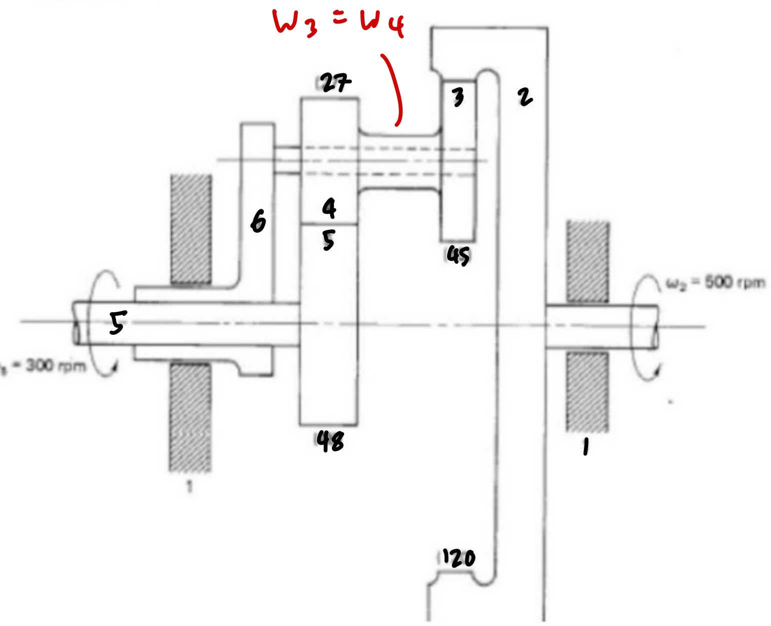
$\omega_{3r} = \omega_3 - \omega_{arm}$

$\omega_2 = -\frac{N_1}{N_2}\omega_1 = \omega_{arm}$

$$\omega_6 = \frac{N_5}{N_6}\frac{N_3}{N_4}\omega_{3r} + \omega_{arm}$$
$$= \frac{N_5}{N_6}\frac{N_3}{N_4}(\omega_3 - \omega_{arm}) + \omega_{arm}$$
$$= \frac{N_5 N_3}{N_6 N_4}(\omega_3 - \omega_2) + \omega_2$$

$h_j = \frac{\omega_j}{\omega_i} \rightarrow h_6 = \frac{\omega_6}{\omega_3}$

In the gear train below gears 2 and 5 act as input the system. Obtain the angular velocity of the remaining gears and that of the arm. What are the kinematic coefficients




| Gear | arm ($\omega_6$ =) | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| arm fixed | 0 | $\omega_{2r}$ | $\frac{N_2}{N_3}\omega_{2r}$ | $\frac{N_2}{N_3}\omega_{2r}$ | |
| gear locked (arm rot.) | $\omega_6$ | $\omega_6$ | $\omega_6$ | $\omega_6$ | |
| total | $\omega_6$ | $\omega_2$ | $\omega_3$ | $\omega_4$ | |

$$\omega_5 = -\frac{N_4}{N_5}\omega_4 = -\frac{N_4}{N}\frac{N_2}{N_3}\omega_{2r} = -\frac{N_4}{N}\frac{N_2}{N_3}(\omega_2 - \omega_6) + \omega_6$$

$\omega_{2r} = \omega_2 - \omega_6$

$h_3 = \frac{\omega_3}{\omega_2} = h_4$ $\quad h_5 = \frac{\omega_5}{\omega_2}$ $\quad h_6 = \frac{\omega_6}{\omega_2}$

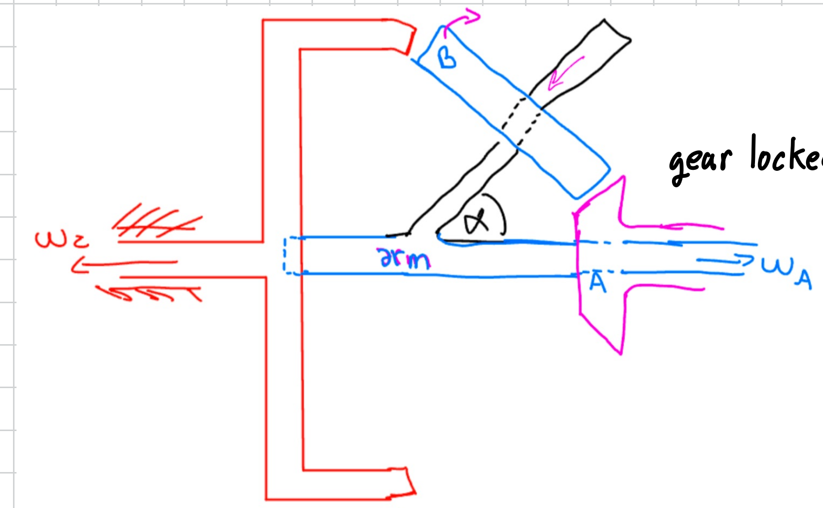



Fig. 1.1

$\omega_{arm}$

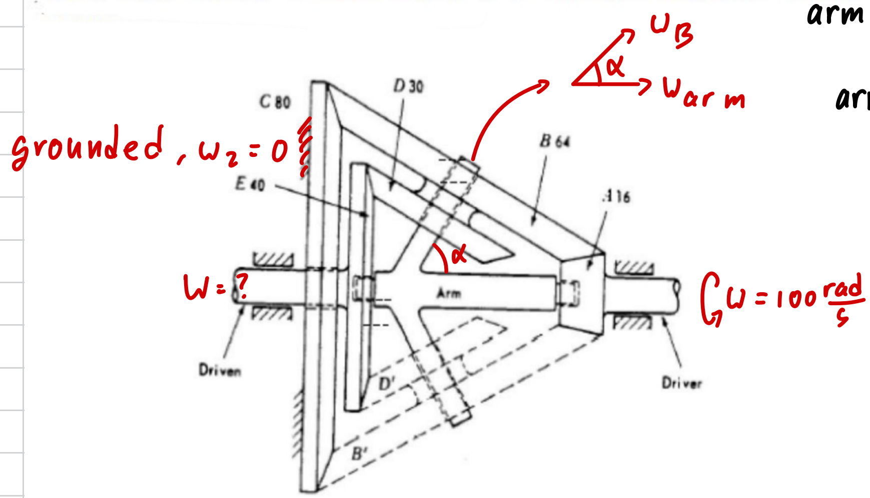
| Gear | A | B | C | arm |
|---|---|---|---|---|
| arm fixed | $\omega_{Ar}$ | $\frac{N_A}{N_B}\omega_{Ar}$ ↙ | $-\frac{N_B N_A}{N_B N_C}\omega_{Ar}$ | 0 |
| gear locked (arm rot.) | $\omega_{arm}$ | $\omega_{arm}\cos\alpha$ ↗ | $\omega_{arm}$ | $\omega_{arm}$ |
| total | $\omega_A$ | $\omega_B$ ↙ | $\omega_C$ | |

$\left.\begin{array}{l}\omega_B \\ \omega_{arm}\end{array}\right\}$ $\omega_B$ are opposite dir. ∴ $\omega_B$ (gear lock) $= -\omega_{arm}\cos\alpha$

$$\omega_C = \omega_{arm} - \frac{N_A}{N_C}(\omega_A - \omega_{arm})$$

$$\omega_B = \frac{N_A}{N_B}(\omega_A - \omega_{arm}) - \omega_{arm}\cos\alpha$$

**Problem 1.** (50) For a driver angular velocity of 100 rad/sec counterclockwise as viewed from the right, determine the angular velocity of the driven gear using tabular method.




| Gear | A | B (B') | C | D |
|---|---|---|---|---|
| arm fixed | $\omega_{Ar}$ | $\frac{N_A}{N_B}\omega_{Ar}$ ↙ | $-\frac{N_A N_B}{N_B N_C}\omega_{Ar}$ | $\frac{N_A}{N_B}\omega_{Ar}$ |
| arm rot. | $\omega_{arm}$ | $\omega_{arm}\cos\alpha$ ↗ | $\omega_{arm}$ | $\omega_{arm}\cos\alpha$ ↗ |
| total | | $\omega_B$ ↙ | 0 | |

$\omega_B = \frac{N_A}{N_B}\omega_{Ar}$ ↙

$\omega_C = -\frac{N_B}{N_C}\omega_{Br}$

| E | arm |
|---|---|
| $-\frac{N_A N_B}{N_B N_C}\omega_{Ar}$ | 0 |
| $\omega_{arm}$ | $\omega_{arm}$ |
| $\omega_E$ | |

$$\omega_A = \omega_{Ar} + \omega_{arm}$$

$$\omega_C = 0 = -\frac{N_A N_B}{N_B N_C}\omega_{Ar} + \omega_{arm} \rightarrow \omega_{arm} = \frac{N_A}{N_C}\omega_{Ar}$$

$$100 = \omega_{Ar} + \frac{N_A}{N_C}\omega_{Ar} = \omega_{Ar}\left(\frac{N_C + N_A}{N_C}\right)$$

1. ↓ solve

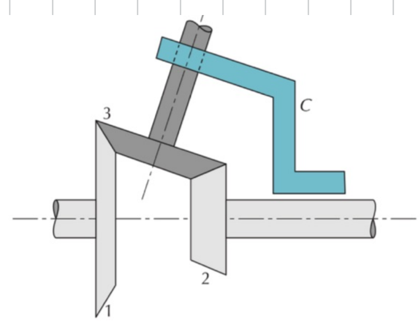
2.

P6.38 Refer to the Figure P6.38.

(a) Determine the rotation of the carrier when gear 1 makes 30 rotations clockwise and gear 2 makes 12 rotations counterclockwise. Also, determine the angular displacement of gear 3 about its own axis.

(b) If gear 1 rotates at $f$ rpm and gear 2 at $g$ rpm, determine the rotational speed of the carrier in terms of $f$ and $g$.

$N_1 = 40$; $N_2 = 30$; $N_3 = 24$




| | 1 | 2 | 3 | C |
|---|---|---|---|---|
| Arm fixed | $\omega_{1r}$ | $-\frac{N_1}{N_2}\omega_{1r}$ | $-\frac{N_1}{N_3}\cdot\omega_{1r}$ | 0 |
| Arm rotates | $\omega_{arm}$ | $\omega_{arm}$ | $\omega_{arm}$ | $\omega_{arm}$ |

★ $\omega_1 = \omega_{1r} + \omega_{arm} = -30 \rightarrow \omega_{1r} + \omega_{arm} = -30$

★ $\omega_2 = \omega_{2r} + \omega_{arm} = 12 \rightarrow -\frac{N_1}{N_2}\omega_{1r} + \omega_{arm} = 12$

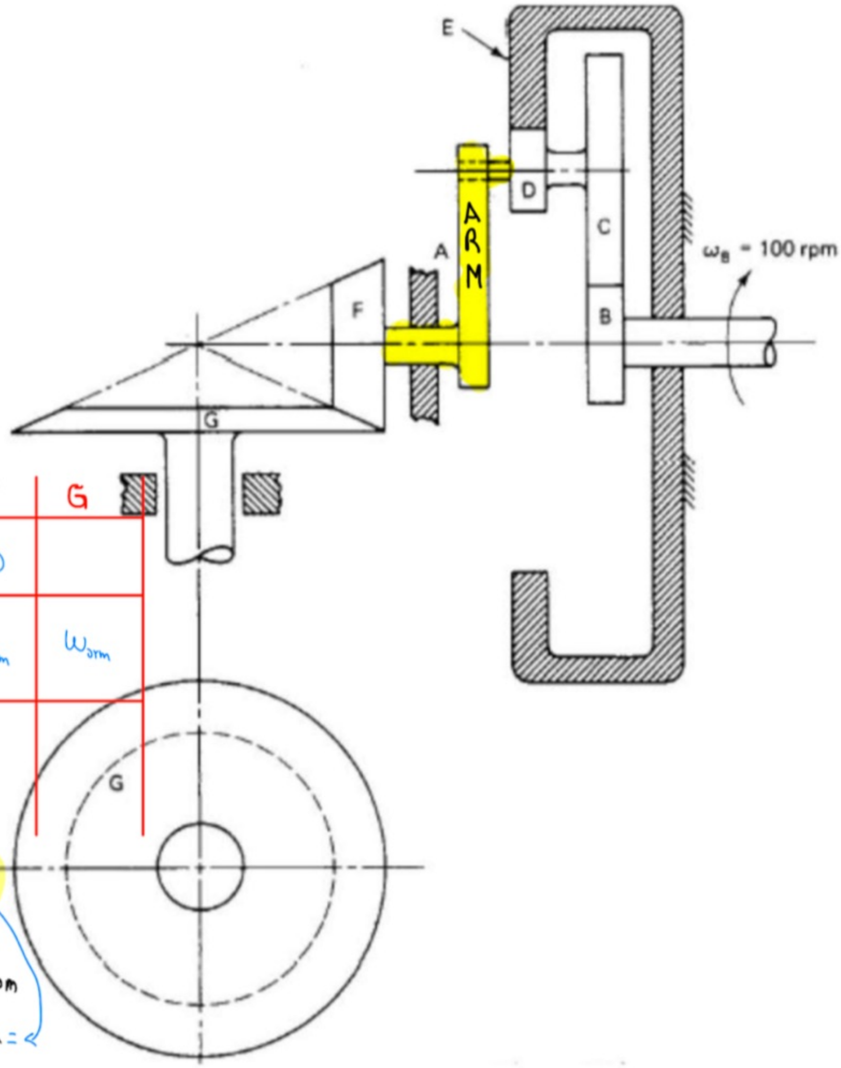
$\boxed{\omega_{1r} = -18 \text{ rpm (cw)};\ \omega_{arm} = -12 \text{ rpm (cw)}}$

★ $\omega_3 = -\frac{N_1}{N_3}\omega_{1r} + \omega_{arm}$

$\boxed{\omega_3 = 18 \text{ rpm ccw}}$

★ E is fixed , Find $\omega_G$

A = arm
B = 24 teeth
C = 60 teeth
D = 18 teeth
E = 102 teeth (fixed)
F = 25 teeth
G = 50 teeth




| Gears | A(arm) | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| Arm fixed | 0 | $\omega_{Br}$ | $-\frac{N_B}{N_C}\cdot\omega_{Br}$ | $-\frac{N_B}{N_C}\cdot\omega_{Br}$ | $-\frac{N_D}{N_E}\cdot\frac{N_B}{N_C}\cdot\omega_{Br}$ | 0 | |
| Gears locked (arm rotates) | $\omega_{arm}$ | $\omega_{arm}$ | $\omega_{arm}$ | $\omega_{arm}$ | $\omega_{arm}$ | $\omega_{arm}$ | $\omega_{arm}$ |
| Total | | | | | | | |

★ $\omega_E = 0 = \frac{N_D}{N_E}\cdot\frac{N_B}{N_c}\cdot\omega_{Br} + \omega_{arm} \rightarrow \omega_{arm} = \frac{N_D}{N_E}\cdot\frac{N_B}{N_c}\cdot\omega_{Br}$

$\omega_B = 100 = \omega_{Br} + \omega_{arm} \rightarrow 100 = \omega_{Br} + \frac{N_D}{N_E}\cdot\frac{N_B}{N_c}\cdot\omega_{Br} \rightarrow \omega_{Br} = 93.41 \text{ rpm}$

$\omega_F = 0 + \omega_{arm}$

$\omega_{arm} = 6.59 \text{ rpm}$

$\omega_G = \frac{N_F}{N_G}\cdot\omega_F = \frac{25}{50}\cdot 6.59 \rightarrow \boxed{\omega_G = 3.296 \text{ rpm}}$ ccw

In the gear train below gear *A* is rotating at *72 rpm* clockwise as viewed form the right. Find angular velocities of gear E and the arm. Find the kinematic coefficient for gear *E*.

$\omega_c = 0 \rightarrow \omega_{arm} = \frac{N_A}{N_c}\cdot\omega_{Ar}$

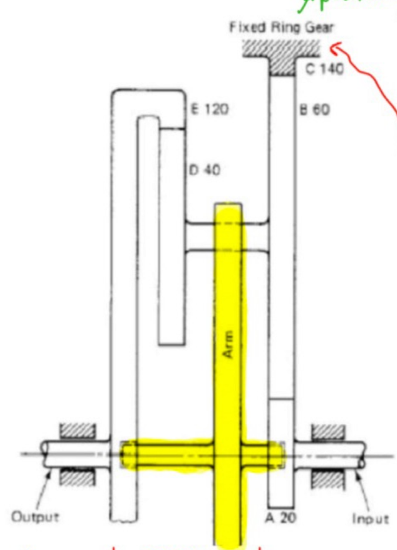
$\omega_A = 72\text{rpm} = \omega_{Ar} + \frac{N_A}{N_c}\cdot\omega_{Ar}$

$\rightarrow \omega_{Ar} = 63 \text{ rpm}$ ; $\omega_{arm} = 9 \text{ rpm}$

$\omega_E = -\frac{N_A}{N_B}\cdot\frac{N_D}{N_E}\cdot\omega_{Ar} + \omega_{arm}$

$\omega_E = 2 \text{ rpm}$

→ planetary gear




★ special case pay attention to this!

| | A | B | C | D | E | ARM |
|---|---|---|---|---|---|---|
| Arm fixed | $\omega_{Ar}$ | $-\frac{N_A}{N_B}\cdot\omega_{Ar}$ | $-\frac{N_A}{N_c}\cdot\omega_{Ar}$ | $-\frac{N_A}{N_B}\cdot\omega_{Ar}$ | $-\frac{N_A}{N_B}\cdot\frac{N_D}{N_E}\cdot\omega_{Ar}$ | 0 |
| Arm rotates | $\omega_{arm}$ | $\omega_{arm}$ | $\omega_{arm}$ | $\omega_{arm}$ | $\omega_{arm}$ | $\omega_{arm}$ |

| | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|
| Arm fixed | $\omega_{2r}$ | $\frac{N_2}{N_3}\cdot\omega_{2r}$ | $\frac{N_2}{N_3}\cdot\omega_{2r}$ | $-\frac{N_2}{N_3}\frac{N_4}{N_5}\omega_{2r}$ | $0$ | $\frac{N_2}{N_3}\cdot\frac{N_4}{N_7}\cdot\omega_{2r}$ |
| Arm rotates | $\omega_{arm}$ | $\omega_{arm}$ | $\omega_{arm}$ | $\omega_{arm}$ | $\omega_{arm}$ | $\omega_{arm}$ |
| Total | | | | | | |

$$\omega_7 = 0 \rightarrow -\frac{N_2}{N_3}\cdot\frac{N_4}{N_7}\cdot\omega_{2r} = \omega_{arm}$$

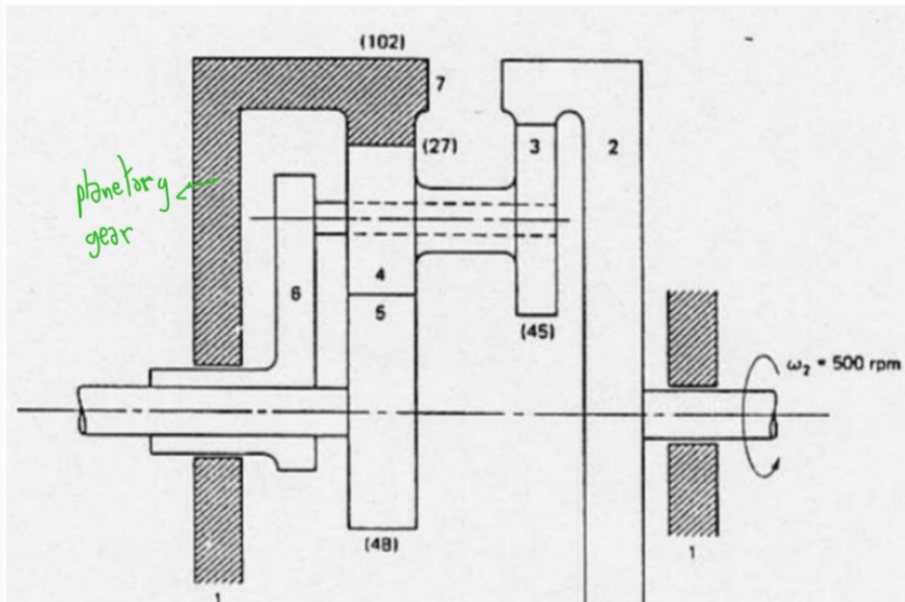
$$500 = \omega_{2r} - \frac{N_2}{N_3}\cdot\frac{N_4}{N_7}\cdot\omega_{2r} \rightarrow$$

$$\omega_{2r} = 1700 \text{ rpm}$$

$$\omega_{arm} = -1200 \text{ rpm}$$

$$\omega_5 = -\frac{N_2 N_4}{N_3 \cdot N_5}\cdot\omega_{2r} + \omega_{arm}$$

$$\rightarrow \omega_5 = -3750 \text{ (cw)}$$




[Problem 3] The figure shown <u>above</u> is a planetary gear train. Find the velocity of gear 5 $\omega_5$. In the figure, the shaded parts are fixed (not moving) and gear 2 is rotating at 500 rpm, i.e. $\omega_2 = 500\ rpm$ [ $N_2$ = 120, $N_3$ = 45, $N_4$ = 27, $N_5$ = 48, $N_7$ = 102 ]

(a) (15 pt.) By using tabular method.

(b) (15 pt.) By using formula method.

$$\omega_D = 0 \rightarrow \frac{N_B}{N_D}\cdot\omega_{Br} = \omega_{arm_L}$$

$$150 = \omega_B = \omega_{Br} + \frac{N_B}{N_D}\cdot\omega_{Br}$$

$$\rightarrow \omega_{Br} = 100 \text{ rpm} ;\ \omega_{arm_L} = \omega_A = 50 \text{ rpm}$$

★ $\omega_{arm_L}$ is input of Right part ★

$$\omega_H = 0 \rightarrow \frac{N_F}{N_H}\cdot\omega_{Fr} = \omega_{arm}$$

$$50 = \omega_{Fr} + \frac{N_F}{N_H}\cdot\omega_{Fr}$$

$$\rightarrow \omega_{Fr} = 35.71 \text{ rpm} ;\ \omega_{arm_R} = 14.29 \text{ rpm}$$

$$\omega_{arm_R} = \omega_E = 14.29 \text{ rpm}$$

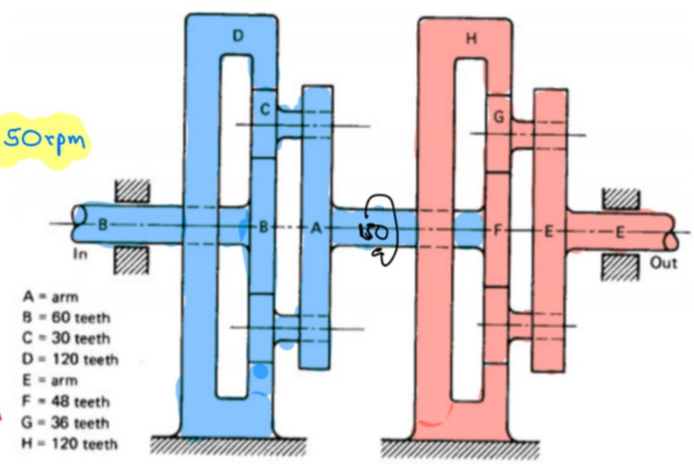



Figure 3: Gear Train 3

| | B | C | D | A-F | G ($\omega_{Fr}$) | H | E |
|---|---|---|---|---|---|---|---|
| Arm fixed | $\omega_{Br}$ | $-\frac{N_B}{N_C}\cdot\omega_{Br}$ | $-\frac{N_B}{N_D}\cdot\omega_{Br}$ | $0$ | $-\frac{N_F}{N_G}\omega_{Fr}$ | $-\frac{N_F}{N_H}\cdot\omega_{Fr}$ | $0$ |
| Arm rotates | $\omega_{arm_L}$ | $\omega_{arm_L}$ | $\omega_{arm_L}$ | $\omega_{arm_L}$ | $\omega_{arm_R}$ | $\omega_{arm_R}$ | $\omega_{arm_R}$ |

**Problem 2.** (25) The differential gear system shown below operates with input from the wheels that drive gears A and B. Use tabular method to obtain the relationship between the arm angular velocity (gear D) and the two input angular velocities. Each gear is designated with a letter and its number of teeth; for example, A-36 indicates gear A containing 36 teeth.

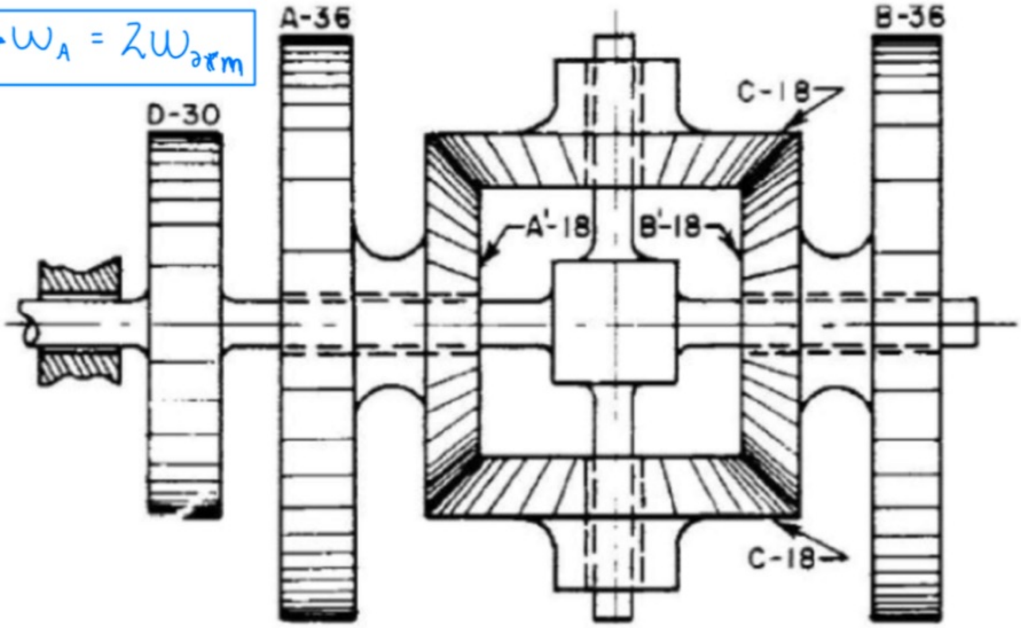
$(\omega_A - \omega_{arm}) = -(\omega_B - \omega_{arm})$

$\omega_A - \omega_{arm} = -\omega_B + \omega_{arm}$

$\omega_A = -\omega_B + 2\omega_{arm}$

$\boxed{\omega_B + \omega_A = 2\omega_{arm}}$

$\star\ \omega_{Ar} = -\omega_{Br}$




| Gears | D | A | A' | C | B' | B |
|---|---|---|---|---|---|---|
| Arm fixed | 0 | $\omega_{Ar}$ | $\omega_{Ar}$ | $\frac{N_{A'}}{N_C} \cdot \omega_{Ar}$ | $-\frac{N_{A'}}{N_{B'}} \cdot \omega_{Ar}$ | |
| Gears locked (arm rotates) | $\omega_{arm}$ | $\omega_{arm}$ | $\omega_{arm}$ | $\omega_{arm}$ | $\omega_{arm}$ | $\omega_{arm}$ |