Numerical Linear Algebra
Programming Assignment #03

2015-17231
박우정

Exercise 2.13.

MATLAB으로 문제에서 요구하는 프로그램을 짠 코드는 다음과 같다.

```
n=input('What is the dimension n? ');
A=zeros(n,n);
matx=eye(n); %x1, x2, ..., xn이 column 단위로 저장된 행렬. 즉 xi=matx(:,i)
check=zeros(n,n);
saveI=matx;
for j=1:n
    A(j,j)=2;
end
for j=1:n-1
    A(j,j+1)=-1;
    A(j+1,j)=-1;
end
saveA=A;
for j=1:n-1
    for k=j+1:n
        m=A(k,j)/A(j,j);
        A(k,j:n)=A(k,j:n)-m*A(j,j:n);
        matx(k,:)=matx(k,:)-m*matx(j,:); %Gauss elimination
    end
end
for j=1:n
for i=n:-1:1
    matx(i,j)=(matx(i,j)-dot(A(i,i+1:n),matx(i+1:n,j)))/A(i,i);
end %find xj=matx(:,j) for j=1:n
end
disp('The solution A^-1 is as follows: ')
disp(matx)
for k=1:n
    for j=1:n
        check(i,j)=saveA(i,:)*matx(:,j);
    end
end
if(abs(check(i,j)-saveI(i,j)>10^-10))
    disp("This inverse is incorrect")
else
    disp("This inverse is correct almost everywhere")
end
```

허용오차는 10^-10으로 주었고, $n = 10$으로 input을 주어 실행하면 다음을 얻는다.

```
>> inverse_tridiagonal
What is the dimension n? 10
The solution A^-1 is as follows:
    0.9091    0.8182    0.7273    0.6364    0.5455    0.4545    0.3636
0.2727    0.1818    0.0909
    0.8182    1.6364    1.4545    1.2727    1.0909    0.9091    0.7273
0.5455    0.3636    0.1818
    0.7273    1.4545    2.1818    1.9091    1.6364    1.3636    1.0909
0.8182    0.5455    0.2727
    0.6364    1.2727    1.9091    2.5455    2.1818    1.8182    1.4545
1.0909    0.7273    0.3636
    0.5455    1.0909    1.6364    2.1818    2.7273    2.2727    1.8182
1.3636    0.9091    0.4545
    0.4545    0.9091    1.3636    1.8182    2.2727    2.7273    2.1818
1.6364    1.0909    0.5455
    0.3636    0.7273    1.0909    1.4545    1.8182    2.1818    2.5455
1.9091    1.2727    0.6364
    0.2727    0.5455    0.8182    1.0909    1.3636    1.6364    1.9091
2.1818    1.4545    0.7273
    0.1818    0.3636    0.5455    0.7273    0.9091    1.0909    1.2727
1.4545    1.6364    0.8182
    0.0909    0.1818    0.2727    0.3636    0.4545    0.5455    0.6364
0.7273    0.8182    0.9091

This inverse is correct almost everywhere
```

100개의 항들이 모두 허용오차를 잘 지키고 있으므로, 알맞게 프로그래밍되었다고 할 수 있다.

Exercise 2.16

MATLAB을 이용하여 inverse of A를 구하는 Gauss-Jordan program을 작성하면 다음과 같다.

```
n=input('What is the dimension n? ');
col=zeros(n,1);row=zeros(1,n);
perm=eye(n); %Permutation matrix exchanging rows.
A=zeros(n,n); AI=zeros(n,n);
check=zeros(n,n);saveI=eye(n);
for j=1:n
    A(j,j)=2;
end
for j=1:n-1
    A(j,j+1)=-1;
    A(j+1,j)=-1;
end
saveA=A;
for j=1:n
    [x,ksave]=max(abs(A(j:n,j))); %Partial pivoting
    k=ksave+j-1;
    perm([j,k],:)=perm([k,j],:); %Row exchange in the permutation matrix
    A([j,k],:)=A([k,j],:);
    m=1/A(j,j); %Start Gauss-Jordran inverse matrix algorithm
    A(j,j)=m;
    col=m*A(:,j);
    row=A(j,:);
    A=A-col*row;
    A(:,j)=col;
    A(j,:)=-1*m*row;
    A(j,j)=m;
end
for i=1:n
    for j=1:n
        AI(i,j)=A(i,:)*perm(:,j);
    end
end %undo the order of their rows.
disp(A)
for i=1:n %check whether the inverse is correct within the tolerance 10e-10
    for j=1:n
        check(i,j)=saveA(i,:)*AI(:,j);
        if(abs(check(i,j)-saveI(i,j))>10^-10)
            disp("This inverse is incorrect")
        end
    end
end
```

이를 실행하면 다음과 같은 결과를 얻을 수 있다.

```
>> tridiagonal_inverse_GaussJordan
What is the dimension n? 10
    0.9091    0.8182    0.7273    0.6364    0.5455    0.4545    0.3636    0.2727
 0.1818    0.0909
    0.8182    1.6364    1.4545    1.2727    1.0909    0.9091    0.7273    0.5455
 0.3636    0.1818
    0.7273    1.4545    2.1818    1.9091    1.6364    1.3636    1.0909    0.8182
 0.5455    0.2727
    0.6364    1.2727    1.9091    2.5455    2.1818    1.8182    1.4545    1.0909
 0.7273    0.3636
    0.5455    1.0909    1.6364    2.1818    2.7273    2.2727    1.8182    1.3636
 0.9091    0.4545
    0.4545    0.9091    1.3636    1.8182    2.2727    2.7273    2.1818    1.6364
 1.0909    0.5455
    0.3636    0.7273    1.0909    1.4545    1.8182    2.1818    2.5455    1.9091
 1.2727    0.6364
    0.2727    0.5455    0.8182    1.0909    1.3636    1.6364    1.9091    2.1818
 1.4545    0.7273
    0.1818    0.3636    0.5455    0.7273    0.9091    1.0909    1.2727    1.4545
 1.6364    0.8182
    0.0909    0.1818    0.2727    0.3636    0.4545    0.5455    0.6364    0.7273
 0.8182    0.9091
```

허용오차를 넘는 메시지가 출력되지 않았으므로, 알맞게 프로그래밍되었다고 할 수 있다.