Single Photon Interference

Woojin Han[1]

[1]Seoul National University, Seoul 151-747, Korea

May 31, 2023

Abstract

In this experiment,

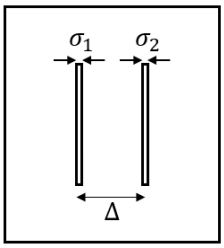
1 Introduction

The experiment of light interference is very significant in research of early quantum mechanics. The interference pattern is used to explain the wave-like nature of light, which is rejected by the photon model. In this experiment, we checked the light interference pattern in the weak amplitude limits of light, to find the quantum duality of light. But, the signal is too small and wide variance the statistical method must be checked. In 1.2 section, I uploaded to the public the analysis module and the fully activating code. Secondly, the data points are tightly measured for high trust in the results.

(results)

1.1 Fitting Functions: Statistical Verification

The results take the form of two different experimental values, the detector voltage in the laser experiment, and the pulse count data in the single photon limit experiment. The natural error of each value is explained at 3.1. The slit has three different significant values to affect the interference pattern. As Fig. 1, $\sigma_{1,2}$, the slit width of right, and left in sight of align each respectively. Δ, the distance between two slits. For convenient naming to fitting variables, I name the slit specification parameters in the Greek alphabet.



(a)

Figure 1: Schematic model of the slit

By simple calculations, the double slit results which assume $\sigma 1 = \sigma 2 = \sigma$ and no spectral dispersion, in each the wavelength of the laser is uniform enough to λ, follows equation 1. The fitting parameters are c, I, d_1, d_2 which are the center of the detector slit, the center intensity, and the spatial parameters of slit width and distance each. Since the values of σ and L have an ambiguity of absolute value, only their ratio matters. Therefore, I select d_1, d_2 instead.

$$I(x;c,I,d_1,d_2) = I\left(\frac{\sin(\pi\frac{x-c}{d_1})}{\pi\frac{x-c}{d_1}}\right)^2(\cos(\pi\frac{x-c}{d_2}))^2$$
$$d_1 = \frac{L\lambda}{\sigma}, \quad d_2 = \frac{L\lambda}{\Delta} \tag{1}$$

For modeling the dispersed nature of the laser wavelength, the Lorentzian distribution is used. Since the laser emits light in band gap resonance, the assumption is valid. Equation 2 shows the Lorentzian profile of the wavelength. γ is the FWHM value of the wavelength, Γ is dimensionless factor of the distribution. To detour the ambiguity of λ and γ, I used Γ as a fitting parameter. d_i, the fitting parameters are linear to λ, the probability density of d while d_i is the central value can be calculated as follows.

$$\begin{aligned} g(\lambda;\gamma) &= \frac{1}{\pi}\frac{\frac{\gamma}{2}}{\lambda^2+(\frac{\gamma}{2})^2} \\ \gamma &= \lambda\times\Gamma \\ P(d|d_i) &= g(\frac{d-d_i}{d_i};\Gamma) \end{aligned} \tag{2}$$

Finally, the fitting equation is 3. This type of fitting is written as Laser Broadening Fitting(LBF). The integrand function is the simple double-slit function in 1.

$$I(x;c,I,d_1,d_2,\Gamma) = \int I(x;c,I,d_1(1+\alpha),d_2(1+\alpha),\gamma)g(\alpha;\Gamma)d\alpha \tag{3}$$

In the asymmetric setting in each case of $\sigma_1 \neq \sigma_2$ changes the single slit interference part either. But, the condition leads to overfitting since there are too many parameters compared to the data points. Therefore, I assume $\sigma_1 \sim \sigma_2$ which gives the difference in the amplitude of light. This type of fitting is written as Asymmetric Correction Fitting(ACF).

$$I(x;c,I,d_1,d_2,I_2) = \left(\frac{\sin(\pi\frac{x-c}{d_1})}{\pi\frac{x-c}{d_1}}\right)^2\frac{1}{4}(I+I_2+2\sqrt{II_2}\cos(2\pi\frac{x-c}{d_2})) \tag{4}$$

In the laser experiment, many photon limits results are fully explained by those functions. But, in the bulb experiment, single photon limit results have different types of measurement occurring. The Photomultiplier Tube (PMT) detector amplifies the small size input to the large signal. And, the measurement data is the count of the pulse which has the amplified signal over a certain threshold. The amplification is controlled by the factor of PMT, High Voltage (HV). I assume two things to make a model of PMT pulse count data. First, the photon detection probability is uniform of the timescale $1s$. Secondly, each photon gives excitation to PMT in uniform amplitude. Let single-photon excites the PMT in the amplitude of V_0 for τ seconds. And, n number of photons emits in unit time $t_0 >> \tau$ by assumption. Then, the probability of the signal V, $P(V;j)$ has a relation of equation 5. Each photon has the probability to excite the PMT in a certain time for $2\tau/t_0$. So, the number of photons follows binomial distribution $B(n, 2\tau/t_0)$.

$$P(V > (threshold);j) \sim Ae^{-2\tau j} \tag{5}$$

In the definition of $j = n/t_0$, the probability of a pulse count is exponentially related to the input signal. By the signal resolution time, the Pulse Counter/Interval Timer(PCIT) value is exponentially dependent on the input signal. Moreover, by the assumption of the binomial distribution, the variance of the PCIT value is proportionally linear to the mean value. In the fitting of single photon limits, I may check the 1σ boundary of multiple experiment results, to check the reliability of the fitting method.

The nonvanishing problem may also be explained by the rejection of the Fraunhofer approximation. Those two slit intensities may differ for the distance between two optical routes. But this opinion is simply neglected by the precision of this experiment. The route distance difference is about $(\frac{\Delta}{L})^2 \sim (\frac{10\mu m}{10cm})^2 = 10^{-8}$. This means that the difference between the route may affect the intensity results, but the effect is negligible with the natural error of 10^{-4}, voltage measurement. Also, the slit size difference $\sigma_1 \neq \sigma_2$ regime is dominating the Fraunhofer correction. So, if the fitting function which considers the optical length difference fits well, the fitting data must not be trusted by this p-value test.

Just like the previous example, the function fits well with many variables. Also, there are a lot of parameters in each fitting function, the fitting results may fall into local minima and lose their physical meaning. To avoid overfitting, I double-checked the parameters in each fitting method. The parameter c, I, d_i must have similar values since they have the physical backgrounds of slit specification. If the parameters are significantly different of 2σ, a p-value of 0.05, I avoid using the fitted results. To help the statistical verification, a specially built module is informed at 1.2.

1.2 Data Analysis Module

To fit in various functions, the codes are too messy in dealing with data sets. Also, the fitting parameters are too many to optimize in strange local minima lots of times. The modulation of the initial parameters is not good enough to try in many data sets. Therefore, I made the module of fitting and plotting. The idea is that the fitting parameters have a relation with the roughly fitted parameters. For example, $[c, I, d_1, d_2]$ results in equation 1 fitting are similar to the results of $[c, I, d_1, d_2, \Gamma]$ in equation 3. If is not, the physical meaning of each parameter is infringed, which means the results are overfitted. Therefore, I made an input of `rough_fitting_functions` and `fitting_param_query` which both take a role in roughly fitting the data itself, to avoid hard manipulation of initial parameters.

```
laser_modified_fig = spi.phys_plot(
        data_set_list,
        lambda x: x.parameters,
        lambda x: x.results[0],
        {'align': 3, 'exp_type': 'double_slit'},
        fitting_function= modified_function,
        rough_fitting_functions = [rough_1,fitting_function],
        fitting_param_query = [None,lambda x: [*x[:4],1e-2]],
        p0_function = laser_double_slit_param_setting,
        truncate = lambda x: True if x<0.7 else False,
        export_param_statics = "export.txt"
    )
```

For example, above is the example usage of `spi.phys_plot` method. The first two inputs are the x, y variables of the plot. So, the function will plot the `x.results[0] - x.parameters`. I labeled the experiment as align trial 3, the experiment type is double slit. And the following `modified_function` is the final fitting equation. The `rough_1, fitting_function` are the roughly fitting functions like equation 1. Sometimes the parameter numbers or the enumerate may change by the functions. For example, there are 4 parameters until `fitting_function`, but we need 5 parameters in `modified_function`. Then the fitting parameter should follow the query, in this case, `lambda x: [*x[:4],1e-2]` which adds the fifth parameter as 10^{-2}. Moreover, the method exports the parameters in the form of LaTeXtabular environment in `export_param_statics`. The exported example is like 2.

1double slit 1	$0.234 \pm 1.54e-08$	$0.042 \pm 6.522e-06$	0.9920

Figure 2: example output of phys plot method

By using this method, I can fit an enormous amount of data points without wasting time setting initial parameters, or writing the parameters down in the report. Every code is uploaded in [2].

2 Methods

3 Results and Discussion

3.1

4 Summary

[1]

References

[1] someone. something.

[2] WoojinHan24. Single photon interference. `https://github.com/WoojinHan24/Single_Photon_Interference`.