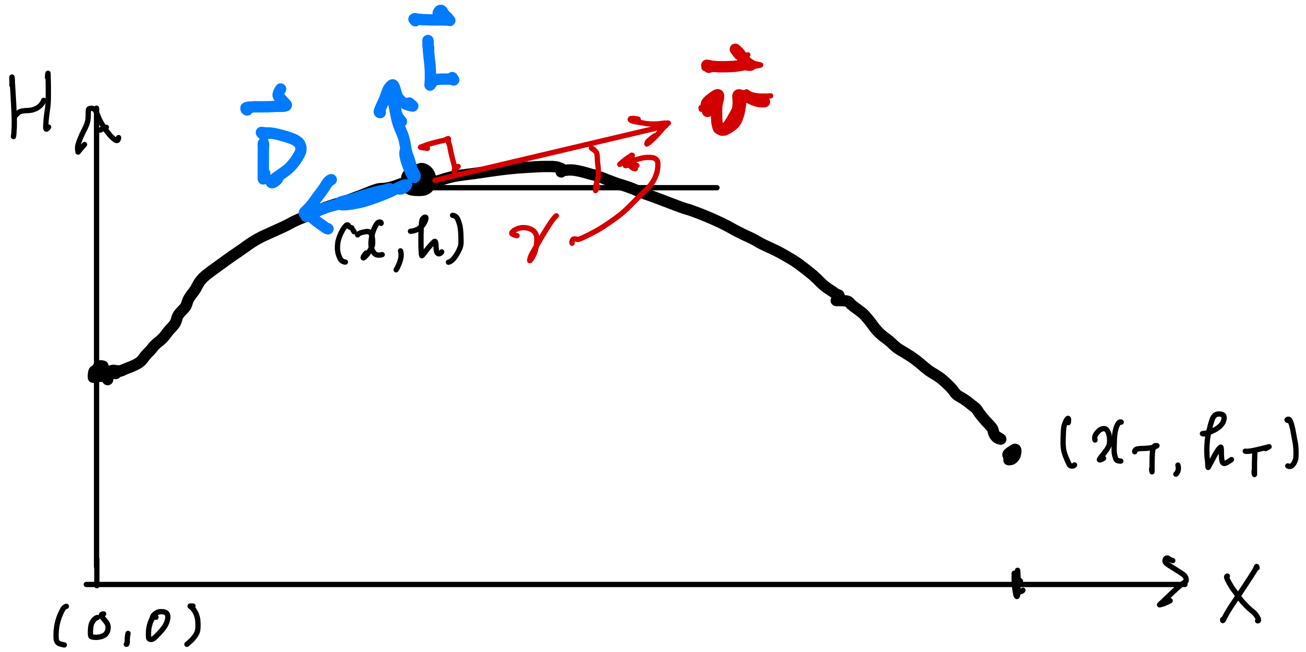
AE 551
Spring 2024

Term Project



Equations of Motion :

$$\dot{x} = v_x , \quad \dot{h} = v_h$$

$$\dot{v}_x = \left(\frac{T-D}{m}\right)\cos\gamma - \left(\frac{L}{m}\right)\sin\gamma$$

$$\dot{v}_h = \left(\frac{T-D}{m}\right)\sin\gamma + \left(\frac{L}{m}\right)\cos\gamma - g$$

$$\dot{m} = -\mu = \text{const.}$$

$$L = C_{L_\alpha} \cdot \alpha \ \frac{1}{2}\rho v^2 \ S_{ref}$$

$$D = (C_{D_0} + k C_L^2)\ \frac{1}{2}\rho v^2 S_{ref}$$

* Check the equations of motion before coding.

$C_{L\alpha}$ (lift derivative) $= 8.5$

C_{D_0} (zero AOA drag coeff.) $= 0.30$

k (induced drag coeff.) $= 0.02$

C_L (lift coefficient) $= C_{L_\alpha} \cdot \alpha$

T (thrust) $= 20{,}000$ N

$$\mu = \frac{m_p \text{ (propellant mass)}}{t_b \text{ (burn time)}} = \frac{60 \text{ kg}}{7 \text{ sec}}$$

ρ (air density) * MATLAB Function

S_{ref} (reference area) $= \frac{\pi}{4} d^2 = 0.026 \text{ m}^2$

α (angle of attack) = control input

$x_0 = 0$ km, $h_0 = 10$ km

$x_T = 100$ km, $h_T = 5$ km

$v_0 = 300$ m/s, $\gamma_0 = 0$ deg

$m_0 = 150$ kg

* You may use v and γ instead of v_x and v_h.

$$\dot{v} = \left(\frac{T-D}{m}\right) - g\sin\gamma$$
$$\dot{\gamma} = \left[\left(\frac{L}{m}\right) - g\cos\gamma\right] / v$$

Check before use!

$$\dot{x} = v\cos\gamma, \qquad \dot{h} = v\sin\gamma$$

* You may use x instead of t as the independent variable:

$$\frac{dt}{dx} = \frac{1}{v\cos\gamma}, \qquad \frac{dh}{dx} = \tan\gamma$$

$$\frac{dv}{dx} = \left(\frac{dv}{dt}\right) / \left(\frac{dx}{dt}\right), \qquad \frac{d\gamma}{dx} = \left(\frac{d\gamma}{dt}\right) / \left(\frac{dx}{dt}\right)$$

This approach is possible if γ is bounded by $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Its merit is that the terminal $x_f = x_T$ is fixed.

Problem:

$$\max \ J = v_f$$

subject to

$$(x_f, h_f) = (x_T, h_T)$$

$$|\alpha| \leq 20 \text{ deg}$$

(1) Convert the problem to a parameter optimization problem using the pseudo-spectral method.
Use 10 collocation points or more.
(You may use the Hermite-Simpson method but there will be some point deduction.)

(2) Calculate the optimal trajectory using a commercial optimization s/w such as SQP or IP of MATLAB.

(3) Calculate the optimal trajectory using the augmented Lagrangian solver developed by yourself.

The term project report should inclue the followings:

- Description of the parameter optimization problem
- Numerical results (graphs) showing the history of α, v, γ and the trajectory shape.
- Comparison of your solver and the commercial solver you use.
- Explanation on the numerical results.
- Code including the pseudo-spectral method and your optimization solver. (the augmented Lagrangian method, the quasi-Newton method, and the line search method.)

* Provide a main program named as 'MAIN_TP.m' that I can run on my PC to check your code.

* Due Date: June 17. Monday 23:59.
  (Late submission will not be accepted.)
* Combine all files as a ZIP file before submission.
* The report document should be a PDF file.