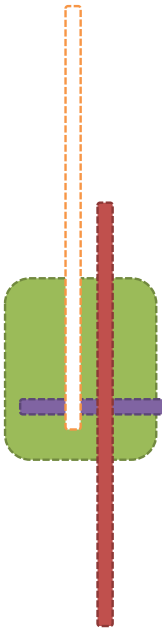
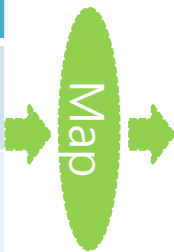


# Big Data Analysis: Matrix calculation



# Word Counting with MapReduce

Documents(t)
Financial, IMF, Economics, Crisis
Financial, IMF, Crisis



Documents(t)
Economics, Harry
Financial, Harry, Potter, Film
Crisis, Harry, Potter

Key	Value
Financial	1
IMF	1
Economics	1
Crisis	1
Financial	1
IMF	1
Crisis	1

Key, that you want to group by in order to achieve GOAL by doing something with each group sharing the same key

# Word Counting with MapReduce

GOAL: output  
the count of  
**each word**

Documents(t)
Financial, IMF, Economics, Crisis
Financial, IMF, Crisis



Documents(t)
Economics, Harry
Financial, Harry, Potter, Film
Crisis, Harry, Potter



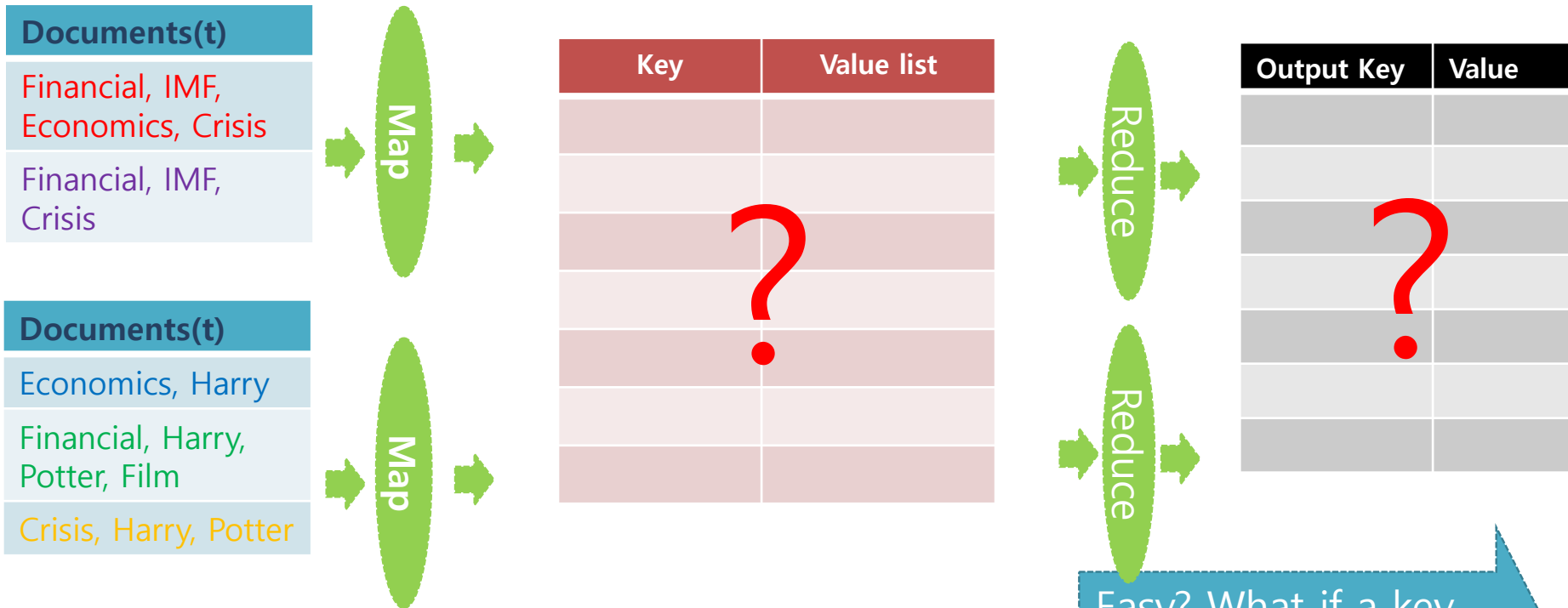
Key	Value list
Financial	1,1, 1
IMF	1,1
Economics	1, 1
Crisis	1,1, 1
Harry	1,1,1
Film	1
Potter	1,1



Output Key	Value
Financial	3
IMF	2
Economics	2
Crisis	3
Harry	3
Film	1
Potter	2

Before reduce functions are called,  
for each distinct key,  
the list of its values are generated

# Problem Solving with MapReduce



Easy? What if a key-value pair is not defined and only; a problem is given to you?

# **MATRIX ADDITION AND MULTIPLICATION**



# Excercise: Matrix Addition

---

## ■ Input

- Two matrices, A and B
- Formatted: a line has a single element
- E.g.,
  - $A, 3, 2, 4.3 \rightarrow A_{32} = 4.3$

## ■ Map

- Input:  $\langle [A|B], i, j, \text{value} \rangle$
- Output:  $\langle \text{key}=\underline{\hspace{2cm}}, \text{value}=\underline{\hspace{2cm}} \rangle$

## ■ Reduce

- Input:  $\langle \text{key}, \text{a list of values} \rangle$
- Output:  $\langle \text{key}=\underline{\hspace{2cm}}, \text{value}=\underline{\hspace{2cm}} \rangle$



# Exercise: Matrix Addition

- Input
  - Two matrices, A and B
- Map
  - Input:  $\langle [A|B], i, j, \text{value} \rangle$
  - Output:  $\langle \text{key}=\{i,j\}, \text{value}=[A_{ij}|B_{ij}] \rangle$
- Reduce
  - Input:  $\langle \text{key}, \text{a list of values}=\{A_{ij}, B_{ij}\} \rangle$
  - Output:  $\langle \text{key}=\{i,j\}, \text{value}=A_{ij}+B_{ij} \rangle$



# Matrix Multiplication: $Av$

- Multiply a matrix and a vector
  - $A * v$

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1k} \\ a_{21} & a_{22} & \dots & a_{2k} \\ \dots & & & \\ a_{n1} & a_{n2} & \dots & a_{nk} \end{pmatrix} \times \begin{pmatrix} v_1 \\ v_2 \\ \dots \\ v_k \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \\ \dots \\ c_n \end{pmatrix}$$



# MapReduce Algorithm

---

## ■ Input

- A n-by-m matrices  $A$  ( $\rightarrow A, [\text{row \#}], [\text{col \#}], [\text{val}]$ )
- A m-dimensional vector  $v$  ( $\rightarrow v, [\text{row \#}], [\text{val}]$ )
- E.g.,
  - $A, 3, 2, 4.3 \rightarrow a_{32} = 4.3$
  - $v, 10, 3.7 \rightarrow v_{10} = 3.7$

## ■ Map

- Input:  $\langle [A|v], i, [j]?, \text{value} \rangle$
- Output:  $\langle \text{key} = ?, \text{value} = ? \rangle$

## ■ Reduce

- Input:  $\langle \text{key}, \text{a list of values} = \{?\} \rangle$
- Output:  $\langle \text{key} = ?, \text{value} = ? \rangle$



# MapReduce Algorithm

---

## ■ Input

- A n-by-m matrices  $A$  ( $\rightarrow A, [\text{row \#}], [\text{col \#}], [\text{val}]$ )
- A m-dimensional vector  $v$  ( $\rightarrow v, [\text{row \#}], [\text{val}]$ )
- E.g.,
  - $A, 3, 2, 4.3 \rightarrow a_{32} = 4.3$
  - $v, 10, 3.7 \rightarrow v_{10} = 3.7$

## ■ Map

- Input:  $\langle [A|v], i, [j]?, \text{value} \rangle$
- Output:  $\langle \text{key} = ?, \text{value} = ? \rangle$

## ■ Reduce

- Input:  $\langle \text{key}, \text{a list of values} = \{?\} \rangle$
- Output:  $\langle \text{key} = \text{the row number } i, \text{value} = (Av)_i \rangle$

# Matrix Multiplication

- Multiply a matrix and a vector

—  $A * v$

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1k} \\ a_{21} & a_{22} & \dots & a_{2k} \\ \dots & & & \\ a_{n1} & a_{n2} & \dots & a_{nk} \end{pmatrix} \times \begin{pmatrix} v_1 \\ v_2 \\ \dots \\ v_k \end{pmatrix} = \begin{pmatrix} \langle 1, (A, 1, a_{11}) \rangle \\ \vdots \end{pmatrix}$$

# Matrix Multiplication

- Multiply a matrix and a vector

—  $A * v$

The diagram illustrates the multiplication of a matrix  $A$  and a vector  $v$ . The matrix  $A$  is represented as a large rounded rectangle containing the elements  $a_{11}, a_{12}, \dots, a_{1k}$  in the first row,  $a_{21}, a_{22}, \dots, a_{2k}$  in the second row, and  $a_{n1}, a_{n2}, \dots, a_{nk}$  in the last row. The vector  $v$  is represented as a smaller rounded rectangle containing the elements  $v_1, v_2, \dots, v_k$ . A large black 'X' symbol is placed between the matrix and the vector, indicating multiplication. A red dashed circle highlights the element  $a_{12}$  in the first row of the matrix and the element  $v_2$  in the vector. A red dashed arrow originates from this circle and points to the right, towards the resulting vector components.

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1k} \\ a_{21} & a_{22} & \dots & a_{2k} \\ \dots & & & \\ a_{n1} & a_{n2} & \dots & a_{nk} \end{pmatrix} \times \begin{pmatrix} v_1 \\ v_2 \\ \dots \\ v_k \end{pmatrix} =$$

$\langle 1, (A, 1, a_{11}) \rangle$   
 $\langle 1, (A, 2, a_{12}) \rangle$

# Matrix Multiplication

- Multiply a matrix and a vector

—  $A * v$

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1k} \\ a_{21} & a_{22} & \dots & a_{2k} \\ \dots & & & \\ a_{n1} & a_{n2} & \dots & a_{nk} \end{pmatrix} \times \begin{pmatrix} v_1 \\ v_2 \\ \dots \\ v_k \end{pmatrix} = \begin{pmatrix} \langle 1, (A, 1, a_{11}) \rangle \\ \langle 1, (A, 2, a_{12}) \rangle \\ \dots \\ \langle 1, (A, k, a_{1k}) \rangle \end{pmatrix}$$

# Matrix Multiplication

- Multiply a matrix and a vector

—  $A * v$

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1k} \\ a_{21} & a_{22} & \dots & a_{2k} \\ \dots & & & \\ a_{n1} & a_{n2} & \dots & a_{nk} \end{pmatrix} \times \begin{pmatrix} v_1 \\ v_2 \\ \dots \\ v_k \end{pmatrix} = \begin{pmatrix} \langle 1, (A, 1, a_{11}) \rangle \\ \langle 1, (A, 2, a_{12}) \rangle \\ \dots \\ \langle 1, (A, k, a_{1k}) \rangle \\ \langle 1, (v, 1, v_1) \rangle \\ \dots \\ \langle 2, (v, 1, v_1) \rangle \\ \dots \\ \langle n, (v, 1, v_1) \rangle \end{pmatrix}$$

A red dashed arrow points from the  $v_1$  element in the vector to the first row of the resulting vector.

# Matrix Multiplication

- Multiply a matrix and a vector

—  $A * v$

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1k} \\ a_{21} & a_{22} & \dots & a_{2k} \\ \dots & & & \\ a_{n1} & a_{n2} & \dots & a_{nk} \end{pmatrix} \times \begin{pmatrix} v_1 \\ v_2 \\ \dots \\ v_k \end{pmatrix} = \begin{pmatrix} \langle 1, (A, 1, a_{11}) \rangle \\ \langle 1, (A, 2, a_{12}) \rangle \\ \dots \\ \langle 1, (A, k, a_{1k}) \rangle \\ \langle 1, (v, 1, v_1) \rangle \\ \langle 1, (v, 2, v_2) \rangle \\ \dots \\ \langle n, (v, 1, v_1) \rangle \\ \langle n, (v, 2, v_2) \rangle \end{pmatrix}$$

A red dashed circle highlights the element  $v_2$  in the vector  $v$ . A red dashed arrow points from this circle to the second element of the first row of the resulting vector,  $\langle 1, (A, 2, a_{12}) \rangle$ .

# Matrix Multiplication

- Multiply a matrix and a vector

—  $A * v$

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1k} \\ a_{21} & a_{22} & \dots & a_{2k} \\ \dots & & & \\ a_{n1} & a_{n2} & \dots & a_{nk} \end{pmatrix} \times \begin{pmatrix} v_1 \\ v_2 \\ \dots \\ v_k \end{pmatrix} = \begin{pmatrix} \langle 1, (A, 1, a_{11}) \rangle \\ \langle 1, (A, 2, a_{12}) \rangle \\ \dots \\ \langle 1, (A, k, a_{1k}) \rangle \\ \langle 1, (v, 1, v_1) \rangle \\ \langle 1, (v, 2, v_2) \rangle \\ \dots \\ \langle 1, (v, k, v_k) \rangle \\ \langle 2, (v, 1, v_1) \rangle \\ \langle 2, (v, 2, v_2) \rangle \\ \dots \\ \langle 2, (v, k, v_k) \rangle \\ \dots \\ \langle n, (v, 1, v_1) \rangle \\ \langle n, (v, 2, v_2) \rangle \\ \dots \\ \langle n, (v, k, v_k) \rangle \end{pmatrix}$$

The diagram illustrates the multiplication of a matrix  $A$  (with elements  $a_{ij}$ ) and a vector  $v$  (with elements  $v_i$ ). The result is a vector where each element is a dot product of a row of  $A$  with the vector  $v$ . A red dashed circle highlights the element  $v_k$  in the vector  $v$ , and a red dashed arrow points from it to the corresponding element in the resulting vector, indicating its contribution to the dot product.

# Matrix Multiplication

- Multiply a matrix and a vector

—  $A * v$

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1k} \\ a_{21} & a_{22} & \dots & a_{2k} \\ \dots & & & \\ a_{n1} & a_{n2} & \dots & a_{nk} \end{pmatrix} \times \begin{pmatrix} v_1 \\ v_2 \\ \dots \\ v_k \end{pmatrix} = \begin{pmatrix} \langle 1, (A, 1, a_{11}) \rangle \\ \langle 1, (A, 2, a_{12}) \rangle \\ \dots \\ \langle 1, (A, k, a_{1k}) \rangle \\ \langle 1, (v, 1, v_1) \rangle \\ \langle 1, (v, 2, v_2) \rangle \\ \dots \\ \langle 1, (v, k, v_k) \rangle \\ \langle 2, (v, 1, v_1) \rangle \\ \langle 2, (v, 2, v_2) \rangle \\ \dots \\ \langle 2, (v, k, v_k) \rangle \\ \dots \\ \langle n, (v, 1, v_1) \rangle \\ \langle n, (v, 2, v_2) \rangle \\ \dots \\ \langle n, (v, k, v_k) \rangle \end{pmatrix}$$

# Matrix Multiplication

- Multiply a matrix and a vector

—  $A * v$

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1k} \\ a_{21} & a_{22} & \dots & a_{2k} \\ \dots & & & \\ a_{n1} & a_{n2} & \dots & a_{nk} \end{pmatrix} \times \begin{pmatrix} v_1 \\ v_2 \\ \dots \\ v_k \end{pmatrix} = \begin{pmatrix} \langle 1, (A, 1, a_{11}) \rangle \\ \langle 1, (A, 2, a_{12}) \rangle \\ \dots \\ \langle 1, (A, k, a_{1k}) \rangle \\ \langle 1, (v, 1, v_1) \rangle \\ \langle 1, (v, 2, v_2) \rangle \\ \dots \\ \langle 1, (v, k, v_k) \rangle \\ \langle 2, (v, 1, v_1) \rangle \\ \langle 2, (v, 2, v_2) \rangle \\ \dots \\ \langle 2, (v, k, v_k) \rangle \\ \dots \\ \langle n, (v, 1, v_1) \rangle \\ \langle n, (v, 2, v_2) \rangle \\ \dots \\ \langle n, (v, k, v_k) \rangle \end{pmatrix}$$



# MapReduce Algorithm

## ■ Map

- Input:  $\langle [A|v], i, [j]?, [a_{ij} \mid v_i] \rangle$
- Output:
  - If "A": Output  $\langle \text{key} = i, \text{value} = ("A", j, a_{ij}) \rangle$
  - Else if "v":
    - ◆ For  $k=1$  to  $n$ :
      - » Output  $\langle \text{key} = k, \text{value} = ("v", i, v_i) \rangle$

## ■ Reduce

- Input:  $\langle \text{key} = ?, \text{a list of values} = \{?\} \rangle$
- Output:  $\langle \text{key} = ?, \text{value} = ? \rangle$



# MapReduce Algorithm

## ■ Map

- Input:  $\langle [A|v], i, [j]?, [a_{ij} \mid v_i] \rangle$
- Output:
  - If "A": Output  $\langle \text{key}=i, \text{value}= ("A", j, a_{ij}) \rangle$
  - Else if "v":
    - ◆ For  $k=1$  to  $n$ :
      - » Output  $\langle \text{key}=k, \text{value}= ("v", i, v_i) \rangle$

## ■ Reduce

- Input:  $\langle \text{key}=i,$   
a list of values  $= \{(A, j, a_{ij}) \mid (v, j, v_j)\}_{j=1, \dots, m} \rangle$
- Output:  $\langle \text{key}=i, \text{value} = \sum_{j=1}^m a_{ij} \cdot v_j \rangle$



# Matrix Multiplication: $A \times B$

---

- Single step map/reduce
  - How can we multiply two matrices with a run of map/reduce?
- Assumption
  - A row can be loaded in memory
  - A reduce for a key is called only once when it have all values for the key



# Single step map/reduce

---

## Map

### — Input

- Two matrices,  $A$  ( $n \times \ell$ ) and  $B$  ( $\ell \times m$ )
- E.g.,
  - ♦  $A, 2, 3, 3.2 \rightarrow a_{23} = 3.2$
  - ♦  $B, 2, 3, 3.2 \rightarrow b_{23} = 3.2$

### — Output

- For  $(A, i, j, a_{ij})$ : ?
- For  $(B, i, j, b_{ij})$ : ?

## Reduce

### — Input

- $\langle \text{key}=\{i,j\}, \{?\} \rangle$

### — Output

- $\langle \text{key}=\{i,j\}, ? \rangle$



# Matrix Multiplication

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1k} \\ a_{21} & a_{22} & \dots & a_{2k} \\ \dots & & & \\ a_{n1} & a_{n2} & \dots & a_{nk} \end{pmatrix} \times \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1m} \\ b_{21} & b_{22} & \dots & b_{2m} \\ \dots & & & \\ b_{k1} & b_{k2} & \dots & b_{km} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & \dots & c_{1m} \\ c_{21} & c_{22} & \dots & c_{2m} \\ \dots & & & \\ c_{n1} & c_{n2} & \dots & c_{nm} \end{pmatrix}$$



# Matrix Multiplication

---

- A matrix: row 1  $\rightarrow$  C matrix: row 1

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1k} \\ a_{21} & a_{22} & \dots & a_{2k} \\ \dots & & & \\ a_{n1} & a_{n2} & \dots & a_{nk} \end{pmatrix} \times \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1m} \\ b_{21} & b_{22} & \dots & b_{2m} \\ \dots & & & \\ b_{k1} & b_{k2} & \dots & b_{km} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & \dots & c_{1m} \\ c_{21} & c_{22} & \dots & c_{2m} \\ \dots & & & \\ c_{n1} & c_{n2} & \dots & c_{nm} \end{pmatrix}$$



# Matrix Multiplication

---

- A matrix: row 1  $\rightarrow$  C matrix: row 1

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1k} \\ a_{21} & a_{22} & \dots & a_{2k} \\ \dots & & & \\ a_{n1} & a_{n2} & \dots & a_{nk} \end{pmatrix} \times \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1m} \\ b_{21} & b_{22} & \dots & b_{2m} \\ \dots & & & \\ b_{k1} & b_{k2} & \dots & b_{km} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & \dots & c_{1m} \\ c_{21} & c_{22} & \dots & c_{2m} \\ \dots & & & \\ c_{n1} & c_{n2} & \dots & c_{nm} \end{pmatrix}$$



# Matrix Multiplication

---

- A matrix: row 1 -> C matrix: row 1

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1k} \\ a_{21} & a_{22} & \dots & a_{2k} \\ \dots & & & \\ a_{n1} & a_{n2} & \dots & a_{nk} \end{pmatrix} \times \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1m} \\ b_{21} & b_{22} & \dots & b_{2m} \\ \dots & & & \\ b_{k1} & b_{k2} & \dots & b_{km} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & \dots & c_{1m} \\ c_{21} & c_{22} & \dots & c_{2m} \\ \dots & & & \\ c_{n1} & c_{n2} & \dots & c_{nm} \end{pmatrix}$$



# Matrix Multiplication

---

- A matrix: row 2  $\rightarrow$  C matrix: row 2

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1k} \\ a_{21} & a_{22} & \dots & a_{2k} \\ \dots & & & \\ a_{n1} & a_{n2} & \dots & a_{nk} \end{pmatrix} \times \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1m} \\ b_{21} & b_{22} & \dots & b_{2m} \\ \dots & & & \\ b_{k1} & b_{k2} & \dots & b_{km} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & \dots & c_{1m} \\ c_{21} & c_{22} & \dots & c_{2m} \\ \dots & & & \\ c_{n1} & c_{n2} & \dots & c_{nm} \end{pmatrix}$$



# Matrix Multiplication

---

- A matrix: row 2  $\rightarrow$  C matrix: row 2

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1k} \\ a_{21} & a_{22} & \dots & a_{2k} \\ \dots & & & \\ a_{n1} & a_{n2} & \dots & a_{nk} \end{pmatrix} \times \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1m} \\ b_{21} & b_{22} & \dots & b_{2m} \\ \dots & & & \\ b_{k1} & b_{k2} & \dots & b_{km} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & \dots & c_{1m} \\ c_{21} & c_{22} & \dots & c_{2m} \\ \dots & & & \\ c_{n1} & c_{n2} & \dots & c_{nm} \end{pmatrix}$$



# Matrix Multiplication

---

- A matrix: row 2  $\rightarrow$  C matrix: row 2

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1k} \\ a_{21} & a_{22} & \dots & a_{2k} \\ \dots & & & \\ a_{n1} & a_{n2} & \dots & a_{nk} \end{pmatrix} \times \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1m} \\ b_{21} & b_{22} & \dots & b_{2m} \\ \dots & & & \\ b_{k1} & b_{k2} & \dots & b_{km} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & \dots & c_{1m} \\ c_{21} & c_{22} & \dots & c_{2m} \\ \dots & & & \\ c_{n1} & c_{n2} & \dots & c_{nm} \end{pmatrix}$$



# Matrix Multiplication

---

- A matrix i-th row  $\rightarrow$  C matrix i-th row  
— key (i, 1), (i, 2), ... (i, m)

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1k} \\ a_{21} & a_{22} & \dots & a_{2k} \\ \dots & & & \\ a_{n1} & a_{n2} & \dots & a_{nk} \end{pmatrix} \times \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1m} \\ b_{21} & b_{22} & \dots & b_{2m} \\ \dots & & & \\ b_{k1} & b_{k2} & \dots & b_{km} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & \dots & c_{1m} \\ c_{21} & c_{22} & \dots & c_{2m} \\ \dots & & & \\ c_{n1} & c_{n2} & \dots & c_{nm} \end{pmatrix}$$



# Matrix Multiplication

---

- B matrix: col 1  $\rightarrow$  C matrix: col 1

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1k} \\ a_{21} & a_{22} & \dots & a_{2k} \\ \dots & & & \\ a_{n1} & a_{n2} & \dots & a_{nk} \end{pmatrix} \times \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1m} \\ b_{21} & b_{22} & \dots & b_{2m} \\ \dots & & & \\ b_{k1} & b_{k2} & \dots & b_{km} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & \dots & c_{1m} \\ c_{21} & c_{22} & \dots & c_{2m} \\ \dots & & & \\ c_{n1} & c_{n2} & \dots & c_{nm} \end{pmatrix}$$



# Matrix Multiplication

---

- B matrix: col 1  $\rightarrow$  C matrix: col 1

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1k} \\ a_{21} & a_{22} & \dots & a_{2k} \\ \dots & & & \\ a_{n1} & a_{n2} & \dots & a_{nk} \end{pmatrix} \times \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1m} \\ b_{21} & b_{22} & \dots & b_{2m} \\ \dots & & & \\ b_{k1} & b_{k2} & \dots & b_{km} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & \dots & c_{1m} \\ c_{21} & c_{22} & \dots & c_{2m} \\ \dots & & & \\ c_{n1} & c_{n2} & \dots & c_{nm} \end{pmatrix}$$



# Matrix Multiplication

---

- B matrix: col 1  $\rightarrow$  C matrix: col 1

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1k} \\ a_{21} & a_{22} & \dots & a_{2k} \\ \dots & & & \\ a_{n1} & a_{n2} & \dots & a_{nk} \end{pmatrix} \times \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1m} \\ b_{21} & b_{22} & \dots & b_{2m} \\ \dots & & & \\ b_{k1} & b_{k2} & \dots & b_{km} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & \dots & c_{1m} \\ c_{21} & c_{22} & \dots & c_{2m} \\ \dots & & & \\ c_{n1} & c_{n2} & \dots & c_{nm} \end{pmatrix}$$



# Matrix Multiplication

---

- B matrix: col 2 -> C matrix: col 2

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1k} \\ a_{21} & a_{22} & \dots & a_{2k} \\ \dots & & & \\ a_{n1} & a_{n2} & \dots & a_{nk} \end{pmatrix} \times \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1m} \\ b_{21} & b_{22} & \dots & b_{2m} \\ \dots & & & \\ b_{k1} & b_{k2} & \dots & b_{km} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & \dots & c_{1m} \\ c_{21} & c_{22} & \dots & c_{2m} \\ \dots & & & \\ c_{n1} & c_{n2} & \dots & c_{nm} \end{pmatrix}$$



# Matrix Multiplication

---

- B matrix: col 2 -> C matrix: col 2

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1k} \\ a_{21} & a_{22} & \dots & a_{2k} \\ \dots & & & \\ a_{n1} & a_{n2} & \dots & a_{nk} \end{pmatrix} \times \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1m} \\ b_{21} & b_{22} & \dots & b_{2m} \\ \dots & & & \\ b_{k1} & b_{k2} & \dots & b_{km} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & \dots & c_{1m} \\ c_{21} & c_{22} & \dots & c_{2m} \\ \dots & & & \\ c_{n1} & c_{n2} & \dots & c_{nm} \end{pmatrix}$$



# Matrix Multiplication

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- B matrix: col 2 -> C matrix: col 2

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1k} \\ a_{21} & a_{22} & \dots & a_{2k} \\ \dots & & & \\ a_{n1} & a_{n2} & \dots & a_{nk} \end{pmatrix} \times \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1m} \\ b_{21} & b_{22} & \dots & b_{2m} \\ \dots & & & \\ b_{k1} & b_{k2} & \dots & b_{km} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & \dots & c_{1m} \\ c_{21} & c_{22} & \dots & c_{2m} \\ \dots & & & \\ c_{n1} & c_{n2} & \dots & c_{nm} \end{pmatrix}$$



# Matrix Multiplication

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- B matrix j-th col  $\rightarrow$  C matrix j-th col
  - key  $(1, j), (2, j), \dots (n, j)$

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1k} \\ a_{21} & a_{22} & \dots & a_{2k} \\ \dots & & & \\ a_{n1} & a_{n2} & \dots & a_{nk} \end{pmatrix} \times \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1m} \\ b_{21} & b_{22} & \dots & b_{2m} \\ \dots & & & \\ b_{k1} & b_{k2} & \dots & b_{km} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & \dots & c_{1m} \\ c_{21} & c_{22} & \dots & c_{2m} \\ \dots & & & \\ c_{n1} & c_{n2} & \dots & c_{nm} \end{pmatrix}$$



# Single step map/reduce

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## Map

### — Input

- Two matrices,  $A$  ( $n \times \ell$ ) and  $B$  ( $\ell \times m$ )
- E.g.,
  - ♦  $A, 2, 3, 3.2 \rightarrow a_{23} = 3.2$
  - ♦  $B, 2, 3, 3.2 \rightarrow b_{23} = 3.2$

### — Output

- For  $(A, i, j, a_{ij})$ : ?
- For  $(B, i, j, b_{ij})$ : ?

## Reduce

### — Input

- $\langle \text{key}=\{i,j\}, \{?\} \rangle$

### — Output

- $\langle \text{key}=\{i,j\}, ? \rangle$