Loop가 존재하는 미로 탐색 방안

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들어가며

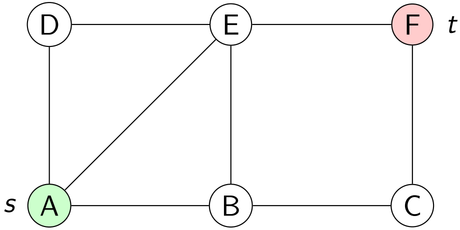
서론 부분 쓰기. 문제의식 및 해결방안.

Definition of a Maze

Maze (Graph Representation)

A maze is an undirected graph $G = (V, E)$ where:

- V: finite set of nodes (cells)
- $E \subseteq \{\{u, v\} \mid u, v \in V,\ u \neq v\}$: set of edges (passages)
- $s \in V$: starting node
- $t \in V$: goal (exit) node



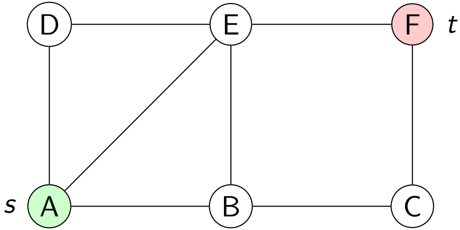
Definition of Path

Exploration Path

A path from s to t is:

$$P = (v_0, v_1, \ldots, v_k)$$

where $v_0 = s,\; v_k = t,\; (v_i, v_{i+1}) \in E \;\forall\; 0 \le i < k.$



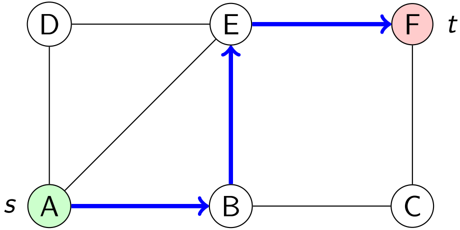
Example of Exploration Path

Exploration Path

A path from s to t is:

$$P = (A, B, E, F)$$

where $A = s$, $F = t$, and each consecutive pair is connected by an edge in E.



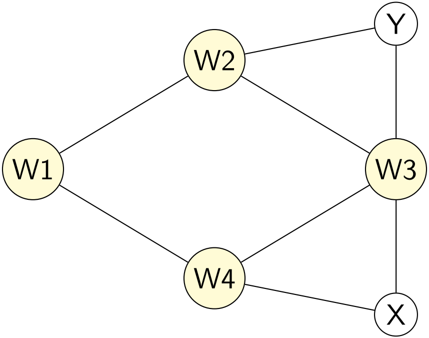
Definition of Loop(cycle)

Loop (Cycle)

A cycle is a sequence:

$$(w_0, w_1, \ldots, w_m, w_0), \quad m \geq 2$$

with distinct nodes (except start/end), where $(w_i, w_{i+1}) \in E \ \forall\ 0 \leq i < m$.



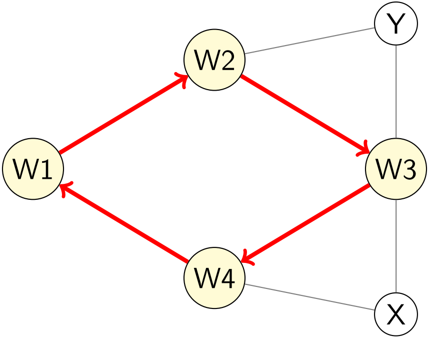
Example of Loop (Cycle)

Cycle in a Maze Graph

A cycle is a sequence:

$$(w_1, w_2, w_3, w_4, w_1)$$

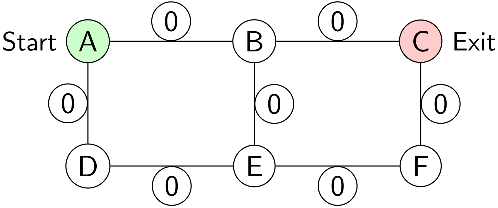
where each node is distinct (except start/end), and each consecutive pair is connected by an edge.



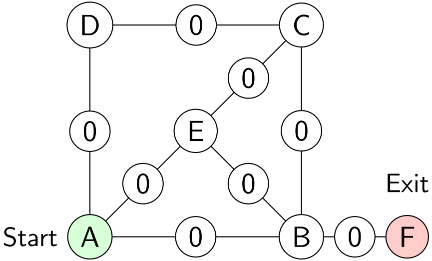
Definition of Trémaux Edge

Trémaux Edge Marking

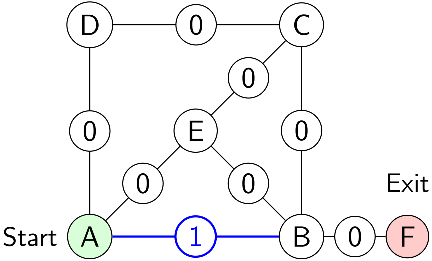
Each edge tracking: $f : E \to \{0, 1, 2\}$ counts visits (up to twice per edge).



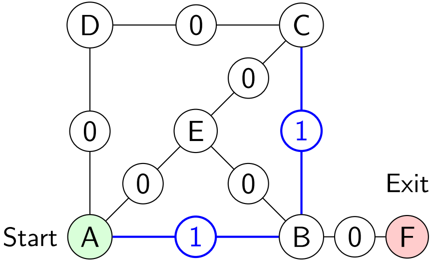
Tremaux Edge Marking: Step 1



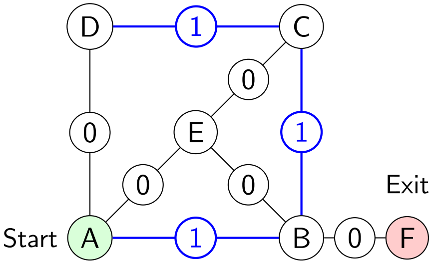
Tremaux Edge Marking: Step 2



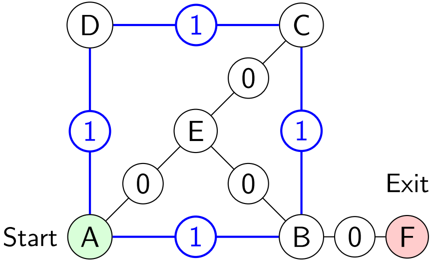
Tremaux Edge Marking: Step 3



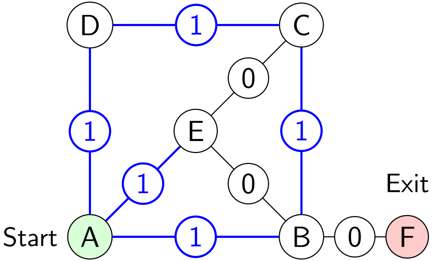
Tremaux Edge Marking: Step 4



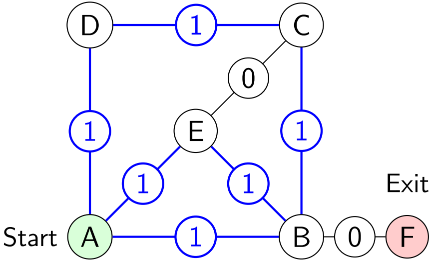
Tremaux Edge Marking: Step 5



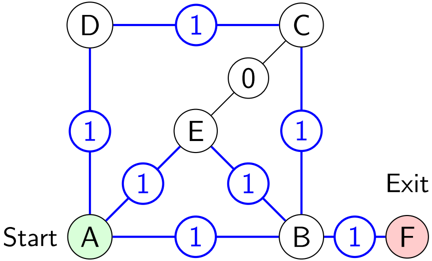
Tremaux Edge Marking: Step 6



Tremaux Edge Marking: Step 7



Tremaux Edge Marking: Step 8



Definition of Simply Connected Maze (No Loops)

Simply Connected Maze

- Maze is an undirected graph $G = (V, E)$.
- The explorer follows the left wall at each step: P_{LH}.
- Definition: $P_{LH} : v_0 = s,\ (v_i, v_{i+1}) \in E,\ v_{i+1}$ is chosen as the leftmost unvisited neighbor (relative to prior orientation).

Classical Left-Hand Rule

Guarantees for the Left-Hand Rule

- If the maze boundary is a single continuous curve (no cycles), the rule guarantees reaching the exit.
- Formally: If G is a tree (connected and acyclic),

$$G \text{ is a tree } \implies \text{ left-hand rule always leads to the exit}$$

- Topologically: $\pi_1(\text{maze}) = 0$ (trivial fundamental group).

Mazes with Loops (Cycles)

Why the Left-Hand Rule Can Fail

- If G contains cycles, the left-hand rule may cause endless tracing of a cycle boundary.
- Formally:

$$\exists \text{ cycle } C \subseteq G \text{ such that the left-hand rule can follow } C \text{ indefinitely}$$

- The explorer may fail to reach the exit depending on the starting position and maze structure.