2. A sky-diver of mass, m, opens her parachute and finds that the air resistance, Fa, is given by the formula Fa= bv, where b is a constant and v is the velocity.

a. Set up, but do not solve a differential equation for her velocity as a function of time.
b. Set up, but do not solve a differential equation for distance as a function of time.
c. Find the terminal velocity in terms of m, b, and g.
d. If in a different situation the formula for air resistance were Fa= bv +cv², where c is another constant find the terminal velocity in terms of the above plus c.
e. **If you are in Calc 2**, solve the differential equations from parts b and ~~c~~ a.

$F_a = bv$

$F_g = mg$

a. $\Sigma = bv - mg = m\frac{dv}{dt}$

b ~~a.~~ $\boxed{v = \frac{dx}{dt}}$ $\quad b\frac{dx}{dt} - mg = m\frac{d}{dt}\left[\frac{dx}{dt}\right]$

~~b. d $\frac{dx}{dt}$~~

c. terminal velocity

when the acceration = 0

$bv - mg = m\frac{dv}{dt}$

$bv - mg = 0$

$bv = mg$

$\boxed{v = \frac{mg}{b}}$

e $\boxed{b \to}$ $bv - mg = m\frac{dv}{dt}$

$\frac{dv}{\left(\frac{bv}{m} + g\right)} = -dt$

$\frac{\frac{m}{b}dv}{v - v_t} = -dt$

$\to \int_0^{v(t)} \frac{dv}{v - v_t} = \int_0^t \frac{-b}{m}dt$

$\ln|v(t) - v_T| + \ln|-v_T| = \frac{-b}{m}t$

$\ln\left|1 - \frac{v(t)}{v_T}\right| = \frac{-bt}{m}$

$1 - \frac{v(t)}{v_T} = e^{\frac{-bt}{m}}$

$\boxed{v(t) = -(e^{-\frac{bt}{m}} - 1}$

d. $bv + cv^2 - mg = m\frac{dv}{dt}$

$v(b + cv) - mg = 0$

$vb + cv^2 = mg$

$cv^2 + bv - mg = 0$

$\boxed{v = \frac{-b \pm \sqrt{b^2 + 4mgc}}{2c}}$