$\sum bv - mg = m\frac{dv}{dt}$

$\sum F = ma = m\frac{dv}{dt}$ $\therefore F_a = bV$

$F_g = mg$

2. A sky-diver of mass, m, opens her parachute and finds that the air resistance. Fa, is given by the formula Fa= bv, where b is a constant and v is the velocity.

a. Set up, but do not solve a differential equation for her velocity as a function of time.
b. Set up, but do not solve a differential equation for distance as a function of time.
c. Find the terminal velocity in terms of m, b, and g.
d. If in a different situation the formula for air resistance were Fa= bv +cv², where c is another constant find the terminal velocity in terms of the above plus c.
e. **If you are in Calc 2**, solve the differential equations from parts b and c.

a. $\boxed{-mg + bV = \frac{m\,dv}{dt}}$

b. $\boxed{\frac{d^2x}{dt^2} - \frac{b}{m}\frac{dx}{dt} + g = 0}$

c. $\frac{dv}{dt} = 0 = bV - mg$

$\boxed{\frac{mg}{b} = V_t}$

d. $m\frac{dv}{dt} = -mg + bV + cV^2$

$V_t = \frac{-b \pm \sqrt{b^2 + 4mgc}}{2c}$

$\frac{dv}{dt} = -\left(\frac{bv}{m} + g\right)$

$\frac{dv}{\left(\frac{bv}{m} + g\right)} = -dt$

$\frac{\frac{m}{b}dv}{V + \frac{mg}{b}}$

$\frac{\frac{m}{b}dv}{V - V_T} = -dt$

$* \; V_T = -\frac{mg}{b}$

$\int_0^{V(t)} \frac{dv}{(V - V_T)} = -\frac{b}{m}\int_0^t dt$

$\ln(V - V_T) - \ln(0 - V_T) = -\frac{b}{m}t\Big|_0^t = -\frac{b}{m}[t - 0]$

$\ln\left(\frac{V - V_T}{-V_T}\right) = \ln\left(1 - \frac{V(t)}{V_T}\right) = -\frac{bt}{m}$

$1 - \frac{V(t)}{V_T} = e^{-\frac{bt}{m}}$

$V_T - V(t) = V_T e^{-\frac{bt}{m}}$

$V(t) = V_T - V_T e^{-\frac{bt}{m}}$

$\boxed{V(t) = V_T\left(1 - e^{-\frac{bt}{m}}\right)}$