DLI Physics Mechanics Problem Set 1 Applied Derivatives in Mechanics

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1. A honeycrisp apple moves in a straight line with its position, x, given by the following equation:
$x(t) = t^4 - 4t^3 + 2t^2 + 3t + 6$
   a. Find its position after 1 second. 8m
   b. Find its velocity after 2 seconds. -5m/s
   c. Find its acceleration after 3 seconds. 40m/s²
   d. What is the rate of change of the acceleration at 1 second. 0 m/s³
   e. Use Python to graph the position, velocity and acceleration as functions of time from t=0 to t=4 seconds.
   f. Use Python to graph the rate of change of acceleration vs. time.

a. $1 - 4 + 2 + 3 + 6 = 8M$

b. $v(t) = 4t^3 - 12t^2 + 4t + 3 = 4(2)^3 - 12(2)^2 + 4(2) + 3 = -5m/s$

c. $a(t) = 12t^2 - 24t + 4 = 40m/s^2$

d $a'(t) = 24t - 24 = 0 m/s^3$

2. A sky-diver of mass, m, opens her parachute and finds that the air resistance, Fa, is given by the formula Fa= bv, where b is a constant and v is the velocity.

$F_a = bv$

a. Set up, but do not solve a differential equation for her velocity as a function of time.
b. Set up, but do not solve a differential equation for distance as a function of time.
c. Find the terminal velocity in terms of m, b, and g.
d. If in a different situation the formula for air resistance were Fa= bv +cv², where c is another constant find the terminal velocity in terms of the above plus c.
e. **If you are in Calc 2**, solve the differential equations from parts b and c.

$Fa = bv$

$V = \frac{Fa}{b}$

a. $F_a = bv$

$F_g = mg$

$\sum F = ma = \frac{mdv}{dt}$

$\sum = bv - mg = \frac{mdv}{dt}$

$bv - mg = mv' \quad v' = \frac{dv}{dt}$

$bv - mg = m\frac{dv}{dt}$

$\frac{dt(bv-mg)}{m(bv-mg)} = \frac{mdv}{(bv-mg)m}$

$\int \frac{1}{m} dt = \int \frac{1}{bv-mg} dv$

$\ln(m) = \frac{1}{b}\ln[bv-mg]$

$m =$

b. $\sum F = ma = m\frac{d''}{dt}$

$\sum = bd' - mg = \frac{md''}{dt}$

c. $bv - mg = ma$

$bv = ma$

d. $V = \frac{mg}{b}$

e. $\sum F = bv + cv^2 - mg$

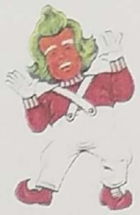
$cv^2 + bv - mg = 0 = ma$

$bv + cv - mg = ma$

$bv + cv^2 - mg = 0$

$V = \frac{-b \pm \sqrt{b^2 - 4cmg}}{2c}$

3. Oompa-Loompas are pulling a 2 kg crate of golden eggs along a rough, but level, surface. In one case it is determined that the position of the block as a function of time is given by : $x(t) = .3t^3 - .1t^2 + .2t$.



a. Find the speed of the block at $t = 2$ sec.
b. Find an expression for acceleration as a function of time.
c. Find an expression for force as a function of time. ($\vec{a} = \frac{\vec{F}}{m}$)
d. Find the initial kinetic energy of the block ($KE = \frac{1}{2}mv^2$)
e. Find the change in kinetic energy of the block from t = 0 to t = 2 sec.
f. Another lab group determines that the Oompa-Loompa force as a function of distance is given by:

$F(x) = x^2 + 2x + 2$ ~~and the block is pulled at an angle of 15° to the horizontal.~~

Find the change in kinetic energy from x = 0 to x = 2 meters.

g. For the above group find a differential equation for power (Power = the time rate of change of kinetic energy).

$x(t) = .3t^3 - .1t^2 + .2t$

$v(t) = .9t^2 - .2t + .2$

$a(t) = 1.8t - .2$

a. $v(2) = .9(2)^2 - .2(2) + .2$

$.9(4) - .4 + .2$

$3.6 - .4 + .2 = \boxed{3.4 \text{ m/s}}$

b. $\boxed{a(t) = 1.8t - .2}$

c. $\vec{a} = \frac{\vec{F}}{m} =$ $F = ma$

$F = (1.8t - .2)m$

$F = (1.8t - .2)2$

$\boxed{F = 3.6t - .4}$

d. $KE = \frac{1}{2}mv^2$

$KE = \frac{1}{2}(2)(.9t^2 - .2t + .2)^2$

$KE = (.9(0)^2 - .2(0) + .2)^2 = \boxed{0.04 \text{ J}}$

e. $KE = \frac{1}{2}(2)(.9t^2 - .2t + .2)^2 = (3.4)^2$

$$\begin{array}{r} 3.4 \\ \times\ 3.4 \\ \hline 136 \\ 1020 \\ 11.56 \\ 0.04 \\ \hline \end{array}$$

$\boxed{11.52 \text{ J}}$

f. $f(x) = x^2 + 2x + 2$

$F(x) \int_0^2 x^2 + 2x + 2 = \frac{1}{3}x^3 + x^2 + 2x \Big|_0^2 =$

$\frac{8}{3} + 4 + 4 - 0 = \boxed{10.67 \text{ J}}$

g. $f(t) = 2(.9t^2 - .2t + .2)(1.8t - .2)$

$(1.8t^2 - .4t + .4)(1.8t - .2)$

$3.24t^3 - 0.72t^2 + 0.72$

$- 0.36t^2 - 0.08t - .04$

$3.24t^3 - 0.36t^2 - 0.08t + 0.$

$\boxed{3.24t^3 - 0.36t^2 - 0.08t + 0.62}$

4. The vector position of a particle is given by

$$\vec{r} = 3\sin(2\pi t)\hat{i} + 2\cos(2\pi t)\hat{j}$$

where t is in seconds, and $\hat{i}$ and $\hat{j}$ are unit vectors in the x, and y directions.

a. Use Python to plot the path of the particle in the x-y plane. (use your parametric heart program as a template if necessary).
b. Find the velocity vector as a function of time. Plot it as an animation.
c. Find the acceleration vector and show that its direction is along r; that is, it is radial.
d. Find the times for which the speed is a maximum or minimum.

See code

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