DLI Physics **Mechanics Problem Set 1** **Applied Derivatives in Mechanics**

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1. A honeycrisp apple moves in a straight line with its position, x, given by the following equation:
$x(t) = t^4 - 4t^3 + 2t^2 + 3t + 6$
   a. Find its position after 1 second.
   b. Find its velocity after 2 seconds.
   c. Find its acceleration after 3 seconds.
   d. What is the rate of change of the acceleration at 1 second.
   e. Use Python to graph the position, velocity and acceleration as functions of time from t=0 to t=4 seconds.
   f. Use Python to graph the rate of change of acceleration vs. time.

$x(t) = t^4 - 4t^3 + 2t^2 + 3t + 6$

$\frac{dx}{dt} = v(t) = 4t^3 - 12t^2 + 4t + 3$

$\frac{dv}{dt} = a(t) = 12t^2 - 24t + 4$

$\frac{da}{dt} = \dot{a}(t) = 24t - 24$

a.) $x(1) = 1^4 - 4\cdot1^3 + 2\cdot1^2 + 3\cdot1 + 6$
$= 1 - 4 + 2 + 3 + 6$
$\boxed{x(1) = 8\,m}$

b.) $v(2) = 4(2^3) - 12(2^2) + 4(2) + 3$
$= 4\cdot8 - 12\cdot4 + 8 + 3$
$v(2) = 32 - 48 + 8 + 3 = \boxed{-5\,m/s}$

c.) $a(3) = 12\cdot3^2 - 24\cdot3 + 4$
$= 12\cdot9 - 24\cdot3 + 4$
$= (12\cdot9 - 12\cdot6) + 4$
$= 12(9-6) + 4 = 36 + 4 = \boxed{40\,m/s^2}$

d.) $\dot{a}(1) = 24\cdot1 - 24 = \boxed{0\,m/s^3}$

2. A sky-diver of mass, m, opens her parachute and finds that the air resistance, Fa, is given by the formula Fa= bv, where b is a constant and v is the velocity.

a. Set up, but do not solve a differential equation for her velocity as a function of time.
b. Set up, but do not solve a differential equation for distance as a function of time.
c. Find the terminal velocity in terms of m, b, and g.
d. If in a different situation the formula for air resistance were Fa= bv +cv², where c is another constant find the terminal velocity in terms of the above plus c.
e. **If you are in Calc 2**, solve the differential equations from parts b and c.

a) $F_a = ma = -mg + bv$

$m\frac{dv}{dt} = -mg + bv$

$$\boxed{\frac{dv}{dt} - \frac{b}{m}v + g = 0}$$

just replace every v with $\frac{dx}{dt}$

b) $$\boxed{\frac{d^2x}{dt^2} - \frac{b}{m}\frac{dx}{dt} + g = 0}$$

c) terminal velocity occurs when $a = 0$

$\frac{dv}{dt} = 0 = \frac{b}{m}v - g$

$$\boxed{V_{Term} = \frac{mg}{b}}$$

d) $m\frac{dv}{dt} = -mg + bv + cv^2$

$$\boxed{V_{Term} = \frac{-b \pm \sqrt{b^2 + 4mgc}}{2c}}$$

e. $\frac{dv}{dt} = \frac{b}{m}v - g$

$\frac{dv}{\frac{b}{m}v - g} = dt$

$\int_0^v \frac{\frac{m}{b}dv}{v - V_T} = \int_0^t dt$

$\ln\left(\frac{V - V_T}{0 - V_T}\right) = \frac{bt}{m}$

$1 - \frac{V}{V_T} = e^{\frac{b}{m}t}$

$V_T - V = V_T e^{\frac{b}{m}t}$

$V(t) = V_T\left(1 - e^{\frac{b}{m}t}\right)$

3. Oompa-Loompas are pulling a 2 kg crate of golden eggs along a rough, but level, surface. In one case it is determined that the position of the block as a function of time is given by : $x(t) = .3t^3 - .1t^2 + .2t$.

a. Find the speed of the block at t = 2 sec.
b. Find an expression for acceleration as a function of time.
c. Find an expression for force as a function of time. ($\bar{a} = \frac{\bar{F}}{m}$)
d. Find the initial kinetic energy of the block ($KE = \frac{1}{2}mv^2$)
e. Find the change in kinetic energy of the block from t = 0 to t = 2 sec.
f. Another lab group determines that the Oompa-Loompa force as a function of distance is given by:

$F(x) = x^2 + 2x + 2$ ~~and the block is pulled at an angle of 15° to the horizontal~~.
Find the change in kinetic energy from x = 0 to x = 2 meters.

g. For the above group find a differential equation for power (Power = the time rate of change of kinetic energy).

g.) $P = \frac{dT}{dt} = \frac{d}{dt}\int F(x)dx$

$= \frac{d}{dt}\left[\frac{1}{2}m[v(t)]^2\right]$

$x(t) = .3t^3 - .1t^2 + .2t$

$v(t) = .9t^2 - .2t + .2$

b.) $\boxed{a(t) = 1.8t - .2}$

a.) $v(2) = .9(2^2) - .2(2) + .2$

$= 3.6 - .4 + .2 = \boxed{3.4\frac{m}{s}}$

c.) $F = ma \rightarrow F(t) = m\,a(t)$

$\rightarrow F = (2kg)(1.8t - .2)$

we're going to use the symbol T for kinetic energy.

d.) $KE_i = T_0 = \frac{1}{2}m(v(0))^2 = \frac{1}{2}(2kg)(.2\,m/s)^2 = \boxed{.04J}$

e) $T(4) = \frac{1}{2}(2kg)(v(2))^2 = \frac{1}{2}(2kg)(3.4\,m/s)^2 = \boxed{11.56J}$

$\Delta T = 11.56J - .04J = \boxed{11.52J}$

3.4
$\times 3.4$
136
1020

f.) $F(x) = x^2 + 2x + 2 \rightarrow W = \Delta(T) = \int F(x)dx = \int_{x=0}^{2}(x^2+2x+2)dx = \frac{x^3}{3} + x^2 + 2x\Big|_0^2$

$= \frac{8}{3} + 4 + 4 = \frac{8}{3} + 8$

$= 8(\frac{4}{3})$

$= \frac{32}{3}$

$\Delta KE = \boxed{10.66J}$

~~$a = \frac{F}{m} = \frac{1}{m}(x^2 + 2x + 2)$~~

~~$2ax = v_f^2 - v_0^2$~~

~~$max = \frac{1}{2}mv_f^2 - v_0^2$~~

Can't do this because $F \neq$ const. $F = F(x)$

4. The vector position of a particle is given by

$$\vec{r} = 3\sin(2\pi t)\hat{i} + 2\cos(2\pi t)\hat{j}$$

where t is in seconds, and $\hat{i}$ and $\hat{j}$ are unit vectors in the x, and y directions.

a. Use Python to plot the path of the particle in the x-y plane. (use your parametric heart program as a template if necessary).

b. Find the velocity vector as a function of time. Plot it as an animation.

c. Find the acceleration vector and show that its direction is along r; that is, it is radial.

d. Find the times for which the speed is a maximum or minimum.

a.)

b.) $\vec{v}(t) = 6\pi\cos(2\pi t)\hat{i} - 4\pi\sin(2\pi t)\hat{j}$

c.) $a(t) = -12\pi^2\sin(2\pi t)\hat{i} - 8\pi^2\cos(2\pi t)\hat{j}$

$\frac{\vec{a}}{\vec{r}} = -4\pi^2 = \text{Const.}$ therefore $\vec{a}(t)$ points along $-\vec{r}(t)$

d.) $|\vec{v}(t)| = \sqrt{v_x^2 + v_y^2} = \sqrt{36\pi^2\cos^2(2\pi t) + 16\pi^2\sin^2(2\pi t)}$

$|v(t)| = 2\pi\sqrt{9\cos^2(2\pi t) + 4\sin^2(2\pi t)}$

$\frac{d|v|}{dt} = 2\pi \cdot \frac{1}{2}\left(9\cos^2(2\pi t) + 4\sin^2(2\pi t)\right)^{-1/2} \cdot \left(-18\cos(2\pi t)\sin(2\pi t)\cdot 2\pi + 8\sin(2\pi t)\cos(2\pi t)\cdot 2\pi\right)$