DLI Physics **Mechanics Problem Set 1** **Applied Derivatives in Mechanics**

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1. A honeycrisp apple moves in a straight line with its position, x, given by the following equation:
$x(t) = t^4 - 4t^3 + 2t^2 + 3t + 6$
   a. Find its position after 1 second.
   b. Find its velocity after 2 seconds.
   c. Find its acceleration after 3 seconds.
   d. What is the rate of change of the acceleration at 1 second.
   e. Use Python to graph the position, velocity and acceleration as functions of time from t=0 to t=4 seconds.
   f. Use Python to graph the rate of change of acceleration vs. time.

a) $x(1) = 1^4 - 4(1)^3 + 2(1)^2 + 3(1) + 6 = \boxed{8m}$

b) $v(t) = 4t^3 - 12t^2 + 4t + 3 \rightarrow v(2) = 4(8) - 12(4) + 4(2) + 3 = \boxed{-5 m/s}$

c) $a(t) = 12t^2 - 24t + 4 \rightarrow v(3) = 12(9) - 24(3) + 4 = \boxed{40 m/s^2}$

d) $a'(t) = 24t - 24 \rightarrow a'(1) = 0$

2. A sky-diver of mass, m, opens her parachute and finds that the air resistance, Fa, is given by the formula Fa= bv, where b is a constant and v is the velocity.

a. Set up, but do not solve a differential equation for her velocity as a function of time.
b. Set up, but do not solve a differential equation for distance as a function of time.
c. Find the terminal velocity in terms of m, b, and g.
d. If in a different situation the formula for air resistance were Fa= bv +cv², where c is another constant find the terminal velocity in terms of the above plus c.
e. **If you are in Calc 2**, solve the differential equations from parts b and c.

a. $\boxed{-bv - mg = mv'}$

b. $\boxed{bd' - mg = md''}$

c. $-bv - mg = ma$ $\qquad \boxed{V = \frac{-mg}{b}}$

$-bv = mg$

d. $cv^2 + bv - mg = 0$

$$V = \frac{-b \pm \sqrt{b^2 - 4(c)(mg)}}{2c} = \boxed{V = \frac{b \pm \sqrt{b^2 - 4cmg}}{2c}}$$

e. $-bv - mg = mv'$

$$\frac{dv}{\left(\frac{bv}{m} + g\right)} = -dt$$

$$\frac{\frac{m}{b}\,dv}{V - V_T} = -dt \rightarrow \int_0^{V(t)} \frac{dv}{V - V_T} = \int_0^t \frac{-b}{m}\,dt$$

$$\ln|V(t) - V_T| - \ln|-V_T| = \frac{-b}{m}t \rightarrow$$

$$\ln\left|\frac{V(t) - V_T}{V_T}\right| = \frac{-b}{m}t$$

$$\frac{V(t) - V_T}{-V_T} = e^{-b/m\,t} \rightarrow \frac{V(t)}{-V_T} + 1 = e^{-b/m\,t}$$

$$\boxed{V(t) = \left(e^{-bt/m} - 1\right)V_T}$$

3. Oompa-Loompas are pulling a 2 kg crate of golden eggs along a rough, but level, surface. In one case it is determined that the position of the block as a function of time is given by : $x(t) = .3t^3 - .1t^2 + .2t$.

a. Find the speed of the block at t = 2 sec.
b. Find an expression for acceleration as a function of time.
c. Find an expression for force as a function of time. ($\bar{a} = \frac{\bar{F}}{m}$)
d. Find the initial kinetic energy of the block ($KE = \frac{1}{2}mv^2$)
e. Find the change in kinetic energy of the block from t = 0 to t = 2 sec.
f. Another lab group determines that the Oompa-Loompa force as a function of distance is given by:

$F(x) = x^2 + 2x + 2$ ~~and the block is pulled at an angle of 15° to the horizontal.~~

Find the change in kinetic energy from x = 0 to x = 2 meters.

g. For the above group find a differential equation for power (Power = the time rate of change of kinetic energy).

a. $.9t^2 - .2t + .2 \rightarrow v(2) = .9(2)^2 - .2(2) + .2 =$

$= 3.6 - .4 + .2 = 3.4164$

b. $1.8t - .2 = a(t)$

c. $F = (1.8t - .2)2 \rightarrow F = 3.6t - .4$

d. $\frac{1}{2}(2)(.9t^2 - .2t + .2)^2 \rightarrow \boxed{KE_i = .04\ J}$

e. $(.9t^2 - .2t + .2)^2 \Big|_0^2 = \boxed{11.52\ J}$

f. $\int_0^2 x^2 + 2x + 2 = \frac{x^3}{3} + x^2 + 2x \Big|_0^2 \rightarrow \frac{8}{3} + 4 + 4 = \frac{32}{3} J$

g. $(1.8t^2 - .4t + .4)(1.8t - .2) = 3.24t^3 - .72t^2 + .72t - .36t^2 + .08t + .0$[illegible]

$\boxed{= 3.24t^3 - 1.08t^2 + .8t + .08}$