INFX 576: Problem Set 7 - Network Dynamics*

Suchitra Sundararaman

Due: Thursday, March 2, 2017

Collaborators:

Instructions:

Before beginning this assignment, please ensure you have access to R and RStudio.

1. Download the `problemset7.Rmd` file from Canvas.
2. Replace the "Insert Your Name Here" text in the `author:` field with your own full name. Any collaborators must be listed on the top of your assignment.
3. Be sure to include well-documented (e.g. commented) code chucks, figures and clearly written text chunk explanations as necessary. Any figures should be clearly labeled and appropriately referenced within the text.
4. Collaboration on problem sets is acceptable, and even encouraged, but each student must turn in an individual write-up in his or her own words and his or her own work. The names of all collaborators must be listed on each assignment. Do not copy-and-paste from other students' responses or code.
5. When you have completed the assignment and have **checked** that your code both runs in the Console and knits correctly when you click `Knit PDF`, rename the R Markdown file to `YourLastName_YourFirstName_ps7.Rmd`, knit a PDF and submit the PDF file on Canvas.

Setup:

In this problem set you will need, at minimum, the following R packages.

```
# Load standard libraries
library(statnet)
```

Problem 1: Social Processes and Structure

Imagine a social group comprised of individuals from two cultures: the "reds" and the "blues." Both groups are alike in their desire for transitive friendships; however, they differ in how they react to intransitivity.

When placed in an arbitrary network, reds tend to act by extending friendship towards those with whom they have a transitive precondition. In other words, the condition $v_i \to v_j$ and $v_j \to v_k$, generates $v_i \to v_k$. Blues, in contrast, generally respond by severing ties which are associated with uncompleted transitive states. In other words, $v_i \to v_j \to v_k$, leads to $v_i \not\to v_j$.

To simulate the above process on a network with `nred` "red" and `nblue` "blue" vertices, use the following function:

```
transsim<-function(nred,nblue,action.prob.red=c(0.9,0.05,0.05),
                   action.prob.blue=c(0.05,0.9,0.05),initial.density=0.2,
                   plot.result=TRUE,return.graph=TRUE,...){
  n<-nred+nblue
  #Draw the initial graph
  g<-rgraph(n,tp=initial.density)
```

*Problems originally written by C.T. Butts (2009)

```r
  rownames(g)<-c(rep("r",nred),rep("b",nblue))
  colnames(g)<-c(rep("r",nred),rep("b",nblue))
  #Now, update in a random order
  uo<-sample(1:n)
  for(i in uo){
    for(j in 1:n)
      for(k in 1:n)
        if((i!=j)&&(j!=k)&&(i!=k)){
          if((g[i,j]*g[j,k])&&!g[i,k]){
            if(i<=nred)
              action<-sample(1:3,1,prob=action.prob.red)
            else
              action<-sample(1:3,1,prob=action.prob.blue)
            if(action==1)
              g[i,k]<-1
            else if(action==2)
              g[i,j]<-0
          }
        }
  }
  #Plot the graph, if necessary
  if(plot.result)
    gplot(g,vertex.col=c(rep(2,nred),rep(4,nblue)),edge.col=rgb(0,0,0,.5))
  #Return, if needed
  if(return.graph)
    return(g)
  invisible()
}
```

The above function should produce a plot of the resulting network. You can return the adjacency matrix by setting the `return.graph` argument to be `TRUE`. The first N vertices will be the red ones, followed by the blue ones.

You can change the probabilities of adding an edge, dropping an edge, or taking no action (respectively) in response to intransitivity for each group with the `action.prob.red` and `action.prob.blue` arguments. For example setting `action.prob.red=c(0.9, 0.05, 0.05)` will set reds to response to intransitivity by adding an edge 90% of the time, dropping an edge 5% of the time, and ignoring the violation 5% of the time. Notice this is the default setting.

`initial.density` is the initial density of the network. `nred` and `nblue` give the number of nodes in each group.

(a) Prediction

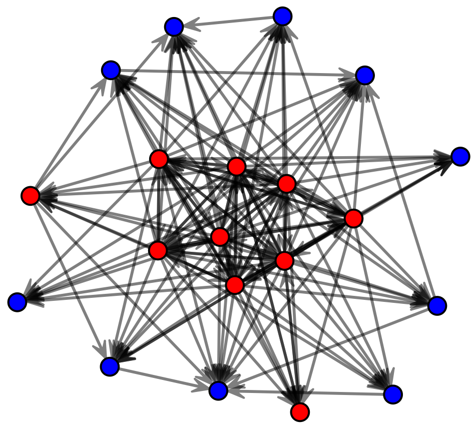
Consider what might happen when persons from both groups are placed together in a single friendship network. Make an initial guess at what you think will happen.

When people from the red and blue groups are placed in a single friendship network, people belonging to the blue group will tend to have greater in- ties as compared to the ones in the red group. This is because, people in the red group will tend to extend friendship in case of a transitive relationship. In this case they will tend to form ties with both people in the blue and the red groups. In case of people belonging to the blue group, they will severe ties whenever there is no transitive relationship, thus reducing the number of out ties directed from the blue vertices. Thus there can a be good amount of edges between the red nodes. The blue nodes will largely have in ties and the number of ties between the blue nodes will be less. This is indicative of a core periphery structure, with the red vertices forming the core and the blue forming the periphery.

(a) Simulation

Design a simple simulation study using the `transsim` function. Consider a group of 10 red nodes and 10 blue nodes. After your experiment you should answer the following questions:

```r
transsim(10,10)
```



```
##   r r r r r r r r r r b b b b b b b b b b
## r 0 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1
## r 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
## r 0 1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
## r 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1
## r 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
## r 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1
## r 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1
## r 1 1 1 0 1 1 1 0 1 0 1 1 0 1 1 1 1 1 1 1
## r 1 1 1 1 1 1 1 1 0 1 1 1 1 0 1 0 1 1 1 1
## r 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1
## b 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
## b 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
## b 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
## b 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0
## b 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0
## b 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0
## b 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
## b 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0
## b 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
## b 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0
```

- What can you conclude about the structural effects of responses to intransitivity on the red and blue nodes? The red nodes extend ties in case of transitive relation, thus in case of intransitivity they might happen to not extend this tie. The blue nodes on the other hand severe ties in the event of intransitivity. Thus this results in reduced number of out ties for the blue nodes. The structural effects of intransitivity is increased number of in ties for blue nodes, which are majorly directed from the red ties and decreased number of out ties.
- How is the total structure affected? The overall structure is affected in the way degree distribution takes place among the nodes.There is significant number of ties between the red nodes, and thus the indegree and outdegree of red nodes are in general high. The blue nodes on the other hand have higher indegree. This results in an overall core-periphery structure, with the cores(red nodes) being densely connected, and the peripheries(blue nodes) sparsely connected.
- How might you describe the overall state which tends to result from the combination of red and blue

behaviors? The combination of red and blue nodes creates an imbalance in the resulting network. There are two groups- red and blue nodes. Their participation in the network vary distictly. Red nodes are a more stronger part of the network as compared to the blue nodes.

- How are these conclusions similar to or different from your initial guess about the outcome of the social process? The conclusions drawn with the help of the graph is similar to my initial guess about the outcome of the social process. The distribution of the tie formation represent a core-periphery structure.The red nodes act as the cores of the network, whereas the blue nodes act as the peripheries.

Problem 2: Network Diffusion

The following is a simple function to simulate cascading behavior in a social network. Write detailed comments for this code to demonstrate you understand the function. Feel free to modify or adjust the functionality.

```
diffusion <- function(network_size=10, network_density=0.2, b_payout=2, #Function definition
  a_payout=3, num_seeds=2, max_steps=10){
  g <- network(rgraph(network_size, tprob=network_density), directed=FALSE)  # Plot an undirected

  vertex_colors <- rep("blue", network_size)                    # Blue color for all the vertices.
  initial_nodes <- sample(1:network_size, num_seeds)            #Take a sample of 2 from the existing

  vertex_colors[initial_nodes] <- "red"                         #Change the color of the selected

   coords <- gplot(g, usearrows=FALSE, vertex.col=vertex_colors, displaylabel=TRUE)
  q = b_payout / (b_payout + a_payout)                          #Threshold value

  par(mar=c(0,0,0,0), mfrow=c(3,2))                           #Divide the window for multiple plots.

  step = 1
  while(step < max_steps && sum(vertex_colors=="red") < network_size){   #To check if all the nodes

    for (i in 1:network_size){
  neighborInds <- get.neighborhood(g, i, type="combined")     # Get neighborhood of the given
  obsA <- sum(vertex_colors[neighborInds]=="red")   #Check which vertices are red and take the sum
  if (obsA/length(neighborInds) >= q){              #Comparing with the threshold value.
  vertex_colors[i] <- "red"                         #Follow the same behavior as neighbor.
  }
  }
  gplot(g, usearrows=FALSE, vertex.col=vertex_colors, coord=coords)  #Plot the graph with the newly
  Sys.sleep(.2)
  step <- step + 1                                              #Repeat for all nodes.

  }
}
```

Adopt a new behavior once a suffiecient proportion of neighbors have adopted.

Experiment with this function to answer the following questions:

- When does this simulation result in everyone switching to A? If initial set of nodes (initial adopters) decide to adopt A's behavior, this might lead to some neighbors of these nodes adopting A's behavior. This is because these neighbors would evaluate the pay-off using the coordination game and this pay-off might be greater than the threshold value.Thus this gives rise to a cascading behavior and ultimately all the nodes in the network might switch to A.
- What causes the spread of A to stop? The spread of A will stop at a point when the number of connected nodes exhibiting behavior A is lesser than the number of connected nodes exhibiting the

other behavior.So if a node is connected to 3 other nodes. One of them is exhibiting behavior A while the other two are exhibiting behavior B.So for the node in question, 1/3 fraction of its neighbors follow A. As $1/3 < 2/5$ (threshold), the node will not switch to B. This is when the spread of A stops.