1 Maintaining Treewidth in Graph Products

1.1 Introduction

With the importance of treewidth and the many applications of product structure theory, a natural question is the following: Can the treewidth be maintained in some meaningful way through taking the product? Solving this problem in any sense would have deep impact on structural graph theory by giving a strong tool to study the treewidth of complex graph classes by using product structure theorems to simplify the problem and give a new approach to studying or improving the treewidth bounds of complicated graph classes.

1.2 Problem 4

Is it true that for every planar graph G, there exists a bounded treewidth graph H and a path P such that $G \subsetneq H \boxtimes P$ and $\text{tw}(H \boxtimes P) \in O(\text{tw}(G))$?

1.3 Related Work

In our paper regarding Problem 1[1], we show that

$\Omega(\min\{|V(H)|, |V(P)|\}) \leq \text{tw}(H \boxtimes P) \leq O(\min\{|V(H)|, |V(P)|\})$. [1, Lemma 3, Equation (2)]

Thus this question really asks whether for every planar graph G, there exists a bounded treewidth graph H and a path P such that $G \subsetneq H \boxtimes P$ and $\min\{|V(H)|, |V(P)|\} \leq O(\text{tw}(G))$.

References

[1] Vida Dujmović, Pat Morin, David R. Wood, and David Worley. Grid minors and products, 2024.