

# PRESENTATION OUTLINE: Distributed Graph Colouring

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## 1 What is Graph Colouring

- Explaining the problem statement for graph colouring
- What is the goal of graph colouring algorithms
- The definition of k-colouring

## 2 An Example

- A diagram showing a proper graph colouring of the peterson graph
- An explanation of what this colouring is (3-colouring of specific graph)

## 3 Distributed Graph Colouring

- Brief explanation of distributed graph colouring
- Pose question: What is the ideal trade-off between minimal colours and minimal runtime?
- Ask why these colourings are useful (set up for next slide)

## 4 Applications

- Applications in job scheduling
- Application in network analysis (social networks, lte networks, etc)

## 5 What Makes a Good Colouring?

- Motivate wanting bounds on the size of the colourings for graph colouring algorithms
- introduce the concept of reducing the number of colours in a larger colouring

## 6 A Lower Bound for Graph Colouring

- Mention the greedy sequential algorithm that generates an  $O(\Delta + 1)$  colouring.
- Introduce this colouring as a suitable lower bound for distributed graph colouring.

## 7 An Upper Bound by Linial

- Introduce Linial's paper presenting the  $O(\Delta^2)$  colouring algorithm in  $O(\log^* n)$
- Discuss how colour reduction algorithms can use Linial's algorithm to generate a colouring to reduce
- Motivate where this can lead regarding future improvements

## 8 Linial's Algorithm

- Short explanation of how Linial's algorithm works

## 9 Algorithm Optimality

- Discuss Linial's result that  $O(\log^* n)$  is optimal for distributed graph colouring algorithm
- Talk about how this has shifted research towards colour reduction algorithms

## 10 Colour Reduction

- Detail colour reduction, the typical round based approach to it, and its goal

## 11 Improving Colour Reduction

- Discuss the SV barrier and the algorithm that breaks it

## 12 A round-based scalable algorithm for colour reduction

- Introduce Maus's algorithm that scales between the two bounds

## 13 Maus's Algorithm

- Discuss how the algorithm works

## 14 Explanation of Algorithm

- Walkthrough of how Maus's algorithm reduces a colouring for a given small  $\Delta$  and  $k$

## 15 Implementation

- Short description of how the algorithm was implemented

## 16 Example of Algorithm

- multiple slides showing a colour reduction using Maus's algorithm on a simple example  
This will stretch over multiple slides

## 17 Experimental Results

- Detail the execution time of the algorithm and reduction amount on large colourings in practice.

## 18 Questions For Audience

- Why is research focusing on colour reduction algorithms instead of colouring algorithms?
- Why do we have a lower bound of  $O(\Delta + 1)$  for our colourings?
- Could an  $O(1)$  colouring algorithm (not colour reduction) be possible?

## 19 Thanks

- Thank the audience and prompt for questions

## 20 References

- Slide showing references used throughout the presentation