习题四.

1. (1) 解：$y^{(0)} = x^{(0)} = (1,0,0)^T$

$x^{(1)} = Ay^{(0)} = (-4,-5,-1)^T,\ \alpha = 5$

$y^{(1)} = \frac{x^{(1)}}{\alpha} = (-\frac{4}{5},-1,-\frac{1}{5})^T$

$x^{(2)} = Ay^{(1)} = (-10.8,-9,0.4)^T,\ \alpha = 10.8$

如此继续下去~~（取精度为10^{-3}）~~.

k	$x_1^{(k)}$	$x_2^{(k)}$	$x_3^{(k)}$	α	$y_1^{(k)}$	$y_2^{(k)}$	$y_3^{(k)}$
0	1	0	0	1	1	0	0
1	-4	-5	-1	5	$-\frac{4}{5}$	-1	$-\frac{1}{5}$
2	-10.8	-9	0.4	10.8	-1.0	-0.8333	0.0370
3	-7.6667	-5.8333	1.0741	10.8	-1.0	-0.7609	0.1401
4	-6.6522	-4.8913	1.2802	7.6667	-1.0	-0.7353	0.1924
5	-6.2941	-4.5588	1.3849	6.6522	-1.0	-0.7243	0.2200
6	-6.1402	-4.4159	1.4401	6.2941	-1.0	-0.7192	0.2345
7	-6.0685	-4.349	1.4691	6.1402	-1.0	-0.7167	0.2421
8	-6.0339	-4.3112	1.4842	6.0685	-1.0	-0.7155	0.2460

取$\lambda_1 \approx 6.0685$. 相应的特征向量为$u \approx (-6.0339,-4.3112,1.4842)^T$

实际上：A的最大特征值$\lambda_1 = 6$，与λ_1相对应的特征向量为$(-6,-4.2857,1.5)^T$.

(2) 解：

k	$x_1^{(k)}$	$x_2^{(k)}$	$x_3^{(k)}$	α	$y_1^{(k)}$	$y_2^{(k)}$	$y_3^{(k)}$
0	1	0	0	1	1	0	0
1	4	-1	1	4	1	-0.25	0.25
2	4.5	-2	2.25	4.5	1	-0.4444	0.5000
3	4.9444	-2.8333	3.3889	4.9444	1	-0.5730	0.6854
4	5.2584	-3.4045	4.2022	5.2584	1	-0.6474	0.7991
5	5.4466	-3.7415	4.6923	5.4466	1	-0.6869	0.8615
6	5.5485	-3.9223	4.9584	5.5485	1	-0.7069	0.8937
7	5.6006	-4.0144	5.0948	5.6006	1	-0.7166	0.9097
8	5.6265	-4.0601	5.1627	5.6265	1	-0.7216	0.9176
9	5.6392	-4.0824	5.1959	5.6392	1	-0.7239	0.9214

取$\lambda_1 \approx 5.6392$. 相应的特征向量为$u \approx (5.6392,-4.0824,5.1959)^T$

实际上：A的最大特征值$\lambda_1 = 5.6511$，与λ_1相对应的特征向量为$(5.6511,-4.1033,5.2272)^T$

2.解：(1)

k	$x_1^{(k)}$	$x_2^{(k)}$	$x_3^{(k)}$	α_k	$\lambda^{(k)}$
0	1	0	0	1	
1	-4	-5	-1	5	-0.3333
2	-10.8000	-9.0000	0.4000	10.8000	-7.8889
3	-7.6667	-5.8333	1.0741	7.6667	8.7657
4	-6.6522	-4.8913	1.2802	6.6522	6.1664
5	-6.2941	-4.5588	1.3849	6.2941	6.0988
6	-6.1402	-4.4159	1.4401	6.1402	6.0241
7	-6.0685	-4.3493	1.4691	6.0685	6.0060
8	-6.0339	-4.3172	1.4842	6.0339	6.0015

取$\lambda_1 \approx 6.0015$. 相应的特征向量为$(-6.0339, -4.3172, 1.4842)^T$.

(2)

k	$x_1^{(k)}$	$x_2^{(k)}$	$x_3^{(k)}$	α_k	$\lambda^{(k)}$
0	1	0	0	1	
1	4	-1	4	4	-0.5000
2	4.5000	-2.2500	2.2500	4.5000	4.6000
3	5.000	-3.5000	3.5000	5	—
4	5.4000	-4.5000	4.5000	5.4000	7
5	5.6667	-5.1667	5.1667	5.6667	6.2000
6	5.8235	-5.5588	5.5588	5.8235	6.0476
7	5.9091	-5.7727	5.7727	5.9091	6.0118
8	5.9538	-5.8846	5.8846	5.9538	6.0029
9	5.9767	-5.9419	5.9419	5.9767	~~5.9767~~ 6.0007

取$\lambda_1 \approx 6.0007$, 相应的特征~~相~~向量为$(5.9767, -5.9419, 5.9419)^T$.

3、解：用幂法计算其按模最大的特征值及对应向量，取 $x^{(0)}=(1,0,0)^T$

k	$x_1^{(k)}$	$x_2^{(k)}$	$x_3^{(k)}$	α	$y_1^{(k)}$	$y_2^{(k)}$	$y_3^{(k)}$
0	1	0	0	1	1	0	0
1	1	2	3	3	$\frac{1}{3}$	$\frac{2}{3}$	1
2	4.6667	6.6667	8.6667	8.6667	0.5385	0.7692	1.0000
3	5.0769	7.3846	9.6923	9.6923	0.5238	0.7619	1.0000
4	5.0476	7.3333	9.6190	9.6190	0.5248	0.7624	1.0000
5	5.0495	7.3366	9.6238	9.6238	0.5247	0.7623	1.0000
6	5.0494	7.3364	9.6235	9.6235	0.5247	0.7623	1.0000
7	5.0494	7.3364	9.6235	9.6235	0.5247	0.7623	1.0000
8	5.0494	7.3364	9.6235	9.6235	0.5247	0.7623	1.0000

则接近 9.6 的特征值为 9.6235.

特征向量约为 $(5.0494, 7.3364, 9.6235)^T$.

4.解：对 $A-\lambda I$ 进行部分选主元的三角分解：

$$P\cdot(A-\lambda I)=LU$$

$$\Rightarrow L=\begin{bmatrix}1&0&0\\0&1&0\\0.7321&-0.26807&1\end{bmatrix}\quad U=\begin{bmatrix}1&1.7321&1\\0&1&2.7321\\0&0&0.29405\times10^{-3}\end{bmatrix}$$

$$P=\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}$$

由 $Uv_1=(1,1,1)^T$ 得： $v_1=(12692,-9290.3\ \ 3,3400.8)^T$

$u_1=(1,-0.732198, 0.26795)^T$

由 $LUv_2=Pu_1$ 得： $v_2=(20404,-14937, 5467.4)^T$

$u_2=(1,-0.73206, 0.26796)^T$

λ_3 对应的特征向量为： $x_3=(1,1-\sqrt{3},3-\sqrt{3})^T\approx(1,-0.73205,0.26795)^T$

由此看出 u_2 是 x_3 的相当好的近似

特征值 $\lambda_3\approx1.2679+\frac{1}{u_2}=1.26794901$

5. (1) 解：取 $\lambda^*=0$，$x^{(0)}=(0,0,1)^T$

对A作三角分解得：

$$A=\begin{bmatrix}-0.2857 & 1.000 & 0\\ 1.000 & 0 & 0\\ 0.5714 & 0.8000 & 1.000\end{bmatrix}\begin{bmatrix}-21.00 & -3.000 & 24.00\\ 0 & -12.8571 & 12.8571\\ 0 & 0 & 27.00\end{bmatrix}$$

$=LU$.

~~取整数~~

按算法4.3：取 $u=1$.

第一次迭代：$x_3=1$，令 $\alpha_0=1$. $\vec{y}_1=\frac{\vec{x}^{(0)}}{\alpha_0}=(0,0,1)^T$ $\vec{z}_1=L^{-1}\vec{y}_1=(0,0,1)^T$，$\vec{x}_1=(0.037, 0.037, 0.03$

$x_1=0.037$. $\beta=0.037$，此时 $\left|\frac{1}{\beta}-\frac{1}{u}\right|=26$，则令 $u=0.037$，$\lambda=27.027$

第二次迭代：$x_2=0.037$. $\vec{y}_2=\frac{\vec{x}_1}{\alpha_1}=(1,1,1)$. $\vec{z}_2=L^{-1}\vec{y}_2=(1,1.2857,-0.6000)$

$\vec{x}_2=(-0.0556,-0.1222,-0.0222)$，$x_2=-0.1222=\beta$ ~~$=-0.0556$~~，~~$\beta=-0.0556$~~

此时 $\left|\frac{1}{\beta}-\frac{1}{u}\right|=35.2103$，则令 $u=-0.1222$. $\lambda=-8.1833$

相应特征向量为 $u\approx\frac{\vec{x}_2}{\alpha_2}=(0.4550,\ 1,\ 0.1817)$

(2) 令 $\lambda_1^*=-8.1833$、用反幂法计算过程如下：

得 $\lambda_{min}=-9$.

相应特征值为 $(0.5345,\ 0.8018\ \ 0.2673)^T$

6. 证明：因A的特征值满足$\lambda_1 > \lambda_2 \geq \cdots \lambda_{n-1} > \lambda_n$

则不论P为何值，$B = A - PI$的主特征值为$\lambda_1 - P$或$\lambda_n - P$.

当我们计算λ_1及x_1时，要使$|\lambda_1 - P| > |\lambda_n - P|$

且使收敛速的比值为：

$$W = \max\left\{ \left| \frac{\lambda_2 - P}{\lambda_1 - P} \right|, \left| \frac{\lambda_n - P}{\lambda_1 - P} \right| \right\} = \min$$

则当$\lambda_2 - P = -(\lambda_n - P)$时.

$P = \frac{\lambda_2 + \lambda_n}{2}$ 时，W最小.

此时收敛速度的比值为：

$$r = \frac{\lambda_2 - \lambda_n}{2\lambda_1 - \lambda_2 - \lambda_n}$$

即收敛速度最快

7. 证明：不妨设A的n个特征值满足：$\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_{n-1} \leq \lambda_n$.

则$\lambda_1 = \min \lambda(A)$. $\lambda_n = \max \lambda(A)$.

令$P(x) = \frac{\langle Ax, x \rangle}{\langle x, x \rangle}$.

则原命题可变为求证：$\lambda_1 \leq P(x) \leq \lambda_n$.

下面进行证明：

由于任意向量可表示为：
$$\begin{aligned} x &= \alpha_1 x_1 + \alpha_2 x_2 + \cdots + \alpha_n x_n \\ &= \sum_{i=1}^{n} \alpha_i x_i \\ &= [x_1, \cdots x_n] \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_n \end{bmatrix} \\ &= [X][\alpha] \end{aligned}$$

则$x^T = [\alpha]^T \cdot [X]^T$

$[X]$为特征向量阵、正交矩阵.

正交条件：$[X]^T[A][X] = [\Lambda]$，$[X]^T[X] = [I]$

$[\alpha]^T = [\alpha_1 \cdots \alpha_n]$ （下面以$[\cdot]$表示为"·".

代入$P(x)$定义，即$P(x) = \frac{x^T A x}{x^T x} = \frac{\alpha^T \cdot X^T A X \alpha}{\alpha^T \cdot X^T X \alpha} =$

$$= \frac{\alpha^T \begin{bmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_n \end{bmatrix} \alpha}{\alpha^T \alpha}$$

$$= \frac{\sum_{i=1}^{n} \alpha_i^2 \lambda_i}{\sum_{i=1}^{n} \alpha_i^2}$$

可见：$\rho(x)$是$\lambda_1 \cdots \lambda_n$的加权平均值，权系数为$\alpha_i^2$

即 $\rho(x) = \frac{\sum_{i=1}^{n} \alpha_i^2 \lambda_i}{\sum_{i=1}^{n} \alpha_i^2}$

因为$\lambda_1 \le \lambda_2 \le \cdots \le \lambda_n$

若用λ_1代入上式，显然 $\lambda_1 \le \rho(x)$

若用λ_n代入上式，显然 $\rho(x) \le \lambda_n$

$\therefore \lambda_1 \le \rho(x) \le \lambda_n$.

证毕.

8. (1) 解：$A^{(0)} = \begin{bmatrix} 4 & 2 & 2 \\ 2 & 5 & 1 \\ 2 & 1 & 6 \end{bmatrix}$，首先取 $i=1, j=2$. $a_{12}=2$.

计算旋转矩阵：

$a = \cot 2\varphi = \frac{a_{11}-a_{22}}{2a_{12}} = -\frac{1}{4}$. $\quad b = \tan\varphi = \mathrm{sign}(a)(\sqrt{a^2+1} - |a|) = \frac{1-\sqrt{17}}{4} \approx -0.7808$.

$c = \cos\varphi = \frac{1}{\sqrt{1+b^2}} = 0.7882$. $\quad \sin\varphi = -0.6154$

则 $V^{(0)} = V_{12}(\varphi) = \begin{bmatrix} 0.7882 & -0.6154 & 0 \\ 0.6154 & 0.7882 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$A^{(1)} = V^{(0)} A^{(0)} V^{(0)T} = \begin{bmatrix} 2.4384 & 0 & 0.9610 \\ 0 & 6.5614 & 2.0190 \\ 0.9610 & 2.0190 & 6 \end{bmatrix}$$

再取 $i=2$ $j=3$. $a_{23}=2.0190$.

$a = \cot 2\varphi = 0.1390$ $\quad b = \tan\varphi = 0.8706$. $\quad c = \cos\varphi = 0.7542$. $\quad \sin\varphi = 0.6566$.

$V^{(1)} = V_{23}(\varphi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.7542 & 0.6566 \\ 0 & -0.6566 & 0.7542 \end{bmatrix}$

$$A^{(2)} = V^{(1)} A^{(1)} V^{(1)T} = \begin{bmatrix} 2.4384 & 0.6310 & 0.7248 \\ 0.6310 & 8.3186 & 0 \\ 0.7248 & 0 & 4.2420 \end{bmatrix}$$

再取 $i=1$，$j=3$　$a_{ij}=0.7248$

$a=\cot 2\varphi=-1.2442$　　$b=\tan\varphi=0.3521$　$c=\cos\varphi=0.9433$　　$d=0.3321$.

则 $V^{(2)}=V_{13}(\varphi)=\begin{bmatrix}0.9433 & 0 & -0.3321\\ 0 & 1 & 0\\ 0.3321 & 0 & 0.9433\end{bmatrix}$

则 $A^{(3)}=V^{(2)}A^{(2)}V^{(2)T}=\begin{bmatrix}2.1835 & 0.5952 & 0\\ 0.5952 & 8.3186 & 0.2096\\ 0 & 0.2096 & 4.4976\end{bmatrix}$

此时 $E(A^{(3)})=0.7964$

要使精度提高，重复上述操作，可得到：

$A^{(n)}=\begin{bmatrix}2.1259 & 0 & 0\\ 0 & 4.4865 & 0\\ 0 & 0 & 8.3876\end{bmatrix}$

则 $\lambda_1\approx 8.3876$，$\lambda_2=4.4865$　$\lambda_3=2.1259$

(2) 解：$A^{(0)}=\begin{bmatrix}2 & -1 & 0\\ -1 & -2 & -1\\ 0 & -1 & 2\end{bmatrix}$，首先取 $i=1$，$j=2$. $a_{12}=-1$

$a=\cot 2\varphi=-2$，$b=\tan\varphi=0.2361$，$c=\cos\varphi=0.9732$　$d=\sin\varphi=-0.2298$

则 $V^{(0)}=V_{12}(\varphi)=\begin{bmatrix}0.9732 & -0.2298 & 0\\ 0.2298 & 0.9732 & 0\\ 0 & 0 & 1\end{bmatrix}$

$A^{(1)}=V^{(0)}A^{(0)}V^{(0)T}=\begin{bmatrix}2.2359 & 0.0003 & 0.2298\\ 0.0003 & -2.2359 & -0.9732\\ 0.2298 & -0.9732 & 2.000\end{bmatrix}$

再取 $i=1$，$j=3$. $a_{13}=0.2298$.

$a=\cot 2\varphi=0.5133$　$b=\tan\varphi=0.6108$　$c=\cos\varphi=0.8534$　$d=\sin\varphi=0.5213$

则 $V^{(1)}=V_{13}(\varphi)=\begin{bmatrix}0.8534 & 0 & 0.5213\\ 0 & 1 & 0\\ -0.5213 & 0 & 0.8534\end{bmatrix}$

$A^{(2)}=V^{(1)}A^{(1)}V^{(1)T}=\begin{bmatrix}2.3764 & -0.5071 & 0\\ -0.5071 & -2.2359 & -0.8307\\ 0 & -0.8307 & 1.8597\end{bmatrix}$

重复上述操作. 最终得到

$A^{(n)}=\begin{bmatrix}-2.4495 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 2.4495\end{bmatrix}$，则 $\lambda_1=-2.4495$　$\lambda_2=2$　$\lambda_3=2.4495$.

9. (1) 解：$a_1=(2,4)^T$，为使 $\tilde{H}_1 a_1=\tilde{H}_1\begin{bmatrix}2\\4\end{bmatrix}=\alpha\begin{bmatrix}1\\0\end{bmatrix}$，取

$$w'=(2,4)^T+2\sqrt{5}(1,0)^T\approx(6.4721,4)^T$$

$$w=\frac{w'}{\|w'\|_2}=(0.8506,0.5257)^T$$

$$\tilde{H}_1=I-2ww^T=\begin{bmatrix}-0.4470 & -0.8943\\ -0.8943 & 0.4473\end{bmatrix}$$

$$H_1=\begin{bmatrix}1 & 0 & 0\\ 0 & -0.4470 & -0.8943\\ 0 & -0.8943 & 0.4473\end{bmatrix}$$

于是有 $H=H_1AH_1=\begin{bmatrix}-4 & 7.6011 & -0.4482\\ -4.4712 & 7.7968 & -0.4012\\ 0 & -0.4012 & 2.1998\end{bmatrix}$

(2) 解：$a_1=(3,0,4)^T$ 为使 $\tilde{H}_1\begin{bmatrix}3\\0\\4\end{bmatrix}=\alpha\begin{bmatrix}1\\0\\0\end{bmatrix}$，取

$$w'=(3,0,4)^T+5(1,0,0)^T=(8,0,4)^T$$

$$w=\frac{w'}{\|w'\|_2}=\left(\frac{2}{\sqrt{5}},0,\frac{1}{\sqrt{5}}\right)^T\approx(0.8944,0,0.4472)^T$$

$$\tilde{H}_1=I-2ww^T=\begin{bmatrix}-0.6 & 0 & -0.8\\ 0 & 1 & 0\\ -0.8 & 0 & 0.6\end{bmatrix}$$

$$H_1=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & -0.6 & 0 & -0.8\\ 0 & 0 & 1 & 0\\ 0 & -0.8 & 0 & 0.6\end{bmatrix}$$

记 $A_{22}^{(1)}=\begin{bmatrix}5 & -1 & 2\\ -1 & 2 & 1\\ 2 & 1 & -1\end{bmatrix}$，取 $w'=(-1,2)^T-\sqrt{5}(1,0)^T=(-3.2361,2)^T$

$$w=\frac{w'}{\|w'\|_2}=(-0.8507,0.5257)^T$$

$$\tilde{H}_2=I-2ww^T=\begin{bmatrix}-0.4474 & 0.8944\\ 0.8944 & 0.4473\end{bmatrix},\quad H_2=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & -0.4474 & 0.8944\\ 0 & 0 & 0.8944 & 0.4473\end{bmatrix}$$

于是有 $H=H_2H_1AH_1H_2$

$$=\begin{bmatrix}4 & -5 & 0 & 0\\ -5 & 3.08 & 3.4458 & 0\\ 0 & 3.4458 & 0.7614 & 1.3280\\ 0 & 0 & 1.3280 & 2.1586\end{bmatrix}$$

10. 证明：因为 $Pu=e_1=(1,0,\cdots,0)^T$，P又是正交阵．

则 $P_1^T=u$，$B_1=PAP_1^T=PAu=\lambda Pu=\lambda e_1$，

又 $B=PAP^T$ 是对称矩阵．

则B的第一行除 λ 外均为零，第一列也是如此．

11. 解：(1)首先将A化为拟上三角形矩阵．可用Householder变换得：

$$H=\begin{bmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 4 \end{bmatrix}$$

下面将H进行QR分解：记 $H^{(1)}=H$．

$r_1=\sqrt{2^2+(-1)^2}=\sqrt5 \quad \cos\varphi_1=\frac{2}{\sqrt5} \quad \sin\varphi_1=\frac{-1}{\sqrt5}$．

取 $V_{21}=\begin{bmatrix} \frac{2}{\sqrt5} & \frac{-1}{\sqrt5} & 0 \\ \frac{1}{\sqrt5} & \frac{2}{\sqrt5} & 0 \\ 0 & 0 & 1 \end{bmatrix}$ 于是 $V_{21}H^{(1)}=\begin{bmatrix} \sqrt5 & -\sqrt5 & \frac{1}{\sqrt5} \\ 0 & \sqrt5 & \frac{-2}{\sqrt5} \\ 0 & -1 & 4 \end{bmatrix}$

再取 $r_2=\sqrt{5+1}=\sqrt6$，$\cos\varphi_2=\frac{\sqrt5}{\sqrt6} \quad \sin\varphi_2=\frac{-1}{\sqrt6}$

$V_{32}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt5}{\sqrt6} & \frac{-1}{\sqrt6} \\ 0 & \frac{1}{\sqrt6} & \frac{\sqrt5}{\sqrt6} \end{bmatrix}$ 于是 $V_{32}V_{21}H^{(1)}=\begin{bmatrix} \sqrt5 & -\sqrt5 & \frac{\sqrt5}{5} \\ 0 & \sqrt6 & -\sqrt6 \\ 0 & 0 & \frac{3\sqrt{30}}{5} \end{bmatrix}=R_1$

$$Q_1=V_{21}^T V_{32}^T=\begin{bmatrix} 0.8944 & 0.4082 & 0.1826 \\ -0.4472 & 0.8165 & 0.3651 \\ 0 & -0.4082 & 0.9129 \end{bmatrix}$$

第一次迭代得：$H^{(2)}=R_1Q_1=\begin{bmatrix} 3 & -1.0954 & 0 \\ -1.0954 & 3 & -1.3416 \\ 0 & -1.3416 & 3 \end{bmatrix}$

同上：迭代10次得 $H^{(10)}=\begin{bmatrix} 1.2679 & 0.0008 & 0 \\ 0.0008 & 3 & 0 \\ 0 & 0 & 4.7321 \end{bmatrix}$

所以A的特征值为 $\lambda_1=1.2679$

$\lambda_2=3$

$\lambda_3=4.7321$

(2)解：将A化为拟上三角形矩阵，

$$H=\begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

将 H QR分解：记 $H^{(1)}=H$

$r_1=\sqrt{3^2+1^2}=\sqrt{10}$ $\cos\varphi_1=\frac{3}{\sqrt{10}}$ $\sin\varphi_1=\frac{1}{\sqrt{10}}$

取 $V_{21}=\begin{bmatrix} \frac{3}{\sqrt{10}} & \frac{1}{\sqrt{10}} & 0 \\ -\frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$，于是 $V_{21}H^{(1)}=\begin{bmatrix} 3.1623 & 1.5811 & 0.3162 \\ 0 & 1.5811 & 0.9487 \\ 0 & 1 & 1 \end{bmatrix}$

再取 $r_2=\sqrt{1.5811^2+1^2}=1.8708$ $\cos\varphi_2=0.8451$ $\sin\varphi_2=0.5345$.

$V_{32}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.8451 & 0.5345 \\ 0 & -0.5345 & 0.8451 \end{bmatrix}$ 于是 $V_{32}H_{21}H^{(1)}=\begin{bmatrix} 3.1623 & 1.5811 & 0.3162 \\ 0 & 1.8707 & 1.3362 \\ 0 & 0 & 0.3380 \end{bmatrix}=R_1$

$Q_1=V_{21}^TV_{32}^T=\begin{bmatrix} 0.9487 & -0.2672 & 0.1690 \\ 0.3162 & 0.8017 & -0.5071 \\ 0 & 0.5345 & 0.8451 \end{bmatrix}$

第一次迭代得 $H^{(2)}=R_1Q_1=\begin{bmatrix} 3.5 & 0.5915 & 0 \\ 0.5915 & 2.2140 & 0.1806 \\ 0 & 0.1807 & 0.2856 \end{bmatrix}$

重复迭代10次得 $H^{(10)}=\begin{bmatrix} 3.7321 & -0.0008 & 0 \\ -0.0008 & 2 & 0 \\ 0 & 0 & 0.2679 \end{bmatrix}$

则 $\lambda_1=3.7321$

$\lambda_2=2$

$\lambda_3=0.2679$