Instance-Based Learning

www.biostat.wisc.edu/~dpage/cs760/

Nearest-neighbor classification

learning task

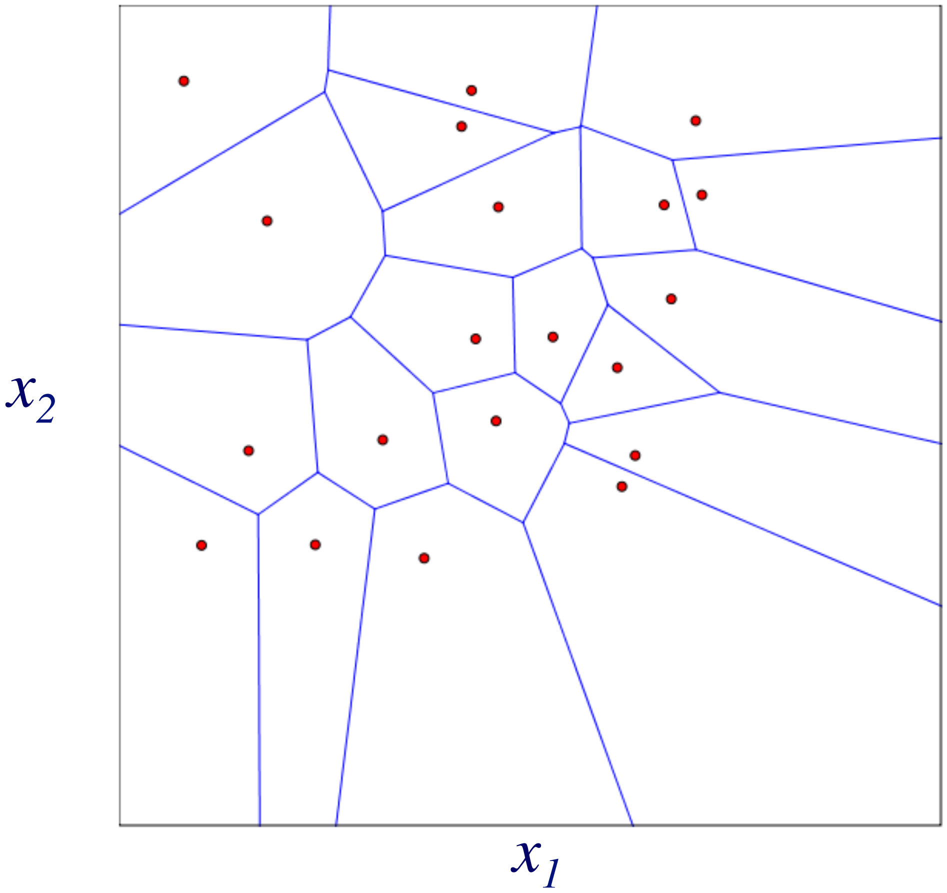
- given a training set $(\boldsymbol{x}_1, y_1) \ldots (\boldsymbol{x}_n, y_n)$, do nothing (it's sometimes called a *lazy learner*)

classification task

- **given**: an instance $\boldsymbol{x}_q$ to classify
- find the training-set instance $\boldsymbol{x}_i$ that is most similar to $\boldsymbol{x}_q$
- return the class value y_i

The decision regions for nearest-neighbor classification

Voronoi Diagram: Each polyhedron indicates the region of feature space that is in the nearest neighborhood of each training instance



k-nearest-neighbor classification

classification task

- **given**: an instance $\boldsymbol{x}_q$ to classify
- find the k training-set instances $(\boldsymbol{x}_1, y_1) \ldots (\boldsymbol{x}_k, y_k)$ that are most similar to $\boldsymbol{x}_q$
- return the class value

$$\hat{y} \leftarrow \underset{v \in \text{values}(Y)}{\arg\max} \sum_{i=1}^{k} \delta(v, y_i) \qquad\qquad \delta(a,b) = \begin{cases} 1 \text{ if } a = b \\ 0 \text{ otherwise} \end{cases}$$

(i.e. return the class that the plurality of the neighbors have)

How can we determine similarity/distance

suppose all features are nominal (discrete)

- Hamming distance: count the number of features for which two instances differ

suppose all features are continuous

- Euclidean distance:

$$d(\boldsymbol{x}_i, \boldsymbol{x}_j) = \sqrt{\sum_f \left(x_{if} - x_{jf}\right)^2}$$

where x_{if} represents the f^{th} feature of $\boldsymbol{x}_i$

- Could also use Manhattan distance: sum the differences in feature values – continuous analog to Hamming distance

How can we determine similarity/distance

- if we have a mix of discrete/continuous features:

$$d(\boldsymbol{x}_i, \boldsymbol{x}_j) = \sum_f \begin{cases} |x_{if} - x_{jf}| & \text{if } f \text{ is continuous} \\ 1 - \delta(x_{if}, x_{jf}) & \text{if } f \text{ is discrete} \end{cases}$$

- If all feature are of equal importance, want to apply to continuous features some type of normalization (values range 0 to 1) or standardization (values distributed according to standard normal)

k-nearest-neighbor *regression*

learning task

- given a training set $(\boldsymbol{x}_1, y_1) \ldots (\boldsymbol{x}_n, y_n)$, do nothing

prediction task

- **given**: an instance $\boldsymbol{x}_q$ to make a prediction for
- find the k training-set instances $(\boldsymbol{x}_1, y_1) \ldots (\boldsymbol{x}_k, y_k)$ that are most similar to $\boldsymbol{x}_q$
- return the value

$$\hat{y} \leftarrow \frac{1}{k} \sum_{i=1}^{k} y_i$$

Distance-weighted nearest neighbor

We can have instances contribute to a prediction according to their distance from $\boldsymbol{x}_q$

classification:

$$\hat{y} \leftarrow \underset{v \in \text{values}(Y)}{\arg\max} \sum_{i=1}^{k} w_i \, \delta(v, y_i) \qquad w_i = \frac{1}{d(x_q, x_i)^2}$$

regression:

$$\hat{y} \leftarrow \frac{\sum_{i=1}^{k} w_i \, y_i}{\sum_{i=1}^{k} w_i}$$

Strengths of instance-based learning

- simple to implement
- “training” is very efficient
- adapts well to on-line learning
- robust to noisy training data (when $k > 1$)
- often works well in practice

Limitations of instance-based learning

- sensitive to range of feature values

- sensitive to irrelevant and correlated features, although…
  - there are variants that learn weights for different features
  - later we'll talk about *feature selection* methods

- classification can be inefficient, although edited methods and k-d trees can help alleviate this weakness

- doesn't provide much insight into problem domain because there is no explicit model