Homework 1: Firm Dynamics

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Part I - Dynare: RBC Model

Exercise 1-4 See Matlab folder.

Exercise 5 If we introduce habit formation into the model, the Lagrangian $\mathcal{L}$ will be

$$\mathcal{L} = \mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[\frac{(C_t - bC_{t-1})^{1-\gamma}}{1-\gamma} - \chi\frac{N_t^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} - \lambda_t\left[C_t + (K_{t+1} - (1-\delta)K_t) - W_tN_t - r_tK_t\right]\right] \tag{1}$$

hence the FOCs are,

$$\frac{\partial\mathcal{L}}{\partial N_t} = -\chi N_t^{\frac{1}{\eta}} + \lambda_t w_t = 0 \tag{2}$$

$$\frac{\partial\mathcal{L}}{\partial C_t} = -\lambda_t + \left[\frac{1}{(c_t - bc_{t-1})^{\gamma}} - \mathbb{E}\beta\frac{b}{(c_{t-1} - bc_t)^{\gamma}}\right] = 0 \tag{3}$$

$$\frac{\partial\mathcal{L}}{\partial K_{t+1}} = -\lambda_t + \mathbb{E}\beta^t\lambda_{t+1}\left[r_{t+1} + 1 - \delta\right] = 0 \tag{4}$$

The main difference is the definition of the lagrangian. In order to solve this, it is easier not to plug it in (as before), but to add an additional variable to the system defined by the lagrangian as a function of consumption. The full implementation and results are in the corresponding .mod file.

Part II - Firm Entry and Exit

Exercise 1 The *incumbent firms* have a production function

$$y_{jt} = e^{\varepsilon_{jt}}k_{jt}^{\theta}n_{jt}^{\nu}, \tag{5}$$

where y_{jt} is output, ε_{jt} is idiosyncratic productivity, k_{jt} is the firm's capital stock, n_{jt} is the firm's labour input, and $\theta + \nu < 1$. Idiosyncratic productivity ε_{jt} follows the AR(1) process $\varepsilon_{jt+1} = \rho\varepsilon_{jt} + \omega_{jt+1}$ (as approximated by a Markov chain using Tauchen's method), where $\omega_{jt+1} \sim N(0, \sigma^2)$.

At the beginning of each period, incumbent firms must pay a fixed cost c_f units of output to remain in operation. A firm that does not pay this fixed cost permanently exits the economy immediately and sells its entire capital stock with value $(1-\delta)k$, i.e. $V_x(k) = (1-\delta)k$.

Then, the start-of-period value of an incumbent firm is dictated by the function $V(\lambda, k, s)$ which solves the following functional equation:

$$V(\varepsilon, k) = \max\left\{V_x(k), \widetilde{V}(\varepsilon, k) - c_f\right\} \tag{6}$$

Given that firms accumulate capital according to the accumulation equation $k_{jt+1} = (1-\delta)k_{jt} + i_{jt}$ and capital accumulation incurs the adjustment cost $-\frac{\varphi}{2}\left(\frac{i_{jt}}{k_{jt}}\right)^2 k_{jt}$, in units of output

$$\tilde{V}(\varepsilon, k) = \max_{k',n}\left\{e^{\varepsilon}k^{\theta}n^{\nu} - wn - (k' - (1-\delta)k) - \frac{\varphi}{2}\left(\frac{k'}{k} - (1-\delta)\right)^2 k + \beta\mathbb{E}\left[\tilde{V}\left(\varepsilon', k'\right)\right]\right\} \tag{7}$$

The prospective value of an *entrant* is

$$V(\varepsilon, 0) = \max\left\{0, \tilde{V}(\varepsilon, 0) - c_e\right\} \tag{8}$$

hence, she will invest and start operating if and only if $c_e \leq \int V(\varepsilon, 0)d\varepsilon$.

For given Γ_0, a *recursive competitive equilibrium* consists of (i) value functions $V(\varepsilon, k)$, $\tilde{V}(\varepsilon, k)$ and $V_e(\varepsilon, 0)$, (ii) policy functions $n(\varepsilon, k), k'(\varepsilon, k)$, and (iii) bounded sequences of wages $\{w_t\}_{t=0}^{\infty}$, incumbents' measures $\{g_t\}_{t=1}^{\infty}$, and entrants' measures $\{m_t\}_{t=0}^{\infty}$ such that, for all $t \geq 0$,

1. $V(\varepsilon, k), \tilde{V}(\varepsilon, k)$, and $n(\varepsilon, k)$ solve the incumbent's optimization problem;
2. $V_e(\varepsilon, 0)$ and $k'(\lambda, 0)$ solve the entrant's optimization problem;
3. The representative household chooses consumption and labour such that $\frac{w(g)}{C(g)} = \frac{a}{N(g)}$;
4. The labour market clears: $N(w_t) = \int n(\varepsilon_t, k)\, g_t(\varepsilon, k) d\varepsilon dk \;\forall\; t \geq 0$;
5. The goods market clears: $C(g_t) = \int \left[y(\varepsilon_t, k) - i(\varepsilon_t, k)\right] g_t(\varepsilon, k) d\varepsilon dk \;\forall\; t \geq 0$.

The steady state version of the recursive competitive equilibrium is summarized below for next next exercise.

Exercise 2 We begin by analysing the steady state equilibrium of the model in which there is a representative firm and productivity is equal to the mean value of ε. In this scenario, the steady state recursive competitive equilibrium is characterized by a set $V^*(\bar{\varepsilon}, k)$, C^*, N^*, w^* and $g(\bar{\varepsilon}, k)^*$ such that

1. $V^*(\bar{\varepsilon}, k)$ solves the representative firm's optimization problem (i.e. Bellman eq.);
2. Taking N^* as given, the representative household's optimization is satisfied by $\frac{w^*}{C^*} = \frac{a}{N^*}$;

3. Labour market clearing follows from $N^* (w_t) = \int n (\bar{\varepsilon}, k)\, g(\bar{\varepsilon}, k) dk \; \forall \; t \geq 0$;
4. The goods market satisfies $C^* = \int [y (\bar{\varepsilon}, k) - i (\bar{\varepsilon}, k)]\, g(\bar{\varepsilon}, k) dk \; \forall \; t \geq 0$.

Now, assume that steady state labour supply is $N^*_{rep} = 0.6$. Then, we can use the following system of equations to solve for K^*_{rep} and w^*_{rep}:

$$
\begin{aligned}
\bar{n} &= 0.6 \\
\bar{r} &= \frac{1}{\beta} - (1 - \delta) \\
\bar{k} &= \left(\frac{r}{\theta \bar{n}^{\nu}} \right)^{\frac{1}{\theta - 1}} \\
\bar{y} &= \bar{k}^{\theta} \bar{n}^{\nu} \\
\bar{i} &= \delta \bar{k} \\
\bar{w} &= \nu \bar{k}^{\theta} \bar{n}^{\nu - 1} \\
\bar{c} &= \bar{k}^{\theta} \bar{n}^{\nu} - \bar{i} \\
\bar{a} &= \frac{w^* n^*}{c^*}
\end{aligned}
$$

All the corresponding values are reported in the accompanying Jupyter notebook. In particular, $K^*_{rep} = 1.09$ and $w^*_{rep} = 0.78$.

Exercise 3-4 The corresponding solutions are reported in the accompanying Jupyter notebook.