Easy method to establish the dispersion relation of capillary waves on water jets - Supplementary Material A.

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I. SIMULATION DETAILS

The simulation that was used in the appendix to illustrate the formation of the capillary wave pattern uses assumptions that makes it unsuitable for making theoretical predictions. It should therefore only be considered as a simulation for illustrative and educational purposes. For quantitative predictions and fitting, we refer the reader to either the theoretical models or specialist physics simulation software.

The following details can be used to replicate this visualization. A Python notebook to do so can also be found in the online supplementary material.

At the point where the stream collides with the rod, a wide range of wavelengths is produced. Firstly, only waves traveling parallel to the cylinder axis are considered thus reducing the problem to a 1D-problem by making use of cylindrical symmetry. Secondly, the amplitude distribution $a(\lambda)$ of these wavelengths should be a distribution such that the amplitude approaches zero for both very short and very long wavelengths. We used the following gamma distribution:

$$a\left(\lambda\right) = \frac{\lambda^{k-1}e^{-\frac{\lambda}{\theta}}}{\theta^{k}\Gamma(k)}, \tag{1}$$

where $\Gamma(k)$ is the gamma function and $k = 4$ and $\theta = 1$ were chosen as the shape and scale parameters respectively. This was chosen to generate a characteristic wavelength amplitude which is not too short. The resulting pattern would otherwise not dominate over simulation errors (due to discretization of the gamma function). Each wavelength was then assumed to have a phase velocity according to the planar relation, as given by the second equation in the paper. For this, $\gamma = 1$ and $\rho = 1$ were used. Note that no units are given since the simulation was chosen to be dimensionless. The stream velocity was chosen to be $v_{\text{stream}} = 1.5$. Based on the planar model, $\lambda_{\text{char}} = 2.79$ would be expected. The displacement in function of height and time $f(h,t)$ for a spreading wave packet is then calculated by adding cosines for every wavelength λ_{n} from a uniformly spaced set of 1000 wavelengths between $\lambda = 0.02$ and $\lambda = 20$:

$$f(h,t) = \sum_{\text{n}=1}^{1000} a\left(\lambda_{\text{n}}\right)\cos\left[\frac{2\pi}{\lambda_{\text{n}}}\left[h - h_0 - v_{\text{p}}(t-t_0)\right]\right], \tag{2}$$

where h_0 and t_0 are height and time at which the waves were emitted respectively and v_{p} is given by the second equation in the paper. The obtained wave patterns were then duplicated and mirrored around the vertical axis to represent a 2D cross-section of the stream. Note that the radius is irrelevant because the planar dispersion relation approximation was utilized.

For the first two figures in the appendix, the three time steps $t_1 = 0$, $t_2 = 5.5$ and $t_1 = 18$ were used. The three emission heights follow from these time steps and v_{stream}. The first figure in the appendix then follows by adding all 1000 waves evaluated at t_1 and t_2. For t_1, a simple delta peak was plotted as the exact disturbance geometry is unknown. For the second figure in the appendix, the second set of waves are shifted by height h_2 and time t_2.

The combined wave pattern in the third figure of the appendix was created using a summation of 500 consecutive emitted wave packets between $t_{\text{init}} = -100$ and $t_{\text{final}} = 20$. For reference, the expected λ_{char} was plotted next to the resulting wave pattern.

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