Non-classical search algorithms
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modified by Jacek Malec for LTH lectures
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Chapter 4 of AIMA

Outline

- ♢ Hill-climbing
- ♢ Simulated annealing (briefly)
- ♢ Genetic algorithms (briefly)
- ♢ Local search in continuous spaces (briefly)
- ♢ (NEW!) Searching with nondeterministic actions (briefly)
- ♢ (NEW!) Searching with partial observations (briefly)
- ♢ (NEW!) Online search and unknown environments (briefly)

Iterative improvement algorithms

In many optimization problems, **path** is irrelevant;
the goal state itself is the solution

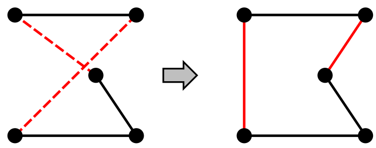
Then state space = set of "complete" configurations;
find **optimal** configuration, e.g., TSP
or, find configuration satisfying constraints, e.g., timetable

In such cases, can use iterative improvement algorithms;
keep a single "current" state, try to improve it

Constant space, suitable for online as well as offline search

Example: Travelling Salesperson Problem

Start with any complete tour, perform pairwise exchanges

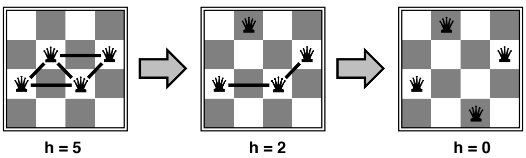


Variants of this approach get within 1% of optimal very quickly with thousands of cities

Example: n-queens

Put n queens on an $n \times n$ board with no two queens on the same row, column, or diagonal

Move a queen to reduce number of conflicts



Almost always solves n-queens problems almost instantaneously
for very large n, e.g., $n = 1 million$

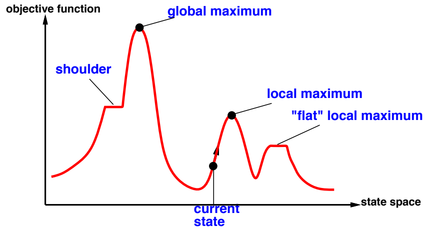
Hill-climbing (or gradient ascent/descent)

"Like climbing Everest in thick fog with amnesia"

```
function HILL-CLIMBING(problem) returns a state that is a local maximum
   inputs: problem, a problem
   local variables: current, a node
                    neighbor, a node
   current ← MAKE-NODE(INITIAL-STATE[problem])
   loop do
      neighbor ← a highest-valued successor of current
      if VALUE[neighbor] ≤ VALUE[current] then return STATE[current]
      current ← neighbor
   end
```

Hill-climbing contd.

Useful to consider state space landscape



Random-restart hill climbing overcomes local maxima—trivially complete

Random sideways moves ☺escape from shoulders ☹loop on flat maxima

Simulated annealing

Idea: escape local maxima by allowing some "bad" moves
but gradually decrease their size and frequency

```
function SIMULATED-ANNEALING(problem, schedule) returns a solution state
   inputs: problem, a problem
           schedule, a mapping from time to "temperature"
   local variables: current, a node
                    next, a node
                    T, a "temperature" controlling prob. of downward steps

   current ← MAKE-NODE(INITIAL-STATE[problem])
   for t ← 1 to ∞ do
       T ← schedule[t]
       if T = 0 then return current
       next ← a randomly selected successor of current
       ΔE ← VALUE[next] – VALUE[current]
       if ΔE > 0 then current ← next
       else current ← next only with probability e^(ΔE/T)
```


Properties of simulated annealing

At fixed "temperature" T, state occupation probability reaches Boltzman distribution

$$p(x) = \alpha e^{\frac{E(x)}{kT}}$$

T decreased slowly enough $\Longrightarrow$ always reach best state x^*
because $e^{\frac{E(x^*)}{kT}}/e^{\frac{E(x)}{kT}} = e^{\frac{E(x^*)-E(x)}{kT}} \gg 1$ for small T

Is this necessarily an interesting guarantee??

Devised by Metropolis et al., 1953, for physical process modelling

Widely used in VLSI layout, airline scheduling, etc.

Local beam search

Idea: keep k states instead of 1; choose top k of all their successors

Not the same as k searches run in parallel!
Searches that find good states recruit other searches to join them

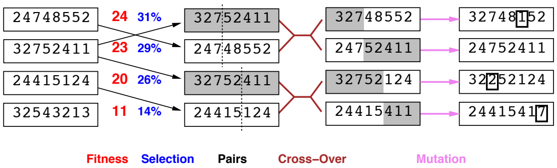
Problem: quite often, all k states end up on same local hill

Idea: choose k successors randomly, biased towards good ones

Observe the close analogy to natural selection!

Genetic algorithms

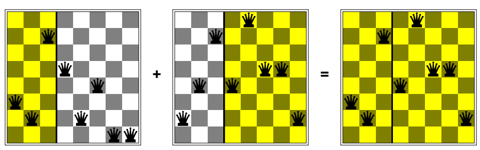
= stochastic local beam search + generate successors from **pairs** of states



Genetic algorithms contd.

GAs require states encoded as strings (GPs use programs)

Crossover helps **iff substrings are meaningful components**



GAs $\neq$ evolution: e.g., real genes encode replication machinery!

Continuous state spaces

Suppose we want to site three robot battery loading stations in the hospital:
- 6-D state space defined by (x_1, y_2), (x_2, y_2), (x_3, y_3)
- objective function $f(x_1, y_2, x_2, y_2, x_3, y_3) =$
  sum of squared distances from each location to nearest loading station

Discretization methods turn continuous space into discrete space,
e.g., empirical gradient considers $\pm\delta$ change in each coordinate

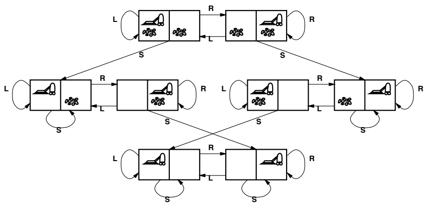
Gradient methods compute

$$\nabla f = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial y_1}, \frac{\partial f}{\partial x_2}, \frac{\partial f}{\partial y_2}, \frac{\partial f}{\partial x_3}, \frac{\partial f}{\partial y_3}\right)$$

to increase/reduce f, e.g., by $\mathbf{x} \leftarrow \mathbf{x} + \alpha\nabla f(\mathbf{x})$

Sometimes can solve for $\nabla f(\mathbf{x}) = 0$ exactly (e.g., with one location).
Newton–Raphson (1664, 1690) iterates $\mathbf{x} \leftarrow \mathbf{x} - \mathbf{H}_f^{-1}(\mathbf{x})\nabla f(\mathbf{x})$
to solve $\nabla f(\mathbf{x}) = 0$, where $\mathbf{H}_{ij} = \partial^2 f/\partial x_i \partial x_j$

Searching with nondeterministic actions



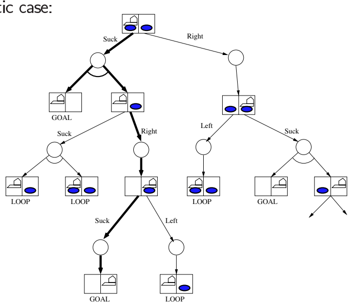
Erratic vacuum world: modified **Suck**;

Slippery vacuum world: modified **Right** and **Left**.

Searching with nondeterministic actions

And-or search trees

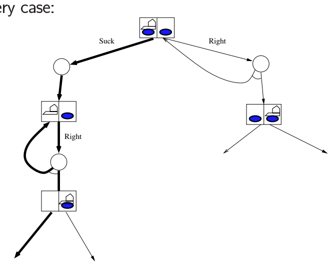
For the erratic case:



Searching with nondeterministic actions

And-or search trees

For the slippery case:



Searching with nondeterministic actions

function AND-OR-GRAPH-SEARCH(*problem*) **returns** *a cond. plan, or failure*
  OR-SEARCH(*problem*.INITIAL-STATE,*problem*,[])

function OR-SEARCH(*state*,*problem*,*path*) **returns** *a conditional plan or failure*
  if *problem*.GOAL-TEST(*state*) **then return** the empty plan
  if *state* is on *path* **then return** *failure*
  for each *action* **in** *problem*.ACTIONS(*state*) **do**
    $plan$ ← AND-SEARCH(RESULTS(*state*,*action*),*problem*,[*state* | *path*])
    if *plan* ≠ *failure* **then return** [*action* | *plan*]
  return *failure*

function AND-SEARCH(*states*,*problem*,*path*) **returns** *a conditional plan or failure*
  for each s_i **in** *states* **do**
    $plan_i$ ← OR-SEARCH(s_i,*problem*,*path*)
    if $plan_i = failure$ **then return** *failure*
  return [**if** s_1 **then** $plan_1$ **else if** s_2 **then** $plan_2$ **else if** ... $plan_{n-1}$ **else** $plan_n$]

Searching with partial observations

◇ no-information case:

  sensorless problem, or

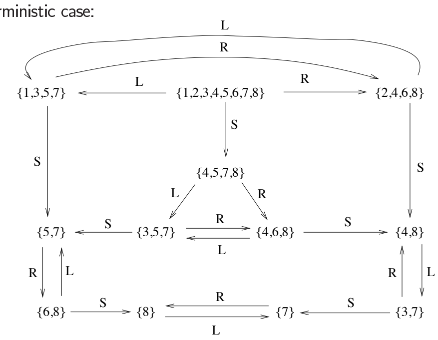
  conformant problem

◇ state-space search is made in **belief space**

◇ Problem solving: and-or search!

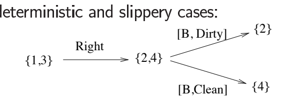
Searching with partial observations

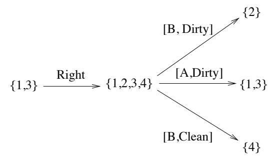
Deterministic case:



Searching with partial observations

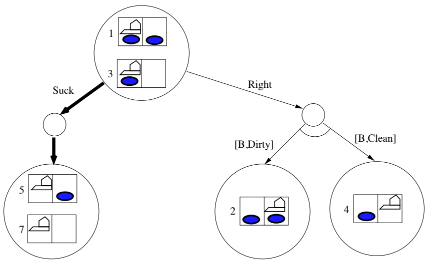
Local sensing, deterministic and slippery cases:





Searching with partial observations

Planning for the local sensing case:



Online search and unknown environments

Interleaving computations and actions:

◇ act

◇ observe the results

◇ find out (compute) next action

Useful in dynamic domains.

Online search usually exploits locality of depth-first-like methods.

◇ random walk

◇ modified hill-climbing

◇ Learning Real-Time A* (LRTA*)

optimism under uncertainty
(unexplored areas assumed to lead to goal with least possible cost)