FMNF05 - Computational Project 1

# Numerical Crime Scene Investigation

See course web page for instructions on how to write your report.
To be handed in before **Friday, February 2, 2018 at 12 noon.**

January 18, 2018

**Description of the Problem.** Professor Bill Grissom has been found dead in his office. At 8:00 p.m., the county coroner determined the core temperature of the corpse to be 32° C. One hour later, the core temperature had dropped to 29.5° C. Maintenance reported that the building's air conditioning unit broke down at 4:00 p.m. The temperature in the professor's office was 20° C at that time. The computerized climate control system recorded that the office temperature rose at a rate of 0.5° C per hour after the air conditioning stopped working.

Lieutenant Columbo from the Lund University Police Department LUPD, believes that the infamous Instructor Sutherland killed the professor. Instructor Sutherland, however, claims that he has an alibi. Famous TV0 anchorman Mo Fateman was interviewing him at the Student Center Building, just across the street from the professor's office. The receptionist at the Student Center Building checked Instructor Sutherland into the building at 5:55 p.m., and the interview tapes confirm that he was interviewed from 6:00 p.m. until 6:50 p.m. Can Lieutenant Columbo be right?

To answer this question, we need to determine the time of death from the information we have at hand. We will assume the core temperature of the corpse was 37° C at the time of death and began decreasing immediately following death. We will further assume that the decrease in core temperature proceeded according to Newton's Law of Cooling. This principle states that the temperature of an object will change at a rate proportional to the difference between the temperature of the object and that of its surroundings.

To explicitly formulate our model, let $T(t)$ denote the core temperature of the corpse as a function of time, with time measured in hours. Take $t = 0$ to correspond to 8:00 p.m. Using this coordinate system, we know $T(0)$ and $T(1)$. Furthermore, the office temperature is given by $T_{\text{office}}(t) = 22 + 0.5t$. Applying Newton's Law of Cooling, we obtain

$$\frac{\mathrm{d}T}{\mathrm{d}t} = -k(T - T_{\text{office}})$$

where $k$ is a positive constant of proportionality. To complete our analysis, we must first determine the solution of this equation that satisfies the conditions stated above. Then, using this solution, we must determine the time when the core temperature of the corpse was 37° C. While Lieutenant Columbo knows a

great deal of mathematics, he finds that he cannot manage to do the computations himself, and that he needs the assistance of some competent computer scientists, to ensure that computations and programs are as accurate as possible. Columbo is quite convinced that Sutherland dunnit, but Columbo needs the last piece of evidence.

**Task 1.** The linear, first–order differential equation may be solved analytically. Show that its general solution is

$$T(t) = 22 + 0.5t - \frac{1}{2k} + c\mathrm{e}^{-kt},$$

where $c$ is a constant of integration.

**Task 2.** Show that using the conditions at $t = 0$ and $t = 1$ we find that

$$T(t) = 22 + 0.5t - \frac{1}{2k} + \left(10 + \frac{1}{2k}\right)\mathrm{e}^{-kt}$$

and

$$7 + \frac{1}{2k} = \left(10 + \frac{1}{2k}\right)\mathrm{e}^{-k}.$$

The last equation cannot be solved explicitly for $k$, so we will have to obtain an approximate solution.

**Task 3.** Construct a Matlab algorithm for the bisection method, with the following inputs: function whose zero is to be located, $f$; left and right endpoints of interval, $a, b$; convergence tolerance, $\varepsilon$; and maximum number of iterations, $Nmax$. Use your algorithm to approximate $k$ to an accuracy of 2 decimal places. Justify your input values.

**Task 4.** Construct an algorithm for the fixed point method. Check your algorithm by finding an approximation to $k$ to an accuracy of 6 decimal places.

**Task 5.** Construct an algorithm for the Newton-Raphson method. Use the approximate value of $k$ to compute the approximate time of death $t_d$ using Newton's method and iterate until you get a minimal residual. Justify your input values.

**Task 6.** Solve the same problem using the fixed point and bisection methods to the same accuracy. Show the work (number of iterations, error estimates) required for each method. Study the speed of convergence of all methods by looking at the quotient $e_n/e_{n-1}$, where $e_n$ is the error in the $n$:th approximation. Discuss your results.

**Task 8.** Compare your result with the result obtained with Matlab's built-in function `fzero`.

**Task 9.** According to your results, could Instructor Sutherland have murdered Professor Bill Grissom, or is his alibi good enough?

Based on a problem in B. Bradie, *A Friendly Introduction to Numerical Analysis*, Pearson International Edition, 2006.

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