FMN011 — Seminar 5 — The Fourier Transform, the Discrete Cosine Transform, Huffman coding.

1. How many primitive fifth roots of unity are there? How many primitive sixth roots of unity?

2. What is the computational complexity of the DFT? What is the best case? What is the worst case?

3. Which of the following can be described as trigonometric polynomials?

   (a) $T(x) = 3 + 2\cos 3x + \cos^{2/3} 3x$

   (b) $T(x) = 3 + \cos 3x + 5\sin 5x$

   (c) $T(x) = 3 + x\cos x$

4. True or false:

   (a) The FFT algorithm can compute both the discrete Fourier transform and its inverse with the same efficiency.

   (b) The Discrete Fourier Transform of a set of real data is complex.

   (c) The DFT of a set of real data always has at least one real component.

   (d) The DFT of a set of real data always has at least two real components.

5. True or false:

   (a) The Discrete Cosine Transform of a set of real data may be complex

   (b) The DCT of a sum of two vectors is the sum of the two separate DCT (dct(v+w) =dct(v) + dct(w))

   (c) The DCT can be computed with a fast algorithm of complexity $n \log_2 n$.

6. Why is orthogonality an important property when doing least squares with trigonometric polynomials?

7. On what method is audio filtering by the DFT based?

8. Can a function of the form $f(x) = c_1 + c_2 \cos(\pi x)$ interpolate data points $(0, 1)$ and $(2, 1/2)$? If so, write the matrix equation that must be solved to find the coefficients of the interpolating function. Otherwise, justify your answer.

9. What properties are assumed for the data when applying the DFT to trigonometric interpolation?

10. What is the advantage of using FFT to do trigonometric interpolation vs using normal equations and QR factorization?

11. (a) What is the DFT of a pure cosine wave $f(t) = A\cos 2t$ sampled at 8 equally spaced points on $[0, 2\pi)$?

(b) What is the DFT of a pure sine wave $f(t) = B \sin 3t$ wave sampled at 8 equally spaced points on $[0, 2\pi)$?

(c) What is the DFT of the sum of the two waves, $g(t) = A \cos 2t + B \sin 3t$, sampled at 8 equally spaced points on $[0, 2\pi)$?

12. Give two applications for the DCT.

13. What are the normal equations for solving least squares approximation with the DCT?

14. How can you get a compression of 4:1 on a file containing 1 048 576 data points?

15. Which type of transformation is used in JPEG compression?

16. True or false: The DFT implies a periodic extension of the function defined over a finite interval, and the DCT implies a periodic even extension of the function.

17. What kind of basis functions should you use to approximate a periodic odd function with real values?

18. True or false: The discrete cosine transform is a linear transformation that is not necessarily invertible.

19. What is the difference between compressing by taking the average and by quantization?

20. True or false: Quantization and Huffman coding are examples of lossy compression.

21. Explain why it is good practice to subtract 128 from an 8 x 8 image matrix before compressing.

22. Draw a Huffman tree for the message DO THE PROBLEMS ASSIGNED, including spaces, and convert to Huffman coding. Compute the Shannon information and compare.