T. Sauer, Numerical Analysis. Pearson, Addison Wesley, 2006/2009/2012.

FMN011 Exercises Chapters 0 and 1

Answers for odd numbered problems are given in the book. Relevant answers for even numbered problems are given here in parenthesis. Computer exercises do not have "answers".

0.4 C1 Calculate the expressions that follow for $x = 10^{-1}, \ldots, 10^{-14}$ (using MATLAB). Then, using an alternative form of the expression that doesn't suffer from subtracting nearly equal numbers, repeat the calculation and make a table of results. Report the number of correct digits in the original expression for each x.

$$(a)\,\frac{1-\sec x}{\tan^2 x} \qquad (b)\,\frac{1-(1-x)^3}{x}$$

1.1 E5 Consider the equation $x^4 = x^3 + 10$.

(a) Find an interval $[a, b]$ of length one inside which the equation has a solution.

(b) Starting with $[a, b]$, how many steps of the Bisection Method are required to calculate the solution within 10^{-10}?

1.1 C6 Write an M-file for the bisection method and use it to calculate the solution of $\cos x = \sin x$ in the interval $[0, 1]$ within 6 correct decimal digits. Use `fzero` to solve the problem and compare.

1.2 E11 Express each equation as a fixed-point problem $x = g(x)$ in three different ways.

(a) $x^3 - x + e^x$

(b) $3x^{-2} + 9x^3 = x^2$

1.2 C1 Write an M-file for the Fixed-Point Iteration and use it to calculate the solution of each equation to 8 correct decimal digits.

(a) $x^3 = 2x + 2$

(b) $e^x + x = 7$

(c) $e^x + \sin x = 4$

1.2 C5 Apply Theorem 1.6 to show that $g(x) = \cos x$ is a convergent Fixed-Point Iteration. Is the same true for $g(x) = \cos^2 x$? Find the fixed point to 6 correct decimal digits, and report the number of iterations needed. Discuss local convergence using Theorem 1.6.

1.4 E6 Sketch a function f and initial guess for which Newton-Raphson's method diverges.

1.4 C7 Consider the function $f(x) = e^{\sin^3 x} + x^6 - 2x^4 - x^3 - 1$ on the interval $[-2, 2]$. Plot the function and find all three roots to 6 correct decimal digits using the Newton-Raphson method. Determine which roots converge quadratically and find the multiplicity of the roots that converge linearly. Use `fzero` and `fsolve` and compare.

1.5 E1/C1 Apply the secant method to the following equations with initial guesses $x_0 = 1$ and $x_1 = 2$.

(a) $x^3 = 2x + 2$

(b) $e^x + x = 7$

(c) $e^x + \sin x = 4$