T. Sauer, Numerical Analysis. Pearson, Addison Wesley, 2006/2009/2012.

FMN011 Exercises Chapter 3

The problems are based on the designated problem from Sauer's book, but may not be identical.

3.1 E1(a) Use Lagrange interpolation to find a polynomial that passes through the points (0,1), (2,3) and (3,0).

3.1 C1 Apply the following world population figures to estimate the 1980 population, using (a) the cubic curve through the first 4 data points; (b) the quartic curve through all 5 data points. Compare with the 1980 estimate of 4452584592.

year	population
1960	3039585530
1970	3707475887
1990	5281653820
2000	6079603571
2010	6895889000

3.2 C3 The total world oil production in millions of barrels per day is shown in the table that follows. Determine and plot the degree 9 polynomial through the data. Use it to estimate 2010 oil production. Does the Runge phenomenon occur in this example? In your opinion, is the interpolating polynomial a good model of the data? Explain.

year	bbl/day($\times 10^6$)
1994	67.052
1995	68.008
1996	69.803
1997	72.024
1998	73.400
1999	72.063
2000	74.669
2001	74.487
2002	74.065
2003	76.777

3.3 C1 Build a sine calculator key using the Chebyshev interpolating polynomial with 4 nodes on the interval $[0, \pi/2]$. Plot the polynomial and the sine function on the interval $[-2, 2]$.

3.3 C5 Let $f(x) = e^{-x^2}$. Compare evenly spaced interpolation with Chebyshev interpolation by plotting degree n polynomials of both types on the interval

$[-1, 1]$, for $n = 10$ and 20. For evenly spaced interpolation, the left and right interpolation base points should be -1 and 1. By sampling at a 0.01 step size, create the empirical interpolation errors for each type, and plot a comparison. Can the Runge phenomenon be observed in this problem?

3.4 E1 Decide whether the equations form a cubic spline.

(a)

$$S(x) = \begin{cases} x^3 + x - 1 & x \in [0,1] \\ -(x-1)^3 + 3(x-1)^2 + 3(x-1) + 1 & x \in [1,2] \end{cases}$$

(b)

$$S(x) = \begin{cases} 2x^3 + x^2 + 4x + 5 & x \in [0,1] \\ (x-1)^3 + 7(x-1)^2 + 12(x-1) + 12 & x \in [1,2] \end{cases}$$

3.4 C13 In a single plot, show the natural, not-a-knot, and parabolically terminated cubic splines through the world oil production data given above.

3.4 E4 Find k_1, k_2, k_3 in the following cubic spline. Which of the three end conditions–natural, parabolically terminated, or not-a-knot–if any, are satisfied? (Parabolically terminated means that the first and last pieces are quadratic polynomials).

$$S(x) = \begin{cases} 4 + k_1 x + 2x^2 - \frac{1}{6}x^3 & x \in [0,1] \\ 1 - \frac{4}{3}(x-1) + k_2(x-1)^2 - \frac{1}{6}(x-1)^3 & x \in [1,2] \\ 1 + k_3(x-2) + (x-2)^2 - \frac{1}{6}(x-2)^3 & x \in [2,3] \end{cases}$$

3.5 E3 Find the three-piece Bézier curve forming the triangle with vertices (1,2), (3,4), and (5,1).

3.5 E6 Implement de Casteljau's algorithm to draw a Bézier curve given its control points. Input variables: value of the parameter and control ponts; output parameter: coordinates of a point on the curve. Use it to draw the 3-piece cubic Bézier given by the control points

(0,1) (0,1) (0,0) (0,0)
(0,0) (0,1) (1,1) (1,0)
(1,0) (1,1) (2,1) (2,0)