T. Sauer, Numerical Analysis. Pearson, Addison Wesley, 2006/2009/2012.

FMN011 Exercises Chapters 2 and 4

The problems are based on the designated problem from Sauer's book, but may not be identical.

2.1 E5 Use the approximate operation count $2n^3/3$ for Gaussian elimination to estimate how much longer it takes to solve n equations in n unknowns if n is tripled.

2.3 E1, E2 Find the norm $\|A\|_\infty$ and the condition number for

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

2.3 C1, C5 Consider the $n \times n$ matrix with entries $A_{ij} = 5/(i + 2j - 1)$, set $x = [1, \dots, 1]^T$ and $b = Ax$. Find the max-norm of the matrix. Solve the system $Ax = b$ using the backslash operator for $n = 6, 7, \dots, 14, 15$. For each value of n, find the condition number of matrix A and the relative error of the computed solution. For what values of n does the solution have no correct significant digits?

2.2 E7 Assume that your computer can solve 100 problems of type $Ux = c$, where U is an upper-triangular 50×50 matrix, per second. Estimate the time it will take to solve a full 500×500 matrix problem, $Ax = b$.

2.4 E5 Write down a 5×5 matrix P such that PA results in the second and fourth rows of A to be exchanged. What happens for AP?

2.4 E7 Change four entries of the leftmost matrix to make the matrix equation correct:

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 5 & 6 \\ 5 & 6 & 7 & 8 \\ 7 & 8 & 9 & 0 \end{bmatrix} = \begin{bmatrix} 5 & 6 & 7 & 8 \\ 3 & 4 & 5 & 6 \\ 7 & 8 & 9 & 0 \\ 1 & 2 & 3 & 4 \end{bmatrix}$$

4.1 E12 Given data points $(x, y, z) = (0, 0, 3), (0, 1, 2), (1, 0, 3), (1, 1, 5), (1, 2, 6)$, find the plane $z = c_0 + c_1 x + c_2 y$ that best fits the data.

4.1 C5 A company test-markets a new soft drink in 22 cities of approximately equal size. The selling price (in dollars) and the number sold per week in the cities are listed as follows:

city	price	sales/week	city	price	sales/week
1	0.59	3980	12	0.49	6000
2	0.80	2200	13	1.09	1190
3	0.95	1850	14	0.95	1960
4	0.45	6100	15	0.79	2760
5	0.79	2100	16	0.65	4330
6	0.99	1700	17	0.45	6960
7	0.90	2000	18	0.60	4160
8	0.65	4200	19	0.89	1990
9	0.79	2440	20	0.79	2860
10	0.69	3300	21	0.99	1920
11	0.79	2300	22	0.85	2160

(a) First, the company wants to find the "demand curve": how many it will sell at each potential price. Let P denote price and S denote sales per week. Find the line $S = c_1 + c_2 P$ that best fits the data. Find the normal equations and the coefficients c_1 and c_2. Plot the fitted line along with the data, and calculate the root mean square error.

(b) After studying the results of the test marketing, the company will set a single selling price P throughout the country. Given a manufactoring cost of \$0.23 per unit, the total profit (per city, per week) is $S(P - 0.23)$ dollars. Use the results of the least squares approximation to find the selling price for which the company's profit will be maximized.

4.1 C9 Let $x_1, \ldots, x_{11}$ be 11 equally spaced points in $[2, 4]$ and $y_i = 1 + x_i + x_i^2 + \cdots + x_i^d$. Use the normal equations to compute the best degree d polynomial, where (a) $d = 5$, (b) $d = 6$, (c) $d = 8$. How many correct decimal places of the coefficients can be computed? Use condition number to explain the results.

4.1 C10 The following data lists the year-over-year percent change in the mean disposable personal income in the U.S. during election years. Also, the proportion of the U.S. electorate that voted for the incumbent party's presidential candidate is listed. Find the best least squares linear model for incumbent party vote as a function of income change. Plot this line along with the 15 data points. How many percentage points of vote can the incumbent party expect for each additional percent of change in personal income?

year	% income change	% incumbent vote
1952	1.49	44.6
1956	3.03	57.8
1960	0.57	49.9
1964	5.74	61.3
1968	3.51	49.6
1972	3.73	61.8
1976	2.98	49.0
1980	-0.18	44.7
1984	6.23	59.2
1988	3.38	53.9
1992	2.15	46.5
1996	2.10	54.7
2000	3.93	50.3
2004	2.47	51.2
2008	-0.41	45.7