Inverse Kinematic Solution for ProbeArm

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1 Introduction

This report describes closed form inverse kinematics solutions for ProbeArm. The solution was automatically generated by the IK-BT package from the University of Washington Biorobotics Lab. The IK-BT package is described in `https://arxiv.org/abs/1711.05412`. IK-BT derives your inverse kinematics equations using `Python 2.7` and the `sympy` module for symbolic mathematics.

2 Kinematic Parameters

The kinematic parameters for this robot are

$$[\alpha_{i-1}, \quad a_{i-1}, \quad d_i, \quad \theta_i]$$

$$\begin{bmatrix} 0 & a_1 & l_1 & th_1 \\ \frac{\pi}{2} & 0 & 0 & th_2 \\ 0 & a_2 & 0 & th_3 \\ 0 & 0 & l_3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{1}$$

3 Forward Kinematic Equations

The forward kinematic equations for this robot are:

$$\begin{bmatrix} r_{11} & r_{12} & r_{13} & Px \\ r_{21} & r_{22} & r_{23} & Py \\ r_{31} & r_{32} & r_{33} & Pz \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_1 c_{23} & -c_1 s_{23} & s_1 & a_1 + a_2 c_1 c_2 + l_3 s_1 \\ c_{23} s_1 & -s_1 s_{23} & -c_1 & a_2 c_2 s_1 - c_1 l_3 \\ s_{23} & c_{23} & 0 & a_2 s_2 + l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

4 Unknown Variables:

The unknown variables for this robot are (in solution order):

1. θ_1
2. θ_2
3. θ_3
4. θ_{23}

5 Solutions

The following equations comprise the full solution set for this robot.

5.1 θ_1

Solution Method: atan2(y,x)

$$\theta_1 = \operatorname{atan2}\left(r_{13}, \frac{r_{11}}{r_{32}}\right) \tag{3}$$

5.2 θ_2

Solution Method: atan2(y,x)

$$\theta_{2s1} = \operatorname{atan2}\left(\frac{1}{a_2}\left(Pz - l_1\right), \frac{Py + l_3\cos\left(\theta_1\right)}{a_2\sin\left(\theta_1\right)}\right) \tag{4}$$

5.3 θ_3

Solution Method: atan2(y,x)

$$\theta_{3s1} = \operatorname{atan2}\left(-r_{11}\sin\left(\theta_{2s1}\right)\cos\left(\theta_1\right) - r_{21}\sin\left(\theta_1\right)\sin\left(\theta_{2s1}\right) + r_{31}\cos\left(\theta_{2s1}\right), r_{11}\cos\left(\theta_1\right)\cos\left(\theta_{2s1}\right) + r_{21}\sin\left(\theta_1\right)\cos\left(\theta_{2s1}\right) + r_{31}\text{ si} \tag{5}$$

5.4 θ_{23}

Solution Method: algebra

$$\theta_{23s1} = \theta_{2s1} + \theta_{3s1} \tag{6}$$

6 Solution Graph (Edges)

The following is the abstract representation of solution graph for this manipulator (nodes with parent -1 are roots):

```
Edge from child: th_2s1 to parent: th_1
Edge from child: th_3s1 to parent: th_2s1
Edge from child: th_23s1 to parent: th_3s1
Edge from child: th_1 to parent: -1
```

7 Solution Sets

The following are the sets of joint solutions (poses) for this manipulator:

```
(th_1, th_2s1, th_3s1, th_23s1)
```

8 Equations Used for Solutions

8.1 θ_1

Solution Method: atan2(y,x)

$$0 = -r_{13} + \sin\left(\theta_1\right) \tag{7}$$

$$0 = -r_{11} + r_{32}\cos\left(\theta_1\right) \tag{8}$$

8.2 θ_2

Solution Method: atan2(y,x)

$$0 = -Pz + a_2\sin\left(\theta_2\right) + l_1 \tag{9}$$

$$0 = -Py + a_2\sin\left(\theta_1\right)\cos\left(\theta_2\right) - l_3\cos\left(\theta_1\right) \tag{10}$$

8.3 θ_3

Solution Method: atan2(y,x)

$$0 = r_{11}\sin(\theta_2)\cos(\theta_1) + r_{21}\sin(\theta_1)\sin(\theta_2) - r_{31}\cos(\theta_2) + \sin(\theta_3) \tag{11}$$

$$0 = -r_{11}\cos(\theta_1)\cos(\theta_2) - r_{21}\sin(\theta_1)\cos(\theta_2) - r_{31}\sin(\theta_2) + \cos(\theta_3) \tag{12}$$

8.4 θ_{23}

Solution Method: algebra

$$0 = \theta_2 - \theta_{23} + \theta_3 \tag{13}$$