FIT3139 Assignment 3

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1 Specification Table

Base Model	Evolutionary Game Theory
Extension Assumptions	Simulate stochastic growth of multiple different strategies
Techniques Showcased	Game Theory Markov Chains Montecarlo Simulations
Modelling Question 1	Why are there species unique to a particular place?
Modelling Question 2	How does good strategies evolve from just a small population using it?

2 Introduction

Question 1: For animals with different strategies in a closed environment, what determines a dominant strategy? Australia is known for its unique animals that are not found elsewhere in the world. Why is the evolution of animals different compared to other places?

Question 2: Does a good mutated strategy dominate the population over time? A similar real-life case study will be the hundredth monkey effect, where monkeys washing sweet potatoes before consuming dominates the population.

To answer the two questions above, we will use an extension to simulate how the same strategy can dominate in one environment but gets dominated in another, to show why evolution can happen differently based on different closed environment. We can also use it to simulate how a small population of a good strategy that dominates the current strategies grow overtime. Strategies here can also mean different subspecies (like Neanderthals and Denisovans) evolving into the same species (Modern Humans).

Using Game Theory, we can understand how each strategy fares against others and therefore determine the payoff of each interaction. We can then use the payoff to decide the probability of how the strategy changes.

Montecarlo simulations will be used to determine random pairings for different strategies as they compete for food.

To calculate the probability of each strategy, and it is determined by the previous iteration, so we use Markov Chain as well. We will be able to know how each strategy's population grows.

3 Model Description

3.1 Extension

Assume an environment where each strategy competes for the same food. Each strategy has three stats, Strength, Agility and Endurance. How much food they receive is based on how good their Strength and Agility stats are, and Endurance stats determine their survivability without food. At the end of the iteration, everyone determines what strategy is good for them based on the each strategy's payoff compared to the rest. Over the iterations, we will be able to know which strategy among them is the best based on how much the strategy is picked.

3.2 Variables

t - time in (thousand) years
S_x - Strength stat of strategy x
A_x - Agility stat of strategy x
E_x - Endurance stat of strategy x
T_x - Stat total of strategy x
$\alpha(x, y)$ - Payoff based on S_x with respect to S_y
$\beta(x, y)$ - Payoff based on A_x with respect to A_y
$\gamma(x)$ - Payoff based on E_x
$p(x, y)$ - Payoff for strategy x with respect to strategy y $(\alpha(x, y) + \beta(x, y) + \gamma(x))$
P_x - Sum of all payoffs made by strategy x in the iteration
$Pr(x)$ - Probability of choosing strategy x for the next iteration

3.3 Assumptions

- Each iteration is in t.
- Each strategy has 3 variables, Strength(S), Agility(A), Endurance(E)
- Each stat has a maximum of 100, with $T = 150$.
- Every iteration, each strategy competes with one another (including itself) with food.
- Food payoff will be one of the payoff factors here, and is set to 1 for all models in this report
- A percentage of Endurance(E) will be added after competing for food as payoffs, denoted by γ
- When competing for food, the strategy with a higher Agility(A) gets a percentage of the payoffs proportionate to the other, capped at 1(food payoff), denoted by β
- The strategy with a higher Strength(S) then gets the remaining food payoff, is denoted by α (regardless of its proportion.)
- The payoff after the above calculation $(\alpha + \beta + \gamma)$ is denoted by p.
- The better the total payoff of a strategy, the higher chance of the strategy being chosen

3.4 Equations

Assuming the set of strategies to be $[x, y, z]$, and food payoff of 1. Then,

$$\beta(x,y) = \begin{cases} \min(\frac{A_x - A_y}{A_y}, 1), & A_x > A_y, \\ 0, & A_x \leq A_y. \end{cases}$$

$$\alpha(x,y) = \begin{cases} 1 - \max(\beta(x,y), \beta(y,x)), & S_x > S_y, \\ 0, & S_x < S_y, \\ \frac{1-\max(\beta(x,y),\beta(y,x))}{2}, & S_x = S_y. \end{cases}$$

$$\gamma(x) = \frac{E_x}{100}$$

$$p(x,y) = \alpha(x,y) + \beta(x,y) + \gamma(x)$$

$$P_x = \sum p(x,x) + \sum p(x,y) + \sum p(x,z)$$

$$Pr(x) = \frac{P_x}{P_x + P_y + P_z}$$

$$\text{Payoff Matrix*} = \begin{pmatrix} p(x,x) & p(x,y) & p(x,z) \\ p(y,x) & p(y,y) & p(y,z) \\ p(z,x) & p(z,y) & p(z,z) \end{pmatrix}$$

*omitted the other direction as it is simply the transpose

3.5 Class of Model

The model

- uses Numerical methods as figures are approximate
- uses Non-linear equations as current state is dependant on previous state
- is Discrete as model updates on every iteration t
- is Stochastic as pairings are random

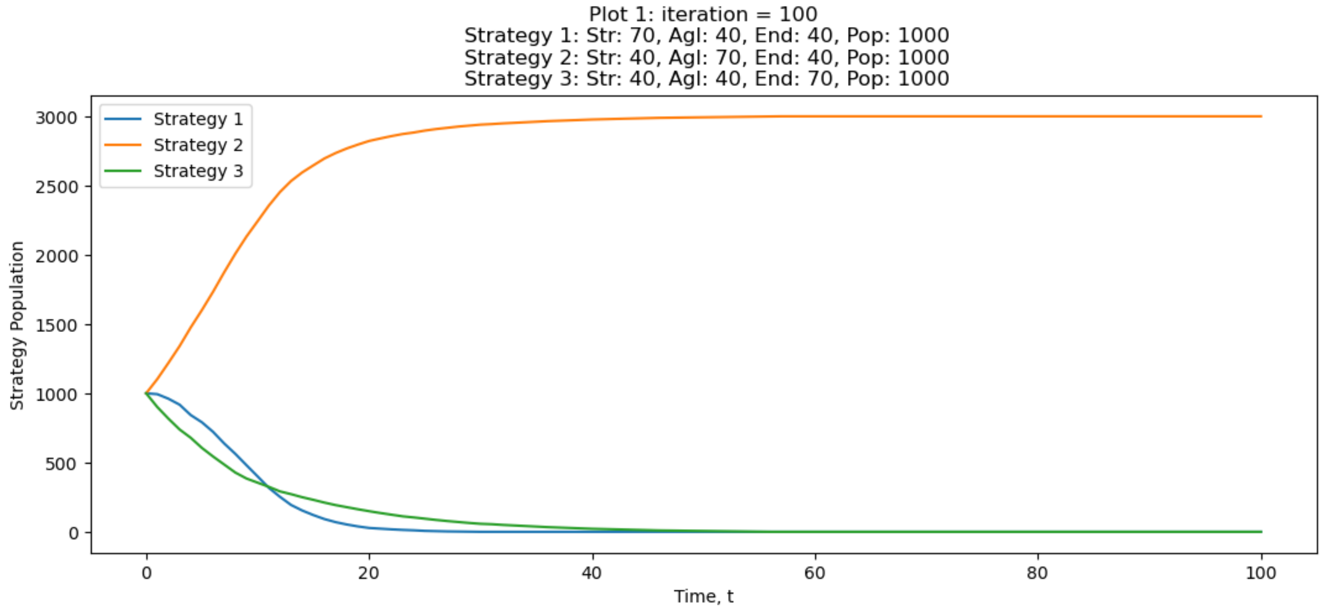
3.6 Algorithms

Game Theory will be used to derive a payoff for each strategy against another. It will also be used to analyse why is a particular strategy dominant. Markov Chain is used to calculate the probability for the change in strategy. Stochastic simulation is used on the random pairings of the strategy, giving us a stochastic solution to the problem. Numpy and matplotlib is also used.

Further algorithms can be found under “Approach Description” in the docstrings of the code in jupyter notebook.

4 Results

4.1 Plot 1



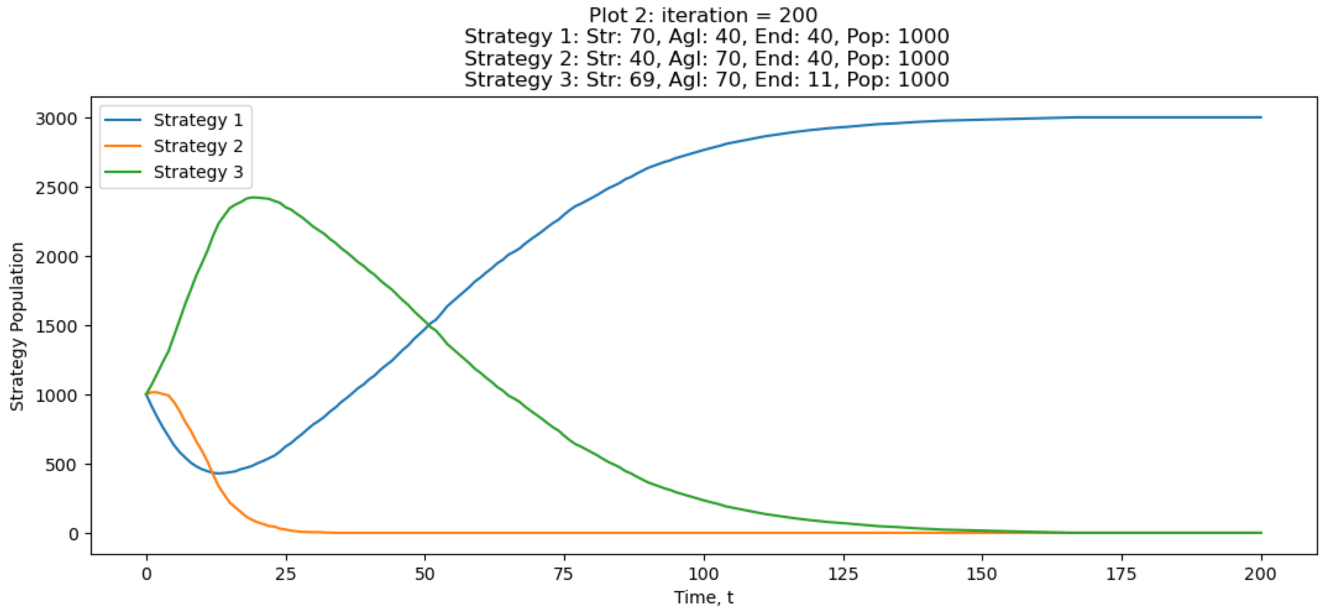
Plot 1 would what be a normal plot looks like. One strategy simply dominates others. I call it the normal plot because most of the plot looks like this. One strategy simply dominates the rest throughout the iterations. Let's have a look at the payout matrix.

$$\text{Payout Matrix} = \begin{pmatrix} 0.9 & 0.65 & 1.4 \\ 1.15 & 0.9 & 1.275 \\ 0.7 & 0.825 & 1.2 \end{pmatrix} = \begin{pmatrix} 0.9 & 0.65 & 0 \\ 1.15 & 0.9 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0.9 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Clearly, row 3 is dominated by row 2, so we can remove that. We omitted the column player because it is just the transpose. This means we can remove the column of whatever row we remove (This will be the case for all payout matrix in this report). We remove column 3, and we realised row 1 is now dominated by row 2. Removing both row 1 and column 1, we achieve Nash Equilibrium with Strategy 2 remaining. Hence we see only see Strategy 2 as per the plots. No surprises here.

*Dominance will be denoted by >> from here, such that if $A >> B$, A dominates B, or in other words, B is dominated by A.

4.2 Plot 2

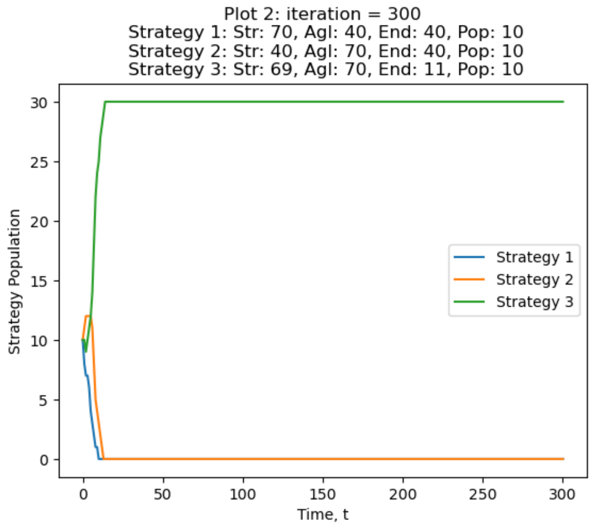


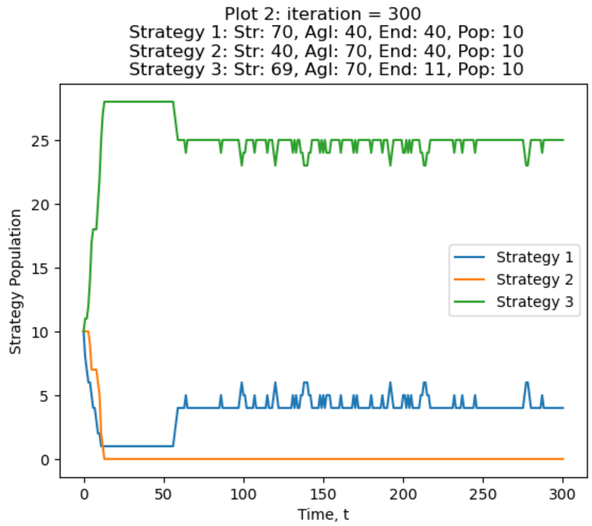
Now we see that we have the same Strategy 1 and 2 from the Plot 1. However, while 1 >> 2 in Plot 1, this time round, we have 1 >> 2.

Another interesting to note is that Strategy 1 was actually losing at the start, but overtook after Strategy 3 starts dominating. This interaction is due to every strategy dominating one another. In this case, 3 >> 2 >> 1 >> 3.

There are two factors to how this could happen. Firstly, while 2 >> 1 and 3 >> 2, the population of Strategy 1 decreases and Strategy 3 increases. This increases the chance Strategy 2 gets paired up with Strategy 3, drastically dropping the population of Strategy 2. Once Strategy 2 dies off, everyone start to tend towards Strategy 1.

The second reason will be the initial population. The lower the initial population, the higher the chance that Strategy 2 gets paired with Strategy 1, which may result in Strategy 1 dying off much earlier. The following observation is shown below. Note that there is also a chance where Strategy 1 and 3 does not converge to 0, given the population is low enough.



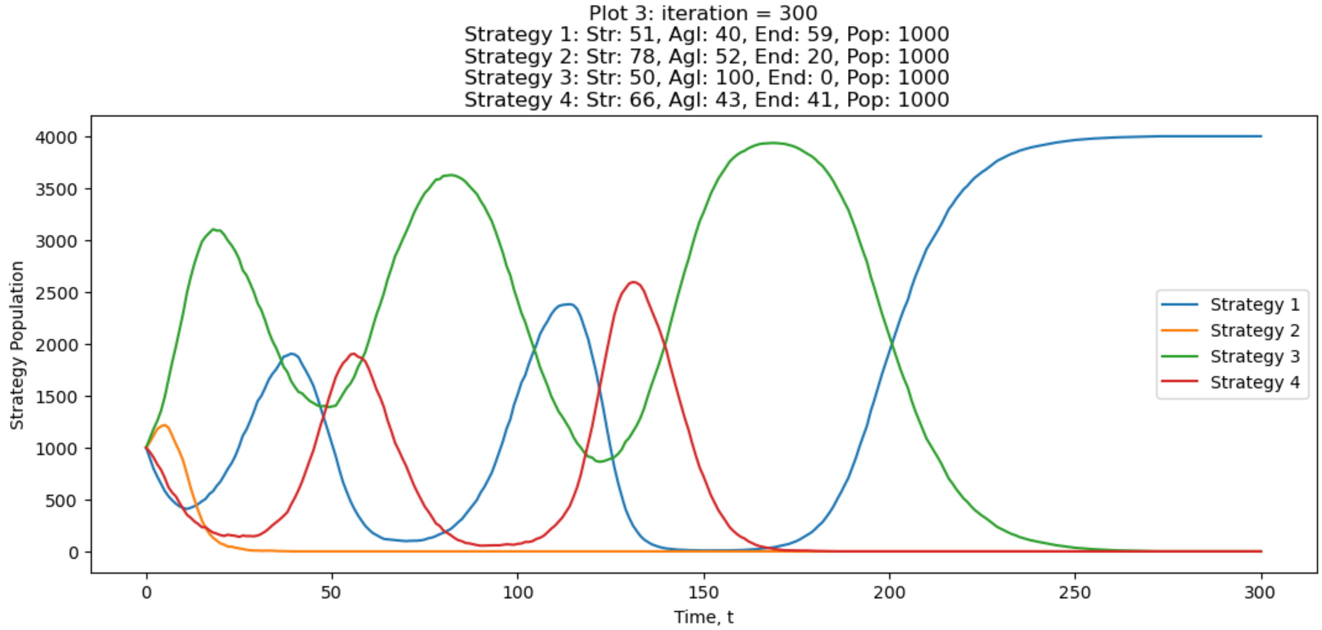


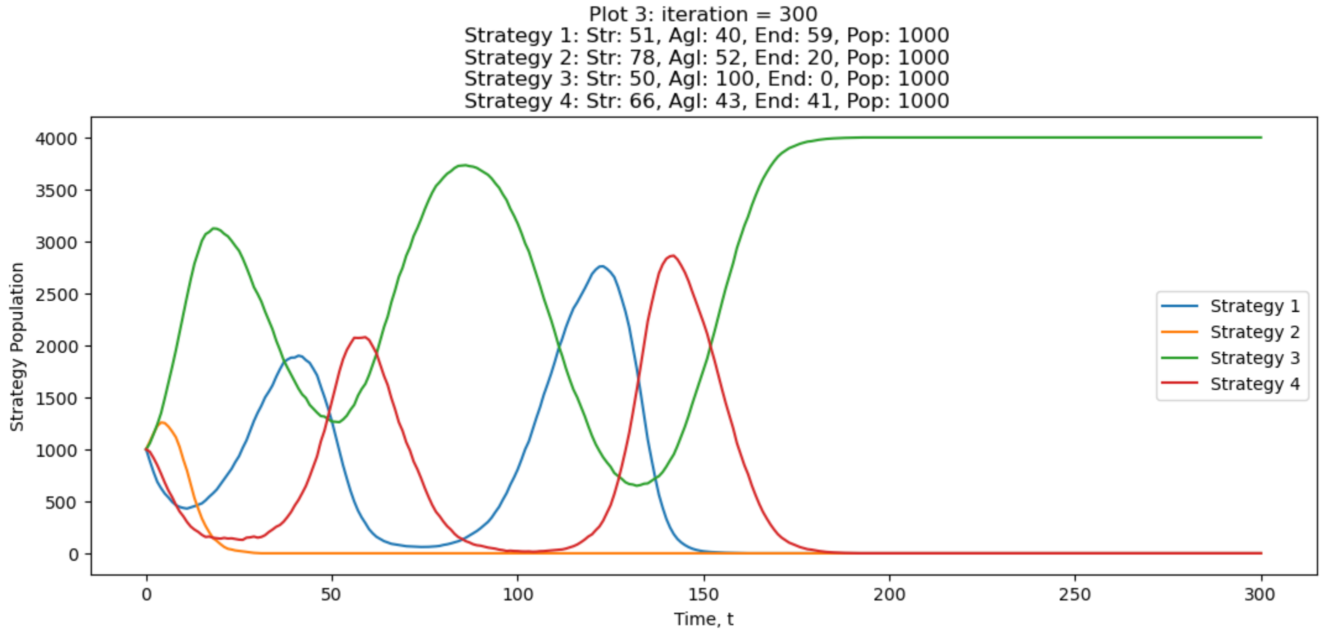
$$\text{Payout Matrix} = \begin{pmatrix} 0.9 & 0.65 & 0.65 \\ 1.15 & 0.9 & 0.4 \\ 0.86 & 1.11 & 0.61 \end{pmatrix}$$

$$P(1,2) = \begin{pmatrix} 0.9 & 0.65 \\ 1.15 & 0.9 \end{pmatrix} \qquad P(2,3) = \begin{pmatrix} 0.9 & 0.4 \\ 1.11 & 0.61 \end{pmatrix} \qquad P(1,3) = \begin{pmatrix} 0.9 & 0.65 \\ 0.86 & 0.61 \end{pmatrix}$$

We find our Nash Equilibrium for each individual pair, and we see that $3 >> 2 >> 1 >> 3$, as mentioned previously, which explains what we see in Plot 2.

4.3 Plot 3





Taken to the extreme would be Plot 3, as we can see the beautiful way that the plots intertwine before eventually reaching a steady state. Note that all strategies have been changed.

However, the more interesting thing about these two plots is that the results are totally different. The first plot has Strategy 1 dominating and the second plot has Strategy 3 dominating. After plotting multiple times, it was around 35% for the first plot to happen and 65% for the second. Strategy 4 seems like it could dominate as well, but sadly, it did not happen.

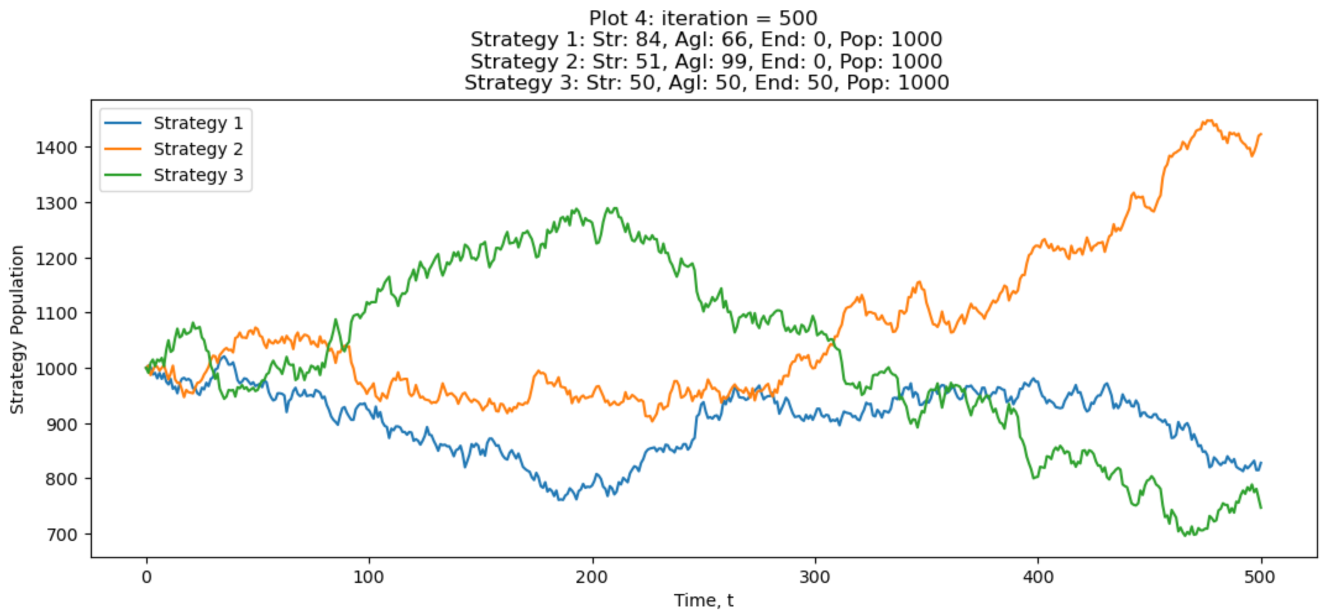
$$\text{Payout Matrix} = \begin{pmatrix} 1.09 & 0.59 & 0.59 & 0.59 \\ 1.2 & 0.7 & 0.277 & 1.2 \\ 1 & 0.923 & 0.5 & 1 \\ 1.41 & 0.41 & 0.41 & 0.91 \end{pmatrix}$$

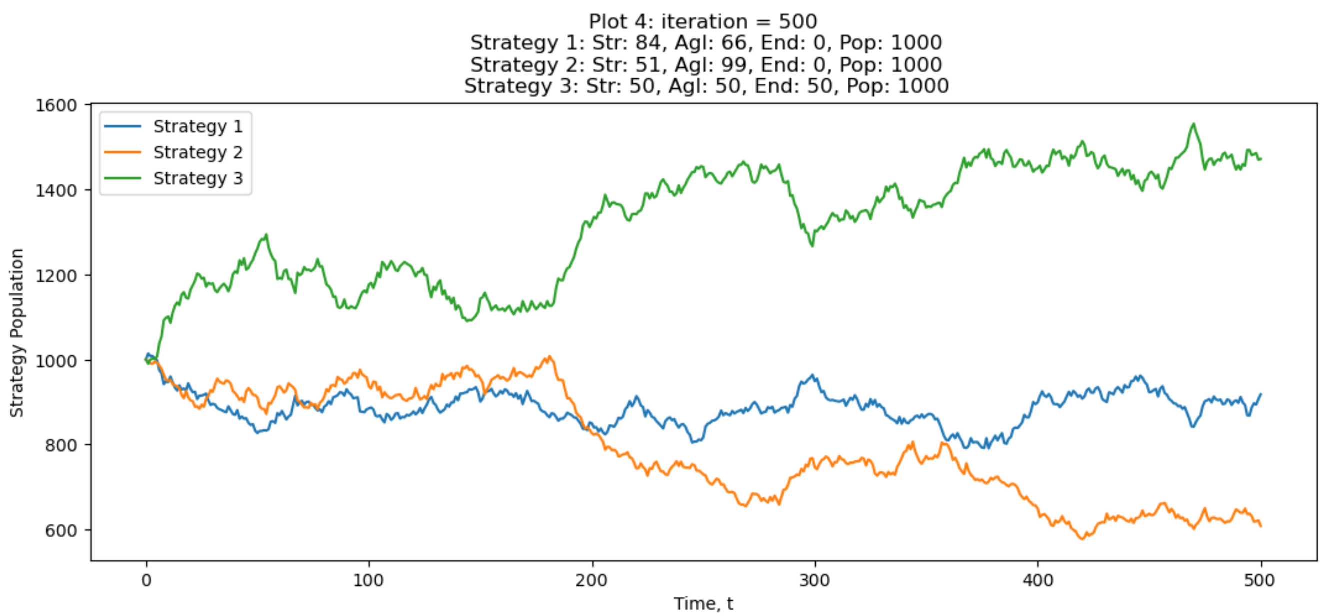
$$P(1,2) = \begin{pmatrix} 1.09 & 0.59 \\ 1.2 & 0.7 \end{pmatrix} \qquad P(1,3) = \begin{pmatrix} 1.09 & 0.59 \\ 1 & 0.5 \end{pmatrix} \qquad P(1,4) = \begin{pmatrix} 1.09 & 0.59 \\ 1.41 & 0.91 \end{pmatrix}$$

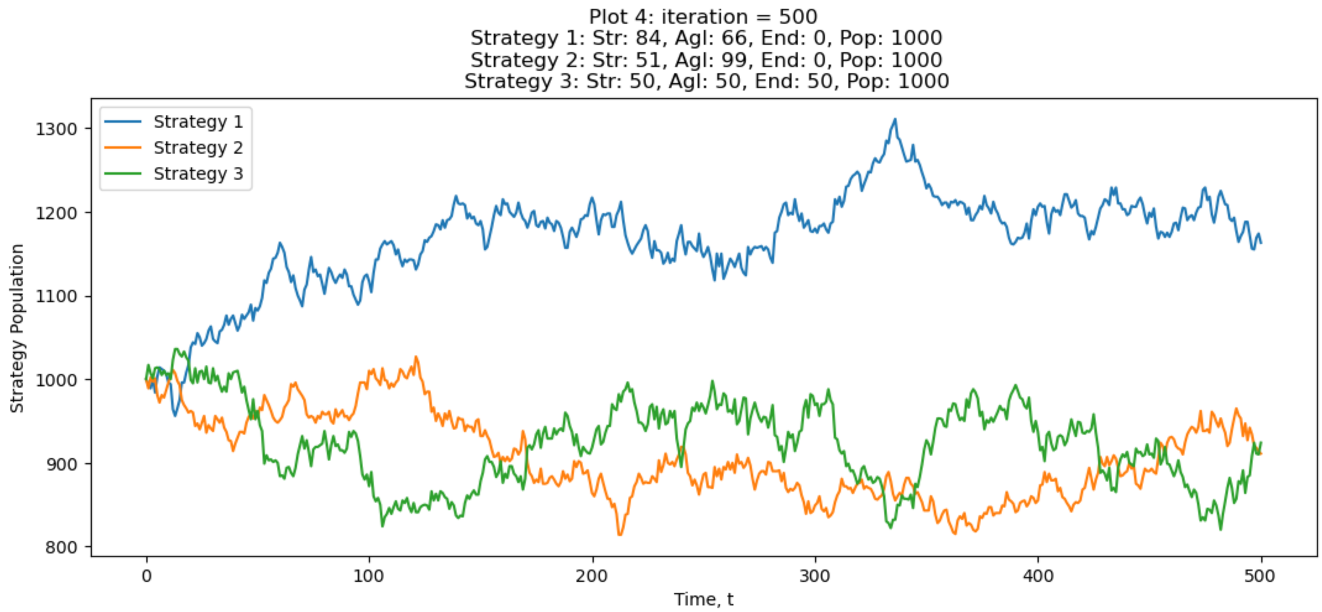
$$P(2,3) = \begin{pmatrix} 0.7 & 0.277 \\ 0.923 & 0.5 \end{pmatrix} \qquad P(2,4) = \begin{pmatrix} 0.7 & 1.2 \\ 0.41 & 0.91 \end{pmatrix} \qquad P(3,4) = \begin{pmatrix} 0.5 & 1 \\ 0.41 & 0.91 \end{pmatrix}$$

Looking at the payout matrix, we can find our nash by splitting up. We see that 2 >> 1, 1 >> 3, 4 >> 1, 3 >> 2, 2 >> 4, 3 >> 4. Similar to before, Strategy 2 plunged as Strategy 3 inflates. After Strategy 2 is gone, we have a cycle similar to before again, 1 >> 3 >> 4 >> 1. We see that 1 and 4 seems to dip dangerously low, only for them to rise again. However when Strategy 1 comes very close, Strategy 4 dies off, as 4 >> 1 which means 4 thrive only when 1 thrives. But as 3 >> 4,, 4 dies off eventually, leaving only Strategy 1 and 3 alive, and the obvious result is Strategy 1 dominating. There is also some chance that 1 dies off with 4, which result in Strategy 3 left.

4.4 Plot 4


Plot 4: iteration = 500
Strategy 1: Str: 84, Agl: 66, End: 0, Pop: 1000
Strategy 2: Str: 51, Agl: 99, End: 0, Pop: 1000
Strategy 3: Str: 50, Agl: 50, End: 50, Pop: 1000
Strategy 1
Strategy 2
Strategy 3
Strategy Population
Time, t

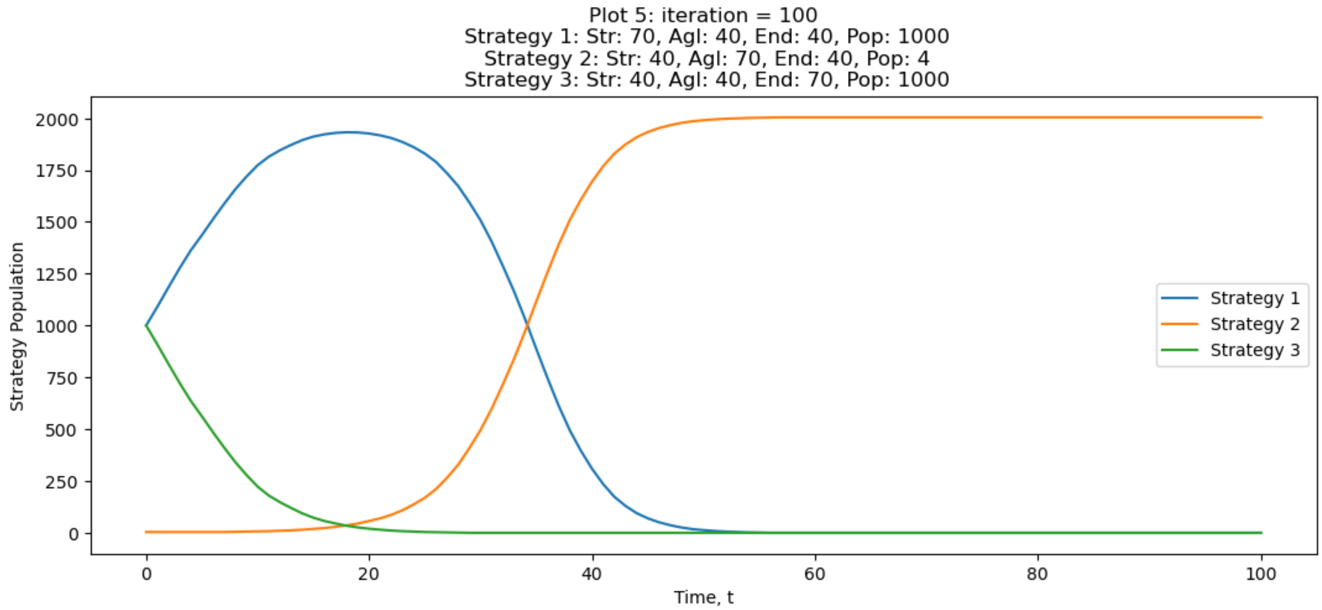

Plot 4: iteration = 500
Strategy 1: Str: 84, Agl: 66, End: 0, Pop: 1000
Strategy 2: Str: 51, Agl: 99, End: 0, Pop: 1000
Strategy 3: Str: 50, Agl: 50, End: 50, Pop: 1000
Strategy 1
Strategy 2
Strategy 3
Strategy Population
Time, t



Plot 4 has 3 different strategies, yet it has an erratic plot unlike any other, with any three strategy capable of dominating, but without the other dying off. When looking at the payout matrix, we realise that every strategy has the exact same payout! As we are using a stochastic simulation, it is only natural that the plots are erratic; every strategy is equally good.

$$\text{Payout Matrix} = \begin{pmatrix} 0.5 & 0.5 & 1 \\ 0.5 & 0.5 & 1 \\ 0.5 & 0.5 & 1 \end{pmatrix}$$

4.5 Plot 5



We have the exact same strategy setting as Plot 1, the only difference is Strategy 2 only has a population of 4 as compared to 1000 for the other two. It seems that what is dominating will eventually dominates. (A population of lesser than 4 actually stays the same without increasing, due to rounding off probability)

4.6 Conclusion

So to answer why does Australia have unique animals, we have to look at Plots 2 and 3. What is considered good in one place may not translate into being good in another. This explains why the giant panda or extinct animals exists in the first place. New strategies may emerge due to mutation (that is not covered in this report) that results in a loop of dominance, giving us a constant evolution of living things.

There is also the case where a set of strategies have payoffs like in Plot 4, where all strategies coexist with one another. However, due to this model being stochastic, we are not able to get a set of strategies that coexist and oscillate around each other nicely. One strategy might accidentally die off due to chance and will break the equilibrium. However, this is more likely to be the case, as there will be external factors that is not considered in this model.

From Plot 5, we can also see that good strategies can spread like wildfire, and can quickly dominate. This would mean good strategies are replicated rapidly, just like how trends can take over the internet. It also shows the dangers of invasive species, which is why Australia takes its quarantine laws so seriously.

There are actually no best strategy based on my assumptions of this model. Assuming strategy x has $S_x,\ A_x,\ E_x$, you can always choose strategy y such that $S_y = S_x + 1,\ A_x = A_y - 1,\ E_y = E_x$. This ensures y gets the remaining while minimising food loss due to agility. If $S_x = 100$, then A_x has a maximum of 50. In this case, choose $S_y = 0,\ A_y = 100,\ E_y = 50$. Since A_y is at least twice of A_x, strategy y gets all the food by means of agility, with added bonus in endurance.

5 List of algorithms and concepts

- Game Theory
- Nash Equilibrium
- Markov Chains
- Montecarlo/Stochastic Simulations
- Iterative methods
- Random Sampling
- Non-linear Equations
- Evolutionary Computation