INSTITUT FÜR VOLKSWIRTSCHAFTSLEHRE
FINANZWISSENSCHAFT

PROF. DR. CARSTEN HELM

CARL VON OSSIETZKY universität OLDENBURG
FAKULTÄT II
INFORMATIK, WIRTSCHAFTS- UND RECHTSWISSENSCHAFTEN

Advanced Microeconomics, winter term 2025/26

Exercise 1

Please solve the exercises below by Wednesday, October 22th. We will discuss them in our exercises (see Stud.IP). To obtain a bonus point, you need to upload your answers as a single pdf in the StudIP folder "Student Solutions Exercise 1". The **document name should start with your surname**. If you prepared the answer in a group of up to 3 students, please only submit one document that contains the names of all contributing students. The DEADLINE is October 22th, 7:30 so that we have a chance to quickly scroll through your submissions to suggest one for presentation.

Question 1 (A starter on rationality: the wine problem)

Imagine you needed to choose how much wine (α) to drink (in litre). Imagine that your payoff function is given by $\theta\alpha - 4\alpha^2$, where θ is a parameter that depends on your physique. Every person may have a different value of θ, and it is known that in the population (1) the smallest θ is 0.2; (2) the largest θ is 6; and (3) larger people have higher θ than smaller people.

a) Can you find an amount that no person should drink?
b) How much should you drink if your $\theta = 1$? If $\theta = 4$?
c) Show that in general smaller people should drink less than larger people.
d) Should any person drink more than one 1-liter bottle of wine?

Question 2 (Working on a joint project)

You are working with a friend on a joint project. Each of you can either work hard or goof off. If your friend works hard then you prefer to goof off (the outcome of the project would be better if you worked hard too, but the increment in its value to you is not worth the extra effort). You prefer the outcome of your both working hard to the outcome of your both goofing off (in which case nothing gets accomplished), and the worst outcome for you is that you work hard and your friend goofs off (you hate to be "exploited"). If your friend has the same preferences, then the game that models the situation you face is given in the figure.

Player 2

1 Player

	Work hard	*Goof off*
Work hard	2, 2	0, 3
Goof off	3, 0	1, 1

Player2 no work, work done by Player 1

the best scenario

Player 1 no work but work done by Player 2

a) Does the game have an equilibrium in dominant strategies?
b) Formulate a strategic game (i.e. a matrix like the one above) that models a situation in which two people work on a joint project in the case that their preferences are the same as

those in the game in the figure except that each person prefers to work hard than to goof off when the other person works hard.

c) Does this alternative game have an equilibrium in dominant strategies?

Question 3 (The odd couple)

Felix and Oscar live in a shared apartment. They have different ideas about cleaning and consequently also about the number of hours they are willing to spend on cleaning the apartment. Assume that at least 12 working hours (per week) are needed to make the apartment sparkling clean, 9 working hours to get it acceptably clean, and anything less than 9 working hours means that the apartment remains dirty. Further, assume that each of the two people can devote 3, 6, or 9 hours of their time to cleaning the apartment.

Felix and Oscar agree that an acceptably clean apartment has a utility index of 2. However, they do not agree on the value of a properly clean apartment; for Felix, in this case, it has a utility index of 10, while for Oscar, the utility is only 5 units. They also disagree on the repulsiveness of a dirty apartment; while for Felix in this case the apartment has a utility index of -10, for Oscar the utility is -5.

Each person's payoff is the difference between the utility of the apartment and the hours spent cleaning it; e.g., a spotlessly clean apartment for which each person worked for 6 hours gives Felix a payoff of 4, while for Oscar the payoff in this case is -1. u - h

a) Represent the game in normal form.
b) Determine the outcome of the game using the procedure of "repeated elimination of dominated strategies".
c) Is the result from b) the only Nash equilibrium in this game?

Question 3

Players $N = 1,2$

(strategies avaliable for each player) cleaning h

Felix: $S_{iF} = S_{i1}, S_{i2}, S_{i3}$
3h, 6h, 9h

Oscar
$S_{iO} = S_{i1}$ 3h, 6h, 9h

(Payoffs) $U_{iF} = 10 - h$

$10 - 6h = 4$ [FOR FELIX!]

$Value_F = 10$ of properly clean appartment

$U_{iO} = 5 - h$

$5 - 6h = -1$

$Value_O = 5$

Player 2

Oscar

[5 - h = ...]

?

Value

Player 1

Filix

[10-h = ...]

	3h (12h)	6h	9h
9h	1, 2 3	1, -1 0	1, -4 -3
6h	4, 2 6	4, -1 3	4, -4 0
3h	7, 2 9	7, -1 6	7, -4 3

Question 3

a) Normal form — players, strategies, payof

① $J = \{Felix, Oscar\}$
② $S_i = \{3h, 6h, 9h\}$
③ set of payoff functions:

12h perfectly clean 9h clean <9h dirty

Utility Felix: 10 Oscar: 5
2 2
-10 -5

Felix:

$U_F(9h, 9h) = 1$	$U_O(9, 9) = -4$
$U_F(6h, 9h) = 10 - 6 = = 4$ (is it 10-3h?)	$U_O(9, 6) = -4$
$U_F(3h, 9h) = 7$	$U_O(9, 3) = -4$
$U_F(9h, 6h) = 1$	$U_O(6, 9) = -1$
$U_F(6h, 6h) = 4$	$U_O(6, 6) = -1$
$U_F(3h, 6h) = -1$	$U_O(6, 3) = -4$
$U_F(9h, 3h) = 1$	$U_O(3, 9) = 2$
$U_F(6h, 3h) = -4$	$U_O(3, 6) = -1$
$U_F(3h, 3h) = -13$	$U_O(3, 3) = -8$

Puting it into the matrix

Oscar

Felix

Felix \ Oscar	9h	6h	3h
9h	1, -4	1, -1	1, 2
6h	4, -4	4, -1	-4, -1
3h	7, -4	-1, -4	-13, -8

WINNER

comparing it with

it is weakly dominated bc it is equal

$$S_{Oscar}(6h) \geq S_{Oscar}(9h)$$

→ Oscar strategy to play 9h is weakly dominated by 6h. Therefore we can erase the left column of

$$S_{Felix}(6h) > S_{Felix}(3h)$$

Therefore we can erase 3h row

$$S_{Oscar}(3) > S_{Oscar}(6h)$$

Therefore we can erase collumn 6h

$S_{Felix}(9h) > S_{Felix}(6h)$

$\Rightarrow$ Outcome of the game: $(9h, 3h)$

$\Rightarrow (1, 2)$

c) Matrix:

		Oscar		
		9h	6h	3h
Felix (the highest value)	9h	1 -4	1 -1	1 2
	6h	4 -4	4 -1	-4 -1
	3h	7 -4	-1 -4	-13 -8

The best mutual response

Question 1

Question 1 (A starter on rationality: the wine problem)

Imagine you needed to choose how much wine (α) to drink (in litre). Imagine that your payoff function is given by $\theta\alpha - 4\alpha^2$, where θ is a parameter that depends on your physique. Every person may have a different value of θ, and it is known that in the population (1) the smallest θ is 0.2; (2) the largest θ is 6; and (3) larger people have higher θ than smaller people.

a) Can you find an amount that no person should drink?
b) How much should you drink if your $\theta = 1$? If $\theta = 4$?
c) Show that in general smaller people should drink less than larger people.
d) Should any person drink more than one 1-liter bottle of wine?

Question 2 (Working on a joint project)

b) $(\theta = 1,\ \theta = 4)$

$\theta\alpha - 4\alpha^2$

1 - $1\alpha - 4\alpha^2 = 0$

$\alpha - 4\alpha^2 = 0$

$\alpha = 4\alpha^2$

$1 = 4\alpha$

$\frac{1}{4}L = \alpha$

If $\theta \geq 1$ the person should drink $\geq 0{,}25$ Litre

2 - $4\alpha - 4\alpha^2 = 0$

$4\alpha = 4\alpha^2$

$4 = 4\alpha$

$1L = \alpha$

If $\theta \geq 4$ the person should drink ≥ 1 Litre

c) showing that smaller people should drink less = smallest value of θ

$\theta = 0{,}2 \rightarrow$ $0{,}2\alpha - 4\alpha^2 = 0$

$0{,}2\cancel{\alpha} = 4\alpha^{\cancel{2}}$

$0{,}2 = 4\alpha$

$\frac{0{,}2}{4} = \alpha$

$\alpha = 0{,}05$ Litre

d) $\alpha = 1L$ $\theta - 4 = 0$

$\theta = 4 \Rightarrow$ No, not everyone, only when your $\theta \geqslant 4$

a) should solve for $\theta > 6$

solve for $\theta = 6 \rightarrow$ $6\alpha - 4\alpha^2 = 0$

$6\alpha = 4\alpha^2$

$6 = 4\alpha$

$\alpha = \frac{6}{4}$

$\alpha = 1{,}5L$

so no one should drink more than 1,5 Litre o- wine

Question 1

class $\theta\alpha - 4\alpha^2$

a) maximize the function

$$V = \theta \cdot \alpha - 4\alpha^2$$

$$V = \cancel{0} + 8\alpha$$

The utility from drinking 0 is equal to 0.

If a decisionmaker drinks $\alpha = 2$, then if he/she has the largest $\theta = 6$, then payoff is $V = 6 \cdot 2 - 4 \cdot 2^2 = -4$ and it is visible that decision makes with smaller value of θ will obtain an even more negative payoff from consuming $\alpha = 2$

b) $V(\alpha) = \theta\alpha - 4\alpha^2$

defined by

$: -\theta\alpha$

$V(\alpha) = \theta\alpha - 4 \cdot 2\alpha$

$\theta\alpha = 8\alpha$

FOC:

$$\theta = 8\alpha = 0$$

$$\alpha = \frac{\theta}{8}$$

$\theta = 1$: $\alpha = \frac{1}{8}$

$\theta = 4$: $\alpha = \frac{1}{2}$

it is

$\Delta y = f'(x_0) \cdot \Delta x$

$f'(x) = \frac{1}{2\sqrt{x}}$ — pochodna

c) This follows from solution b) $\Rightarrow$ for every type of person θ, the solution is $\alpha = \frac{\theta}{8}$ which is necessary in θ and larges people have higher values of θ

$9x^{3}+$ x^3 $\frac{8^2}{}$

$f(x) = a$ $f'(x) = 0$ $f(a) =$

$f(x)$	$f'(x), \frac{\partial f}{\partial x}, \Delta f$
mx^n	$m \cdot n \cdot x^{n-1}$ e.g., $x^2 \xrightarrow{f'(x)} 2x$, $3x^2 \xrightarrow{f'(x)} 6x$, $4x^3 \rightarrow 12x^2$
mx	$5x \rightarrow 5x^{1-1} \rightarrow 5x^0 \rightarrow 5 \cdot 1 = 5$
e^x	e^x
a	any other letter beside the function, f'(x) of a = 0
x	1

$5x^n = n \cdot 5x^{n-1}$

$$V(\alpha) = 5\alpha - 4\alpha^3$$

$$V'(\alpha) = 5\alpha^1 - 12\alpha^2$$

$$V'(\alpha) = 5 \cdot 1 - 12\alpha^2$$

$$V'(\alpha) = 5 - 12\alpha^2$$

$$V''(\alpha) = 0 - 24\alpha$$

$$V'''(\alpha) = 0 - 24$$

$$f(x) = 5x - 5 + 3x^2 - e^x$$

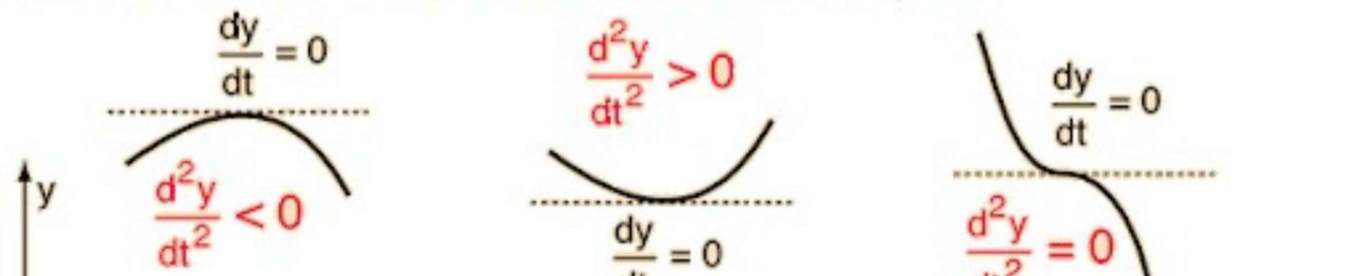
$$f'(x) = 5 - 0 + 6x - e^x$$

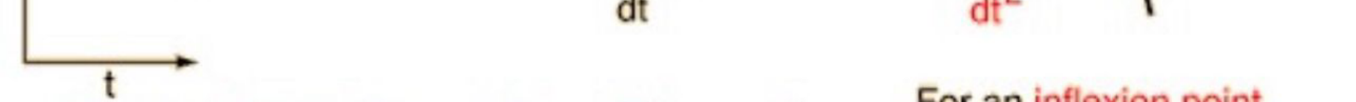
$$f''(x) = 0 - 0 + 6 - e^x$$

$$f'''(x) = 0 - 0 + 0 - e^x$$

$$f'''(x) = -e^x$$

The second derivative demonstrates whether a point with zero first derivative is a maximum, a minimum, or an inflexion point.





For a maximum, the second derivative is negative. The slope of the curve (first derivative) is at first positive, then goes through zero to become negative.

For a minimum, the second derivative is positive. The slope of the curve = first derivative is at first negative, then goes through zero to become positive.

For an inflexion point, the second derivative is zero at the same time the first derivative is zero. It represents a point where the curvature is changing its sense. Inflexion points are relatively rare in nature.

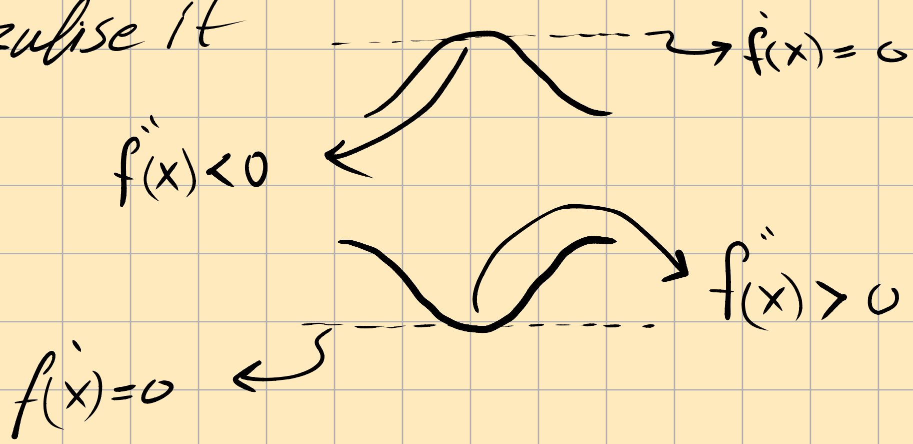
↳ what this is trying to say: with every function, finding the maximum point means to follow these steps:-

1- Find the derivative of the given function & set it $= 0$

2- Find the 2nd derivative of the function & check,

* If $f''(x) < 0$ → this means, you found a maximum
* If $f''(x) > 0$ → this means, you found a minimum

to vizulise it



$v(\alpha) = \theta\alpha - 4\alpha^2$

$v'(\alpha) = \theta - 8\alpha \rightarrow \theta - 8\alpha = 0$

$\theta = 8\alpha \rightarrow \alpha = \frac{\theta}{8}$

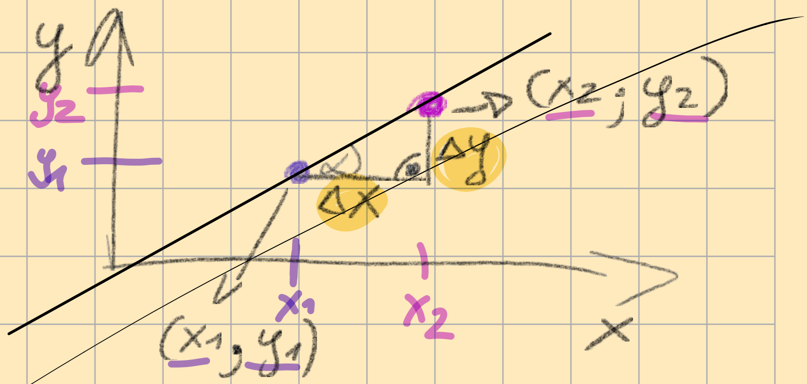
$v''(\alpha) = -8$ → negative < 0 → so, we found maximum

d) No, Even the largest type of person with ⊕ 6 should only consume $\frac{3}{4}$

Pochodne:

$$a = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$

nachylenie prostej



Jak zmienia się y w odniesieniu do x

How y is changing in regarding to X

Pochodna: $f'(x) = \frac{\Delta y}{\Delta x} =$

$$f'(x = \lim_{h \to 0} \frac{f(x+h) - f(x)}{x+h-x}$$

Question 2

$1-1\alpha-4a=0$

Question 2 (Working on a joint project)

You are working with a friend on a joint project. Each of you can either work hard or goof off. If your friend works hard then you prefer to goof off (the outcome of the project would be better if you worked hard too, but the increment in its value to you is not worth the extra effort). You prefer the outcome of your both working hard to the outcome of your both goofing off (in which case nothing gets accomplished), and the worst outcome for you is that you work hard and your friend goofs off (you hate to be "exploited"). If your friend has the same preferences, then the game that models the situation you face is given in the figure.

Player 2

1 Player	Work hard	Goof off
Work hard	2, 2	0, 3
Goof off	3, 0	1, 1

Player2 no work, work done by Player 1

the bad scenario

Player 1 no work but work done by Player 2

a) Does the game have an equilibrium in dominant strategies?

b) Formulate a strategic game (i.e. a matrix like the one above) that models a situation in which two people work on a joint project in the case that their preferences are the same as

	Wh	Go
Wh	44	03
Go	30	11

those in the game in the figure except that each person prefers to work hard than to goof off when the other person works hard.

c) Does this alternative game have an equilibrium in dominant strategies?

class

Question 2

a)

2 person

best

1 person	Work hard	Goof off
Work hard	2, 2	0, 3
Goof off	3, 0	1, 1

better than 2

better than 1

2 strategies < Work hard, goof off

1	2	
	2, 2	0 3
	3 0	$\boxed{1\ 1}$

Goof off is the equilibrium in dominant strategies, and that Goof off results in 1, 1

b)

	work hard	Goof off
Work hard	4, 4	0, 3
goof off	3, 0	1, 1

anything bigger then 3

bigger than 3, 0, 1

No equilibrium

Payoff has to be bigger than 3 ⇒ (4, 4) each person prefers to work hard if the other person works hard

c) We do NOT have an equilibrium in dominant strategies

But

there is a goof of equilibrium

Why is there NO eqilibrium in dominant strategies?