

Advanced Microeconomics, winter term 2025/26

Exercise 5

Please solve the exercises below by Wednesday, December 17th. We will discuss them in our exercises (see Stud.IP). To obtain a bonus point, you have to upload your answers as a single pdf in the StudIP folder “Student Solutions Exercise 5”. The **document name should start with your surname**. If you prepared the answer in a group of up to 3 students, please only submit one document that contains the names of all contributing students. The DEADLINE is December 17th, 7:30 so that we have a chance to quickly scroll through your submissions to suggest one for presentation.

Question 1 (First-price sealed bid auction with n players)

Suppose you want to buy a house that is sold by way of a first-price sealed bid auction. In contrast to the model in the lecture, there are more than 2 players. Players simultaneously and independently submit bids $b_i, i = 1, \dots, n$. The house is awarded to the highest bidder who must pay his bid, denoted x . Your conjecture is that each player bids a fraction a of her valuation, i.e. $b_i = a v_i$, where v_i is a player's valuation. Moreover, suppose that you have the following information: You know the number of persons n that bid for the house. You believe the other bidders' values are independently and uniformly distributed over the interval $[0, 1]$, where 1 stands for €1.000.000.

- a) What is the probability that your bid x exceeds the bid of player 2?
- b) What is the probability that your bid x exceeds the bids of player 2 and 3?
- c) Now, what is the probability that your bid exceeds that of all other bidders?
- d) What bid should you choose?
- e) Explain intuitively how your bid changes with the number of bidders n .
- f) Now suppose that your own valuation for the house is €800.000 and that there are 5 other bidders. How much should you bid?

Question 2 (Signalling - Sequential Games of Incomplete Information)

Consider a firm offering a product – let's say an advent calendar – that may be either high quality or low quality (probability 0.3 vs. 0.7). Only the firm knows its own type, but it is common knowledge that 30% of the products are of high quality and 70% are of low quality. The consumer earns a payoff (benefits minus costs) of 100 from purchasing the high-quality product but 0 from purchasing a low-quality product, and she earns 50 if she buys something else with known quality. Furthermore, the firm earns a profit of 100 from selling a high-quality product (since it will have gained a loyal customer), 30 from selling a low-quality product (since it will be a onetime sale and the customer avoids purchasing from the firm ever again), and 0 from not making a sale. Prior to the customer making the purchase, the firm can take a costly action: it can pay 80 for a 30-second Super Bowl ad. Assume that advertising does not increase product awareness, and *a priori* customers do not consider advertising to be particularly credible. Hence the only purpose of the Super Bowl ad is to serve as a signal.

- a) Write the game in extensive form, similarly as we have done it in the job market signaling model in the lecture.
- b) Find any pooling perfect Bayesian equilibria.
- c) Is there a separating perfect Bayesian equilibrium in which the high-quality firm advertises and the low-quality firm does not?
- d) What exactly does the Super Bowl ad signal, and why does the signal work?