INSTITUT FÜR VOLKSWIRTSCHAFTSLEHRE
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FAKULTÄT II
INFORMATIK, WIRTSCHAFTS- UND RECHTSWISSENSCHAFTEN

# Advanced Microeconomics, winter term 2025/26

## Exercise 1

Please solve the exercises below by Wednesday, October 22th. We will discuss them in our exercises (see Stud.IP). To obtain a bonus point, you need to upload your answers as a single pdf in the StudIP folder "Student Solutions Exercise 1". The **document name should start with your surname**. If you prepared the answer in a group of up to 3 students, please only submit one document that contains the names of all contributing students. The DEADLINE is October 22th, 7:30 so that we have a chance to quickly scroll through your submissions to suggest one for presentation.

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**Question 1 (A starter on rationality: the wine problem)**

Imagine you needed to choose how much wine ($\alpha$) to drink (in litre). Imagine that your payoff function is given by $\theta\alpha - 4\alpha^2$, where $\theta$ is a parameter that depends on your physique. Every person may have a different value of $\theta$, and it is known that in the population (1) the smallest $\theta$ is 0.2; (2) the largest $\theta$ is 6; and (3) larger people have higher $\theta$ than smaller people.

a) Can you find an amount that no person should drink?
b) How much should you drink if your $\theta = 1$? If $\theta = 4$?
c) Show that in general smaller people should drink less than larger people.
d) Should any person drink more than one 1-liter bottle of wine?

**Question 2 (Working on a joint project)**

You are working with a friend on a joint project. Each of you can either work hard or goof off. If your friend works hard then you prefer to goof off (the outcome of the project would be better if you worked hard too, but the increment in its value to you is not worth the extra effort). You prefer the outcome of your both working hard to the outcome of your both goofing off (in which case nothing gets accomplished), and the worst outcome for you is that you work hard and your friend goofs off (you hate to be "exploited"). If your friend has the same preferences, then the game that models the situation you face is given in the figure.

| | *Work hard* | *Goof off* |
|---|---|---|
| *Work hard* | 2, 2 | 0, 3 |
| *Goof off* | 3, 0 | 1, 1 |

a) Does the game have an equilibrium in dominant strategies?
b) Formulate a strategic game (i.e. a matrix like the one above) that models a situation in which two people work on a joint project in the case that their preferences are the same as

those in the game in the figure except that each person prefers to work hard than to goof off when the other person works hard.

c) Does this alternative game have an equilibrium in dominant strategies?

**Question 3 (The odd couple)**

Felix and Oscar live in a shared apartment. They have different ideas about cleaning and consequently also about the number of hours they are willing to spend on cleaning the apartment. Assume that at least 12 working hours (per week) are needed to make the apartment sparkling clean, 9 working hours to get it acceptably clean, and anything less than 9 working hours means that the apartment remains dirty. Further, assume that each of the two people can devote 3, 6, or 9 hours of their time to cleaning the apartment.

Felix and Oscar agree that an acceptably clean apartment has a utility index of 2. However, they do not agree on the value of a properly clean apartment; for Felix, in this case, it has a utility index of 10, while for Oscar, the utility is only 5 units. They also disagree on the repulsiveness of a dirty apartment; while for Felix in this case the apartment has a utility index of -10, for Oscar the utility is -5.

Each person's payoff is the difference between the utility of the apartment and the hours spent cleaning it; e.g., a spotlessly clean apartment for which each person worked for 6 hours gives Felix a payoff of 4, while for Oscar the payoff in this case is -1.

a) Represent the game in normal form.
b) Determine the outcome of the game using the procedure of "repeated elimination of dominated strategies".
c) Is the result from b) the only Nash equilibrium in this game?

E1

**Question 1 (A starter on rationality: the wine problem)**

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c) Show that in general smaller people should drink less than larger people.
d) Should any person drink more than one 1-liter bottle of wine?

E2

**Question 1** (Synergies)

Two division managers can invest time and effort in creating a better working relationship. Each invests $e_i \geq 0$, and if both invest more then both are better off, but it is costly for each manager to invest. In particular, the payoff function for player $i$ from effort levels $(e_i, e_j)$ is $v_i(e_i, e_j) = (a + be_j)e_i - \frac{1}{2}e_i^2$, where $a > 0$ and $0 < b < \frac{1}{2}$.

(a) What is the best response function of each player?

(b) Find the Nash equilibrium of this game.

(c) Find the efficient solution of this game and compare it to the Nash equilibrium.

(a) $V_i(e_i, e_j) = (a + be_j)e_i - \frac{1}{2}e_i^2$

$\left(\frac{dV_i(e_i, e_j)}{de_i}\right)$ FOC: $\frac{dV_i}{de_i} = a + be_j - e_i = 0$ This is the best function

$e_i = a + be_j$

$e_j = a + be_i$

(b) $e_i$ and $e_j$ are symmetric(대칭)으로 $e_i = e_j$

FOC: $a + be_j - e_i = 0$ $(e_i = e_j)$

$\Rightarrow a + be_i - e_i = 0$, $e_i(1 - b) = a$

$\therefore e_i = \frac{a}{1-b}$ is Nash Equilibrium

(c) Efficient Solution is Maximize the overall payoff.

$$\max = \underbrace{(a + be_j)e_i - \tfrac{1}{2}e_i^2}_{\text{Player } i} + \underbrace{(a + be_i)e_j - \tfrac{1}{2}e_j^2}_{\text{Player } j}$$

FOC(max) $= \frac{d\max}{de_i} = a + be_j - e_i + be_j = 0$

$(e_i = e_j)$, $a + be_i - e_i + be_i = 0$

$(1 - 2b)e_i = a$

$e_i = \frac{a}{1-2b}$ is the efficient Solution.

We can conclude that $e_i = \frac{a}{1-2b} > e_i = \frac{a}{1-b}$ (Nash Equilibrium)

due to the Lower dominator. (Lower effect in Nash Equilibrium)

**Question 2** (Monitoring and mixed strategies)

An employee (player 1) who works for a boss (player 2) can either work ($W$) or shirk ($S$), while his boss can either monitor the employee ($M$) or ignore him ($I$). Like most employee-boss relationships, if the employee is working then the boss prefers not to monitor, but if the boss is not monitoring then the employee prefers to shirk. The game is represented in the following matrix:

| player 1 \ Player 2 | $M$ | $I$ |
|---|---|---|
| $W$ | 1, 1 | 1, 2 |
| $S$ | 0, 2 | 2, 1 |

a) Find all Nash equilibria of this game (i.e. equilibria in pure and in mixed strategies). Do you find the result "realistic"?

(a)

| | | $P_2$: $r$ M | $1-r$ I |
|---|---|---|---|
| $P_1$ | $p$ W | 1, 1 | 1, 2 |
| | $1-p$ S | 0, 2 | 2, 1 |

- In Pure strategies, have no Nash Equilibrium
- Mixed Strategies Nash Equilibrium

$p$ : Probability that $P_1$ does W (work)
$1-p$ : Probability that $P_1$ does S (Shirk)

$E_2(M) = E_2(I)$

$p \cdot 1 + (1-p) \cdot 2 = p \cdot 2 + (1-p) \cdot 1$

$2 - p = p + 1$, $2p = 1$, $p = \frac{1}{2}$

$r$ : Probability that $P_2$ does M (Monitoring)
$1-r$ : Probability that $P_2$ does I (Ignoring)

$E_1(M) = E_1(I)$

$1 \cdot r + (1-r) = 0 \cdot r + 2(1-r)$, $1 = 2 - 2r$, $2r = 1$, $r = \frac{1}{2}$

Nash Equilibrium $(p^*, r^*) = (0.5, 0.5) = ((0.5, 0.5), (0.5, 0.5))$

employee choose $p$ such that Boss indifferent between his pure strategies.

**Question 3** (Trembling hand)

Consider the following sequential game between Player 1 and Player 2:

- Player A moves first and chooses L1 or R1.
- If A chooses R1, the game ends immediately with the payoff (2, 2).
- If A chooses L1, then Player B moves and chooses between L2 and R2:

  L2 leads to a payoff of (1, 0).

  R2 leads to a payoff of (3, 1).

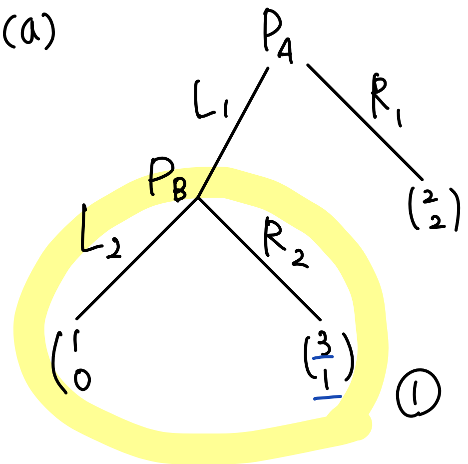
a) Draw the extensive form (game tree) of the game.

b) Which Nash equilibria exist, and which solution results from backward induction?

c) Which Nash equilibrium is Pareto optimal?

d) Is the solution obtained by backward induction in part (a) also trembling-hand perfect?

(a)




(b) Backward induction

$(L_1, R_2)$ is Nash Equilibrium

$(3, 1)$

(c)

| $P_A$ \ $P_B$ | $L_2$ | $R_2$ |
|---|---|---|
| $L_1$ | 1, 0 | 3, 1 |
| $R_1$ | 2, 2 | 2, 2 |

(3,1), (2,2)

Both are pareto optimal.

None is dominated.

We can't make sure one player better off, without making the other worse off.

(d) (3,1) is always trembling-hand perfect. Since $R_2$ is dominant strategy

So, Looking at $P_A$.

$\varepsilon$ : Probability that $P_B$ does not play $R_2$

$E_A(L_1) = (1-\varepsilon)\cdot 3 + \varepsilon \cdot 1 = 3 - 2\varepsilon$

$E_A(R_1) = (1-\varepsilon)\cdot 2 + \varepsilon \cdot 2 = 2$

$E_A(L_1) > E_A(R_1)$ , $3 - 2\varepsilon > 2 \rightarrow \frac{1}{2} < \varepsilon$

$L_1, R_2$ is trembling-hand perfect for $P_A$ if $P_B$ trembling with $\varepsilon < \frac{1}{2}$

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# Advanced Microeconomics, winter term 2025/26

## Exercise 3

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**Question 1 (Bargaining over two indivisible objects)**
Consider a variant of the ultimatum game, with indivisible units. Two people use the following procedure to allocate two desirable identical indivisible objects. Person 1 proposes an allocation (both objects go to person 1, both go to person 2, one goes to each person), which the other person then either accepts or rejects. In the event of rejection, neither person receives either object. Each person cares only about the number of objects she obtains. Hence the payoffs are (2,0) if both objects go to person 1, (0,2) if both go to person 2, and (1,1) if one goes to each person.

a) Represent the game in normal form and find all pure-strategy Nash equilibria.
   (Hint: If you find it difficult to write down the strategies, you may start by drawing the game tree for question b.)
b) Construct an extensive game that models this situation.
c) What kind of game is it (static/dynamic, complete/incomplete, perfect/imperfect information)?
d) How many terminal nodes and how many information sets does the game have?
e) How many pure strategies does each player have?
f) Suppose that if person 2 is indifferent, she accepts the proposal. Find the subgame perfect equilibria.
g) Now suppose instead that if person 2 is indifferent, she rejects the proposal. Find the subgame perfect equilibria.
h) Is there any outcome that is generated by a Nash equilibrium but not by any subgame perfect equilibrium (from questions f. and g.)?

Q1 ultimatum game

$P_1, P_2$

A $(2,0)$ B $(0,2)$ C $(1,1)$

reject $(0,0)$

| | | $P_2$: A | |
|---|---|---|---|
| $P_1$ | A | 2,0 | 0,0 |
| | B | 0,2 | 0,0 |
| | C | 1,1 | 0,0 |

$S_1 = \{(2,0), (0,2), (1,1)\}$

$S_2 = \{(A,A,A), (A,A,R), (A,R,A), (A,R,R), (R,A,A), (R,A,R) (R,R,A), (R,R,R)\}$

$\{(2,0); (AAA), (A.RA), (AAR). (ARR)\}$

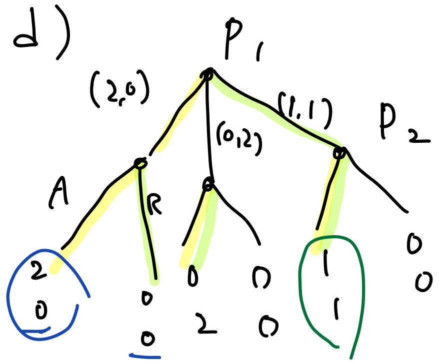
$\{(0,2); (R.A,R)\}$

$\{(1,1)\} (R,A,A), (RR A)$

7

c) Dynamic complete perfect

d)




4 nodes → 4 info set

e) $P_1 \rightarrow 3$ pure strategies

$P_2 \rightarrow 2^3$ pure strategies

f) $(2,0)$

$P_2$ is accept indifference, so $P_1$ $(2,0)$ since want to maximize payoff,

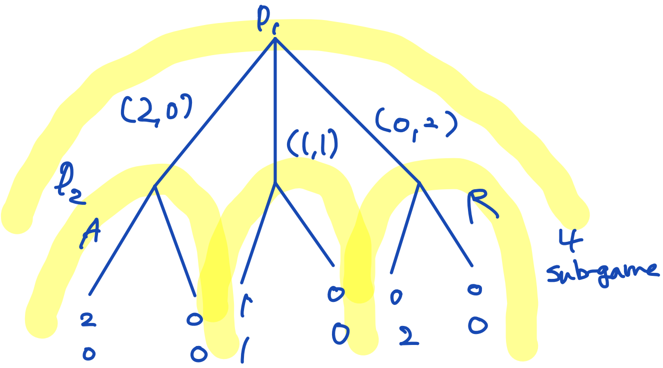
g) $(1,1)$

$P_2$ reject is indifference, $P_1$ choose $(1,1)$ that is the maximized payoff to $P_1$

h) $(0,2)$

$(0,2)$ is NE but not SPE (from question f. and. g)

a) Represent the game in normal form and find all pure-strategy Nash equilibria.
(Hint: If you find it difficult to write down the strategies, you may start by drawing the game tree for question b.)




$S_1 = \{(2,0),(1,1),(0,2)\}$

$S_2 = \{(A,A,A),(A,A,R),(A,R,A),(A,R,R)$
$(R,R,R),(R,R,A)\ (R,A,R),(R,A,A)\}$

↳ $2^3 = 8$ 개의 pure strategies

| $P_1$ \ $P_2$ | AAA | AAR | ARA | ARR | RAA | RAR | RRA | RRR | |
|---|---|---|---|---|---|---|---|---|---|
| (2,0) | 2,0 | 2,0 | 2,0 | 2,0 | 0,0 | 0,0 | 0,0 | 0,0 | 2SPNE |
| (1,1) | 1,1 | 1,1 | 0,0 | 0,0 | 1,1 | 1,1 | 0,0 | 0,0 | |
| (0,2) | 0,2 | 0,0 | 0,2 | 0,0 | 0,2 | 0,0 | 0,2 | 0,0 | 9NE |

c) Dynamic game : Have 2 stages, not playing simultaneously. (not at the same time)

Complete information : know all of the pay off.
(possible action, possible outcome)

Perfect information : Every nodes are singleton
(Imperfect : 2 or more nodes are sharing same info)

* Static game can't be perfect info but can be complete info

d) 6 terminal nodes

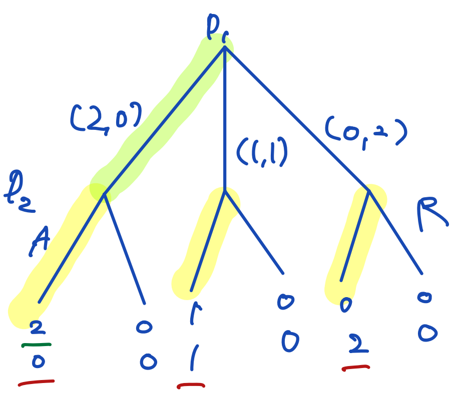
4 info-sets

e) $S_1 = \{(2,0),(1,1),(0,2)\}$

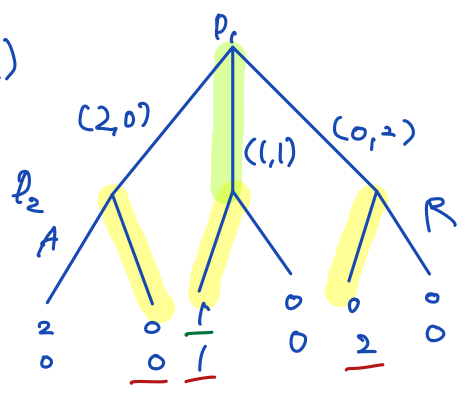
$S_2 = \{(A,A,A),(A,A,R),(A,R,A),(A,R,R)$
$(R,R,R),(R,R,A)\ (R,A,R),(R,A,A)\}$

f) (SPNE $\Rightarrow$ NE, $\not\Leftarrow$)

SPNE = {(2,0), AAA}
→ (2,0)




g)




SPNE = {(1,1), RAA} → (1,1)

h) (0,0), (0,2)

**Question 2 (Price guarantee)**

Consider two hardware stores that are in competition with each other. Both decide at the same time whether to charge a high price ($p_H$) or a low price ($p_L$) (for simplicity, we assume that the hardware stores sell only one product). Overall, the companies make the highest profits if they both charge high prices. Hardware store 1 is the dominant company in the market, hardware store 2 is a smaller competitor. If hardware store 2 undercuts the price of the other hardware store, it gains many new customers and therefore profits from this. If, on the other hand, hardware store 1 undercuts the competitor's price, it only gains a few new customers and these cannot compensate for the losses due to the lower price. This results in the following payout matrix:

| | | Hardware store 2 $p_H$ | Hardware store 2 $p_L$ |
|---|---|---|---|
| Hardware store 1 | $p_H$ | 3,2 | 0,3 |
| | $p_L$ | 2,0 | 1,1 |

a) Determine the Nash equilibrium of this game. Interpret it briefly from the customer's point of view.

Now consider the following extension. In stage 1 of the game, hardware store 1 can give a price guarantee that it will never charge a higher price for its product than the other hardware store. In stage 2, the two hardware stores again decide on their prices simultaneously, with the payouts for the respective price combinations being the same as in the matrix above. If hardware store 1 has given a price guarantee and still has a higher price than the competitor, then customers can (and will) demand the price difference back from hardware store 1. Therefore, the payoffs will be the same as if both stores had charged a low price.

b) Present this game in extensive form.
c) How many proper sub-games does the game have?
d) Write the (pure-) strategy sets for both players.
e) What kind of game is it (static/dynamic, complete/incomplete, perfect/imperfect information) and which equilibrium concept should be used?
f) What is the equilibrium of the game? Interpret it briefly.

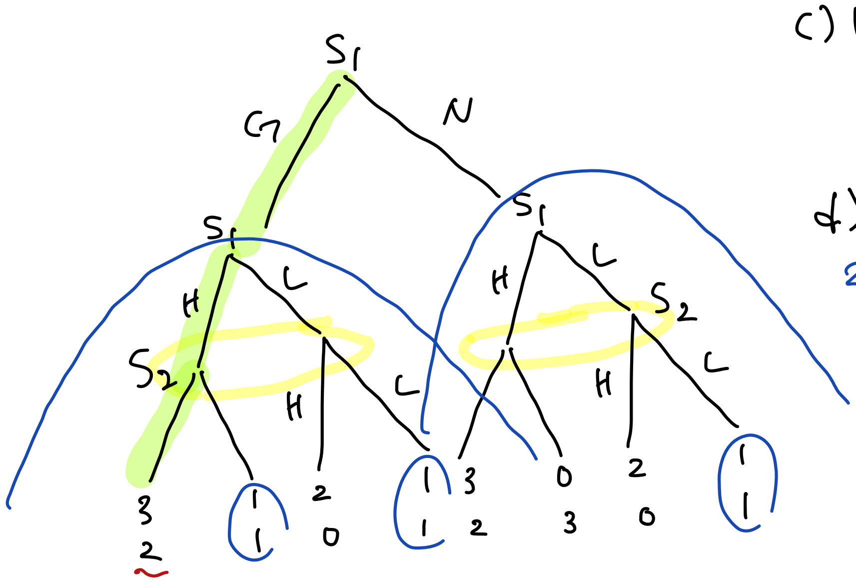
Q2

| | | $S_2$ | |
|---|---|---|---|
| | | H | L |
| $S_1$ | H | 3 2 | 0 3 |
| | L | 2 0 | 1 1 |

a) (L, L) NE

customers view, it is also better situation since both stores are choose lower price. they can buy as same and low price.

b)




c) proper sub-game : 2 (without whole game)

include whole game ⇒ 3 sub-games

d) $S_1 = ($ Stage1 $\in \{G, N\}$,

$2 \cdot 2^2 = 8$

Stag2_G $\in \{H, L\}$,

Stage_N $\in \{H, L\}$ )

$S_2 = ($ Stage3_G $\in \{H, L\}$,

$2^2 = 4$

Stag3_N $\in \{H, L\}$ )

$S_2 = \{P_H P_H, P_H P_L, P_L P_H, P_L P_L\}$

$S_1 = \{G P_H P_H, G P_H P_L, G P_L P_H, G P_L P_L, N P_H P_H, N P_H P_L, N P_L P_H, N P_L P_L\}$

e) Dynamic : Before choose the High or Low profit, by both stores $S_1$ decide price guarantee first. (total 2 step)

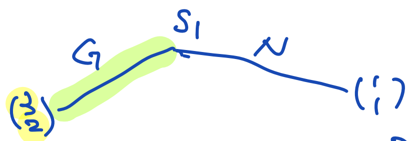
complete info : Both stores know each others payoff

imperfect info : stage3 is sharing info, $S_2$ doesn't know which has chosen in stage2 by $S_1$ ( $S_1$ and $S_2$ are decide at the same time)

SPNE : This is dynamic game therefore find a equilibrium through the backward induction. But each sub-games has decided at the same time. In sub-game have to use Nash Equilibrium.

G : sub game (L, L) (H, H)       NG : sub game (L, L)

(H, H) is pareto superior to (L, L) → more plausible




→ SPNE ( $G P_H P_L$ , $P_H P_L$ )

→ (3, 2)

f) In this game (L, L) is the equilibrium.

whether $S_1$ choose Price Guarantee or Not, always (L, L) is the equilibrium.

If $S_1$ decided do the Price Guarantee, choose the 'L' is always better option to $S_2$.

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# Advanced Microeconomics, winter term 2025/26

## Exercise 4

Please solve the exercises below by Wednesday, December 3rd. We will discuss them in our exercises (see Stud.IP). To obtain a bonus point, you have to upload your answers as a single pdf in the StudIP folder "Student Solutions Exercise 3". The **document name should start with your surname**. If you prepared the answer in a group of up to 3 students, please only submit one document that contains the names of all contributing students. The DEADLINE is December 3rd, 7:30 so that we have a chance to quickly scroll through your submissions to suggest one for presentation.

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**Question 1 (Grim Trigger)**

Consider the infinitely repeated game with discount factor $\delta < 1$ of the following variant of the Prisoner's Dilemma:

| | | Player 2 | | |
|---|---|---|---|---|
| | | $L$ | $C$ | $R$ |
| | $T$ | 6, 6 | −1, 7 | −2, 8 |
| Player 1 | $M$ | 7, −1 | 4, 4 | −1, 5 |
| | $B$ | 8, −2 | 5, −1 | 0, 0 |

a) For which values of the discount factor $\delta$ can the players support the pair of actions $(M, C)$ played in every period?
b) For which values of the discount factor $\delta$ can the players support the pair of actions $(T, L)$ played in every period?
c) Why is your answer different from that for (a)?

시험범위 제외

**Question 2 (Cournot Duopoly and Bayesian Nash-Equilibrium)**

Consider a Cournot duopoly operating in a market with inverse demand $P(Q) = a - Q$, where $Q = q_1 + q_2$ is the aggregate quantity on the market.

Both firms have constant unit costs and no fixed costs. The unit costs of each firm can be either high, $c_H$, or low, $c_L$, where $c_H > c_L$. Each firm knows its own unit costs. However, each firm only knows that the other firm has probability $\frac{1}{2}$ of having unit costs of $c_H$ and probability $\frac{1}{2}$ of having unit costs of $c_L$. All of this is common knowledge. The two firms simultaneously choose their quantities.

a) Specify the type space $T$, from which nature draws.
b) Specify firm 1's expected payoff for the cases where it has high costs and low costs. (Hint: You can do that separately, but it is recommended to use an indicator $j = H, L$ so that you have to write only one profit function that covers both cases.)
c) Determine the best-response function of firm 1 for the cases where it has high costs and low costs. Do the same for firm 2. (Hint: See previous hint, determine the first order condition of the expected payoff function and exploit the fact that the two firms are symmetric.)
d) Find the Bayesian Nash-Equilibrium of this game. (Hint: Symmetry also helps when you determine the solution from the 4 best-response functions).

Q1

$P_2$

$P_1$

| | L | C | R |
|---|---|---|---|
| T | $\boxed{6,6}$ | -1, 7 | -2, 8 |
| M | 7, -1 | 4, 4 | -1, 5 |
| B | 8, -2 | 5, -1 | 0, 0 |

$\delta < 1$

(M,C)=(4,4) 항상 이 보상을 받기 위한 δ discount factor

a) $4+\delta 4+\delta^2 4+\cdots = 4\sum_{t=1}^{\infty}\delta^{t-1} = 4\alpha = 4\frac{1}{1-\delta}$ , $\alpha = \frac{1}{1-\delta}$

suppose both stick to the grim trigger strategy

Seq payoff : 4, 4, 4 ···

present value $4+4\delta+4\delta^2+\cdots = 4\sum^{\infty}\delta^{t-1} = \frac{4}{\delta-1}$

b) ( B,R ) = ( 0,0 )

( 5, 0, 0, ··· )

$5+0\delta+0\delta^2 = 5$

$\frac{4}{1-\delta} \le 5$

a) Seq of payoff 5, 0, 0 ···

P.V $5+0\delta+0\delta^2+\cdots = 5$

deviation does not payoff if $\frac{4}{1-\delta} \geq 5$ , $\frac{1}{5} \le \delta$

1) if $\delta \geq \frac{1}{5}$ then grim trigger is a NE (proof Step1)

2) playing (B,R) in every period one sided deviation does not payoff

(Pl. 2) : worse off when playing L or C

Pl. 1 : M or T

→ grim trigger is a SPNE if $\delta \geq \frac{1}{5}$

c) From (b) Each player's pay-off difference is smaller than (a).

(a was '4', b was '2')

depends on pay-off difference of coorporate and deviate is affect of discount factor

b) $6.6.6 \cdots$ / $6 + 6\delta + 6\delta^2 + \cdots = 6\sum^{\infty} \delta^{t-1} = 6 \cdot \frac{1}{1-\delta}$

$8.0.0 \cdots$ / $8 + 0\delta + 0\delta^2 + \cdots = 8$

Deviation does not payoff is $\frac{6}{1-\delta} \geq 8$, $\frac{1}{4} \leq \delta$

1) If $\delta \geq \frac{1}{4}$ then grim trigger is a NE (proof step 1)

2) playing (B,R) in every period one sided deviation does not payoff
(Pl.2): worse off when playing L or C
Pl.1 : M or T
→ grim trigger is a SPNE if $\delta \geq \frac{1}{4}$

c)

Q2.

a) type $T = \{(C_H, C_H), (C_H, C_L), (C_L, C_H), (C_L, C_L)\}$

b) Firm 1's expected pay off, given its own type $C_j$ ($j = H$ or $L$)

$$E\{\pi_1(q_1)\} = \underset{P(C_L)}{\frac{1}{2}}\{(a - q_1 - q_2^L)q_1 - C_j q_1\} + \underset{P(C_H)}{\frac{1}{2}}\{(a - q_1 - q_2^H)q_1 - C_j q_1\}$$

$q_2^L$, $q_2^H$ are the equilibrium outputs of firm 2 when it has low or high cost, respectively.

$\pi = P \cdot q_{1j} - C_j q_{1j}$ $\qquad Q = q_1 + q_2$ $\qquad P(Q) = a - Q = a - q_1 - q_2$

$$\pi_1^j = P(C_L) \cdot P \cdot q_{1j} + P(C_H) \cdot P \cdot q_{1j} - C_j q_{1j}$$

$$= \underset{\frac{1}{2}}{P(C_L)}\, \underbrace{(a - q_{1j} - q_{2L})\, q_{1j}}_{aq_{1j} - q_{1j}^2 - q_{2L}q_{1j}} + \underset{\frac{1}{2}}{P(C_H)}\,(a - q_{1j} - q_{2H})\, q_{1j} - C_j q_{1j}$$

FOC $\frac{d\pi_1^j}{d q_{1j}} = \frac{1}{2}(a - 2q_{1j} - q_{2L}) + \frac{1}{2}(a - 2q_{1j} - q_{2H}) - C_j = 0$ $\quad )\times 2$

$$2a - 4q_{1j} - q_{2L} - q_{2H} - 2C_j = 0$$

$$q_{1j} = \frac{2a - q_{2L} - q_{2H} - 2C_j}{4}, \quad q_{2j} = \frac{2a - q_{1L} - q_{1H} - 2C_j}{4}$$

c) Firm 1 (maximizes the expected payoff with respect to $q_1$)

$$q_1^*(c_j) = \arg\max E\{\pi_1(q_1)\}$$

$$= \frac{a - c_j - \frac{1}{2}(q_2^L + q_2^H)}{2}$$

Firm 2 -> apply the same structure, flipping indices

d) The Bayesian Nash Equilibrium occurs where each type's best response is optimal, given beliefs about the other firm's cost type.
Using symmetry, solve the coupled best-response functions:

$$q_L = \frac{a - c_L - \frac{1}{2}(q_L^* + q_H^*)}{2} \qquad q_H = \frac{a - c_H - \frac{1}{2}(q_L^* + q_H^*)}{2}$$

These linear equations can be solved for $q_L^*$ and $q_H^*$, yielding the Bayesian Nash Equilibrium quantities for each type.

unknowns : $q_{1L}, q_{1H}, q_{2L}, q_{2H}$, (symmetry) $q_{1L} = q_{2L}$ $q_{1H} = q_{2H}$

$$q_{1H} = \frac{2a - q_{1H} - q_{1L} - 2c_H}{4} \qquad q_{1L} = \frac{2a - q_{1L} - q_{1H} - 2c_L}{4}$$

$$4q_{1H} = 2a - q_{1H} - q_{1L} - 2c_H \qquad 5q_{1L} = 2a - q_{1H} - 2c_L$$

$$(1)\ 5q_{1H} = 2a - q_{1L} - 2c_H \qquad q_{1H} = 2a - 5q_{1L} - 2c_L$$

$$(2)\ 5q_{1H} = 10a - 25q_{1L} - 10c_L$$

$$(2)-(1) \rightarrow 0 = 8a - 24q_{1L} - 10c_L + 2c_H$$

$+24q_{1L}$

$$24q_{1L} = 8a - 10c_L + 2c_H$$

$\frac{1}{24}$

$$q_{1L}^* = \frac{1}{3}a - \frac{5}{12}c_L + \frac{1}{12}c_H = q_{2L}^*$$

insert in (1)

$$5q_{1H} = 2a - \left(\frac{1}{3}a - \frac{5}{12}c_L + \frac{1}{12}c_H\right) - 2c_H$$

$$5q_{1H} = \frac{5}{3}a + \frac{5}{12}c_L - \frac{25}{12}c_H$$

$\frac{1}{5}$

$$q_{1H}^* = \frac{1}{3}a + \frac{1}{12}c_L - \frac{5}{12}c_H = q_{2H}^*$$

INSTITUT FÜR VOLKSWIRTSCHAFTSLEHRE
FINANZWISSENSCHAFT

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# Advanced Microeconomics, winter term 2025/26

## Exercise 5

Please solve the exercises below by Wednesday, December 17th. We will discuss them in our exercises (see Stud.IP). To obtain a bonus point, you have to upload your answers as a single pdf in the StudIP folder "Student Solutions Exercise 5". The **document name should start with your surname**. If you prepared the answer in a group of up to 3 students, please only submit one document that contains the names of all contributing students. The DEADLINE is December 17th, 7:30 so that we have a chance to quickly scroll through your submissions to suggest one for presentation.

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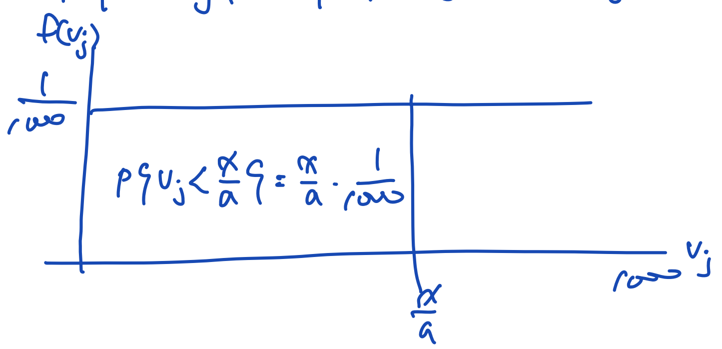
### Question 1 (First-price sealed bid auction with $n$ players)

Suppose you want to buy a house that is sold by way of a first-price sealed bid auction. In contrast to the model in the lecture, there are more than 2 players. Players simultaneously and independently submit bids $\boldsymbol{b_i}, \boldsymbol{i = 1, \dots, n}$. The house is awarded to the highest bidder who must pay his bid, denoted $x$. Your conjecture is that each player bids a fraction $a$ of her valuation, i.e. $\boldsymbol{b_i = a v_i}$, where $\boldsymbol{v_i}$ is a player's valuation. Moreover, suppose that you have the following information: You know the number of persons $\boldsymbol{n}$ that bid for the house. You believe the other bidders' values are independently and uniformly distributed over the interval $[\boldsymbol{0}, \boldsymbol{1}]$, where 1 stands for €1.000.000.

a) What is the probability that your bid $x$ exceeds the bid of player 2?
b) What is the probability that your bid $x$ exceeds the bids of player 2 and 3?
c) Now, what is the probability that your bid exceeds that of all other bidders?
d) What bid should you choose?
e) Explain intuitively how your bid changes with the number of bidders $n$.
f) Now suppose that your own valuation for the house is €800.000 and that there are 5 other bidders. How much should you bid?

$P\{x > b_j\} = P\{x > a v_j\} = P\{v_j < \frac{x}{a}\} = \frac{x}{1000\, a}$



$$\lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{1}{n}\right) f\left(\frac{k}{n}\right) = \int_0^1 f(x)\,dx$$

more than 2 players / simultaneously / independently submit bids $b_i$, $i = 1 \cdots n$ denoted $x$

$b_i = a v_i$ $\quad v_i$ = players valuation (type)

$[0,1]$ € 1,000,000

a) What is the probability that your bid $x$ ($b_1$) exceeds the bid of player 2 ($b_2$)? $P(b_1 > b_2)$ $n=2$ (include me)

$b_1 = x$, $b_2 = a v_2$

$$P(x > \overset{b_2}{a v_2}) \;/\; P\left(\frac{x}{a} > v_2\right) = \int_0^{\frac{x}{a}} f(v_2)\,dv_2 = \int_0^{\frac{x}{a}} 1\,dv_2 = \left(\frac{x}{a} - 0\right) = \frac{x}{a} = P\left(v_2 < \frac{x}{a}\right) = P(x > a v_2) = \frac{x}{a}$$

$f(v_2) = 1$, $v_2 \in \{0,1\}$

$$P\left(v_2 < \frac{b_1}{a}\right)^{n-1} = \left(\frac{b_1}{a}\right)^{n-1}$$

$$P(x > b_2) = \frac{x}{a} = \frac{x}{\frac{n-1}{n}} = \frac{nx}{n-1}$$

$$E\{\pi_1(b_1|v_1)\} = (v_1 - b_1)\left(\frac{b_1}{a}\right)^{n-1}$$

FOC

$$D \quad \left(\frac{1}{a}\right)^{n-1}\{v_1(n-1)b_1^{n-2} - n b_1^{n-1}\}$$

$$v_1(n-1) - n b_1 = 0$$

$$b_1 = \frac{n-1}{n} v_1$$

$$b_1 = a v_1 \quad \therefore a = \frac{n-1}{n}$$

b) What is the probability that your bid $x$ exceeds the bids of player 2 and 3? $n=3$

$b_1 = x$, $b_2$, $b_3$

assume all players follow the Symmetric Nash Equilibrium (SNE) strategy

$b_i = a v_i$, $a = \frac{n-1}{n}$

$$P(b_1 > b_2),\ P(b_1 > b_3) \Rightarrow P(x > a v_2,\ x > a v_3) = P\left(\frac{x}{a} > v_2,\ \frac{x}{a} > v_3\right)$$

$$P\left(\frac{x}{a} > v_2\right) \times P\left(\frac{x}{a} > v_3\right) = \frac{x}{a}\cdot\frac{x}{a} = \left(\frac{x}{a}\right)^2 = \left(\frac{nx}{n-1}\right)^2$$

c) Now, what is the probability that your bid ($b_1$) exceeds that of all other bidders?

$n-1$ = other players (except me)

$$P(b_1 > b_2, b_3, \cdots b_n) = \left(\frac{x}{a}\right)^{n-1}$$

$$\therefore \left(\frac{nx}{n-1}\right)^{n-1}$$

d) What bid should you choose?

maximize my payoff, SNE $= b_i = a v_i$

$$\max_x \; \underbrace{(v_i - x)}_{\text{surplus}}\underbrace{\left(\frac{x}{a}\right)^{n-1}}_{\text{Prob of winning}}$$

$$E\{\pi(x|v_1)\} = \underbrace{(v_1 - x)}_{\text{payoff if win}} \cdot \underbrace{P(x)}_{P(\text{win with bid } x)}$$

$$= (v_1 - x)\cdot\left(\underbrace{\frac{nx}{n-1}}_{\frac{x}{a}}\right)^{n-1}$$

$$\left(\frac{n}{n-1}\right)^{n-1}\{(v_1-x)\cdot x^{n-1}\} \xrightarrow{FOC} \left(\frac{n}{n-1}\right)^{n-1}\{(-x^{n-1})+(v_1-x)(n-1)\cdot x^{n-2}\}=0$$

$$(v_1-x)\left(\frac{x}{a}\right)^{n-1}$$

FOC

$$-1\cdot\left(\frac{x}{a}\right)^{n-1}+(v_1-x)(n-1)\left(\frac{x}{a}\right)^{n-2}=0$$

$$\div\left(\frac{x}{a}\right)^{n-2} \Big|\ -\left(\frac{x}{a}\right)+(v_1-x)(n-1)=0$$

$$\times a \Big|\ -x+(v_1-x)(n-1)=0$$

$$(v_1-x)(n-1)=x$$

$$v_1(n-1)-x(n-1)=x$$

$$v_1(n-1)=x(n-1)+x$$

$$\frac{v_1(n-1)}{n}=x^*$$

$$0=\cancel{\left(\frac{n}{n-1}\right)^{n-1}}(-x^{n-1})+(v_1-x)(n-1)x^{n-2}$$

$$\frac{1}{(x^{n-2})} \Rightarrow -x^{n-1}+(v_1-x)(n-1)\cancel{x^{n-2}}=0$$

$$\Rightarrow -x+(v_1-x)(n-1)=0$$

$$-x+v_1(n-1)-x(n-1)=0$$

$$v_1(n-1)=x(n-1)+x$$

$$v_1(n-1)=nx$$

$$\underline{x^*=\frac{v_1(n-1)}{n}} \quad \left(\frac{n-1}{n}=a\right)$$

Bayesian NE

whe the number of bidders (n) increase, the fraction $\frac{n-1}{n}$ is get closer to 1. meaning the optimal bid gets closer to my valuation.

This is because competition increase, and the necessity to bid aggresively to win forces the bid higher.

e) Explain intuitively how your bid changes with the number of bidders $n$.

$$n\to\infty, \quad \lim_{n\to\infty}\left(\frac{n-1}{n}\right)=1$$

option 1 : $n=2$, $x^*=\frac{1}{2}v_i$

$n=3$, $x^*=\frac{2}{3}v_i$

$n=100$, $x^*=\frac{99}{100}v_i$

option 2 :

$$\frac{\partial x}{\partial n}=n^{-2}v_i=\frac{v_i}{n^2}>0$$

In this scenario, to win must bid my true valuation ($v_1=x^*$)
If I bid anything less, there will almost certainly be an opponent whose valuation is between my bid and my true value, and that opponent will win.

f) Now suppose that your own valuation for the house is €800.000 and that there are 5 other bidders. How much should you bid? $n=6$

$v_1=800{,}000$, $n=6$ $\qquad 1=1{,}000{,}000 \Rightarrow 800{,}000=0{,}8$

$$x^*=\frac{n-1}{n}v_1 \Rightarrow \frac{5}{6}\times\frac{8}{10}=\frac{4}{6}=\frac{2}{3}$$

optimal bid is $\frac{2}{3}$

$$\frac{2}{3}\times 1{,}000{,}000 \approx €666{,}666$$

Should bid about €666,666

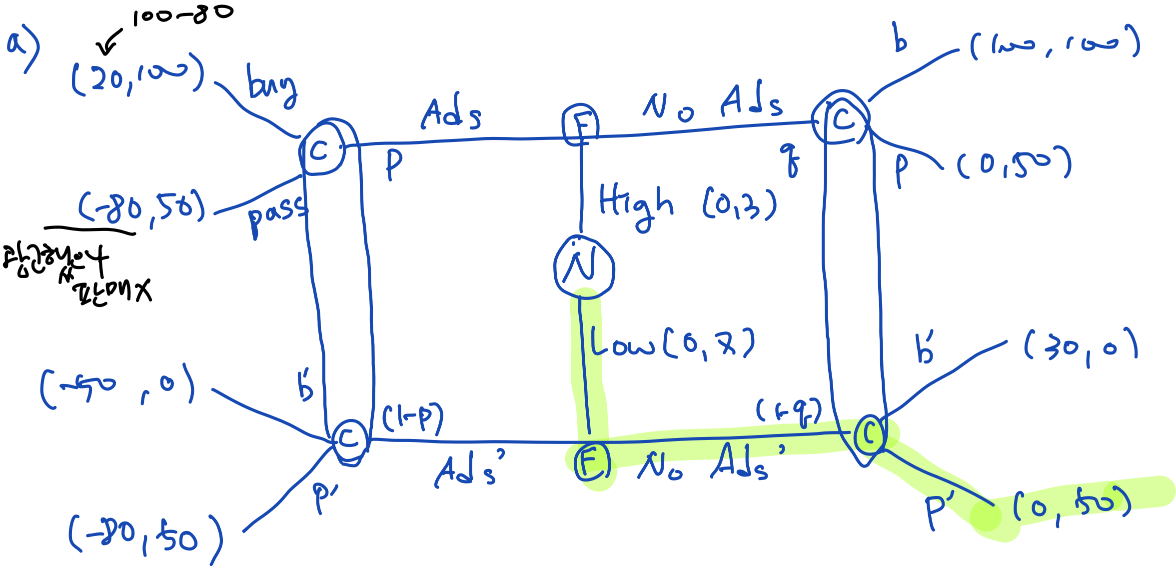
**Question 2 (Signalling - Sequential Games of Incomplete Information)**

30% high
70% Low

Consider a firm offering a product – let's say an advent calendar – that may be either high quality or low quality (probability 0.3 vs. 0.7). Only the firm knows its own type, but it is common knowledge that 30% of the products are of high quality and 70% are of low quality. The consumer earns a payoff (benefits minus costs) of 100 from purchasing the high-quality product but 0 from purchasing a low-quality product, and she earns 50 if she buys something else with known quality. Furthermore, the firm earns a profit of 100 from selling a high-quality product (since it will have gained a loyal customer), 30 from selling a low-quality product (since it will be a onetime sale and the customer avoids purchasing from the firm ever again), and 0 from not making a sale. Prior to the customer making the purchase, the firm can take a costly action: it can pay 80 for a 30-second Super Bowl ad. Assume that advertising does not increase product awareness, and a priori customers do not consider advertising to be particularly credible. Hence the only purpose of the Super Bowl ad is to serve as a signal.

a) Write the game in extensive form, similarly as we have done it in the job market signaling model in the lecture.
b) Find any pooling perfect Bayesian equilibria.
c) Is there a separating perfect Bayesian equilibrium in which the high-quality firm advertises and the low-quality firm does not?
d) What exactly does the Super Bowl ad signal, and why does the signal work?

a)




광고없이 판매라서
이익100

$p=1$
$q=1$

b)

1) pooling Equilibrium candidate (Ads, Ads')

Pooling NE 에서 는,

$p=0.3$ ← (Nature 선택이 30%, Nature 이후에 광고 할 일 결정에서 광고한다는 확률은 1
(0,3×1)
p=(…)

$V_c(b) = 0.3 \times 100 + 0.7 \times 0 = 30$ (고객이 받게되는 기대이익?)
buy

$V_c(p) = 0.3 \cdot 50 + 0.7 \cdot 50 = 50$
pass

→ Best response of C: pass

→ Firm has incentive to deviate to No Ads since $0 > -80$

→ (Ads, Ads) no equilibrium

2) Pooling Equilibrium (No Ads, No Ads')

$q = 0.3 = (0.3 \times 1)$

$V_c(b) = 30$

$V_c(p) = 50$ ← best response of C: pass

⇒ (No Ads, No Ads') (pass, pass') is a pooling NE (0,50)

c)

(Consumer)

| | buy | Pass | |
|---|---|---|---|
| high / Ads | 100 | 50 | → Choose to buy if seeing an ad |
| Low / No Ads | 0 | 50 | → passes if no ad |

→ Consumer buys only if seeing ad.

(Firm)

| | Ad | No Ads |
|---|---|---|
| high | 20 | 0 |
| Low | −80 | 0 |

→ Firm doesn't have incentive to switch

⇒ seperating eq {(Ad, no Ads'), (buy, pass')}

Seperating is a pareto improblem

(0,50) ⟶ {(0,100)
(No Ads, No Ads') ⟶ (20, 100)}

d)

1.Q : ad = high quality

2.Q : For the high quality firm, advertising is strictly better : 20>0

Low quality firm, advertising is strictly worse ; -50<0

ad carries signal for high quality

# Advanced Microeconomics, winter term 2025/26

# Exercise 6

Please solve the exercises below by Wednesday, January 14th. We will discuss them in our exercises (see Stud.IP). To obtain a bonus point, you need to upload your answers as a single pdf in the StudIP folder "Student Solutions Exercise 6". The **document name should start with your surname**. If you prepared the answer in a group of up to 3 students, please only submit one document that contains the names of all contributing students. The DEADLINE is January 14th, 7:30 so that we have a chance to quickly scroll through your submissions to suggest one for presentation.

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**Question 1** (Principal Agent)
The laundry "Cleanex" is located at a river downstream to the sewage plant "Dirtex". Last week, part of the clothes were spoiled during the laundry process so that the customers had to be compensated, leading to additional costs $d$. Cleanex is convinced that a defect in the sewage plant Dirtex is responsible for the problem. Therefore, Cleanex authorizes the law firm "Lawex" to sue Dirtex for damages.

We make the following assumptions: The effort cost of the of the law firm are a convex function $c(e) = e^2$ and efforts $e \in [0; 0.5d]$. Cleanex pays Lawex conditional on the indemnification $x$ that the court grants to Cleanex, where $x$ is uniformly distributed in the interval $[0, 2e]$. In particular, payments are $w(x) = a + bx, b \in [0, 1]$. Furthermore, $w(x) \geq 0$, i.e. the law firm cannot be hold liable for a bad job. Both Cleanex and Lawex are risk-neutral. Finally, if Lawex does not accept the contract offered by Cleanex, it gets a reservation income of 0.

a) Set up the Principal-Agent problem.
b) Why does the liability constraint $w(x) \geq 0$ imply that $a \geq 0$?
c) Solve the Principal-Agent problem for the optimal contract elements $(a, b, e)$.
d) Calculate the rent which Lawex receives due to the liability constraint.
e) Compare the size of this rent with the effort cost.

**Question 2** (Risk and Incentives in Contracting)
Consider a principal–agent relationship where a risk-neutral principal hires an agent whose effort is not observable.

We make the following assumptions: The agent's effort level is $e \geq 0$ and effort costs are given by $c(e) = 0.5 \cdot e^2$. Output is $Q = 10 \cdot e + u$ where $u$ is a random variable with $E[u] = 0$. Only output $Q$ is observable. The principal offers a **linear incentive contract** $p = s + r \cdot Q$ where $s$ is a fixed payment and $r$ measures the strength of incentives. The agent is **risk-averse**. His expected utility (certainty-equivalent form) is $EU = E[p] - 0.5 \cdot e^2 - Kr^2$ where $K > 0$ measures the agent's degree of risk aversion. If the agent does not accept the contract, he obtains a reservation utility $u_0$.The principal maximizes expected profits.

a) Set up the Principal-Agent problem.
b) Solve the agent's incentive constraint. Derive the agent's optimal effort choice $e(r)$.
c) Show that the participation constraint binds in the optimal contract and derive the fixed payment $s(r)$.
d) Determine the optimal incentive parameter $r^*$chosen by the principal and the induced effort level $e^*$.
e) Calculate and discuss how the strength of incentives, $r$, depends on the agent's degree of risk aversion.

Q1

a) principal = cleanex Agent = Lavex

Cost of agent : $c(e) = e^2$ , $\in \{0, \frac{1}{2} \cdot d\}$

Cost & outcome : $x \in \{0, 2 \cdot e\}$

Payment to agent : $w(x) = a + b \cdot x$ , $b \in \{0, 1\}$
$w(x) \geq 0$

since x is uniform distributed
in intries $\{0, 2e\}$
$\rightarrow E[x] = e$
$\rightarrow E[w(x)\} = a + b \cdot e$

a) agent's problem
$\max_e \; E[w(x)] - c(e)$
$\Rightarrow \max_e \; a + b \cdot e - e^2$

" constraint "

participation constraint
Agent must at least get reservation income.
$a + b \cdot e - e^2 \geq 0$ "PC"

Limited liability
$w(x) = a + b \cdot x \geq 0$ "LL"

Principal's problem
principal gets cost & outcome but pays $w(x)$
$\max \; E[x] - E[w(x)]$
$\max \; e - (a + b \cdot e)$

b) $\rightarrow w = a + b \cdot x$
if $x = 0$ , then "LL" would violated for any $a < 0$

c) step 1 : solve agent "IC"
Agent maximize
$\max_e \; a + b \cdot e - e^2$
FOC : $b - 2e = 0$
$e = \frac{b}{2}$

step 2 : use "LL" and "participation costraint"
to choose a
PC : $a + be - e^2$

Now use $e = \frac{b}{2}$ and assume $a = 0$ since that is best for the principal

$0 + b \cdot \frac{b}{2} - (\frac{b}{2})^2 \geq 0$

$\frac{b^2}{2} - \frac{b^2}{4} \geq 0$

$\frac{b^2}{4} \geq 0$ → even with zero fixed payment ($a \geq 0$), the agent wants to participate.

$a^* = 0$

step3: pricipal chooses $b$:

with $a=0$ and $e=\frac{b}{2}$, expected profits

$\max_b \; e - (a + be)$

$\max_b \; \frac{b}{2} - b \cdot \frac{b}{2} = \frac{b}{2} - \frac{b^2}{2}$

FOC: $\frac{1}{2} - b = 0$ , $b^* = \frac{1}{2}$

d) Rent that agent receives due to "LL"

we knows $a^* = 0$ , $e = \frac{b^*}{2}$

$a + be - e^2 \Rightarrow 0 + b \cdot \frac{b}{2} - (\frac{b}{2})^2 \Rightarrow \frac{b}{2} - \frac{b^2}{4} \Rightarrow \frac{b^2}{4}$

rent of the agent $\frac{b^2}{4}$

$\frac{(\frac{1}{2})^2}{4} = \frac{1}{16} > 0$

e) compare size of rent with effort cost:

$c(e) = e^2 = (\frac{b}{2})^2 = \frac{b^2}{4}$ ⇒ cost = rent

→ income is double the cost (rent = income − cost)

Q2.

a) $Q = 10e + u$ , $E[u] = 0$ , $e \geq 0$

$P = s + r \cdot Q$

$Eu = E[P] - 0.5e^2 - r^2k$ , $k \geq 0$

Since $E[u] = 0$

$E[Q] = 10e$

$E[P] = s + 10er$

(agent expect utility)

$Eu = s + 10er - 0.5e^2 - r^2k$

$s + 10er - 0.5e^2 - r^2k \geq u$

↳ "PC"

b) Incentive constraint

$\max_e \; s + 10er - 0.5e^2 - r^2k$ → "IC"

FOC: $10r - e = 0$ , $e^* = 10r$

c) Participation constraint : $Eu \geq u_0$

using : $e = 10r$

$u_0 = S + 10r \cdot (10r) - 0{,}5 \cdot (10r)^2 - r^2 k$

$= S + 100r^2 - 50r^2 - r^2 k$

$S = u_0 + r^2 \cdot (k - 50)$

d) principal chooses r:

$\max_S \; E[Q] - E[P]$

$= 10e - (S + 10er)$

using: $e = 10r$, $S = u_0 + r^2(k-50)$

$\max_r \; 10 \cdot 10r - (u_0 + r^2(k-50) + 10r \cdot 10r)$

$= 100r - 100r^2 - u_0 - r^2(k-50)$

$= 100r - 100r^2 - u_0 + 50r^2 - r^2 k$

$= 100r - 50r^2 - u_0 - r^2 k$

FOC: $100 - 100r - 2rk = 0$

$r(100 + 2k) = 100$

$r^* = \frac{100}{100 + 2k} = \frac{50}{50 + k}$

$e^* = 10r^* = \frac{500}{50 + k}$

e) $r = \frac{50}{50+k}$ $\rightarrow$ r is strictly decreasing in k

$\Rightarrow$ higher risk aversion means contract with more insurance.