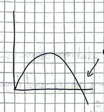
Exercise 1

1\. a) The utility from drinking nothing (0) is 0.
If a decision maker drinks $a=2$ then, if he has the largest $\theta=6$ the pay-off is: $6 \cdot 2 - 4 \cdot 2^2 = -4$
and it is easy to see that a decision maker with smaller values of θ will obtain a more negative payoff from consuming $a=2$

graphically:



analytically:

$V(a) = \theta a - 4a^2 = 0$

plug in $\theta = 6$ for the heaviest drinker

$6a - 4a^2 = 0 \quad | + 4a^2$

$6a = 4a^2 \quad | : a$

$6 = 4a \quad | : 4$

$a = \frac{6}{4} = 1{,}5$

solve for a

=> only wors off if he drinks more than 1,5 litres.

b) $V(a) = \theta a - 4a^2$

FOC: $\frac{\partial V}{\partial a} = \theta - 8a \overset{!}{=} 0 \quad | + 8a \; | : 8$

$a^* = \frac{\theta}{8}$

plug in: $\theta = 1$, $a^* = \frac{1}{8}$ $\qquad$ $\theta = 4$, $a^* = \frac{4}{8} = \frac{1}{2}$

c) This follows from the solution above in $a^* = \frac{\theta}{8}$ is increasing in θ and large persons have larger θ.

d) No. Even the largest person with $\theta = 6$ should only consume $a = \frac{6}{8} = \frac{3}{4}$ litres of a bottle of wine.

just plugging in

	Work hard (p2)	Goof off
Work hard	2, 2	0, 3
(p1) Goof off	3, 0	1, 1

2\. a) -> 3, 1 > 2, 0 goof off is dominant for P1

-> 3, 1 > 2, 0 goof off is dominant for P2

=> goof, goof is the equilibrium of dominant strategies

b)

P1 \ P2	W	G
W	4, 4	0, 3
G	3, 0	1, 1

mutual best responses

c) no equilibrium in dominant strategies.

Exercise 2

1\. a) $\max_{e_i, e_j} V = (a + be_j)e_i - \frac{1}{2}e_i^2$

player 1:

FOC: $\frac{\partial V}{\partial e_i} \overset{!}{=} 0$

$\frac{\partial V}{\partial e_i} = a + be_j - e_i = 0 \quad | + e_i$

$e_i^* = a + be_j \qquad R_i(e_j) = a + be_j$

player 2:

$\frac{\partial V}{\partial e_j} \overset{!}{=} 0$

Assuming symmetry:

$e_j^* = a + be_i \qquad R_j(e_i) = a + be_j$

b) insert and solve:

$e_i^* = a + b(a + be_i)$

$e_i = a + ba + b^2 e_i \quad | (\;)$

$e_i = a(1+b) + b^2 e_i \quad | - b^2 e_i$

$e_i - b^2 e_i = a(1+b) \quad | (\;)$

$e_i(1 - b^2) = a(1+b) \quad | : (1-b^2)$

$e_i = \frac{a(1+b)}{1-b^2} \quad$ | 3rd binomial formula $a^2 - b^2 = (a-b)(a+b) \Rightarrow 1 - b^2 = (1-b)(1+b)$

$e_i = \frac{a\cancel{(1+b)}}{(1-b)\cancel{(1+b)}}$

$e_i = \frac{a}{1-b}$ $\quad$ assuming symmetry => $e_j = \frac{a}{1-b}$

Nash-Equilibrium $\left(\frac{a}{1-b} ; \frac{a}{1-b}\right)$

b) alternative, easier solution

b) The game is symmetric so that in equilibrium $e_1 = e_2$

$e_i = a + be_i \quad | - be_i$

$e_i - be_i = a \quad | (\;) e_i$

$e_i(1-b) = a \quad | : (1-b)$

$e_i = \frac{a}{1-b}$ -> Nash equilibrium

c) $W = V_i + V_j \quad$ | Sum of payoffs

$W = (a + be_j)e_i - \frac{1}{2}e_i^2 + (a + be_i)e_j - \frac{1}{2}e_j^2$

simplify:

$W = ae_i + be_j e_i - \frac{1}{2}e_i^2 + ae_j + be_i e_j - \frac{1}{2}e_j^2$

$W = ae_i + 2be_i e_j - \frac{1}{2}e_i^2 + ae_j - \frac{1}{2}e_j^2$

FOC: $\frac{\partial W}{\partial e_i} \overset{!}{=} 0$

$\frac{\partial W}{\partial e_i} = a + 2be_j - e_i = 0 \quad | + e_i$

$e_i^* = a + 2be_j$

symmetrical problem => $e_i^* = e_j^* = e^*$

$e_j^* = a + 2be_i$

solve for e:

$e^* = a + 2be^* \quad | - 2be$

$e^* - 2be^* = a \quad | (\;)$

$e^*(1 - 2b) = a \quad | : (1-2b)$

$e^* = \frac{a}{1-2b}$

Nash: $\frac{a}{1-b}$ $\qquad$ Efficient: $\frac{a}{1-2b}$

$0 < b < \frac{1}{2}$, $\quad 1 - b \; \cancel{>} \; 1 - 2b$ $\quad <$

In the Nash equilibrium players dont invest enough effort compared to the efficient solution. This happens as a result of individual utility maximization which ignores the positive externality of one players effort on the other players payoff given in the function

----

Exercise 3

1\. a) Normal Form -> simultaneous

player 1: proposes $S1 = \{A, B, C\}$

A: both to P1 (2, 0)

B: both to P2 (0, 2)

C: one each (1, 1)

player 2: responds by accepting (A) or rejecting (R)

$S2 = \{AAA, RRR, AAR, \ldots, RRA\}$ (strat. needs to be complete response, invisible acts = simultaneous)

-> complete plan of play

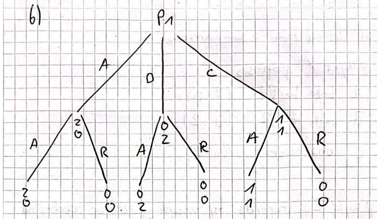
1.) AAA
2.) RRR
3.) AAR
4.) ARA
5.) ARR
6.) RAA
7.) RAR
8.) RRA

-> because p1 goes first:

-> to find equilibria given P2 strat would P1 decide?

Player 1	AAA	RRR	AAR	ARA	ARR	RAA	RAR	RRA
A (2,0)	2,0	0,0	2,0	2,0	2,0	0,0	0,0	0,0
B (0,2)	0,2	0,0	0,2	0,0	0,0	0,2	0,2	0,0
C (1,1)	1,1	0,0	0,0	1,1	0,0	1,1	0,0	1,1

b)



c) - dynamic game -> moves by the players are sequential

- complete information - both players have info about:
  - ↳ game structure
  - ↳ all possible actions by each player
  - ↳ both payoff functions
  - ↳ preferences (utility max)

- perfect information - at every decision node the player knows the complete history of all prior moves.

d) -> 6 terminal nodes

-> 4 info sets: P1's initial set of 3 options followed by P2's responses dependent on which option P1 picked (because of perfect information)

e) pure strat = complete game - plan

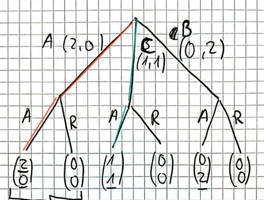
-> P1 has 3 pure strategies

-> P2 has 8 possible responses and therefore 8 pure strats -> see a)

f) SPNE $\underset{\not\leftarrow}{\Rightarrow}$ NE

every subgame perfect equilibrium is a Nash-equilibrium but not the other way around.

-> we have 4 subgames



-> P2 is indifferent here

g) P2 rejects now when indifferent

-> P1 chooses B to max his own utility because his payoff for B would be 0.

=> SPNE (C, RAA) = (1, 1)

---

3. a)

Hrs	U_F	U_O
12	10	5
9-12	2	2
< 9	-10	-5

1) set of players
$J = \{$ Felix, Oskar $\}$

2) set of pure strategies
$S_i = \{3h, 6h, 9h\}$

3.) set of payoff functions

$U_F(9,9) = 1$
$U_F(6,9) = 4$
$U_F(3,9) = 7$
$U_F(9,6) = 1$
$U_F(6,6) = 4$
$U_F(3,6) = -1$
$U_F(9,3) = 1$
$U_F(6,3) = -4$
$U_F(3,3) = -13$

$U_O(9,9) = -4$
$U_O(9,6) = 4$
$U_O(9,3) = -4$
$U_O(6,9) = -1$
$U_O(6,6) = -1$
$U_O(6,3) = -4$
$U_O(3,9) = 2$
$U_O(3,6) = -4$
$U_O(3,3) = -8$

Felix \ Oskar	9h	6h	3h
9h	1, -4	1, -1	1, 2
6h	4, -4	4, -1	-4, -1
3h	7, -4	-1, 4	-13, -8

---

2. a) pure strategies:

Employee \ Boss	M	I
W	1, 1	1, 2
S	0, 2	2, 1

Boss M -> Employee W
Boss I -> Employee S
Employee W -> Boss I
Employee S -> Boss M

=> There is no Nash equilibrium in pure strategies.

① \ ②		p: M	1-p: I
r	W	1, 1	1, 2
1-r	S	0, 2	2, 1

mixed:

① chooses r such that ② is indifferent between his pure strategies

$E_2(M) = E_2(I)$
$r1 + (1-r)2 = r2 + (1-r)$
$r + 2 - 2r = 2r + 1 - r$
$2 - r = r + 1$
$r = \frac{1}{2}$

② chooses p such that ① is indifferent between his pure strategies

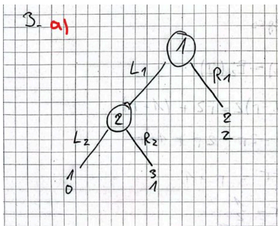
$E_1(W) = E_1(S)$
$p1 + (1-p)\cdot 1 = p\cdot 0 + (1-p)\cdot 2$
$p + 1 - p = 2 - 2p$
$p = \frac{1}{2}$

Nash-eq. $(r^*, p^*) = (\frac{1}{2}, \frac{1}{2})$
$= [(0{,}5; 0{,}5), (0{,}5; 0{,}5)]$

d) everything works if the arguments are good

---

3. a)



b)

	L_2	R_2
L_1	1, 0	3, 1
R_1	2, 2	2, 2

-> Nash equilibria: L_1, R_2 (3,1)
R_1, L_2 (2,2)

Backwards-Induction: only (3,1).

h) again: SPNE $\Rightarrow$ / $\not\Leftarrow$ NE
-> Nash-equilibria which are not subgame perfect are not credible.
-> (0,2) and (0,0) are NE which are not SPNE
=> answer is yes.

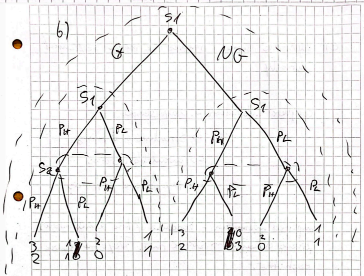
---

2. a)

1 \ 2	P_H	P_L
P_H	3, 2	0, 3
P_L	2, 0	1, 1

-> The nash-eq. is great for customers. Competitive pricing ensures they get low prices in both stores.

b)



c) 3 subgames, one starts after G is chosen, one after NG is chosen and then the entire game itself.

d) The price guarantee only affects payoffs not strategies

Store 1: (initial choice) (price choice) $\{$ G $P_L P_L$, G $P_H P_H$, G $P_H P_L$, G $P_L P_H$, NG $P_L P_L$, NG $P_H P_H$, NG $P_L P_H$, NG $P_H P_L$ $\}$

Store 2: two info-sets, G set, NG set.
$\{ P_H P_H$,
P_H, P_L, - high if G, low if NG
P_L, P_H, - low if G, high if NG
$P_L P_L \}$

e) dynamic game - moves are made sequential initially

complete information - players know:
↳ structure and timing
↳ available actions
↳ payoffs and preferences

imperfect information - because of the simultaneous pricing decision in stage two. They cant observe each other.

-> solve by backward induction, repeatedly elimination, to find subgame perfect Nash equilibrium.

f) NG-subgame -> (1,1) see a)

G-subgame:

	P_H	P_L
P_H	3, 2	1, 1
P_L	2, 0	1, 1

-> $P_H P_H$ (3,2) > $P_L P_L$ (1,1)
-> $P_H P_H$ is pareto superior to $P_L P_L$ and therefore more plausible.

b) $S_O(6h) \geq S_O(9h)$

↳ Oskars strategy to claim 9h is weakly dominated by 6h

↳ therefore erase Oskars 9h column as he would never choose it

$S_F(6h) > S_F(3h)$

↳ erase 3h row for Felix

$S_O(3h) \geq S_O(6h)$

↳ erase Oskar 6h

$S_F(9h) > S_F(6h)$

=> Outcome of the game is (9,3) with a corresponding utility of (1,2).

Oskar

Felix	9h	6h	3h
9h	1, -4	1, -1	1, 2
6h	4, -4	4, -1	-4, -1
3h	7, -4	-1, 4	-13, -8

Oskar

mutual best responses

Felix	9h	6h	3h
9h	1, -4	1, -1	$\boxed{1, 2}$
6h	4, -4	$\boxed{4, -1}$	-4, -1
3h	$\boxed{7, -4}$	-1, 4	-13, -8

c) no equilibrium, there are other mutual best responses.

c) Both are pareto optimal, none is dominated. You cannot make one player better off without making the other worse off

d) trembling hand perfect: (3, 1) – L_1, R_2

L_1, R_2 is always trembling hand perfect for player two since R_2 is a dominant strategy.

So looking at player 1:

ε = probability that player 2 does not play R_2.

$E_1(L_1) = (1-\varepsilon)\cdot 3 + \varepsilon 1$
$= 3 - 2\varepsilon$

$E_1(R_1) = (1-\varepsilon)\cdot 2 + \varepsilon \cdot 2$
$= 2 - 2\varepsilon + 2\varepsilon$
$= 2$

when is $E_1(L_1) > E_1(R_1)$

$3 - 2\varepsilon > 2 \quad | + 2\varepsilon ; -2$

$2\varepsilon < 1 \quad | : 2$

$\varepsilon < \frac{1}{2}$

=> L_1, R_2 is trembling hand perfect for player 1 if player 2 trembles with $\varepsilon < \frac{1}{2}$.

-> by backwards induction S1 choose G because the utility is greater

=> SPNE $(\underbrace{G\,P_H P_L}_{S1}, \underbrace{P_H P_L}_{S2})$

-> given the guarantee both stores charge high prices, without the guarantee low prices.

-> The price guarantee removes Store 2's incentive to undercut Store 1, allowing both firms to sustain high prices.
-> not cool for the customers.

c) - dynamic game -> moves by the players are sequential

- complete information - both players have info about:
  - ↳ game structure
  - ↳ all possible actions by each player
  - ↳ both payoff functions
  - ↳ preferences (utility max)

- perfect information - at every decision node the player knows the complete history of all prior moves.

Exercise 4

Micro Exercise 4.

a)

P_1 \ P_2	L	C	R
T	6,6	-1,7	-2,8
M	7,-1	4,4	-1,5
B	8,-2	5,-1	$\boxed{0,0}$

1.) find NE of the stage game

stage game NE = (B,R) = (0,0)

→ Grim Trigger strat.:

player play (M,C) as long as everyone cooperates if anyone deviates they revert to the Nash eq. of the stage game forever.

as long as they cooperate payoffs are:

player 1:

$V_1 = 4 + \delta 4 + \delta^2 4 + \dots + \delta^n 4$ | infinite geometric series

rewrite as: $4 \sum_{t=1}^{\infty} \delta^{t-1} = \frac{4}{1-\delta} = \frac{1}{4} \frac{1}{(1-\delta)}$

⇒ payoffs are (4,4) so its symmetrical

player 2:

$V_2 = \frac{4}{1-\delta}$

Player 1 best deviation is B = 5

$V_1 = 5 + \delta 0 + \delta^2 0 + \dots$

player 2 doesnt cooperate anymore → Nash eq.

player 1 doesnt deviate if:

$\frac{4}{1-\delta} \geq 5 \quad | \cdot (1-\delta)$

$4 \geq 5(1-\delta)$

$4 \geq 5 - 5\delta \quad | -5 \,;\, (-1) \to$ switches sign

$1 \leq 5\delta \quad | :5$

$\frac{1}{5} \leq \delta$

one sided deviation from the grim trigger strategy does not pay off for d >= 1/5 grim trigger strat. is SPNE

b) TL (6 6) → symmetrical

$V_1 = \frac{6}{1-\delta}$

$V_2 = \frac{6}{1-\delta}$

player 1 best deviation is B = 8

so he doesnt deviate if:

$\frac{6}{1-\delta} \geq 8 \quad | \cdot (1-\delta)$

$6 \geq 8 - 8\delta \quad | -8$

$-2 \geq -8\delta \quad | \cdot (-1)$

$2 \leq 8\delta \quad | :8$

$\frac{2}{8} \leq \delta$

$\frac{1}{4} \leq \delta$ → stronger temptation to deviate than a)

if d >= 1/4 then grim trigger strat. is a SPNE

c) The temptation to deviate / cheat is stronger in b) than in a) because the relative gain from deviating is greater

2 > 1

d smaller -> impatient
d bigger -> more patient
therefore: in b) the short term benefit is greater, we have to be a bitmore patient deviating

2. inverse demand: $P(Q) = a - Q$

$Q = q_1 + q_2$

a) $T = \{(c_H, c_H); (c_H, c_L); (c_L, c_H); (c_L; c_L)\}$

→ each type space has a probability of $\frac{1}{4}$

static game, incomplete information

b) general profit:

$\Pi_i = \underbrace{q_i \cdot P(Q)}_{\text{price} \cdot \text{quantity}} - \underbrace{c_i q_i}_{\text{cost}}$

insert:

$\Pi_i = q_i(a - Q) - c_i q_i \quad | \, Q = q_1 + q_2$

Exercise 5

1. a) $b = a v_i \qquad v_i \sim \text{uniform}[0,1]$

looking for $x > \underbrace{b_2}_{= a v_2}$

$x > a v_2 \quad | :a$

$v_2 < \frac{x}{a}$

uniformly distributed:

$P(v_2 < \frac{x}{a}) = \frac{x}{a}$ assuming $\frac{x}{a} < 1$ (probability)

b) looking for $x > a v_2 \cap x > a v_3 \quad | \Leftrightarrow$

$v_2 < \frac{x}{a} \cap v_3 < \frac{x}{a}$

due to independence in the distribution:

$P(A \cap B) = P(A) \cdot P(B)$

$P(v_2 < \frac{x}{a} \cap v_3 < \frac{x}{a}) = \underbrace{P(v_2 < \frac{x}{a})}_{= \frac{x}{a}} \cdot \underbrace{P(v_3 < \frac{x}{a})}_{= \frac{x}{a}}$

$\frac{x}{a} \cdot \frac{x}{a} = \left(\frac{x}{a}\right)^2$

c) $n-1$ other bidders

looking for: $x > a v_1 \cap x > a v_2 \cap x > a v_3 \cap \dots \cap x > a v_{n-1}$

$\Leftrightarrow v_1 < \frac{x}{a} \cap v_2 < \frac{x}{a} \cap \dots \cap v_{n-1} < \frac{x}{a}$

independence:

$P(v_1 < \frac{x}{a}) \cdot P(v_2 < \frac{x}{a}) \cdot \dots \cdot P(v_{n-1} < \frac{x}{a})$

$= \frac{x}{a} \cdot \frac{x}{a} \cdot \frac{x}{a} \cdot \dots \cdot \frac{x}{a} = \left(\frac{x}{a}\right)^{n-1}$

d) $E(\Pi) = P(\text{win}) \cdot \text{payoff} + P(\text{lose}) \cdot \text{payoff}$

$= \left(\frac{x}{a}\right)^{n-1} \cdot \underbrace{(v_i - x)}_{\text{valuation} - \text{price (bid)}} + \cancel{P(\text{lose})} \; 0$

FOC: $\frac{\partial E(\Pi)}{\partial x} = 0$ | product rule: $f'(x) \cdot g(x) + f(x) \cdot g'(x)$ and chain rule: outer · inner

$= \frac{1}{a} n-1 \cdot \left(\frac{x}{a}\right)^{n-2} \cdot (v_i - x) + \left(\frac{x}{a}\right)^{n-1} \cdot (-1) = 0$

$= \frac{n-1}{a} \left(\frac{x}{a}\right)^{n-2} (v_i - x) - \left(\frac{x}{a}\right)^{n-1} = 0 \quad | () \left(\frac{x}{a}\right)^{n-2}$

rewritten $\Leftrightarrow \left(\frac{x}{a}\right)^{n-2} \cdot \left(\frac{x}{a}\right)^1$

$= \underbrace{\left(\frac{x}{a}\right)^{n-2}}\underbrace{\left(\frac{n-1}{a}(v_i - x) - \frac{x}{a}\right)} = 0$

one of the factors must be 0 so the equation works.

assuming: $\left(\frac{x}{a}\right)^{n-2} \neq 0$

then: $\frac{n-1}{a}(v_i - x) - \frac{x}{a} = 0 \quad | \cdot a$

$(n-1)(v_i - x) - x = 0$

$(n-1)v_i - (n-1)x - x = 0$

$(n-1)v_i - nx + x - x = 0 \quad | + nx$

$(n-1)v_i = nx \quad | :n$

$\frac{n-1}{n} v_i = x^*$

recall: $x = a v_i \Leftrightarrow a = \frac{n-1}{n}$

e) $n = 20 \qquad v_i = 1{,}000{,}000 € = 1$

$x^* = \frac{20-1}{20} \cdot 1 = \frac{19}{20} = 0{,}95 = 95\%$ of the valuation

$n = 2$

$x^* = \frac{1}{2} = 0{,}5 = 50\%$

couldve also taken the derivative and see that it is > 0

→ bid more the more competitors there are as the fraction of the value that we are bidding optimally rises in n. But with rising n the profit margin is decreasing.

Exercise 6

Micro Exercise 6

1.

principal: Cleanex (min expected costs)

agent: Lawex (choose effort)

1.) principal offers contract to agent
$w(x) = a + bx \qquad b \in [0,1]$

2.) agent accepts or rejects (reservation utility = 0)

3.) agent chooses effort $e \in [0; 0{,}5d]$
with associated costs $c(e) = e^2$

4.) court decides $x \sim U[0, 2e]$

5.) agent receives payment $w(x) = a + bx$

simplification

since $x \sim U(0, 2e) \qquad E(x \sim U) = \frac{a+b}{2}$

$E(x) = \frac{0 + 2e}{2} = \frac{\cancel{2}e}{\cancel{2}} = e$

$E[w(x)] = E[a + bx] = \underbrace{E[a]}_{= a} + E[bx]$

$= a + b \underbrace{E[x]}_{e} = a + be$

a) Agents problem:

expected utility = revenue - costs
choose effort e to max. expected utility

$\max_e E[w(x)] - c(e)$

$\Rightarrow \max_e a + be - e^2$ | IC (incentive constraint)

Participation constraint (PC):

Agent must atleast get reservation income

$E[w(x)] - C(e) \geq 0$ here u=0

limited liability / payment constraint (LL)

$w(x) = a + bx \geq 0$

→ cant have payment of zero

principals problem:

→ gets court outcome x but pays w(x)

$\max E(x) - E[w(x)]$

$\max \; e - (a + be)$

b) substitute payment function w(x) into the liability constraint (LL)

$w(x) \geq 0$

$a + bx \geq 0$

→ constraint must hold for all values of $x \in [0, 2e]$ and $b \in [0,1]$ rewritten as $0 \leq b \leq 1$

insert in $x = 0$ situation: court grants nothing

$w(0) = a + b(0) = a$

or $b = 0$

$w(0) = a + 0 \cdot 0 = a$

= $a \geq 0$ is required for the (given) liability constraint to hold

⇒ the payment must still be non-negative when the court grants the worst outcome. a is a fixed salary.

alternative b) solution from the tutorial

b) $w \neq a + bx$ i.e. no grant

if $x = 0$, then LL would be violated for any $a < 0$

c) step 1: solve agents IC

$$= q_i(a - q_i - q_j) - c_i q_i$$

own other

$$= q_i a - q_i^2 - q_i q_j - c_i q_i \quad | ( ) q_i$$

$$\Pi_i = q_i(a - q_i - q_j - c_i)$$

Firm 1 high cost c_H: probability $\frac{1}{2}$ for $c_H = c$

$$E(\Pi_1) = \underbrace{\frac{1}{2} q_1 (a - q_1 - q_2^H - c_H)}_{\text{prob. other firm has high cost } c_H} + \underbrace{\frac{1}{2} q_1 (a - q_1 - q_2^L - c_H)}_{\text{prob. other firm has } c_L}$$

Firm 1 low cost $c = c_L$

$$E(\Pi_1) = \frac{1}{2} q_1 (a - q_1 - q_2^H - c_L) + \frac{1}{2} q_1 (a - q_1 - q_2^L - c_L)$$

c) maximize $E(\Pi_i)$ with respect to q_i

simplify: $E(\Pi_1 | c_1 = c_j) = \frac{1}{2} q_1 (a - q_1 - q_2^H - c_j) + \frac{1}{2} q_1 (a - q_1 - q_2^L - c_j) \; | ( ) \frac{1}{2} q_1$

$$= \frac{1}{2} q_1 \left[(a - q_1 - q_2^H - c_j) + (a - q_1 - q_2^L - c_j) \right]$$

$$= \frac{1}{2} q_1 (2a - 2q_1 - q_2^H - q_2^L - 2c_j)$$

$$= q_1 \left(a - q_1 - \frac{q_2^H - q_2^L}{2} - c_j\right)$$

$$= q_1 a - q_1^2 - q_1 \frac{q_2^H - q_2^L}{2} - q_1 c_j$$

FOC:

$$\frac{\partial E(\Pi_1)}{\partial q_1} = a - 2q_1 - \frac{q_2^H - q_2^L}{2} - c_j = 0 \quad | + 2q_1$$

$$2q_1 = a - \frac{q_2^H - q_2^L}{2} - c_j \quad | : 2$$

$$q_1 = \frac{a - \frac{q_2^H - q_2^L}{2} - c_j}{2} = \frac{a - c_j}{2} - \frac{q_2^H - q_2^L}{4}$$

-> symmetric, same for Firm 2:

$$q_2 = \frac{a - c_j}{2} - \frac{q_1^H - q_1^L}{4}$$

d) due to symmetry we can assume

$q_1^H = q_2^H$
$q_1^L = q_2^L$ -> both firms produce the same when they have the same costs

$\Rightarrow q_1^H = q_2^H = q^H$
$q_1^L = q_2^L = q^L$

insert and solve:

$$q^H = \frac{a - c_H}{2} - \frac{q^H + q^L}{4} \quad | \cdot 4$$

$$4q^H = 2(a - c_H) - (q^H + q_L)$$

$$4q^H = 2a - 2c_H - q^H - q_L \quad | + q^H$$

$$5q^H = 2a - 2c_H - q_L \quad | : 5$$

$$q^H = \frac{2a - 2c_H - q_L}{5}$$

symmetry:

$$q_L = \frac{2a - c_L - q_H}{5}$$

substitute q_L into q^H: (easier with earlier equations)

~~[illegible]~~ $5q^H = 2a - 2c_H - q_L$

$$5q^H = 2a - 2c_H - \frac{2a - 2c_L - q^H}{5} \quad | \cdot 5$$

$$25q^H = 5(2a - 2c_H) - (2a - 2c_L - q_H)$$

$$25q^H = 10a - 10c_H - 2a + 2c_L + q^H \quad | - q_H$$

$$24q^H = 8a - 10c_H + 2c_L \quad | : 24$$

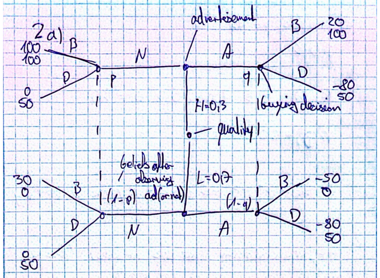
$$q^H = \frac{8a - 10c_H + 2c_L}{24} = \frac{4a - 5c_H + c_L}{12}$$

f) $n = 6$ $\quad V_i = 800{,}000 = 0{,}8$ in the distribution: „0,8 of 1"

$$x^* = \frac{6-1}{6} \cdot 0{,}8 = \frac{2}{3} \approx 0{,}667 = 66{,}7\%$$

$$0{,}667 \cdot 1{,}000{,}000 € = 666{,}667 €$$

2a)



b) pooling: type hiding

1.1 (Ads, Ads)

$p = 0{,}3$ $\quad p \quad (1-p)$

$V_c(B) = 0{,}3 \cdot 100 + 0{,}7 \cdot 0 = 30$

payoff consumer buys

$V_c(D) = 0{,}3 \cdot 50 + 0{,}7 \cdot 50 = 50$

don't buy

-> consumers best response is passing $50 > 30$

-> firm wants to deviate to no ads since $0 > -80$

↳ (Ads, Ads) can't be equilibrium.

2.) (no ads, no ads)

$q = 0{,}3$

$V_c(B) = 0{,}3 \cdot 100 + 0{,}7 \cdot 0 = 30$

$V_c(D) = 50$

$50 > 30$ -> best consumer response is don't buy

-> equilibrium (no ads, no ads), (don't buy, don't buy) -> (0, 50)

pooling

c) separating: type revealing

consumer

	buy	don't
high q.	100	50
low q.	0	50

-> buy when ad. observed
-> don't buy when no ad

firm:

	ad	no ad.
high q.	20	0
low q.	-50	0

-> firm does not have an incentive to switch

~~separating eq.~~

-2-

-> Separating eq.

$$[\overbrace{(ad, no\ ad)}^{\text{firm}}, \overbrace{(buy, no\ buy)}^{\text{consumer}}]$$

high q. $\quad$ low q. $\quad$ when ad $\quad$ no ad.

-> separating eq. is pareto improvement.

$$(0, 50) > \begin{Bmatrix} 0, 50 \\ 20, 100 \end{Bmatrix}$$

pooling eq.

d) ad -> high q., signal of confidence in their product

-> for the high q. firm advertising is strictly better $20 > 0$

-> for low q. firm it is strictly worse to advertise $-50 < 0$

-> signal of ad is credible

c) agent choose e to max. utility

$$\max_e U_A = a + be - e^2$$

FOC $\quad \frac{\partial U_A}{\partial e} = b - 2e = 0 \quad | + 2e \; | : 2$

$$e = \frac{b}{2} \quad \text{-> incentive constraint IC}$$

step 2: use LL and PC to choose a

substitute $e = \frac{b}{2}$ into the agents participation constraint (PC)

$$a + be - e^2 \geq 0$$

$$= a + b \cdot \frac{b}{2} - \left(\frac{b}{2}\right)^2 \geq 0$$

$$= a + \frac{b^2}{2} - \frac{b^2}{4} \geq 0$$

$$= a + \frac{2b^2}{4} - \frac{b^2}{4} \geq 0$$

-4-

$$= a + \frac{b^2}{4} \geq 0$$ | => cheapest way to satisfy PC is $a = 0$

-> even with zero fixed payment, the agent wants to participate

Step 3: principal chooses b:

with $a = 0$ and $e = \frac{b}{2}$

expected profits: $\max_b e - (a + be)$

principals payoff: $(a + be) - e = \Pi$

insert $a = 0$ and $e = \frac{b}{2}$

simplification, not necessarily required

$$\Pi = \left(0 + b \cdot \frac{b}{2}\right) - \frac{b}{2}$$

$$= \frac{b^2}{2} - \frac{b}{2} \quad | ( ) \frac{b}{2}$$

$$= \frac{b}{2}(1 - b)$$

maximize over b:

$$\Pi = \frac{b}{2}(1 - b) = \frac{b}{2} \cdot \frac{b^2}{2} \quad | ( ) \frac{1}{2}$$

$$\Pi = \frac{1}{2}(b - b^2) \quad | \text{ keep factor on inner derivative}$$

$$\frac{\partial \Pi}{\partial b} = \frac{1}{2}(1 - 2b) = 0$$

$$= \frac{1}{2} - b = 0 \quad | + b$$

$$b = \frac{1}{2}$$

recall $e^* = \frac{b}{2}$ insert

$$e^* = \frac{\frac{1}{2}}{2} = \frac{1}{4}$$

=> optimal contract is therefore:

$$a^* = 0, \; e^* = \frac{1}{4} \quad b^* = \frac{1}{2}$$

d) Rent that agent receives due to "LL"

agents utility / revenue / rent $\quad a + be - e^2$

$$= 0 + \frac{1}{2} \cdot \frac{1}{4} - \left(\frac{1}{4}\right)^2$$

$$= \frac{1}{8} - \frac{1}{16} = \frac{1}{16}$$

e) effort cost $\quad c(e) = e^2$

$$c(e) = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$$

=> rent = effort cost

income is double the cost, since rent = income - cost

2.a) $\quad Q = 10e + u \quad , \quad E[u] = 0$

$e \geq 0$

$$p = s + rQ$$

$$EU = E[p] - 0{,}5\, e^2 - r^2 K$$

now for q_L: <u>insert!</u>

$$5q_L = 2a - 2c_L - \frac{4a - 5c_H + c_L}{12} \quad | \cdot 12$$

$$60q_L = 12(2a - 2c_L) - (4a - 5c_H + c_L)$$

$$60q_L = 24a - 24c_L - 4a + 5c_H - c_L$$

$$60q_L = 20a - 25c_L + 5c_H \quad | : 60$$

$$q_L = \frac{20a - 25c_L + 5c_H}{60} = \frac{4a - 5c_L + c_H}{12}$$

$K > 0$

since $E[u] = 0$

$$E[Q] = 10e$$

$$E[p] = s + 10er$$

$$EU = s + 10re - 0{,}5e^2 - r^2K$$

$$\underbrace{s + 10re - 0{,}5e^2 - r^2K \geq u_0}_{PC}$$

→ incentive constraint (IC): agent chooses e to max. his own utility EU

→ participation constraint (PC): $EU \geq u_0$
agents utility needs to exceed reservation utility

→ liability constraint (LL): $s \geq 0$
→ no negative payment to the agent

b) optimal effort by the agent:

$$\max EU = \underbrace{E[p]}_{= s + r10e} - 0{,}5e^2 - Kr^2$$

$$EU = s + 10re - 0{,}5e^2 - Kr^2$$

$$FOC: \frac{\partial EU}{\partial e} = 10r - e \overset{!}{=} 0 \quad | + e$$

$$e = 10r$$

c) principal looks for the lowest possible s where the agent barely participates: $EU = u_0$

PC: $EU \geq u_0$ how the tutorial puts it

set equal:

$$s + 10re - 0{,}5e^2 - Kr^2 = u_0$$

substitute $e = 10r$

$$s + 10r(10r) - 0{,}5(10r)^2 - Kr^2 = u_0$$

$$s + 100r^2 - 50r^2 - Kr^2 = u_0$$

$$s + 50r^2 - Kr^2 = u_0 \quad | -50r^2; +Kr^2$$

$$s = Kr^2 - 50r^2 + u_0$$

d) optimal incentive r and effort e

insert $e = 10r$ and s into principals profit (Π)

$$\Pi = 10e - s - r10e$$

$$= 10(10r) - (Kr^2 - 50r^2 + u_0) - 10r(10r)$$

$$= 100r - Kr^2 + 50r^2 - u_0 - 100r^2$$

$$= 100r - Kr^2 - 50r^2 - u_0$$

$$FOC: \frac{\partial \Pi}{\partial r} = 100 - 2Kr - 100r = 0 \quad | +2Kr; +100r$$

$$100 = 2Kr + 100r \quad | ( )r$$

$$100 = r(2K + 100) \quad | : (2K+100)$$

$$r = \frac{100}{2K + 100} = \frac{50}{K + 50}$$

we know $e = 10r$, therefore

$$e = 10\left(\frac{50}{K+50}\right) = \frac{500}{K+50}$$

-9-

e) derive r with respect to K for marginal effects:

$$r = \frac{50}{K+50} = 50 \cdot \frac{1}{K+50} = 50 \cdot (K+50)^{-1}$$

$$\frac{\partial r}{\partial K} = -1 \cdot 50(K+50)^{-2} \cdot 1$$

$$= -50(K+50)^{-2}$$

$$= \frac{-50}{(K+50)^2} < 0$$

→ inverse relationship, r decreases in K.
This means as the agent becomes more risk-averse ($K\uparrow$) the incentive rate r decreases in the optimal contract.

↳ principal has to pay higher fixed/base salary

↳ trade-off where the principal may accept lower effort to avoid the high fix costs of a risk averse agent.