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INFORMATIK, WIRTSCHAFTS- UND RECHTSWISSENSCHAFTEN

Advanced Microeconomics, winter term 2025/26

Exercise 3

Please solve the exercises below by Wednesday, November 19th. We will discuss them in our exercises (see Stud.IP). To obtain a bonus point, you have to upload your answers as a single pdf in the StudIP folder "Student Solutions Exercise 3". The **document name should start with your surname**. If you prepared the answer in a group of up to 3 students, please only submit one document that contains the names of all contributing students. The DEADLINE is November 19th, 7:30 so that we have a chance to quickly scroll through your submissions to suggest one for presentation.

Question 1 (Bargaining over two indivisible objects)
Consider a variant of the ultimatum game, with indivisible units. Two people use the following procedure to allocate two desirable identical indivisible objects. Person 1 proposes an allocation (both objects go to person 1, both go to person 2, one goes to each person), which the other person then either accepts or rejects. In the event of rejection, neither person receives either object. Each person cares only about the number of objects she obtains. Hence the payoffs are (2,0) if both objects go to person 1, (0,2) if both go to person 2, and (1,1) if one goes to each person.

a) Represent the game in normal form and find all pure-strategy Nash equilibria. (Hint: If you find it difficult to write down the strategies, you may start by drawing the game tree for question b.)
b) Construct an extensive game that models this situation.
c) What kind of game is it (static/dynamic, complete/incomplete, perfect/imperfect information)?
d) How many terminal nodes and how many information sets does the game have?
e) How many pure strategies does each player have?
f) Suppose that if person 2 is indifferent, she accepts the proposal. Find the subgame perfect equilibria.
g) Now suppose instead that if person 2 is indifferent, she rejects the proposal. Find the subgame perfect equilibria.
h) Is there any outcome that is generated by a Nash equilibrium but not by any subgame perfect equilibrium (from questions f. and g.)?

Question 2 (Price guarantee)

Consider two hardware stores that are in competition with each other. Both decide at the same time whether to charge a high price (p_H) or a low price (p_L) (for simplicity, we assume that the hardware stores sell only one product). Overall, the companies make the highest profits if they both charge high prices. Hardware store 1 is the dominant company in the market, hardware store 2 is a smaller competitor. If hardware store 2 undercuts the price of the other hardware store, it gains many new customers and therefore profits from this. If, on the other hand, hardware store 1 undercuts the competitor's price, it only gains a few new customers and these cannot compensate for the losses due to the lower price. This results in the following payout matrix:

		Hardware store 2: p_H	Hardware store 2: p_L
Hardware store 1	p_H	3,2	0,3
	p_L	2,0	1,1

a) Determine the Nash equilibrium of this game. Interpret it briefly from the customer's point of view.

Now consider the following extension. In stage 1 of the game, hardware store 1 can give a price guarantee that it will never charge a higher price for its product than the other hardware store. In stage 2, the two hardware stores again decide on their prices simultaneously, with the payouts for the respective price combinations being the same as in the matrix above. If hardware store 1 has given a price guarantee and still has a higher price than the competitor, then customers can (and will) demand the price difference back from hardware store 1. Therefore, the payoffs will be the same as if both stores had charged a low price.

b) Present this game in extensive form.
c) How many proper sub-games does the game have?
d) Write the (pure-) strategy sets for both players.
e) What kind of game is it (static/dynamic, complete/incomplete, perfect/imperfect information) and which equilibrium concept should be used?
f) What is the equilibrium of the game? Interpret it briefly.

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a) Normal form - table

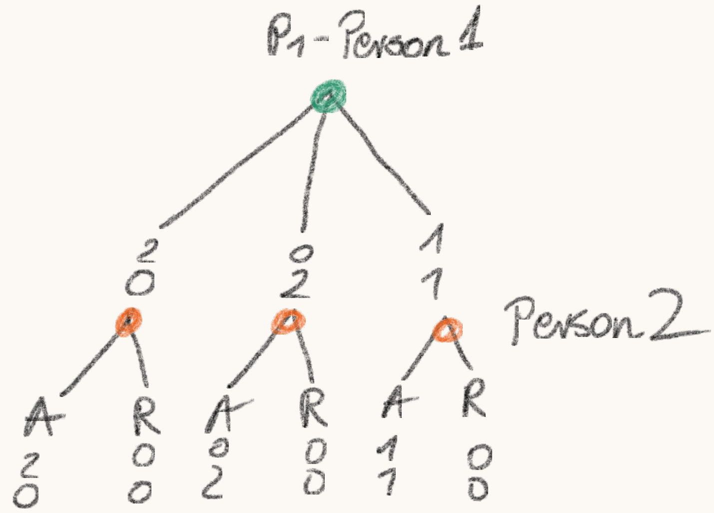
Person 2

Person 1	Accept	Reject
	2,0	0,0
	0,2	0,0
	1,1	0,0

Player 2

P_1								
2,0	2,0	2,0	20	2,0	0,0	0,0	0,0	0,0
1,1	1,1	1,1	00	00	11	11	00	00
0,2	0,2	0,0	0,2	02	02	00	02	00

b)



A - Accept
R - Reject

c) 1 Definicions:

- **Static vs. Dynamic**
  - Static: players move simultaneously or without knowledge of previous moves.
  - Dynamic: players move sequentially (one after the other), so the later players can condition on earlier moves.
- **Complete vs. Incomplete information**
  - Complete information: every player knows the structure of the game (players, strategies, payoffs).
  - Incomplete: some players lack knowledge about some aspect (e.g., types, payoffs).
- **Perfect vs. Imperfect information**
  - Perfect information: at every decision point the player whose turn it is **knows all previous moves** (no hidden moves).
  - Imperfect: some earlier moves are not observed.

This game:

- dynamic - bc 1 Person decides and only after that 2 Person decides
- complete information - both knows the payoffs for both and avaliable actions - it is common knowledge
- perfect information - bc 2 Player sees what proposal 1 Player made before deciding

d) Terminal nodes: each Player have 3 proposalss × 2 responces = 6 nodes

Information sets: 1 Player: 1 decision node

2 Player: 3 decision nodes

Total information sets: 3 + 1 = 4

e) How many pure strategies per person? $\longrightarrow$ what a player will do at every decision point in the whole game

1 Person: 3 pure strategies

2 Person: (For each of 3 proposals of 1 Player there are 2 opions {A, R})

$2 \cdot 2 \cdot 2 = 8$ pure strategies

f) Suppose that if person 2 is indifferent, she accepts the proposal. Find the subgame perfect equilibria. SPNE

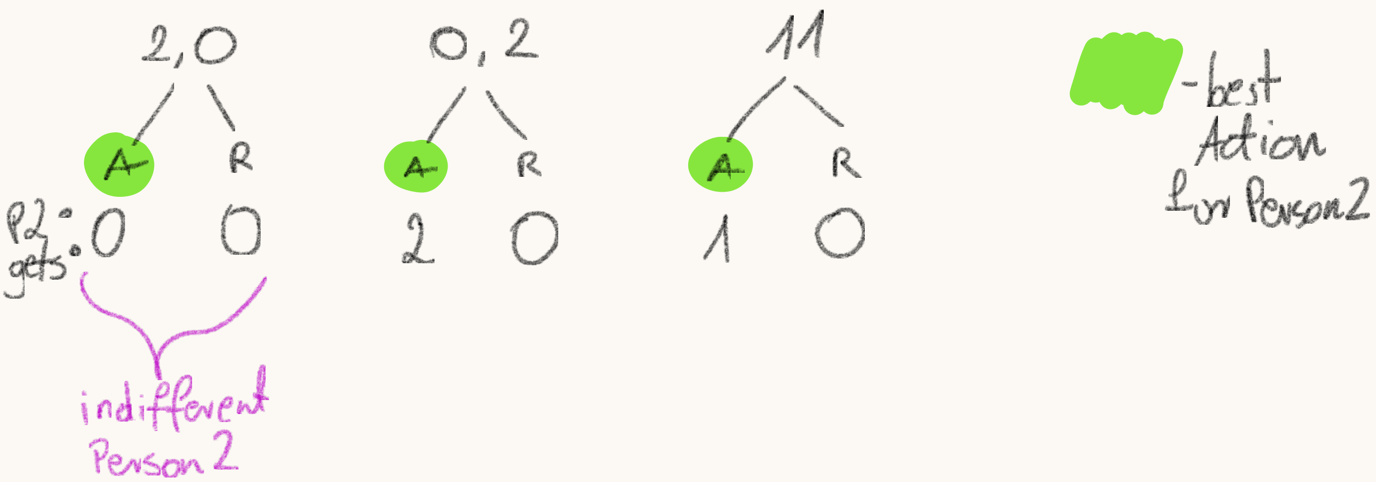
g) Now suppose instead that if person 2 is indifferent, she rejects the proposal. Find the subgame perfect equilibria.

h) Is there any outcome that is generated by a Nash equilibrium but not by any subgame

SPNE → strategy that brings Nash equilibrium in every subgame

Assumption: If person would get the same payoff by accepting or rejecting – in this part we assume "Accept"

Person 1 propositions:

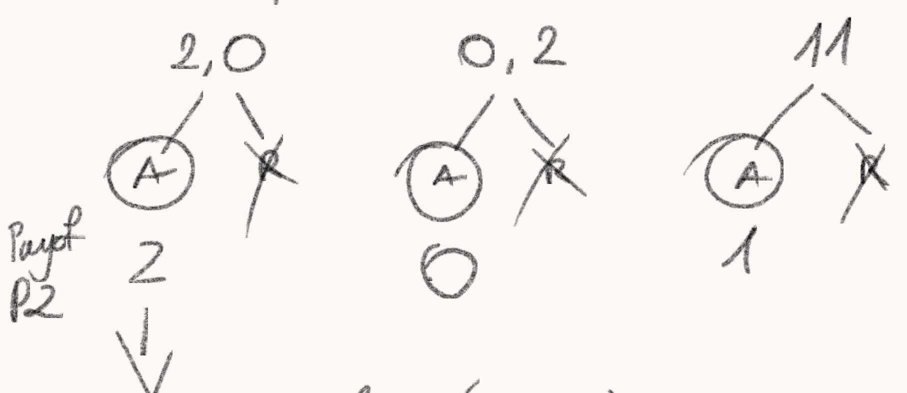


Person 2's best responses are accept for every proposal → strategy (AAA)

Person 1 moves first, anticipates that and chooses the proposal that gives the highest payoff

–

Person 1 propositions:



→ bc here we assume P2 will always Accept

Best proposal is (2, 0) – gives payoff 2

Answer: SPNE: ((2,0), AAA)

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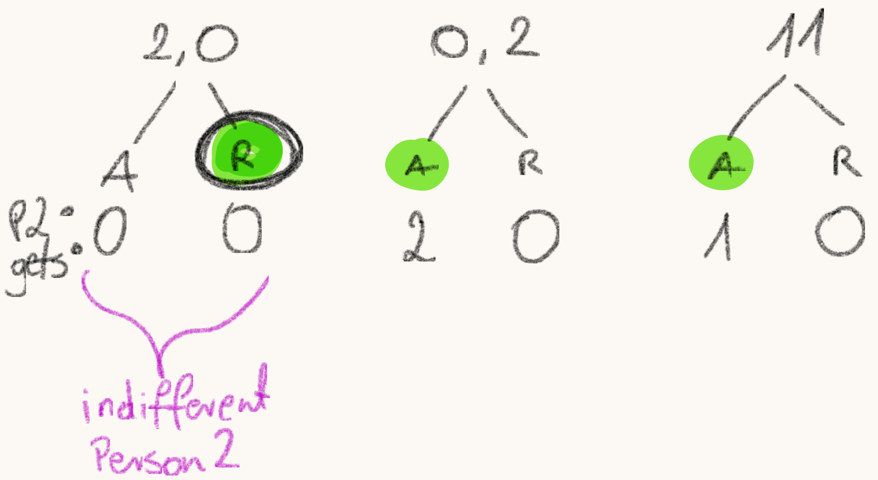
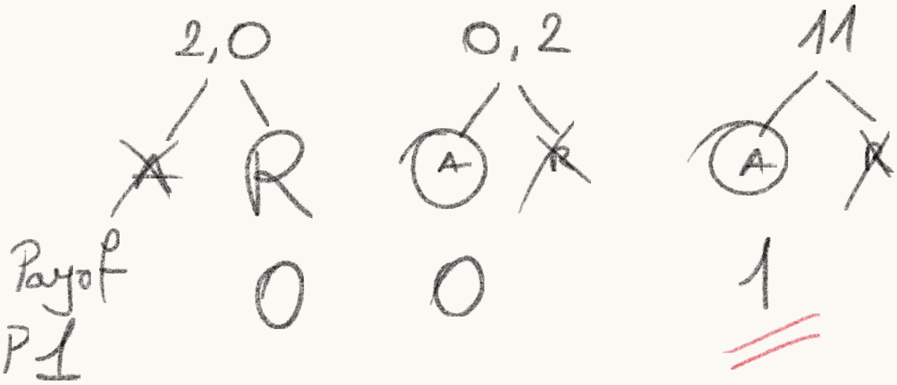
Person 1 propositions:



Person 1 sse A and anticipates:

Person 1 propositions:



Person 2 Rejects on (2,0), Accept (0,2), Accept (1,1)

strategy (RAA)

Answer: SPNE: (1,1, RAA) → (1, 1)

g) Now suppose instead that if person 2 is indifferent, she rejects the proposal. Find the

subgame perfect equilibria.

h) Is there any outcome that is generated by a Nash equilibrium but not by any subgame perfect equilibrium (from questions f. and g.)?

9 eqilibria in a) , 1 eqilibria in f) and g)

SPNE $\overset{\Rightarrow}{\cancel{\Leftarrow}}$ Nash Eqilibrium

h) → SPNE has also 0,0 and 0,2 otion of eqilibrius

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Nash equilibrium

		Hardware store 2 p_H	p_L
Hardware store 1	p_H	3,2	0,3
	p_L	2,0	1,1

3, 2 | 0, 3
2, 0 | 1, 1

a) Determine the Nash equilibrium of this game. Interpret it briefly from the customer's point of view.

Now consider the following extension. In stage 1 of the game, hardware store 1 can give a price guarantee that it will never charge a higher price for its product than the other hardware store. In stage 2, the two hardware stores again decide on their prices simultaneously, with the payouts for the respective price combinations being the same as in the matrix above. If hardware store 1 has given a price guarantee and still has a higher price than the competitor, then customers can (and will) demand the price difference back from hardware store 1. Therefore, the payoffs will be the same as if both stores had charged a low price.

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f) What is the equilibrium of the game? Interpret it briefly.

a) Nash Egilibrium = LL
Interpretation :

ORUGINAL MATRIX

b)
Now consider the following extension. In stage 1 of the game, hardware store 1 can give a price guarantee that it will never charge a higher price for its product than the other hardware store. In stage 2, the two hardware stores again decide on their prices simultaneously, with the payouts for the respective price combinations being the same as in the matrix above. If hardware store 1 has given a price guarantee and still has a higher price than the competitor, then customers can (and will) demand the price difference back from hardware store 1. Therefore, the payoffs will be the same as if both stores had charged a low price.

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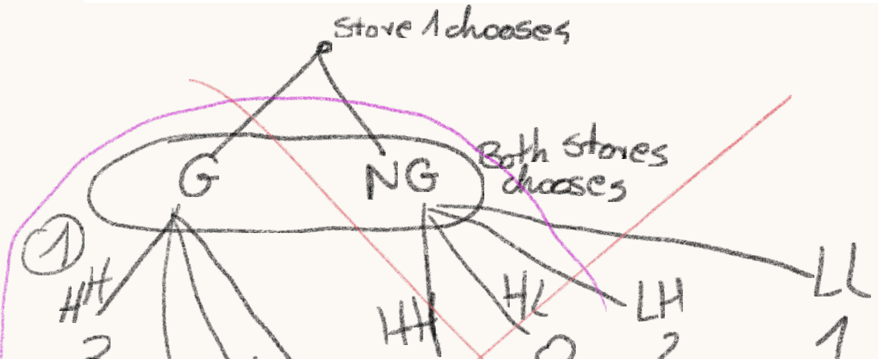

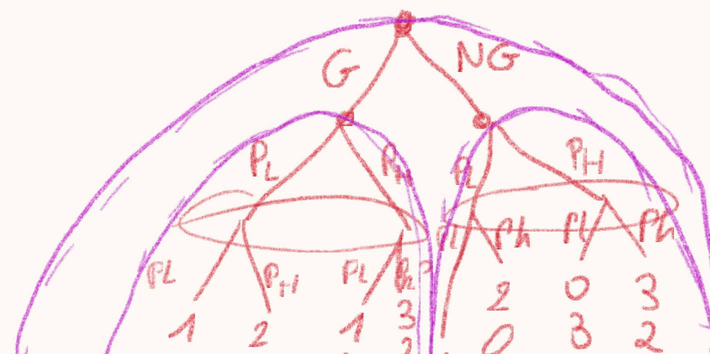

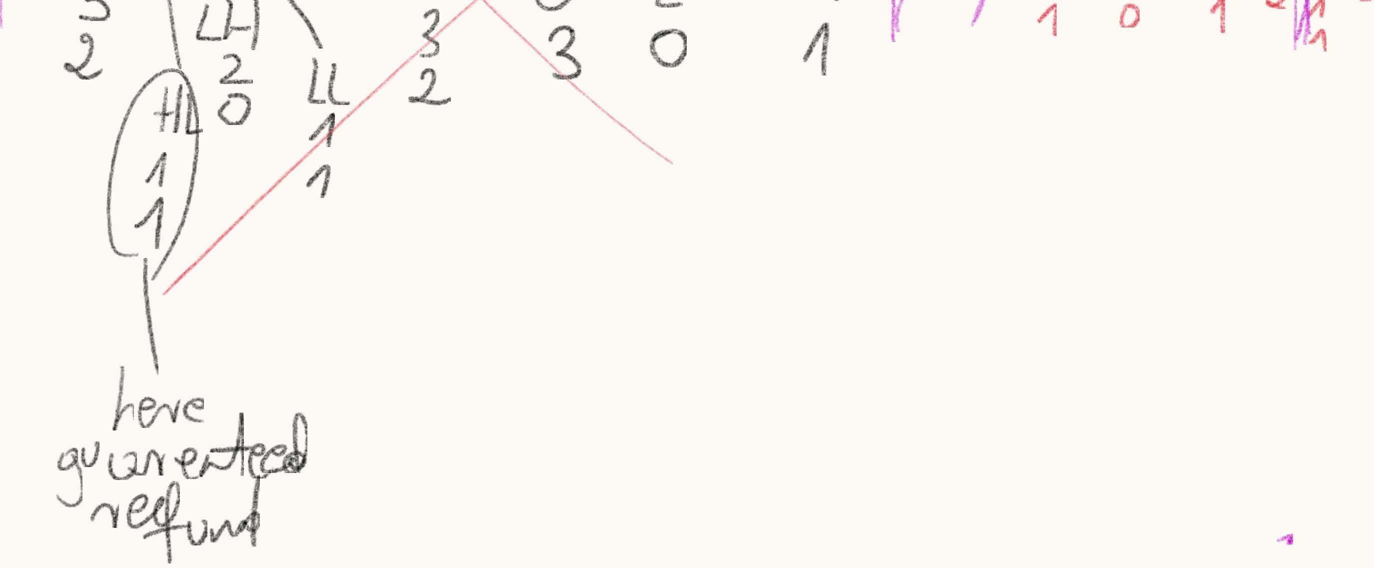

c) Number of subgames: 3 (bc those are simoultaneous choices)

d) Pure strategies

Store 1

Stage 1 choice: G, NG $\rightarrow$ 2 options

Stage 2 choice: Conditional on whether stage 1 is NG/G

Two choices after G and two after NG

$2 \cdot 2 \cdot 2 = 8$

Store 2

Stage 2: Two choices after G and two after NG

(Store 2 moves ONLY in stage 2)

$2 \cdot 2 = 4$

e)

e) What kind of game is it (static/dynamic, complete/incomplete, perfect/imperfect information) and which equilibrium concept should be used?

f) What is the equilibrium of the game? Interpret it briefly.

- Dynamic : sequencial game
- Complete information: both players know payoff matrix
- Imperfect information: Stage 2 happends simultaneusly so at stage 2 player 2 does NOT know other stage 2 choice

Which equilibrium concept schould be used: Solution procedure is backwards induction

		Player 2	
		H	L
Player 1	H	3,2	1,1
	L	2,0	1,1

Two Nash equilibria but would go with HH bc Logic)

Pareto efficient superiour

Answer: HH is Pareto superior to LL equilibrium

f) (f) What is the equilibrium of the game? Interpret it briefly.

sh Eqilibrium = LL

Answer: Our SPNE: $(G\, P_H\, P_L,\ P_H\, P_L)$

$\rightarrow (3,2)$