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Advanced Microeconomics, winter term 2025/26

Exercise 2

Please solve the exercises below by Wednesday, November 5th. We will discuss them in our exercises (see Stud.IP). To obtain a bonus point, you need to upload your answers as a single pdf in the StudIP folder "Student Solutions Exercise 2". The **document name should start with your surname**. If you prepared the answer in a group of up to 3 students, please only submit one document that contains the names of all contributing students. The DEADLINE is November 5th, 7:30 so that we have a chance to quickly scroll through your submissions to suggest one for presentation.

Question 1 (Synergies)

Two division managers can invest time and effort in creating a better working relationship. Each invests $e_i \geq 0$, and if both invest more then both are better off, but it is costly for each manager to invest. In particular, the payoff function for player i from effort levels (e_i, e_j) is $v_i(e_i, e_j) = (a + be_j)e_i - \frac{1}{2}e_i^2$, where $a > 0$ and $0 < b < \frac{1}{2}$.

(a) What is the best response function of each player?

(b) Find the Nash equilibrium of this game.

(c) Find the efficient solution of this game and compare it to the Nash equilibrium.

Question 2 (Monitoring and mixed strategies)

An employee (player 1) who works for a boss (player 2) can either work (W) or shirk (S), while his boss can either monitor the employee (M) or ignore him (I). Like most employee-boss relationships, if the employee is working then the boss prefers not to monitor, but if the boss is not monitoring then the employee prefers to shirk. The game is represented in the following matrix:

Boss

		Player 2	
		M (q)	I ($1-q$)
Employee player 1 (p)	W	1, 1	1, 2
($1-p$)	S	0, 2	2, 1

When each player's action is the best response to the other player's action

a) Find all Nash equilibria of this game (i.e. equilibria in pure and in mixed strategies). Do you find the result "realistic"?

Employee best response
If P_2 plays M → W best
If P_2 plays I → S best

No Nash equilibrium

Boss best response
If P_1 plays W → I best
If P_1 plays S → M best

$\forall$	Best E	Best B
WM	yes	
WI		yes
SM		yes
SI	yes	

Mixed strategies:

B $\quad v_1 (W, S) \qquad v_2 (M, I)$

Boss

E employee player 1		Player 2: M (q)	Player 2: I ($1-q$)
p	W	1, 1	1, 2
$1-p$	S	~~0, 2~~	~~2, 1~~

$$S_E (W, S) \Leftrightarrow (p, 1-p)$$
$$S_B (M, I) \Leftrightarrow (q, 1-q)$$

Formula
$p + 1 - p$

Pl. 2
Boss (M): $p \cdot 1 + 1 - p = 2 - p$

Boss (I): $p \cdot 2 + 1 - p \cdot 1 = 2p + 1 - p = 1 + p$

$$2 - p = 1 + p$$
$$2 - 1 = p + p$$
$$1 = 2p$$
$$p = \frac{1}{2}$$

Pl.1 Employee
If E works (W): $1 \cdot q + 1 + (1-q) = 1$

If E Shrinks (S): $0\cdot q + 2+(1-q) = 2-2q$

$$1 = 2-2q \qquad // -2$$

$$-1 = 2q$$

$$q = \frac{1}{2}$$

Teacher explenation:

No mutual best responce

That means there is no Nash equilibria in pure strategies

E (1) \ B (2)		p M	$1-p$ I
r	W	1, 1	1, 2
$1-r$	S	0, 2	2, 1

Mixed strategies

r : probability that player 1 plays W

p : probability that player 2 plays M

NA KRZYŻ

liczbę jednej linii X drugiej linii

Player 1 chooses r such that 2 is indifferent between his pure strategies

$$E_2(M) = E_2(I)$$

$$r\cdot 1 + (1+r)\cdot 2 = r\cdot 2 + (1-r)\cdot 1$$

$$r + 2 - 2r = 2r + 1 - r$$

$$2 - r = r + 1$$

$$r = 0{,}5$$

$$r + 2 - 2r = \cancel{2r} + 1 - r$$

$$r + 2 = 4r + 1 - r$$

$$2r + 2 = 4r + 1$$

$$2 = 2r + 1$$

$$1 = 2r$$

$$0{,}5 = r$$

Player2 chooses p such as player 1 is indifferent between his pure strategies:

$$E_1(W) = E_1(S)$$

$$p\cdot 1 + (1-p)\cdot 1 = p\cdot 0 + (1-p)\cdot 2$$

$$p + 1 - p = 2 - 2p$$

$$p = 0,5$$

Odp: Nash equilibrium $(r^*, p^*) = (0,5, 0,5)$

$$= \big(\underbrace{(0,5, 0,5)}_{\text{this is player 1}}, \underbrace{(0,5, 0,5)}_{\text{this is player 2}}\big)$$

E		② B: p M	1-p I
①	r W	1,1	1,2
	1-r S	0,2	2,1

$$p + (1-p) = (1-p)\cdot 2$$

$$\cancel{p} + 1 - \cancel{p} = 2 - 2p$$

$$1 = 2 - 2p \quad / -2 \quad \boxed{\pm}$$

$$+1 = +2p$$

$$0,5 = p$$

Q1

Question 1 (Synergies)

Two division managers can invest time and effort in creating a better working relationship. Each invests $e_i \geq 0$, and if both invest more then both are better off, but it is costly for each ~~manager to invest~~. In particular, the payoff function for player i from effort levels (e_i, e_j) is

$v_i(e_i, e_j) = (a + be_j)e_i - \frac{1}{2}e_i^2$, where $a > 0$ and $0 < b < \frac{1}{2}$.

each Menager is Trying to maximize the payoff

cost of effort

(a) What is the best response function of each player?

(b) Find the Nash equilibrium of this game.

(c) Find the efficient solution of this game and compare it to the Nash equilibrium.

Players Menager 1, Menager 2

investment: $e_i \geq 0$

Strategies

$ae_i + be_i$

Payoffs e_i, e_j

e_i " $= 1e_i^1 = 1 \cdot e_i^0 = 1$

PYT maximalizuje wszystko i sod alles

Where the one "e_i" went? przy nawiasie

a) derivative

payoff $\frac{\delta \Delta v_i}{\delta \Delta e_i} = (a + be_j) - e_i$ investment

Marginal benefit

Marginal cost

$-\frac{1}{2}e_i^2 \to \Delta \to -e_i$

$\frac{dv(e_i, e_j)}{\partial e_i} = a + be_i - e_i$

$e_1^* = a + be_2 - e_1$

$e_2^* = a + be_1 - e_2$

Solve for e_i $(a + be_j) - e_i = 0$

$e_i = a + b_j$

bc Synergies:

$e_1 = e_2$

$0 = a + be_2 - e_1$

$e = a + be$

$e - be = a$

$e(1-b) = a$

$e^{NE} = \frac{a}{1-b}$

b) The ~~best~~ Nash eqillibrium:

$e_1 = e_2 = e_3$

the best investment

Best responce: $e^* = a + be$ / $-be$

$e - be = a$

$e(\cancel{b}1 - b)$

Solve for e $e(1-b)=a$

$e = \frac{a}{1-b}$ → The point where both managers best responses are consistent with each other

Nash equilibrium

c) Efficient Solution

→ This means maximizing the sum of both payoffs

Total payoff "W"

$$W(e_i, e_j) = V_i + V_j = (a+be_j)e_i - \frac{1}{2}e_i^2 + (a+be_i)e_j - \frac{1}{2}e_j^2$$

$$= ae_i + be_je_i - \frac{1}{2}e_i^2 + ae_i + be_ie_j - \frac{1}{2}e_j^2$$

$$= a(e_i+e_j) + 2be_ie_j - \frac{1}{2}(e_i^2+e_j^2)$$

$$W = V_1 + V_2$$

$$= ae_1 + be_2e_1 - \frac{1}{2}e_1^2 + ae_2 + be_1e_2 - \frac{1}{2}e_2^2$$

derivative

$$\frac{\partial V}{\partial e_1} = a + 2be_2 - e_1$$

$$\frac{\partial V}{\partial e_2} = a + 2be_1 - e_2$$

$$e_2 = a + 2be \qquad e^* = \frac{a}{1-2}$$

Teacher explenation:

$$e \geq 0$$

$$V_i(e_i, e_j) = (a + b \cdot e_j) \cdot e_i - \frac{1}{2} \cdot e_i^2$$

a) best responce function:

First Outo Condition
FOC:

$$a + b \cdot e_j - e_i = 0$$

$e_i = a + b \cdot e_j$ → best responce function

$$e_1 = a + b \cdot e_2$$
$$e_2 = a + b \cdot e_1$$

b) The game is symetric so that in equilibrium $e_1 = e_2$, $e_i = e_j$

→ $e_i = a + b \cdot e_i$

$$e_i - b \cdot e_i = a$$
$$e_i \cdot (1-b) = a$$
$$e_i = \frac{a}{1-b}$$ → Nash eqilibrium

$(1-b)$

c) Efficient solution:

↳ maximize overall profit

$$\max_{e_1, e_2} \underbrace{(a + b \cdot e_2) \cdot e_1 - \frac{1}{2} e_1^2}_{\text{player 1}} + \underbrace{(a + b \cdot e_1) \cdot e_2 - \frac{1}{2} \cdot e_2^2}_{\text{player 2}}$$

FOC: $a + b \cdot e_j - e_i + b \cdot e_j = 0$

$e_i = e_j$

$$a + b \cdot e_i - e_i + b \cdot e_i = 0$$
$$a = -b \cdot e_i + e_i - b \cdot e_i$$
$$a = e_i \cdot (-b + 1 - b)$$
$$a = e_i \cdot (1 - 2b)$$
$$e_i = \frac{a}{1-2b}$$

$2b > b$

We conclude that $e_i = \frac{a}{1-2b} > e_i = \frac{a}{1-b}$ due to lower denominator

Question 3 (Trembling hand)

Consider the following sequential game between Player 1 and Player 2:

- Player A moves first and chooses L1 or R1.
- If A chooses R1, the game ends immediately with the payoff (2, 2).
- If A chooses L1, then Player B moves and chooses between L2 and R2:
  - L2 leads to a payoff of (1, 0).
  - R2 leads to a payoff of (3, 1).

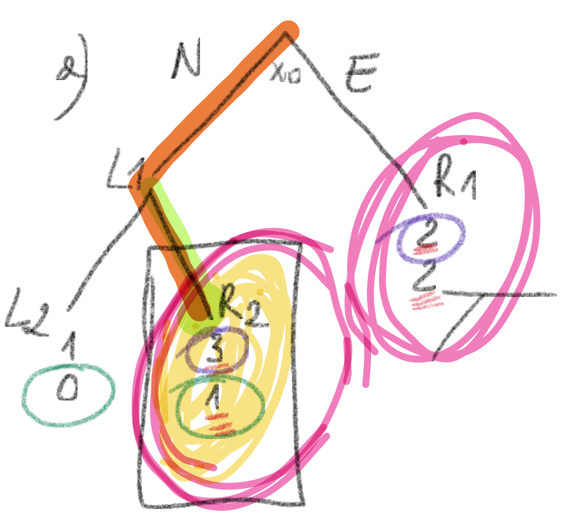
a) Draw the extensive form (game tree) of the game.

b) Which Nash equilibria exist, and which solution results from backward induction?

c) Which Nash equilibrium is Pareto optimal?

d) Is the solution obtained by backward induction in part (a) also trembling-hand perfect?

a)



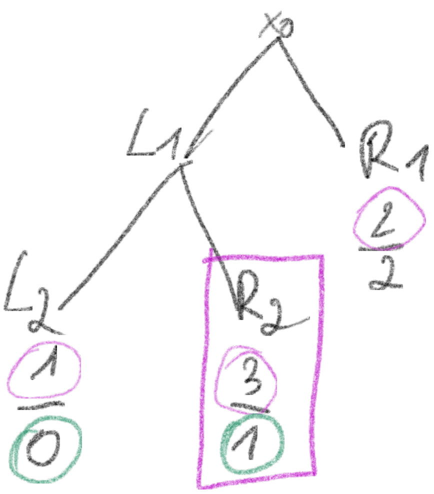
b) Nash equilibria , R_1, $R_2 L_1$

From backwards L_1 R_2

c) – Pareto optimal R_1 and $L_1 R_2$

d) yes bc it uses credible threats

No player uses a weakly dominated or non-credible strategy



Teacher explenation

b) Transfering into the matrix:

(First step look for best responces)

2

1		L_2	R_2
	L_1	1,0	3,1
	R_1	2,2	2,2

Nash eqilibria in pure strategies

(bc both best responce that is)

Backward induction:

Compare for player 2

Compare values player 1

c) Both are pareto optimal none is dominated.

You cannot make one player better off without making the other worse off

d) Trembeling Hand: $(3,1)$ L_1, R_2 > is that trembling hand

L_1, R_2 is always trembling hand perfect for player 2 since R_2 is a dominant strategy.

So looking at player 1:

ε = probability that player 2 does NOT play R_2

Expected payoff of Player 1:

that's payoff of player 1

$$E_1(L_1) = (1-\varepsilon)\cdot 3 + \varepsilon \cdot 1 =$$
$$= 3 - 2\varepsilon$$

$$E_1(R_1) = (1-\varepsilon)\cdot 2 + \varepsilon \cdot 2$$
$$= 2 - 2\varepsilon + 2\varepsilon$$
$$= 2$$

$$E_1(L_1) > E_1(R_1)$$

$$3 - 2\cdot\varepsilon > 2$$

$$\varepsilon < 0{,}5$$

L_1, R_2 is trembling hand perfect for player 1 if player 2 trembles with $\varepsilon < \frac{1}{2}$