Advanced Microeconomics

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Lecture 16: Psychological Game Theory

Essential reading:

- Carpenter, Jeffrey; Robbett, Andrea. Game Theory and Behavior, Chapter 28, MIT Press, 2023.

Introductory experiment

- 1. go to https://classEx.uni-passau.de.
  2. Select your institution: Carl von Ossietzky Universität Oldenburg (Germany)
  3. Choose your account name: Advanced Microeconomics
  4. Select participants
  5. Enter password: yZnH

- Trust game
  - Two participants play together – one could be seen as investor, the other one as trustee
  - Both receive 10 €
  - The investor can decide how much to send to the trustee.
  - The trustee receives the amount multiplied by 3.
  - The trustee can then decide how much he sends back.

Extension of psychological game theory

- Standard game theory assumes that
  - players have preferences only over the ultimate outcomes of the interaction
    - which are uniquely determined by each player's action
- psychological game theory allows the process to matter too
  - preferences may also depend on one's own or others' beliefs
  - allows to incorporate "psychological" or belief-based motives into strategic interactions
    - e.g., self-esteem, reciprocity, and emotions like guilt

Psychological game theory

- Often people don't just care about what others do, but also about what others think, e.g.,
  - we are often motivated to take certain actions purely because we want others to think that we are moral, smart, or cool
  - we view someone as more worthy of punishment if they intentionally rather than inadvertently harmed someone else, even if the ultimate outcome is the same
    - and our justice system often reflects this
  - we assess outcomes relative to what we had initially expected,
  - we worry about letting others down and feel guilty when we do, …
- But keep in mind:
  - models always simplify reality
  - For many problems – like strategic interactions of firms – the "standard" model, which disregards psychological aspects, should be well suited

Psychological game theory

- main characteristic of a psychological game: ultimate payoffs can depend on the players' beliefs about the choices and beliefs of others
  - Example trust game: how much the trustee gives depends partly on what he thinks the investor expects
- typically, analysis restricted to first- and second-order beliefs
  - **first-order beliefs**: probabilities that players assign to others' strategies
    - i.e., what one player thinks the others will do
      - e.g., what the investor thinks how much the trustee sends back
  - **second-order beliefs**: beliefs that players have about others' first-order beliefs
    - i.e., what a player thinks that others think that he will do
      - e.g., what trustee thinks about what the investor expects him to send back

Incorporating guilt into games

- Analysis requires to specify belief-dependent motives that may affect strategic interactions. We distinguish between
  a) a player's material payoff
     - standard game-theoretic payoff from the realized strategy profile
  b) and a player's psychological payoff, which may depend on beliefs

Definition 1 (Guilt-averse preferences) Consider a strategy profile s and let player j's material payoff from s be given by $\pi_j(s)$. Let $E_{ij}(\pi_j)$ denote the payoff that player i believes player j expects to receive. A guilt-averse player i acts to maximize the expression: $u_i(s, E_{ij}(\pi_j)) = \pi_i(s) - \theta_i \max\{0, E_{ij}(\pi_j) - \pi_j\}$, where θ_i measures i's guilt aversion.

- i.e., player i feels guilty if j's payoff, π_j, is lower than what i believes player j expects to receive, $E_{ij}(\pi_j)$

Incorporating guilt into games

- $E_{ij}(\pi_j)$: payoff that player i believes player j expects to receive
- Utility of guilt-averse player i (θ_i measures i's guilt aversion)

$$u_i(s, E_{ij}(\pi_j)) = \pi_i(s) - \theta_i \max\{0, E_{ij}(\pi_j) - \pi_j\}$$

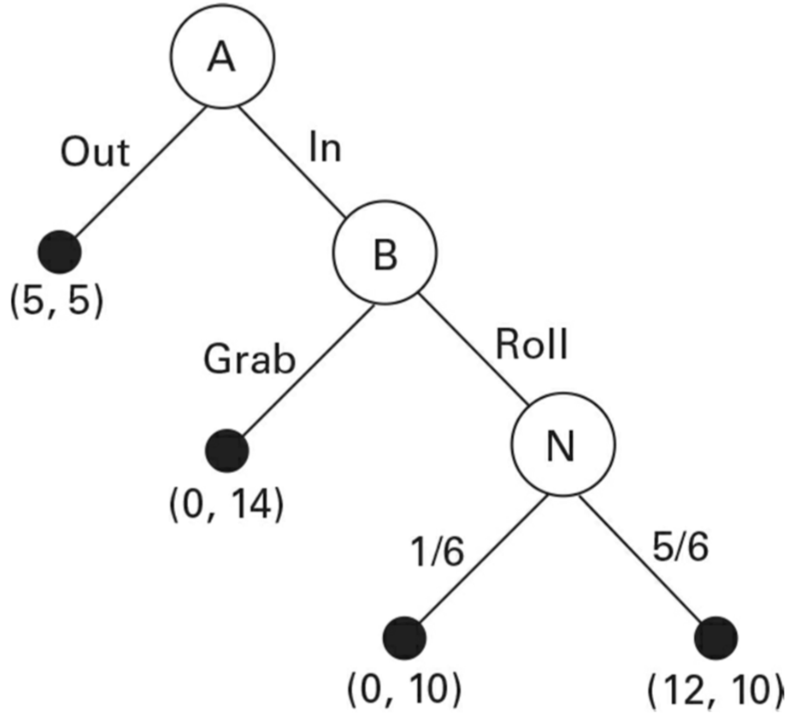
- Remember: in Lecture 8 we already introduced other-regarding preferences:

$$u_i(x_i, x_j) = x_i - \underbrace{\alpha_i \max\{0, x_j - x_i\}}_{\text{Envy}} - \underbrace{\beta_i \max\{0, x_i - x_j\}}_{\text{Empathy}}$$

- Discuss the difference

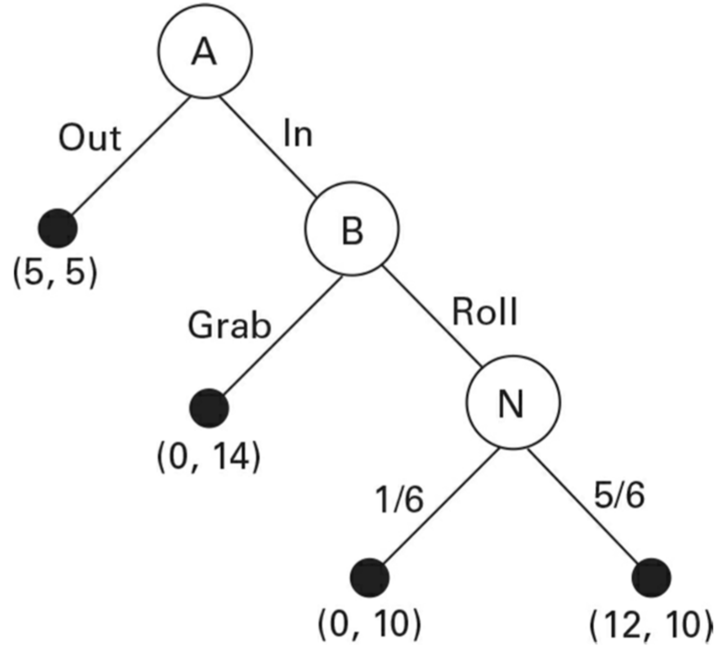
Example: A Trust Game

- Player A first chooses between Out (i.e., not to trust) or In
- For In, player B can choose between Grab or Roll
- Roll means that a die is rolled and
  - A gains 0 if the number 1 comes up and 12 otherwise
  - B always gets 10 from rolling the die
- Subgame perfect Nash equilibrium (SPNE): (Out, Grab) with payoff $(5,5)$
- Mutual Cooperation: (In, Roll) with expected payoff $(10,10)$
  - Player A: $\frac{1}{6} \cdot 0 + \frac{5}{6} \cdot 12 = 10$



Example: A Trust Game

- Due to the die roll component, player A won't know whether
  - he got 0 because B screwed him over or
  - because she tried to reward his trust and they got unlucky
- Assumption: player B is guilt averse
  - She earns psychological disutility whenever her actions cause A to earn less than A had been expecting



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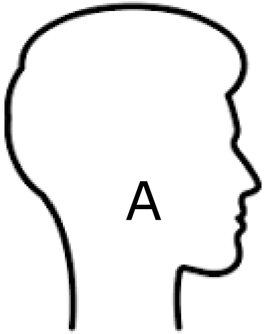
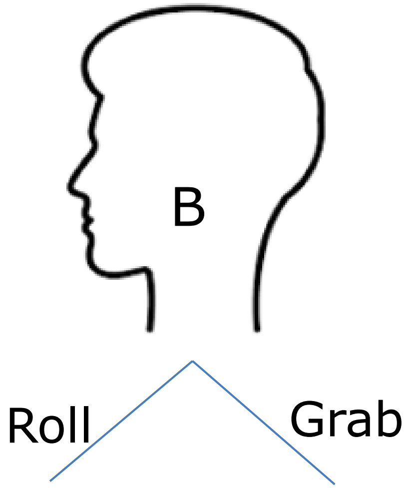
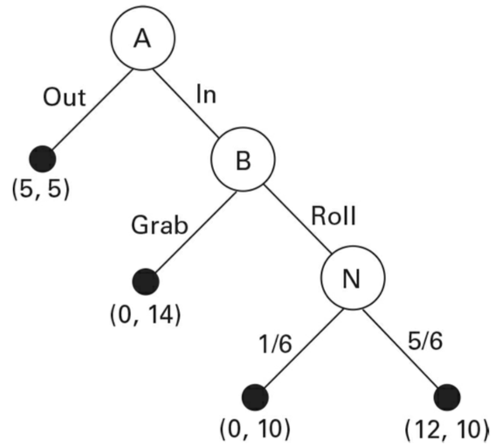
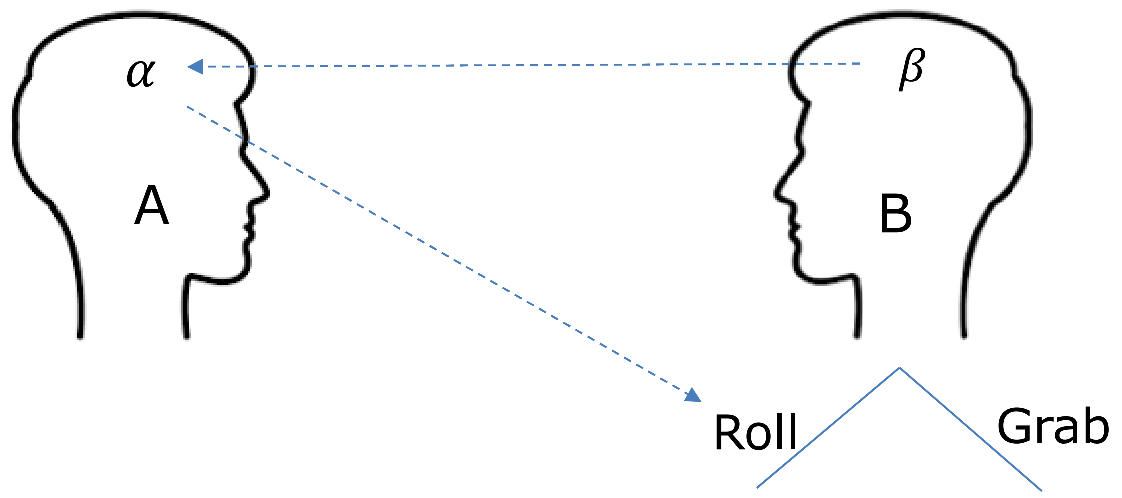


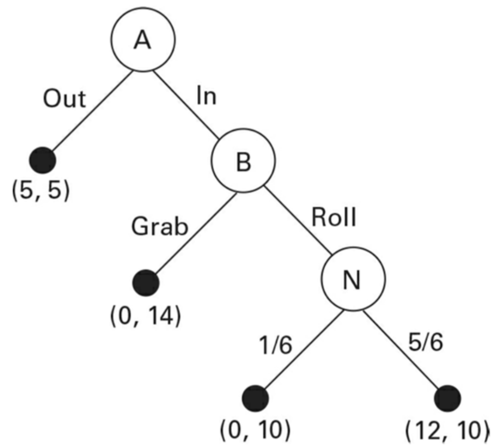




- $\alpha \in [0, 1]$: A's **first-order belief** at the start of the game regarding the probability that B will choose Roll
  - i.e. a belief about the others' *strategy*
- $\beta \in [0, 1]$: B's **second-order belief** (at the time that B moves) about the probability with which A expects him to choose Roll
  - B's belief about α, i.e., about the others' (first-order) belief

Example: A Trust Game





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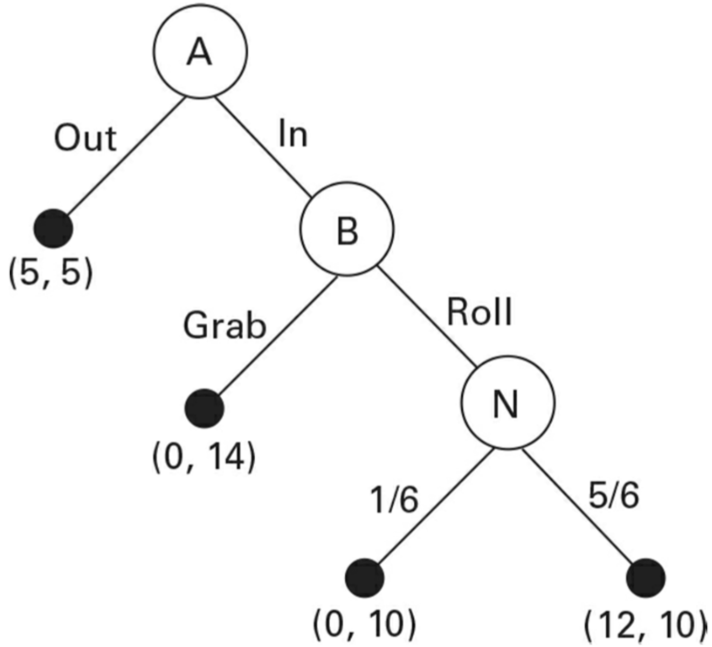
  $u_i(s, E_{ij}(\pi_j)) = \pi_i(s) - \theta_i max\{0, E_{ij}(\pi_j) - \pi_j\}$



- $E_{BA}(\pi_A)$: payoff that player B believes A expects to receive
  - $E_{BA}(\pi_A) = \beta \cdot 10 + (1-\beta) \cdot 0 = 10\beta$
- B's utility from Grab:

$$u_B(Grab, E_{BA}(\pi_A)) = \pi_B(Grab) - \theta_B max\{0, E_{BA}(\pi_A) - \pi_A\}$$

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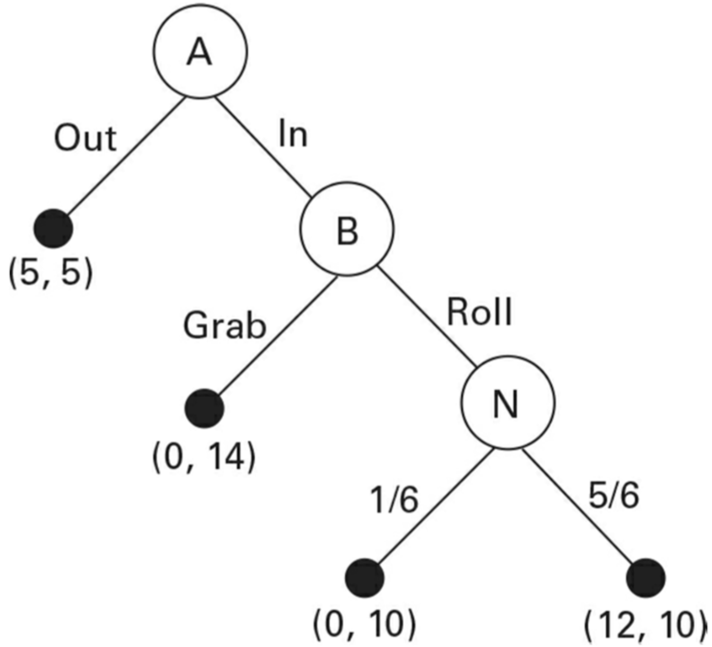

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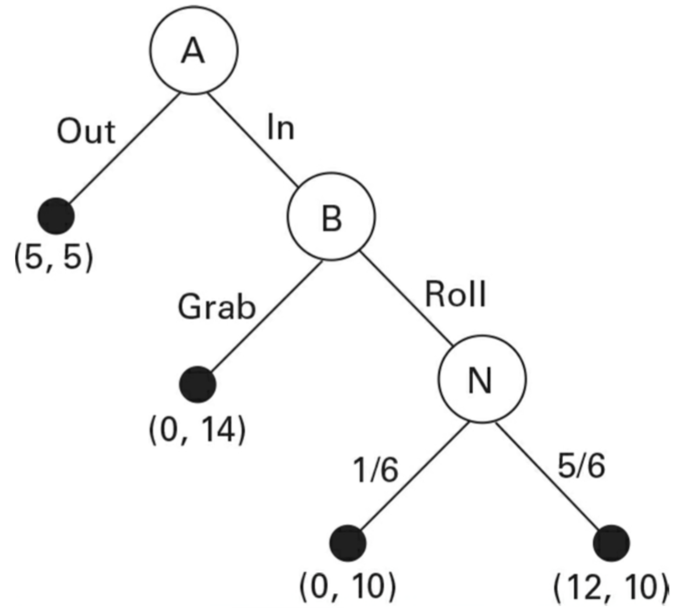
$$u_B(Grab, E_{BA}(\pi_A)) = \pi_B(Grab) - \theta_B max\{0, E_{BA}(\pi_A) - \pi_A\}$$
$$= 14 - \theta_B max\{0{,}10\beta - 0\} = 14 - 10\theta_B\beta$$

Equilibrium of psychological games

- We only consider basic idea of psychological equilibrium concept (formal analysis is beyond the scope of this lecture)
- **psychological equilibria (PE)** require that
  1) players best respond to each other
     - which means they maximize their full payoffs in the psychological game (i.e., the sum of the psychological and material payoffs,
       – i.e., players' payoffs are belief dependent
  2) higher-order beliefs are consistent (or accurate)
     - i.e., each player's first-order beliefs align with the actual strategy profile played by the others, each player's second-order beliefs match the other players' first-order beliefs, and so on.
  3) in dynamic games, players are sequentially rationale
     - i.e., players best respond at every information set, even those off the equilibrium path

Psychological Equilibria in the Trust Game

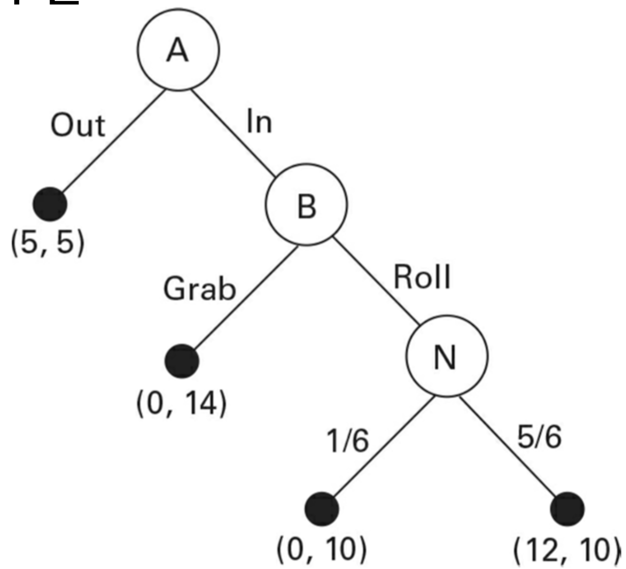
- let p denote the probability with which player B chooses Roll
- In a PE, beliefs must be consistent with the actual strategy profile
  - hence $\beta = \alpha = p$ (1)
- In a PE, players are sequentially rationale and best respond to each other
  - Hence A chooses In only if $5 \leq 10\alpha$ so that using (1) $p \geq 0.5$
  - B chooses Roll if $14 - 10\theta\beta \leq 10$ so that using (1) $\theta p \geq 0.4$
- For low θ (i.e., low guilt aversion), $\theta p \geq 0.4$ is never satisfied
  - hence the PE is the same as the SPNE



Psychological Equilibria in the Trust Game

- Shown: in a PE, A chooses In and B chooses Roll if
  - $\beta = \alpha = p \geq 0.5$
  - $\beta = \alpha = p \geq \frac{0.4}{\theta}$
- If $\frac{0.4}{\theta} \leq 1 \Leftrightarrow \theta \geq 0.4$, there is a PE with (In, Roll, $\beta = \alpha = p = 1$)
- Hence guilt aversion leads to the efficient outcome
  - Note that players never experience guilt in this PE
- Example illustrates effectiveness of social norms (here: guilt)
- Note: there are further PE in mixed strategy with

$$\beta = \alpha = p \in \left[\max\left\{\frac{0.4}{\theta}, 0.5\right\}, 1\right]$$



Experimental Evidence on Trust Game

- Charness and Dufwenberg (2006) conducted trust game in lab
- They included a treatment in which
  - player B could send a message to player A prior to the start of the game,
- they also elicited first- and second-order beliefs
  - by asking players A to guess the proportion of Bs choosing Roll and
  - players B to guess the average guess of As
- They find that
  - communication substantially increases the likelihood that player B chooses Roll, and that
  - messages in which player B explicitly promised to play Roll were especially likely to be followed by (In, Roll)
- They take this as evidence that achievement of cooperative outcome works through beliefs, consistent with the guilt-aversion argument