

Question 1(Principal Agent)

Information from the text:

- Principal= Cleanex
- Agent= Lawex
- Case: Dirtex should pay d to cleanex

- Damage (d)
- Costs for agent $c(e)=e^2$
- Effort $e(0;0,5d)$
- Schadensersatz x vom Gericht zugesprochen $x \sim U(0,2e)$
- Contract: $w(x)=a+bx$, $b \in (0,1)$
- Liability constraint: $w(x) \geq 0$ für alle x

a)

Objective function (Principals): $e - (a + be)$

$$IC = e = B/2$$

$$PC = a + be - e^2 \geq 0$$

$$LL = a \geq 0$$

b)

The liability constraint says that $w(x) \geq 0$, what means Lawex can earn 0 but not less than that. X can be 0 if X is zero A must be ≥ 0 because if not Lawex would pay money, but the liability constraint doesn't allow it

c)

IC = e = B/2 and LL = a = 0

$$\ln PC = 0 + b^*/(b/2) - (b/2)2$$

$$= b^*2/4$$

$$\Pi P = e - (a + be) = e - be = e(1 - b) // e = B/2$$

$$P = B/2(1-b) = b/2 - b^*2/2 //$$

$$Dp/db = 1/2 - b = 0 \rightarrow b^* = 1/2$$

$$e^* = b^*2/4 = 1/4$$

$$a^* = 0$$

d)

rent = überschuss zum minimum

$$rent = a + be - e$$

$$rent = 0 + (1/2 * 1/4) - (1/4)2$$

$$rent = 1/8 - 1/16$$

$$\text{rent} = 1/16$$

e) The rent and effort cost are both 1/16.

Question 2

Text Information's:

Players:

- Principal(risk-neutral)
- Agent(risk-averse)

Effort and cost:

- $E \geq 0$ effort
- $c(e) = 0,5 * e^2$

Output:

- $Q = 10e + u$
- Only Q is observable not e = moral hazard

Contract:

- $P = s + rQ$
- s = fixed salary, r = bonus rate

Agents expected utility:

- $EU = E(p) - 0,5e^2 - Kr^2$
- $E(p)$ = expected payment
- $0,5e^2$ = effort cost
- Kr^2 = risk cost

Reservation utility:

- U_0

Checklist to set up principal agent problem:

1. Principal objective

$$\text{Max}(s,r) E(Q) - E(p) \quad \text{where } E(p) = s + rE(Q)$$

$$= \max(s,r) E(Q) - (s + rE(Q))$$

2. Agents expected utility:

$EU = E(p) - c(e)$ - (risk cost if risk-averse)

3. Write the constraints

IC(Anreiz): e maximizes EU

PC(Teilnahmebedingungen): $EU \geq u_0$

Principal objective:

$\text{Max}(s,r) 10e - s - 10re // \text{Outcome} = \text{Outcome} - \text{Fixum} - \text{bonus\%}$

$IC = \max(e) (s + 10re - 0,5^2 - Kr^2)$

$Pc = s + 10re - 0,5e^2 - Kr^2 \geq u_0$

b)

Agent is maximizing:

$$s + 10re - 0,5e^2 - Kr^2$$

$$= 10re - 0,5e^2$$

$$= f(e) = 10re - 0,5e^2$$

=

Advanced Microeconomics, winter term 2025/26

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We make the following assumptions: The effort cost of the law firm are a convex function $c(e) = e^2$ and efforts $e \in [0; 0.5d]$. Cleanex pays Lawex conditional on the indemnification x that the court grants to Cleanex, where x is uniformly distributed in the interval $[0, 2e]$. In particular, payments are $w(x) = a + bx$, $b \in [0, 1]$. Furthermore, $w(x) \geq 0$, i.e. the law firm cannot be held liable for a bad job. Both Cleanex and Lawex are risk-neutral. Finally, if Lawex does not accept the contract offered by Cleanex, it gets a reservation income of 0.

- a) Set up the Principal-Agent problem.
- b) Why does the liability constraint $w(x) \geq 0$ imply that $a \geq 0$?
- c) Solve the Principal-Agent problem for the optimal contract elements (a, b, e) .
- d) Calculate the rent which Lawex receives due to the liability constraint.
- e) Compare the size of this rent with the effort cost.

Question 2 (Risk and Incentives in Contracting)

Consider a principal–agent relationship where a risk-neutral principal hires an agent whose effort is not observable.

We make the following assumptions: The agent’s effort level is $e \geq 0$ and effort costs are given by $c(e) = 0.5 \cdot e^2$. Output is $Q = 10 \cdot e + u$ where u is a random variable with $E[u] = 0$. Only output Q is observable. The principal offers a **linear incentive contract** $p = s + r \cdot Q$ where s is a fixed payment and r measures the strength of incentives. The agent is **risk-averse**. His expected utility (certainty-equivalent form) is $EU = E[p] - 0.5 \cdot e^2 - Kr^2$ where $K > 0$ measures the agent’s degree of risk aversion. If the agent does not accept the contract, he obtains a reservation utility u_0 . The principal maximizes expected profits.

- a) Set up the Principal-Agent problem.
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Principal Cleanex
Agent Lawex

Cost of agent $C(e) = e^2, e \in [0, \frac{1}{2} \cdot d]$

Court outcome $x, x \in [0, 2e]$

Payment to Agent $w(x) = a + bx$
 $b \in [0, 1]$

Limited liability $w(x) \geq 0$

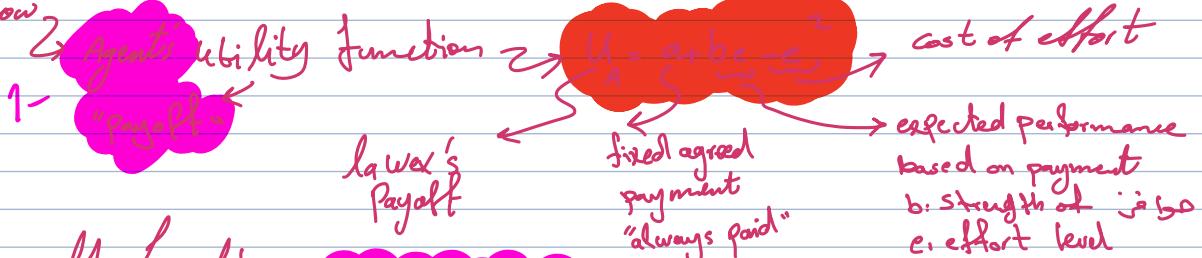
Given:

Lawex is paid: $w(x) = a + bx$ where b is $(0 \leq b \leq 1)$

Effort costs $C(e) = e^2 \rightarrow$ efforts are costly!

Expected court compensation $x \sim U[0, 2e] \rightarrow E[x] = e$

a) should know:



2. Cleanex profit function:

Profit = expected court

Compensation - expected wage paid

$$\Pi_p = e - (a + be)$$

Cleanex expected profit

a) Agent Problem:
 $\max_e E[w(x)] - C(e)$
 $\Rightarrow \max_e a + be - e^2$
 $\text{# incentive constraint}$

$e = \text{expected court compensation}$ $\xrightarrow{\text{Agent must get at least income 0}}$
 $a + be - e^2 \geq 0$

from $x \sim U[0, 2e] \rightarrow E[x] = e$

$a + be = \text{expected payment to Lawex}$

3. Constraints in the problem: 1) Equations: $e \in \arg \max_e \{a + be - e^2\}$

the constraint must make the desired effort optimal for Lawex

Effort is unobservable
Lawex must choose effort voluntarily.

Was ist das?

Incentive compatibility constraint

2) Participation Constraint (PC) $\rightarrow a + b - e^2 \geq 0 \rightarrow$ What? Lawex can reject the

contract & earn 0 elsewhere, so Lawex must at least earn 0 €

3) Limited liability constraint (LL)

Principle Problem:
 Gets court outcome but pays $w(x)$
 $\max_e E[x] - E[w(x)]$

$$\max_e e - (a + b \cdot e)$$

$w(x) \geq 0 \Rightarrow$ lawer x can't be forced to pay money to cleaner

b) why does $w(x) \geq 0$ imply $a \geq 0$?

wage function $w(x) = a + bx$

- Wage Lawyer Receives \downarrow
- fixed payment \downarrow
- Court Compensation \downarrow

$$w(x) \rightarrow x = 0$$

$$w(0) = a \rightarrow a \geq 0$$

$$w(0) = a \rightarrow a \geq 0$$

worst Case outcome $w(0) = a \rightarrow$ If the court gives 0, is the worst outcome, so $a \geq 0$

c) solving agent problem!

lawer x optimal effort \geq maximum effort

maximum? \rightarrow is this derivative again? YES \approx

finding the maximum effort means finding the derivative with respect to e

Agent

$$\frac{d}{de} \rightarrow \frac{\partial}{\partial e} (a + be - e^2) \rightarrow (0 + b - 2e)$$

optimal effort

$$b - 2e = 0 \rightarrow e = \frac{b}{2}$$

Principal
Cleaner reduced problem:

$$\Pi_p = c - (a + be) \\ = \frac{b}{2} - (a + \frac{b^2}{2})$$

$$\frac{\partial}{\partial x} \rightarrow S X$$

$$n \leq X$$

$$n-1$$

$$4x \rightarrow 12x$$

$$2x \rightarrow 8x$$

Fixed Payment \geq -

$$a \geq 0 \rightarrow a = 0$$

$$a + be - e^2 \geq 0$$

$$0 + b \cdot \frac{b}{2} - \left(\frac{b}{2}\right)^2 \geq 0$$

$$\frac{b^2}{2} - \frac{b^2}{4} \geq 0$$

$$\frac{b^2}{4} \geq 0$$

$$\frac{2b^2}{4} - \frac{b^2}{4}$$

zak oligomeric
utramki:

$$U_P = \frac{a}{2} - \left(\theta + \frac{\theta^2}{2} \right)$$

$$= \frac{b}{2} - \frac{b^2}{2}$$

- · + = -
- · - = +

$$\max_b \frac{\partial}{\partial b} \left(\frac{b}{2} - \frac{b^2}{2} \right)$$

$$b^2 \rightarrow 2b$$

$$\left(\frac{1}{2} - \frac{b}{2} \right) = 0$$

$$\frac{1}{2} - b = 0$$

$$b = \frac{1}{2}$$

$$e = \frac{b}{2} \Rightarrow e = \frac{\frac{1}{2}}{2} \Rightarrow \frac{1}{4}$$

optimal contract terms $\Rightarrow a^* = 0, b^* = \frac{1}{2}, e^* = \frac{1}{4}$

* means "optimal" value of a

d) Liability Constraint

$U_A = a + be - e^2 \Rightarrow$ firm can't be punished for low outcomes.

$$= 0 + \frac{1}{2} \cdot \frac{1}{4} - \left(\frac{1}{4} \right)^2 \rightarrow \frac{1^2}{4^2}$$

$$= \frac{1}{8} \cancel{\times \frac{1}{16}} = \frac{1}{16}$$

rent of the Agent $= \frac{b^2}{4}$

$$\hookrightarrow \frac{16 - 8}{8 \cdot 16} = \frac{8 \div 8}{128 \div 8} = \frac{1}{16} > 0$$

e) Effort

$$C(e) = c = \left(\frac{1}{q}\right)^2 = \frac{1}{16}$$

Effort = Rent AKA lower rent due to limited liability.
cost = rent

which means income is double the cost
(rent = income - cost)

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- abstandnahme*
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2)

Principal:

$$\text{payoff: } Q - p$$

Average effort
Average pay

$$Q = 10e + u \quad [E[u] = 0]$$

$$p = s + r \cdot Q$$

Agent: Expected Utility

$$E(u) = E[p] - 0,5e^2 - Kr^2$$

Happiness ($E[u]$) Average Pay Work cost Stress cost

Rules:

- Agent says "yes" when their Happiness is at least U_0

= Principle offers $p = s + rQ$

without Safety pay bonus strength

since $E[u] = 0$

we can state

$$E[Q] = 10e$$

$$E[p] = s + 10 \cdot r \cdot e$$

$$EU = s + 10 \cdot r \cdot e - 0,5 \cdot e^2 - r^2 \cdot K$$

$$s + 10re - 0,5e^2 - r^2K \geq U_0 \rightarrow \text{Participation Constraint}$$

b) How hard will Agent work? (Incentive Constraint)

Agent's HAPPINES FORMULA :

$$EU = S + \tau(10e) - 0,5e^2 - Kr^2$$

Now we look where the „extra money“ matches „extra pain“ of working

We use Derivatives : with respect to e

$$\max_e (10re - 0,5e^2) \Rightarrow 10\tau - e = 0$$

$$10\tau = e$$

$$e^* = 10\tau$$

If the Principal gives the Agent bigger τ , Agent works harder

c)

c) Show that the participation constraint binds in the optimal contract and derive the fixed payment $s(r)$.

~~d) Determine the optimal incentive parameter r^* chosen by the principal and the induced effort level e^* .~~

~~e) Calculate and discuss how the strength of incentives, r , depends on the agent's degree of risk aversion.~~

The Principal ofc wants to pay the Agent the min. amount that keeps the agent to work

Participation Constraint

using $e^* = 10\tau$

$S \rightarrow$ fixed payments

$$U_0 = S + 10\tau \cdot 10\tau - 0,5 \cdot (10\tau)^2 - r^2 K$$

$\tau \rightarrow$ Measures the strength of incentives

$$S + \tau(10e) - 0,5e^2 - Kr^2 = U_0$$

↑ ↑ ↑
 fixed payment incentive strength effort
 ↓ ↓ ↓
 Risk Aversion

Higher K means the Agent gets very stressed if their pay is uncertain

(Minimum bar)
 The amount of happiness that keeps the agent at work

① The bonus pay for Agent:

Agent's bonus is based on output $r \cdot Q$
we know $Q = 10e$

Since Agent chooses to work $e = 10r$ then
the output is $10 \cdot 10r = 100r$

$$\text{Bonus Pay: } r \cdot 100r = \underline{\underline{100r^2}}$$

② Cost of working / Work pain

the cost of working: $0,5e \cdot e^2$

$$e = 10r$$

$$0,5 \times (10r)^2 = 0,5 \cdot 100r^2 = \underline{\underline{50r^2}}$$

③ Solving for „s“ $\rightarrow s$ is the safety pay

We have:

$$100r^2 \quad 50r^2 \quad Kr^2$$

$$s + r(10e) - 0,5e^2 - Kr^2 = U_0$$

$$s + 50r^2 - Kr^2 = U_0$$

$$s = U_0 + Kr^2 - 50r^2 / = U_0 + r^2 \cdot (K - 50)$$

To keep the Agent happy (U_0) the Boss has to
pay enough money (s) to cover the stress (Kr^2)

- d) Determine the optimal incentive parameter r^* chosen by the principal and the induced effort level e^* .

~~Calculate and discuss how the strength of incentives, r , depends on the agent's degree of risk aversion.~~

Here the Principal picks the bonus straight τ

that makes his profit the highest

And see how hard the Agent works bc of it - efforts of Agent

The Principle wants to Maximize their Expected Profit

(value of product sold - payment to the Agent)

① What we have : $Q = 10e$

$$\text{Average work} : 10 \cdot (10r) = 100r$$

from ① Average Payment : $(E[\rho])$ \rightarrow to keep Agent to work the Principal has to pay average

$$U_0 + Kr^2 + 50\tau r^2$$

Profit equation : Principle Profits = work - payment to Agent

$$= 100r - [U_0 + Kr^2 + 50\tau r^2] = \\ 100r - U_0 - (K+50)\tau r^2$$

② Finding the best Bonus :

- we look for value r that makes the equat. win max
- Expected Profit \rightarrow max

(income - costs)

The income: 100τ

$$d) \max_e E[\bar{Q}] - E[\bar{P}]$$

$$= 10e - (s + 10\tau e)$$

$$\text{using } e = 10\tau, s = u_0 + \tau^2 \cdot (K - 50)$$

from b)

The Costs: $u_0 + (K + 50)\tau^2$

minimum happiness for the agent to keep him working

Risk Premium

work pain

$$\max_{\tau} = 10 \cdot 10\tau - (u_0 + \tau^2 \cdot (K - 50) + 10\tau \cdot 10\tau)$$

$$100\tau - 100\tau^2 - u_0 - \tau^2 \cdot (K - 50)$$

$$100\tau - 100\tau^2 - u_0 - \tau^2 K + \tau^2 50$$

$$100\tau - 50\tau^2 - u_0 - \tau^2 K$$

$$\text{FOC: } 100 - 100 \cdot \tau - 2 \cdot \tau \cdot K = 0$$

$$\tau \cdot (100 + 2 \cdot K) = 100$$

$$\tau^* = \frac{100}{100 + 2K} = \frac{50}{50 + K}$$

Profit Formula: $100\tau - u_0 - (K + 50)\tau^2$

gain from " τ " is 100

the increasing cost of the bonus: $2 \cdot (K + 50)\tau$

We set them as equal to find the balance point

$$100 = 2(K + 50)\tau$$

$$50 = (K + 50)\tau$$

$$\tau = \frac{50}{K + 50}$$

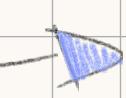
→ This shows that K Risk Aversion of the helper is the most important factor
Large K = bonus smaller

e)

- e) Calculate and discuss how the strength of incentives, r , depends on the agent's degree of risk aversion.

from d)

$$r = \frac{50}{50+K}$$



In any fraction, if the bottom number (denominator) gets larger while the top stay the same, the total value of the fraction decreases.

Low K : The Brave Agent - The Principle gives high bonus bc the agent doesn't need much "stress money" to stay happy

High K Scared Agent - The Principle gives low bonus and higher guaranteed salary to avoid paying a big expensive K risk premium

- **Incentives vs. Stress:** Higher bonuses (r) are good because they motivate the worker to put in more effort (e)

5 6 . However, because the outcome is partly based on luck (u), a higher bonus makes the worker's paycheck "jump around" and become uncertain 6 7 .

- **The Risk Premium:** A risk-averse worker hates this uncertainty. To get them to accept the contract, the Boss has to pay them a **risk premium**—basically "stress money"—to compensate them for the risk they are taking

4 8 .

- **The Cost to the Boss:** The risk premium is calculated as Kr^2 6 8 . If the worker is very risk-averse (high K), this "stress money" becomes very expensive for the Boss 5

8 .

e) $r^* = \frac{50}{50+K}$

$\Rightarrow r$ is strictly decreasing in K .

\Rightarrow Higher risk aversion means a contract with more insurance

$$\frac{dr^*}{dK} < 0$$

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Given:

$$\text{Lawex is payed: } w(x) = a + bx \quad \text{where } b = (0 \leq b \leq 1)$$

$$\text{Effort costs } C(e) = e^2$$

$$\text{Expected court compensation: } x \sim U[0, 2e] = E[x]e$$

a) Agent's utility function:

$$U_A = a + \underbrace{bx}_{\substack{\text{Lawex} \\ \text{payment}}} - \underbrace{e^2}_{\substack{\text{fixed} \\ \text{agreed} \\ \text{payment}}} \quad \begin{matrix} \text{compensation} \\ \downarrow \\ \text{expected} \\ \text{performance} \end{matrix}$$