INSTITUT FÜR VOLKSWIRTSCHAFTSLEHRE
FINANZWISSENSCHAFT
PROF. DR. CARSTEN HELM

CARL VON OSSIETZKY universität OLDENBURG
FAKULTÄT II
INFORMATIK, WIRTSCHAFTS- UND RECHTSWISSENSCHAFTEN

Advanced Microeconomics, winter term 2025/26

Exercise 6

Please solve the exercises below by Wednesday, January 14th. We will discuss them in our exercises (see Stud.IP). To obtain a bonus point, you need to upload your answers as a single pdf in the StudIP folder "Student Solutions Exercise 6". The **document name should start with your surname**. If you prepared the answer in a group of up to 3 students, please only submit one document that contains the names of all contributing students. The DEADLINE is January 14th, 7:30 so that we have a chance to quickly scroll through your submissions to suggest one for presentation.

Question 1 (Principal Agent)
The laundry "Cleanex" is located at a river downstream to the sewage plant "Dirtex". Last week, part of the clothes were spoiled during the laundry process so that the customers had to be compensated, leading to additional costs d. Cleanex is convinced that a defect in the sewage plant Dirtex is responsible for the problem. Therefore, Cleanex authorizes the law firm "Lawex" to sue Dirtex for damages.

We make the following assumptions: The effort cost of the of the law firm are a convex function $c(e) = e^2$ and efforts $e \in [0; 0.5d]$. Cleanex pays Lawex conditional on the indemnification x that the court grants to Cleanex, where x is uniformly distributed in the interval $[0, 2e]$. In particular, payments are $w(x) = a + bx, b \in [0, 1]$. Furthermore, $w(x) \geq 0$, i.e. the law firm cannot be hold liable for a bad job. Both Cleanex and Lawex are risk-neutral. Finally, if Lawex does not accept the contract offered by Cleanex, it gets a reservation income of 0.

a) Set up the Principal-Agent problem.
b) Why does the liability constraint $w(x) \geq 0$ imply that $a \geq 0$?
c) Solve the Principal-Agent problem for the optimal contract elements (a, b, e).
d) Calculate the rent which Lawex receives due to the liability constraint.
e) Compare the size of this rent with the effort cost.

Question 2 (Risk and Incentives in Contracting)
Consider a principal–agent relationship where a risk-neutral principal hires an agent whose effort is not observable.

We make the following assumptions: The agent's effort level is $e \geq 0$ and effort costs are given by $c(e) = 0.5 \cdot e^2$. Output is $Q = 10 \cdot e + u$ where u is a random variable with $E[u] = 0$. Only output Q is observable. The principal offers a **linear incentive contract** $p = s + r \cdot Q$ where s is a fixed payment and r measures the strength of incentives. The agent is **risk-averse**. His expected utility (certainty-equivalent form) is $EU = E[p] - 0.5 \cdot e^2 - Kr^2$ where $K > 0$ measures the agent's degree of risk aversion. If the agent does not accept the contract, he obtains a reservation utility u_0.The principal maximizes expected profits.

a) Set up the Principal-Agent problem.
b) Solve the agent's incentive constraint. Derive the agent's optimal effort choice $e(r)$.
c) Show that the participation constraint binds in the optimal contract and derive the fixed payment $s(r)$.
d) Determine the optimal incentive parameter r^*chosen by the principal and the induced effort level e^*.
e) Calculate and discuss how the strength of incentives, r, depends on the agent's degree of risk aversion.