Box-Jenkins approach I

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Overview

1. Steps of Box-Jenkins approach
2. Stationarity of time series:
   2.1 Determination of difference filter d
   2.2 Unit root test by Dickey-Fuller
3. Empirical example: Dow Jones

The Box-Jenkins Methodology

Overview

The Box-Jenkins methodology is a systematic, iterative framework for building and refining ARIMA models for time series analysis. It consists of four steps:

1. **Identification**
2. **Estimation and Selection**
3. **Model Diagnosis**
4. **Prediction**

Step 1: Identification

Goal: Identify the structure of the ARIMA model (p, d, q).

- **Stationarity:** Verify whether the time series is stationary:
  - Visual inspection (trends, seasonality)
  - Statistical tests (e.g., Augmented Dickey-Fuller test)
- **Differencing (d):** Apply differencing if the series is non-stationary:

$$Y_t' = Y_t - Y_{t-1} \quad \text{(first-order differencing)}$$

- **Order Selection:** Use:
  - **ACF (Autocorrelation Function)** to analyze lag correlations.
  - **PACF (Partial Autocorrelation Function)** to determine direct lag effects.
- Patterns guide p (AR order) and q (MA order).

Step 2: Estimation and Selection

Goal: Estimate model parameters and select the best-fitting model.

- **Parameter Estimation:**
- **Model Selection:**
  - Compare models using:

$$\text{AIC} = -2\ln(L) + 2k \quad \text{and} \quad \text{BIC} = -2\ln(L) + k\ln(n)$$

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Step 3: Model Diagnosis

Goal: Verify the adequacy of the selected model.

- **Residual Analysis:**
- **Statistical Tests:**
- Refine the model if residuals fail these tests.

Step 4: Prediction

Goal: Use the validated model for forecasting future values.

- **Forecasting:**

$$\hat{Y}_t = \phi_1 Y_{t-1} + \cdots + \theta_1 \epsilon_{t-1} + c$$

- **Error Metrics:**
  - Mean Absolute Error (MAE)
  - Root Mean Squared Error (RMSE)
  - Mean Absolute Percentage Error (MAPE)
- **Validation:**
  - Compare forecasts to actual values (in-sample or out-of-sample).
  - Plot forecasts and prediction intervals for visualization.

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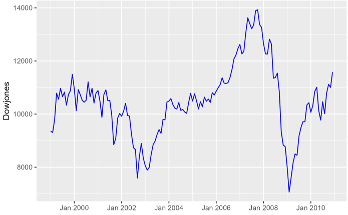
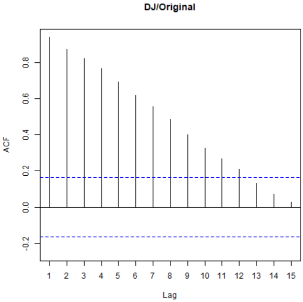
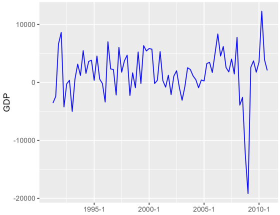
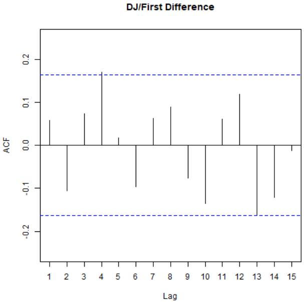
Time series plot Dow Jones / Original / Monthly 1999-2010





Time series plot Dow Jones / First Difference/ Monthly 1999-2010





Unit Root Test by Dickey and Fuller I

- **Unit Root Tests:** These are a set of statistical testing procedures that examine whether the time series data generating process contains a **unit root**, which is indicative of a random walk component (non-stationary behavior).
- The **Dickey-Fuller (DF) Test:** A prominent test within this group, designed to check for stationarity by testing the presence of a unit root.
- The **Simple DF Test:**
  - Null hypothesis (H_0): The time series is **not** stationary, implying the presence of a unit root.
  - Alternative hypothesis (H_a): The time series is stationary.

Unit Root Test by Dickey and Fuller II

Dickey and Fuller propose three estimated equations, where parameters can be determined using **ordinary least squares (OLS)** methods of linear regression:

(i) $X_t = b_1 X_{t-1} + \varepsilon_t$ **(Test for Random Walk)**
- This is the simplest test, where the presence of a unit root implies the series follows a pure random walk.

(ii) $X_t = b_0 + b_1 X_{t-1} + \varepsilon_t$ **(Test for Random Walk with Drift)**
- Adds a constant (b_0) to account for a non-zero mean in the process.

(iii) $X_t = b_0 + b_1 X_{t-1} + b_2 t + \varepsilon_t$ **(Test for Random Walk with Drift and Deterministic Trend)**
- Incorporates a deterministic trend ($b_2 t$) to account for a systematic change in the series over time.

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Unit Root Test by Dickey and Fuller III

For case (i), it applies:

$$\Delta X_t = X_t - X_{t-1} = (b_1 - 1)X_{t-1} + \varepsilon_t = \alpha_1 X_{t-1} + \varepsilon_t$$

In general: With $\alpha_1 := b_1 - 1$, the three equations can be expressed as:

(i) $\Delta X_t = \alpha_1 X_{t-1} + \varepsilon_t$

(ii) $\Delta X_t = \alpha_0 + \alpha_1 X_{t-1} + \varepsilon_t$

(iii) $\Delta X_t = \alpha_0 + \alpha_1 X_{t-1} + \alpha_2 t + \varepsilon_t$

If the null hypothesis $H_0 : \alpha_1 = 0$ is **true**, the parameters are **not** t-distributed. They follow a specific distribution established by Dickey and Fuller.

Dow Jones – Original

	RW	RW with Drift	RW with Drift/Trend
Constant		611.987*	606.858*
		(314.117)	(315.375)
lag(Dowjones)	0.001	−0.057*	−0.059*
	(0.004)	(0.029)	(0.030)
Trend			0.340
			(1.043)
Num.Obs.	143	143	143
R2	0.000	0.029	0.029
R2 Adj.	−0.007	0.022	0.016
AIC	2159.5	2157.2	2159.1
BIC	2165.4	2166.1	2170.9
RMSE	453.85	447.10	446.89

Dow Jones – First differences

	RW	RW with Drift	RW with Drift/Trend
Constant		15.313	16.492
		(38.793)	(87.206)
lag(Dowjones/FD)	-0.941^{***}	-0.941^{***}	-0.941^{***}
	(0.092)	(0.093)	(0.093)
Trend			-0.016
			(1.073)
Num.Obs.	142	142	142
R2	0.468	0.468	0.468
R2 Adj.	0.464	0.464	0.460
AIC	2144.9	2146.8	2148.8
BIC	2150.8	2155.6	2160.6
RMSE	454.66	454.41	454.41