①

TAP → projekt

$$\frac{dV}{dt} = F_H + F_C + F_D - F(h)$$

$$\frac{dT \cdot V}{dt} = F_H \cdot T_H + F_C \cdot T_C + F_D \cdot T_D - F(h) \cdot T$$

$F(h) = \alpha\sqrt{h}$, $V(h) = \frac{\pi h^2}{3} \cdot (3 \cdot r - h)$, $F_H(t) = F_{Hin}(t - \tau_H)$, $F_C(t) = F_{Cin}(t-\tau$

$r = 68\,cm$, $\alpha = 25$

$y \to h, T$

$u \to F_{Hin}, F_{Cin}$

Co jest nieliniowe?

z układu

$F(h) = \alpha \cdot \sqrt{h} \longrightarrow F(h) = \alpha \cdot \left(\sqrt{h_0} - \frac{1}{2\sqrt{h_0}}(h - h_0)\right)$

$V(h) = \pi h^2 r - \frac{\pi h^3}{3}$

liniowe

szereg taylora h^2:

$h^2 = h_0^2 + 2h_0(h - h_0)$

szereg taylora h^3:

$h^3 = h_0^3 + 3h_0^2(h - h_0)$

(zlinearyzowane do pom można !)

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ode45

$$\frac{dV}{dt} \; \frac{\pi h^2}{\;} \; \pi\frac{dh^2}{dt} r - \frac{\pi h^3}{3dt}$$

chcę mieć $\frac{dh}{dt}$ i $\frac{dT}{dt}$

$$\frac{dV}{dt} = F_H + F_C + F_D - \alpha\sqrt{h}$$

$$\left(\frac{\pi h^2}{3}(3r - h)\right)' = \left(\pi h^2 r - \frac{\pi h^3}{3}\right)' = \left(2\pi h r - \pi h^2\right)\frac{dh}{dt} = \frac{dV}{dt}$$

$$\left(2\pi r\frac{dh}{dt} - \pi\frac{dh}{dt}\right)\frac{dh}{dt} = F_H + F_C + F_D - \alpha\sqrt{h}$$

$$\boxed{\frac{dh}{dt} = \frac{F_H + F_C + F_D - \alpha\sqrt{h}}{2\pi r h - \pi h^2}}$$

$$\frac{dTV}{dt} = F_H \cdot T_H + F_C \cdot \cancel{\beta}\, T_{C2} + F_D \cdot T_D - \alpha\sqrt{h} \cdot T$$

$$\cancel{\frac{dT}{dt}} = \frac{F_H \cdot T_H + F_C \cdot T_{C2} + F_D \cdot T_D - \alpha\sqrt{h} \cdot T}{\cancel{\frac{dV}{dt}}}$$

$$\boxed{\frac{dT}{dt} = \frac{F_H \cdot T_H + F_C \cdot T_{C2} + F_D \cdot T_D - \alpha\sqrt{h} \cdot T}{F_H + F_C + F_D - \alpha\sqrt{h}}}$$

Linearyzacja $\beta = F_H + F_C + F_D$ ← Tutaj niewolne

$$\frac{dh}{dt} = \frac{F_H + F_C + F_D - \alpha\sqrt{h}}{2\pi r h - \pi h^2} = \frac{1}{\pi} \cdot \frac{\beta - \alpha\sqrt{h}}{2rh - h^2} = (\beta - \alpha\sqrt{h})$$

$$(2rh - h^2)(2rh + h^2) = 4r^2h^2 - h^4$$

$$\cancel{2\pi r h - \pi h^2}$$

$$\frac{(\beta - \alpha\sqrt{h})(\beta + \alpha\sqrt{h})}{(2rh - h^2)\cancel{(2rh + h)}(\beta + \alpha\sqrt{h})} = \frac{\beta^2 - \alpha^2 \cdot h}{}$$

$$(2rh - h^2)(\beta + \alpha\sqrt{h}) = \cancel{2 2rh \cdot \beta}$$

$$= 2rh \cdot \beta + 2rh\alpha\sqrt{h} - h^2\beta - \alpha h^2\sqrt{h}$$

$$\frac{F_H + F_C + F_D - \alpha\sqrt{h}}{2\pi r h - \pi h^2} = \frac{1}{\pi} \cdot \frac{\beta - \alpha\sqrt{h}}{2rh - h^2}$$

$$\frac{dh}{dt}\left(\frac{\beta - \alpha\sqrt{h}}{2rh - h^2}\right) = (\beta - \alpha\sqrt{h})\left(\frac{1}{2rh - h^2}\right) = (\beta \cancel{- \alpha\sqrt{h}})\cancel{\left(\frac{1}{h}\right)\left(\frac{1}{2r - h}\right)}$$

$$(\beta - \alpha\sqrt{h})'\left(\frac{1}{2rh - h^2}\right) + (\beta - \alpha\sqrt{h})\left(\frac{1}{2rh - h^2}\right)'$$

$=$

$$\left(\overset{0}{\beta} - \alpha\sqrt{h}\right)' = \cancel{\beta} - \alpha \cdot \frac{1}{2\sqrt{h}} = \frac{-\alpha}{2\sqrt{h}}$$

$$\left(\frac{1}{2rh-h^2}\right)' = \left(\left(\frac{1}{h}\right)\left(\frac{1}{2r-h^2}\right)\right)' = \left(\frac{1}{h}\right)'\left(\frac{1}{2r-h^2}\right) + \frac{1}{h}\left(\frac{1}{2r-h^2}\right)'$$

$$\left(\frac{1}{2r-h^2}\right)' = \cancel{\frac{2r+h^2}{4r^2}} = \cancel{2h}$$

$$= \frac{2h}{(2r-h^2)^2}$$

czyli:

~~pochodna~~ $$-\frac{\alpha}{2\sqrt{h}}\left(\frac{1}{2rh-h^2}\right) + (\beta - \alpha\sqrt{h})\left[-\frac{1}{h^2}\cdot\left(\frac{1}{2r-h^2}\right) + \frac{1}{h}\cdot\left\{\frac{2h}{(2r-h^2)^2}\right\}\right]$$

Szereg taylora:

$$\frac{dh}{dt} \approx \frac{\overbrace{F_H + F_C + F_D}^{\beta_H} - \alpha\sqrt{h_0}}{2\pi r h_0 - \pi h_0^2} + \left(-\frac{\alpha}{2\sqrt{h_0}}\cdot\frac{1}{2rh_0 - h_0^2} + (\beta - \alpha\sqrt{h_0})\left[-\frac{1}{h_0^2}\cdot\left(\frac{1}{2r-h_0^2}\right) + \frac{1}{(2r-h_0^2)^2}\right]\right)(h - h_0)$$

Linearyzacja $\frac{dT}{dt}$ (4)

$$\frac{dT}{dt} = \frac{\overbrace{F_H \cdot T_H + F_C \cdot T_{C2} + F_D \cdot T_D}^{\gamma} - \alpha\sqrt{h} \cdot T}{\underbrace{F_H + F_C + F_D}_{\beta} - \alpha\sqrt{h}}$$

$$\frac{dT}{dt} = \frac{\gamma - \alpha\sqrt{h} \cdot T}{\beta - \alpha\sqrt{h}}$$

$$\left(\frac{\gamma - \alpha\sqrt{h}\cdot T}{\beta - \alpha\sqrt{h}}\right)' = \left[\left(\gamma - \alpha\sqrt{h}\cdot T\right) \cdot \left(\frac{1}{\beta - \alpha\sqrt{h}}\right)\right]'$$

$$= \left(\gamma - \alpha\sqrt{h}\cdot T\right)' \cdot \left(\frac{1}{\beta - \alpha\sqrt{h}}\right) + \left(\gamma - \alpha\sqrt{h}\cdot T\right) \cdot \left(\frac{1}{\beta - \alpha\sqrt{h}}\right)'$$

$$\left(\gamma - \alpha\sqrt{h}\cdot T\right)' = \frac{-\alpha T}{2\sqrt{h}}$$

$$\left(\frac{1}{\beta - \alpha\sqrt{h}}\right)' = -\frac{1}{(\beta - \alpha\sqrt{h})^2} \cdot \frac{-\alpha}{2\sqrt{h}} = \frac{\alpha}{(\beta - \alpha\sqrt{h}) \cdot 2\sqrt{h}} =$$

$$\left(\beta - \alpha\sqrt{h}\right)' = \frac{-\alpha}{2\sqrt{h}} \qquad = \frac{\alpha}{2\beta\sqrt{h} - 2\alpha h}$$

$$\cancel{\frac{dT}{dt}} = \boxed{\frac{-\alpha T}{2\sqrt{h}} \cdot \left(\frac{1}{\beta - \alpha\sqrt{h}}\right) + \left(\gamma - \alpha\sqrt{h}\cdot T\right) \cdot \frac{\alpha}{2\beta\sqrt{h} - 2\alpha h}}$$

Szereg Taylora:

$$\frac{dT}{dt} = \left[\frac{-\alpha T}{2\sqrt{h_0}} \cdot \left(\frac{1}{\beta - \alpha\sqrt{h_0}}\right) + \left(\gamma - \alpha\sqrt{h_0}\cdot T\right) \cdot \frac{\alpha}{2\beta\sqrt{h_0} - 2\alpha h_0}\right](h - h_0)$$

$$+ \frac{\gamma - \alpha\sqrt{h_0}\cdot T}{\beta - \alpha\sqrt{h_0}}$$

$$\cdot \frac{\pi h^2}{3}$$

$$\frac{dVT}{dt} = \frac{dV}{dt} \cdot T + \frac{dT}{dt} \cdot V$$

$$\frac{dT}{dt} = \frac{\frac{dVT}{dt} - \frac{dV}{dt} \cdot T}{V} = \frac{F_H \cdot T_H + F_C T_C + F_D T_D - \alpha\sqrt{h} \cdot T - (F_H + F_C + F_D - \alpha\ldots}{\frac{\pi h^2}{3}(3r - h)}$$

$$\frac{d\bar{T}}{dt} = \frac{\overbrace{F_H \cdot \bar{T}_H + F_C \bar{T}_C + F_D \bar{T}_D}^{\gamma} - \alpha\sqrt{h}\,\bar{T}}{\frac{\pi h^2}{3}(3r - h)} - \left(F_H + F_C + F_D - \alpha\sqrt{h}\right)$$

$$\left(\frac{\gamma - \alpha\sqrt{h}\cdot \bar{T}}{\pi h^2 \cdot r - \frac{\pi h^3}{3}}\right)' = \left(\gamma - \alpha\sqrt{h}\cdot \bar{T}\right)'\left(\frac{1}{\pi h^2 r - \frac{\pi h^3}{3}}\right)$$

$$+ \left(\gamma - \alpha\sqrt{h}\,\bar{T}\right)\left(\frac{1}{\pi h^2 r - \frac{\pi h^3}{3}}\right)'$$

$$\left(\gamma - \alpha\sqrt{h}\cdot \bar{T}\right)' = \frac{-\alpha \bar{T}}{2\sqrt{h}}$$

$$\left(\frac{1}{\pi h^2 r - \frac{\pi h^3}{3}}\right)' = \frac{9(h-2r)}{\pi h^3 (h-3r)^2}$$

$$\frac{d\bar{T}}{dt} = \left[\left\{\frac{-\alpha \bar{T}}{2\sqrt{h_0}} \cdot \left(\frac{1}{\pi h_0^2 r - \frac{\pi h_0^3}{3}}\right) + \left(\gamma - \alpha\sqrt{h_0}\,\bar{T}\right) \cdot \frac{9(h_0 - 2r)}{\pi h_0^3 (h_0 - 3r)^2}\right\}\right.$$

$$\left. - \left(\cancel{F_H F}\,\beta + \alpha \frac{1}{2\sqrt{h_0}}\right)\right](h - h_0)$$

$$+ \frac{\gamma - \alpha\sqrt{h_0}\,\bar{T}}{\frac{\pi}{3} h_0^2 (3r + h_0)} - \left(\beta - \alpha\sqrt{h_0}\right)$$