

Figure 2.4: Uniform reconstruction of the sampled signal of Figure 2.2. The time-domain and frequency-domain representations are displayed on the left and on the right, respectively. From top to bottom are the discrete signal a_s , the reconstruction signal r (which is a *sinc* function, here translated and truncated for the display), and the reconstructed signal a.

2.1.2 Reconstruction

Any band-limited signal a with spectrum in $[-\Omega_a, +\Omega_a]$ can then be completely reconstructed from its samples, provided that the Nyquist criterion has been respected during the sampling. However the methods for reconstructing sampled signals are different for the uniform and irregular cases.

From Uniform Sampling

Reconstructing the continuous version of the signal a from its discrete values resulting from uniform sampling is quite easy. In fact a simple low-pass filter will do it, provided that its cutoff frequency is in the $[\Omega_a, \Omega_s - \Omega_a[$ interval. Figure 2.4 illustrates this. The sampled signal a_s is multiplied with the impulse response r of the low-pass filter in the time domain to obtain the reconstructed signal a. In fact, to reduce transients at the beginning and at the end, the signal is mirrored:

$$\begin{cases} s[-i] & = 2 \ s[0] - s[i] \\ s[(N-1)+i] & = 2 \ s[N-1] - s[(N-1)-i] \\ \text{for } 0 < i < N \end{cases}$$

This extrapolation by mirroring the signal ensures the continuity of the signal and of its first derivative at the beginning and at the end, and provided that *r* is symmetric the values of the reconstructed signal match the values of the sampled signal at these positions. The problem is that the theoretical (ideal)